

# International Portfolios with Demand and Supply Shocks

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May 10, 2007

## 1 Introduction

In the past twenty years, both gross and net capital flows have increased to unprecedented levels (see Lane and Milesi-Ferretti b, 2006). In industrialized countries, this has been the case for both stocks and bonds. One consequence of this period of financial integration is that gross financial positions now exceed 100% of GDP in several industrialized countries. This also means that differences in returns on foreign assets held by domestic agents and on domestic assets held by foreign agents can generate sizable wealth transfers between countries. Lane and Milesi-Ferretti (2006 a and b) and Tille (2003).have recently shown the extent of these valuation effects. The financial integration process at work in the past twenty years has not however eliminated the financial home bias in stocks.

The theoretical literature has analyzed these two issues - the financial home bias and the valuation effects - separately. In this paper, we study them jointly in a two country general equilibrium model with portfolio choice and show that they can be related. We analyze them in the context of the stylized fact of the

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home bias in consumption which has remained strong even of the presence of increasing trade flows. A recent literature has revived the interest in the link between the financial and real home biases. In particular Obstfeld and Rogoff (2000), have argued that trade costs can explain the puzzle of the home bias in portfolios. Kollman (2005) and Coeurdacier (2005) have analyzed the conditions for which the home bias in goods could generate a home bias in assets.

The role of various shocks on the demand and supply side of the economy and of the menu of available assets (stocks and bonds) is analyzed. We do this both in the case of complete and incomplete markets. We find that a combination of shocks can qualitatively replicate both a home bias in stocks and positions on bonds that produce qualitatively plausible valuation effects. The intuition of this result is the following: any shock that increases dividends of domestic firms while reducing income of domestic agents can be insured by holding stocks of domestic firms, the home bias observed in the data. The simplest shock that has this property is one that redistributes income from labor to dividends. Given this home bias in stocks, a shock that increases both dividends and terms of trade will induce agents to hold foreign bonds which then generates plausible valuation effects. We can think at least of two types of shocks that have the later property: a shock on margins in a model with imperfect competition, or the one we analyze and call a "i-pod" shock i.e. a demand shock that changes the world relative demand of Home versus Foreign goods. If agents hold a biased portfolio in stocks to insure against the shocks that redistribute income between capital and labor, they will want to insure the second type of shocks by holding foreign bonds that induce a valuation effect when terms of trade change. Indeed, if a shock deteriorates terms of trade and dividends at the same time, holding foreign bonds that are valued more in these states of nature and induce a transfer will be optimal.

An interesting property of this result is therefore that the Home bias in

stocks is indirectly at the origin of a position in bonds that can then produce plausible valuation effects. Hence, from a theoretical point of view we show that the Home bias in stocks and the valuation effects can be related.

In the second section of the paper, we analyze the robustness of this result in more realistic situations where markets are incomplete and where we allow for a larger array of shocks. In the last section, we simulate our model.

## 2 Related literature

To be written

## 3 Set-up of the model

### 3.1 Preferences and trade costs

We consider a discrete time endowment-economy with an infinite horizon.

There are two countries, home ( $H$ ) and foreign ( $F$ ). Each country is producing one good and foreign and home goods. We introduce the aggregate consumption index in country  $i$  in period  $t$  which is defined as the aggregator of home and foreign goods:

$$\begin{aligned} C_{Ht} &= \left[ a^{1/\phi} (\Psi_{Ht} c_{Ht}^H)^{(\phi-1)/\phi} + (1-a)^{1/\phi} (\Psi_{Ft} c_{Ft}^H)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} \quad (1) \\ C_{Ft} &= \left[ a^{1/\phi} (\Psi_{Ft} c_{Ft}^F)^{(\phi-1)/\phi} + (1-a)^{1/\phi} (\Psi_{Ht} c_{Ht}^F)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} \end{aligned}$$

where  $\phi$  is the elasticity of substitution between home and foreign goods, and  $c_{jt}^i$  is the consumption of goods from country  $j$  by country  $i$ ,  $i = H, F$  in period  $t$ .  $\Psi_{it}$ ,  $i = H, F$  are demand or preference shocks with  $E(\Psi_{it}) = 1$ . They reflect a change at the world level in the preference for the good of country  $H$  or  $F$ . To be more illustrative we call them “i-pod shocks”.

We also allow for a preference bias for home goods by assuming  $a \geq \frac{1}{2}$ . When  $a = \frac{1}{2}$ , both agents have identical preferences and we will assume that a decrease

in  $a$  is similar to a deeper integration on the goods market. Note that in the “Cobb-Douglas” case ( $(\phi - 1)/\phi = 0$ ),  $a$  is the share of revenues dedicated to home goods.

Each country has a representative agent with utility functional

$$U_{it} = E_t \left[ \sum_{s=t} \beta^{s-t} \frac{C_{is}^{1-\sigma}}{1-\sigma} \right] \quad (2)$$

where  $C_s^i$  is the consumption rate in country  $i$  and  $\beta$  is the discount factor. As in most of the literature, we take the coefficient of relative risk aversion  $\sigma$  to be equal or superior to one.

The appropriate price index that corresponds to the above specification of preferences is:

$$\begin{aligned} P_{Ht} &= \left[ a (p_{Ht}/\Psi_{Ht})^{1-\phi} + (1-a) (p_{Ft}/\Psi_{Ft})^{1-\phi} \right]^{1/(1-\phi)} \\ P_{Ft} &= \left[ (1-a) (p_{Ht}/\Psi_{Ht})^{1-\phi} + a (p_{Ft}/\Psi_{Ft})^{1-\phi} \right]^{1/(1-\phi)} \end{aligned} \quad (3)$$

where we introduce a second source of home bias in consumption through iceberg-type trade costs ( $\tau$ ). The resource constraints are given by:

$$c_{Ht}^H + c_{Ht}^F = y_{Ht} \quad (4)$$

$$c_{Ft}^F + c_{Ft}^H = y_{Ft} \quad (5)$$

where  $y_{it}$  is the exogenous level of output of country  $i$  in period  $t$ .

We introduce  $q_t$  the relative price of home goods over foreign goods (the terms-of-trade) as:

$$q_t \equiv \frac{p_{Ht}}{p_{Ft}} \quad (6)$$

## 4 Characterization of world equilibrium with complete markets

To build up intuition, we first characterize the equilibrium with complete markets. To do this, we assume the same number of shocks and of assets. The assets

we will analyze are stocks and bonds. Home and Foreign stocks which are claims on the output  $y_{Ht}$  and  $y_{Ft}$  respectively. We normalize the number of shares in each country to one. There are also two bonds, a "home" bond and a "foreign bond" in zero net supply. Bond returns are defined in the following way. Buying one unit of the Home bond at period  $t$  gives  $r_{H,t+1}$  units if the Home good at  $t + 1$ . The Foreign bond. gives  $r_{F,t+1}$  units if the Foreign good at  $t + 1$ .

#### 4.1 Consumption rules and terms-of-trade

Since markets are complete in this case, we can find the equilibrium allocation by solving the planner's problem. The planner maximizes the sum of countries utilities with equal weights subject to the resource constraints (4) and (4):

$$\max_{\{c_H^H, c_H^F, c_F^H, c_F^F\}} E_t \left[ \sum_{s=t}^T \beta^{s-t} \left( \frac{C_{Hs}^{1-\sigma}}{1-\sigma} + \frac{C_{Fs}^{1-\sigma}}{1-\sigma} \right) \right]$$

The competitive equilibrium prices are identified with the Lagrange multipliers associated with the resource constraints. The multiplier on (4) is the price of one unit of the Home good. Similarly, the multiplier on (5) is the price of one unit of the Foreign good. We obtain the following first order conditions for the intratemporal allocation of consumption. Given the stationarity of the problem, we can eliminate all time subscripts:

$$\begin{aligned} c_H^H &= a \Psi_H^{\phi-1} p_H^{-\phi} C_H^{1-\sigma\phi} & (7) \\ c_H^F &= (1-a) \Psi_H^{\phi-1} p_H^{-\phi} C_F^{1-\sigma\phi} \\ c_F^H &= (1-a) \Psi_F^{\phi-1} p_F^{-\phi} C_H^{1-\sigma\phi} \\ c_F^F &= a \Psi_F^{\phi-1} p_F^{-\phi} C_F^{1-\sigma\phi} \end{aligned}$$

Using the market clearing condition (??), we get:

$$\begin{aligned} c_H^H + c_H^F &= \Psi_H^{\phi-1} p_H^{-\phi} \left[ a C_H^{1-\sigma\phi} + (1-a) C_F^{1-\sigma\phi} \right] = y_H \\ c_F^F + c_F^H &= \Psi_F^{\phi-1} p_F^{-\phi} \left[ a C_F^{1-\sigma\phi} + (1-a) C_H^{1-\sigma\phi} \right] = y_F \end{aligned}$$

Taking the ratio and defining  $\Omega(x) = \frac{1+x(\frac{1-a}{a})}{x+(\frac{1-a}{a})}$  we get:

$$\left(\frac{\Psi_H}{\Psi_F}\right)^{\phi-1} q^{-\phi} \Omega \left[ \left(\frac{C_F}{C_H}\right)^{1-\sigma\phi} \right] = \frac{y_H}{y_F}$$

When markets are complete and countries are symmetric, the ratio of Home to Foreign marginal utility of consumption is linked to the real exchange rate by the following, familiar condition:

$$\left(\frac{C_F}{C_H}\right)^{\sigma} = \frac{P_H}{P_F} \quad (8)$$

so that because of perfect insurance, any change in parameter values that raises domestic consumption relative to foreign consumption must be associated with a real depreciation. Then:

$$q^{-\phi} \left(\frac{\Psi_H}{\Psi_F}\right)^{\phi-1} \Omega \left[ \left(\frac{P_H}{P_F}\right)^{\frac{1}{\sigma}-\phi} \right] = \frac{y_H}{y_F} \quad (9)$$

## 4.2 Constant Portfolios

We will assume that portfolios are non-time varying: as shown by Devereux and Sutherland (2006), a first-order approximation of the equations non related to portfolios imply constant variance-covariances matrix, constant risk-premia and constant portfolios. These portfolios are the ones that replicate (up to the first-order) the consumption allocations of the planner found in the previous section. Note also that due to symmetry between countries, and the normalization of the number of shares to one, we can denote by  $\alpha$  the number of Home (Foreign) shares that Home (Foreign) agents hold and  $1-\alpha$ , the number of Foreign (Home) shares that Home (Foreign) agents hold.  $B_H$  are the holdings of Home bonds by Home agents and.  $B_F$  are the holdings of Foreign bonds by Foreign agents. Due to the assumption that both bonds in net zero supply, we know that Home bonds held by Foreign agents are  $-B_H$  and that Foreign bonds held by Home agents are  $-B_F$ . Due to the stationarity of the problem we can abstract from

time and therefore choose to eliminate all time subscripts and write the budget constraints as<sup>1</sup>:

$$\begin{aligned} P_H C_H &= \alpha k_H p_H y_H + (1 - \alpha) k_F p_F y_F + (r_H - 1) B_H - (r_F - 1) B_F + (1 - k_H) p_H y_H \\ P_F C_F &= (1 - \alpha) k_H p_H y_H + \alpha k_F p_F y_F + (r_F - 1) B_F - (r_H - 1) B_H + (1 - k_F) p_F y_F \end{aligned}$$

$k_i = H, F$  is the share of output that goes to capital owners in the form of dividends.  $T_H$  or  $T_F$  are taxes by Home and Foreign agents respectively. Given that shocks are iid, asset prices are constant over time. Also, due to symmetry between countries,  $B_H = B_F = B$ .

Then, using the budget constraints, we can write:

$$P_H C_H - P_F C_F = (2\alpha - 1)(p_H y_H k_H - p_F y_F k_F) + 2B(r_H - r_F) + (1 - k_H)p_H y_H - (1 - k_F)p_F y_F \quad (10)$$

which says that relative expenditures must be equal to relative incomes which depend only on the structure of portfolios. Note in particular that  $\alpha > 1/2$  implies a home bias in the portfolio of shares.

We will focus on four possible sources of disturbances in this economy. The first is a productivity shock which affects the ratio:.

$$y \equiv \frac{y_H}{y_F} \quad (11)$$

We also introduce a source of uncertainty in the form of an exogenous change in the share of output that goes to dividends and the share that goes to labor.

$$k \equiv \frac{k_H}{k_F} \quad (12)$$

We will sequentially introduce two types of demand shocks: preference or i-pod shocks and government spending shocks.

The relative preference shock is:

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<sup>1</sup>Given that shocks are iid, asset prices are constant over time so they do not appear in these equations.

$$\Psi \equiv \frac{\Psi_H}{\Psi_F} \quad (13)$$

We log-linearize expression (9) around the symmetric steady-state (see Coeurdacier (2005)) where the steady state values of  $q$ ,  $y$  and  $k$  are equal to one. We denote with a hat the deviations of the variables from the symmetric steady state values which are denoted with a bar  $\hat{x} = \log(x/\bar{x})$ . We get:

$$\hat{y} = - \left[ \phi(1 - \theta^2) + \theta^2 \frac{1}{\sigma} \right] (\hat{q} - \hat{\Psi}) - \hat{\Psi} \equiv -\lambda \hat{q} + (\lambda - 1) \hat{\Psi} \quad (14)$$

where  $\theta \equiv 2a - 1$  belongs to  $[0; 1]$  and is a monotonic transformation of the reference bias<sup>2</sup>.  $\theta = 0$  means that there are no barriers to trade in goods, and  $\theta = 1$  is equivalent to trade autarky.  $\lambda \equiv \phi(1 - \theta^2) + \frac{\theta^2}{\sigma}$

The equilibrium relative price is then:

$$\hat{q} = -\frac{1}{\lambda} \hat{y} + \frac{\lambda - 1}{\lambda} \hat{\Psi} \quad (15)$$

As expected, terms-of-trade are decreasing in the relative supply shocks  $\hat{y}$  and increasing in the relative demand shock, the i-pod shock  $\hat{\Psi}$ .

We then log-linearize equation (10) around the symmetric steady and use the complete markets implication on the link between consumption and real exchange rates given by (8). Note that the difference in returns of bonds is given by the change in the terms of trade:  $\widehat{r}_H - \widehat{r}_F = \hat{q}$ , we obtain:

$$\theta \left(1 - \frac{1}{\sigma}\right) (\hat{q} - \hat{\Psi}) = \bar{k} (2\alpha - 1) (\hat{q} + \hat{k} + \hat{y}) + (1 - \bar{k}) \left( \hat{q} + \hat{y} - \frac{\bar{k}}{1 - \bar{k}} \hat{k} \right) + 2b\hat{q} \quad (16)$$

where  $b = B/(\beta \bar{P}_i \bar{y}_i)$ ,  $i = H, F$  is related to the gross bond position in terms of steady-state GDP.

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<sup>2</sup>In the presence of trade costs that take the form of iceberg costs between countries, then (see Coeurdacier 2005)  $\theta = \frac{1 - (\frac{1-a}{a})(1+\tau)^{\frac{\rho}{\rho-1}}}{1 + (\frac{1-a}{a})(1+\tau)^{\frac{\rho}{\rho-1}}}$ .  $\theta$  would have the same interpretation of measuring trade restrictions.



### 4.3 Complete markets examples

In order to analyze results in complete markets, we study sequentially situations with the same number of shocks and assets. Given that we restrict ourselves to stocks and bonds, we analyze pairs of (relative) shocks.

#### 4.3.1 Productivity and demand shocks

We start with a situation with productivity shocks  $\hat{y}$  and demand shocks in the form i-pod shocks  $\widehat{\Psi}$ .

Portfolios of stocks (described by  $\alpha$ , the share of Home stocks as a percentage of total stocks held) and of bonds (described by  $b$ , the gross holding of bonds in local currency in percentage of GDP) are found using (16) and (15). In such way that portfolios give the optimal allocation for all shocks. It can be shown that this is the case for:

$$\alpha = 1/2 \left[ 1 - \frac{\theta}{\bar{k}} \frac{(1-1/\sigma)}{\lambda-1} - \frac{1-\bar{k}}{\bar{k}} \right] ; \quad b = 0 \quad (17)$$

In the portfolio of stocks, three terms appear: the first term,  $1/2$ , is the diversification motive to insure uncertainty on dividends. The second term,  $\frac{\theta}{\bar{k}} \frac{(1-1/\sigma)}{\lambda-1}$ , is the hedging of real exchange rate movements as analyzed in Coeurdacier (2005), Kollman (2005), Obstfeld (2007) and van Wincoop and Warnock (2006). This term tends to generate foreign bias (turns negative), for sufficiently high elasticity of substitution between goods (which insures  $\lambda > 1$ ) and sufficiently high risk aversion ( $\sigma > 1$ ). As in these papers, in the absence of trade barriers ( $\theta = 0$ ), the real exchange rate is constant and this term disappears. The last term  $\frac{1-\bar{k}}{\bar{k}}$ , is identical to the one that appears in Baxter and Jerman (1995): here, income from labor and from holding domestic stocks are perfectly correlated and this generates a foreign bias in stocks. With productivity and demand shocks, holding bonds does not serve any purpose because all uncertainty

on real exchange rate can be eliminated through holding stocks whose dividends are proportional to the real exchange rate.

Hence, this case shares some of the usual difficulties of the literature to explain the home bias in stocks. It however makes clear that in order to get more plausible portfolios, one needs a shock that both eliminates the perfect correlation between dividends and the real exchange rate as well as the perfect correlation of labor and capital income. A shock on the share of income that goes to capital and labor is exactly such a shock that we now analyze.

### 4.3.2 Productivity and capital share shocks

We now replace analyze a situation with productivity shocks  $\hat{y}$  and capital share shocks  $\hat{k}$ . Portfolios on stocks (described by  $\alpha$ ) and on bonds (described by  $b$ ) are found using (16) and (15) in such way that portfolios give the optimal allocation for all shocks. It can be shown that this is the case for:

$$\alpha = 1 \quad ; \quad b = \frac{1}{2} [\theta(1 - 1/\sigma) + \lambda - 1] \quad (18)$$

so that a complete (100%) home bias appears in stocks ( $\alpha = 1$ ) for any parameter value on preferences or trade costs and any stochastic structure of the two shocks. This full Home bias is in sharp contrast with the results obtained in the previous literature. The main difference with Baxter and Jermian (1995) is that we consider a world with two goods and the possibility to hedge risk through bonds. With Coeurdacier (2005), Kollman (2005), Obstfeld (2007) and van Wincoop and Warnock (2006), the difference comes from the presence of shocks on the capital share and the existence of bonds. These authors focus on supply shocks, where the financial home bias in stocks depends on trade costs parameters, preference parameters and the elasticity of substitution between Home and Foreign goods.

To gain intuition on the full Home bias we obtain, it is useful to consider the

capital share shock first which is at the origin of the full home bias in stocks. In the presence of such shocks that redistribute domestic income from labor to capital, the best strategy is to hold domestic stocks that provide high dividends in these states of nature where labor incomes are low. Note that such shocks on the share of capital in income does not rule out that labor and capital incomes are positively correlated for example because of productivity shocks that are common to both factors.

Now that the full Home bias in stocks is understood, we can better analyze the position on bonds. The remaining uncertainty to insure is due to movements in the real exchange rate, which here are proportional to movements in terms of trade because of the absence of the i-pod shock. The position in Home bonds ( $b$ ) depends on the net effect of the two terms in the bracket:  $\theta(1 - 1/\sigma)$  and  $\lambda - 1$ . The first effect is due to the hedging of real exchange rate movements. Note that without a home bias in trade ( $\theta = 0$ ) or log utility ( $\sigma = 1$ ), this effect would disappear. When terms of trade increase, the real exchange appreciates, and investors want to hold Home bonds which in effect are indexed on the price of Home goods. When  $\lambda > 1$ , dividends of stocks increase with a supply shock because  $1/\lambda$  is the elasticity of terms of trade to a supply shock. As discussed in Coeurdacier (2005) and Kollman (2005),  $\lambda$  is larger than unity for reasonable trade costs and an elasticity of substitution between Home and Foreign goods larger than one. In such a case, given the full Home bias in stocks, an investor will want to transfer wealth to the rest of world when a positive Home supply shock occurs. Hence, the holding of local bonds allows such a transfer because terms of trade decrease with a positive supply shock. In a sense, the holding of Home bonds compensates for the full Home bias in stocks.

More generally, any shock or combination of shocks that takes away revenues from consumers and redistributes them to private Home firms, without any effect on the terms of trade, would have the same effect on portfolios. Holding stocks

of Home firms insures against these shocks. Redistributive shocks between labor income and profits are the most natural in this context. We have checked that a combination of a demand shock that comes from government expenditure and a productivity shock would have the same qualitative effect on portfolios.

The main conclusion here is therefore that the introduction of redistributive shocks provides an incentive to have a large home bias in stocks. We find it interesting that the introduction of such reasonable shocks provides a strong incentive to hold Home stocks. This result however comes here at the price of a position on bonds that is realistic only for  $\lambda < 1$ . Indeed in the data, industrialized countries hold foreign bonds which translate in our model as a negative  $b$ . For  $\lambda < 1$  (an elasticity of substitution between goods more than one and reasonable trade barriers) such that  $b > 0$ , investors are short in Foreign bonds and long in local bonds. Another way to say it is that a depreciation of terms of trade transfers wealth from the Home to the Foreign country. This is in contrast with the valuation effects that have been described by a recent literature at least in the case of the US (see for example Gourinchas and Rey, Tille and Lane and Milesi-Ferreti).

### 4.3.3 I-pod and capital share shocks

We now investigate what is necessary to obtain both a realistic home bias in stocks and positions in bonds consistent with observed valuation effects. We now show that I-pod  $\hat{\Psi}$  and capital share shocks  $\hat{k}$  are sufficient to deliver these two results jointly. Portfolios on stocks (described by  $\alpha$ ) and on bonds (described by  $b$ ) are found using (16) and (15) in such way that portfolios give the optimal allocation for all shocks.

$$\alpha = 1 \quad ; \quad b = -\frac{1}{2} - \frac{1}{2} \frac{\theta(1-1/\sigma)}{\lambda-1} \quad (19)$$

Hence, the result of the full home bias in stocks remains. The intuition is

similar to the one described above. However, agents are now long in Foreign bonds and short in Home bonds. The gross bond position can be composed in two terms; the first term  $(-\frac{1}{2})$  is the consequence of the full Home bias in stocks. Indeed, following a deterioration in the terms-of-trade, Home agents suffer from a decrease in revenues (dividends fall due to the full Home bias in stocks and labor income also falls) and want to obtain a transfer of wealth from abroad in these states of nature. A short position on home bonds (and a long position on foreign bonds) allows such a transfer because foreign bonds pay more than home bonds when the terms-of-trade deteriorate. The second term  $(-\frac{1}{2} \frac{\theta(1-1/\sigma)}{\lambda-1})$  comes from the hedging of the real exchange rate. Following a negative I-pod shock at home, the home real exchange rate appreciates (but terms of trade deteriorate when  $\lambda > 1$ ) and home agents need a positive transfer from abroad. Again, holding Foreign bonds insures that this transfer exists in these states of nature. Such a position implies qualitatively plausible valuation effects as described in Gourinchas and Rey (), Lane (). An interesting feature of this result is that the Home bias in stocks is indirectly at the origin of the valuation effect on the gross position in bonds.

*Discuss effects on the current account*

## 5 Characterization of world equilibrium with incomplete markets

Given some of the unrealistic implications of complete markets, we now turn to the case of incomplete markets, assuming that available assets are restricted to the two stocks and two bonds in net-zero supply. Around the symmetric steady-state, one can still derive the goods market equilibrium but the equilibrium will depend on portfolio choices. In turn, portfolio choices depends on the goods market equilibrium, which makes the problem rather complicated. However,

following Devereux and Sutherland (2006), we can assume that up to the first-order approximation, portfolios are constant equal to their steady-state value. One can show that market-clearing condition in goods market and intratemporal allocation of both agents across goods imply (see Coeurdacier (2005)):

$$q^{-\phi} \Omega \left[ \frac{C_F}{C_H} \left( \frac{P_H}{P_F} \right)^\phi \right] = \frac{y_H - g_H}{y_F - g_F} \quad (20)$$

where the function  $\Omega$  has the same form as before.

One can show (see appendix) that in the presence of three types of shocks (productivity, i-pod and redistributive shocks), the optimal position for stocks is given by:

$$\begin{aligned} \alpha = & \frac{1}{2} - \frac{1}{2k} \frac{2a\theta(\phi-1)(1-1/\sigma)\sigma_y^2\sigma_\Psi^2}{2a(\phi-1)(\lambda-1)(\sigma_y^2 + \sigma_k^2)\sigma_\Psi^2 + \sigma_y^2\sigma_k^2} \\ & + \frac{1}{2k} \frac{[2a(\phi-1)(\lambda-1)\sigma_\Psi^2 + \sigma_y^2]\sigma_k^2}{2a(\phi-1)(\lambda-1)(\sigma_y^2 + \sigma_k^2)\sigma_\Psi^2 + \sigma_y^2\sigma_k^2} - \frac{1-\bar{k}}{2k} \end{aligned} \quad (21)$$

where  $\sigma_y^2$ ,  $\sigma_\Psi^2$  and  $\sigma_k^2$  are the variances of the productivity, i-pod and redistributive shocks respectively. We assume that shocks are uncorrelated. Note that eliminating one of the shocks (setting one of the variances to zero) brings us back to the complete markets situation analyzed in the preceding section. The structure of the portfolio in stocks depends on four terms. The first is the usual diversification motive to insure uncertainty on dividends. The second term is the hedging of real exchange rate movements that we already analyzed before but which is now richer because of the presence of several sources of disturbances of the real exchange rate, productivity and i-pod shocks. This term tends to generate a foreign bias (it reduces  $\alpha$ ) for sufficiently high elasticity of substitution between goods and sufficiently high risk aversion. Again, in the absence of trade barriers ( $\theta = 0$ ), the real exchange rate is constant and this term disappears. The third term comes from the presence of the redistribution shock and as in the complete market case induces a Home bias in stocks. This means

that when redistributive shocks are sufficiently large ( $\sigma_k^2$  is large), a home bias always exists. The last term, as in the complete markets case, induces a foreign bias because of the correlation of income from labor and from holding domestic stocks.

Note that with an elasticity of substitution of unity, a full Home bias ( $\alpha = 1$ ) is obtained for all configurations of parameters as long as the variance of the distribution shock is not zero.

The gross position in bonds is given by:

$$b = -\bar{k}(1 - \alpha) \frac{\sigma_k^2}{\sigma_\Psi^2} \left[ \frac{\sigma_\Psi^2}{\sigma_y^2} - \frac{1}{2a(\phi - 1)} \right] \quad (22)$$

where  $\alpha$  is given by (21). This expression first shows the relation between the Home bias in stocks as measured by  $\alpha$  and the extent and sign of the valuation effect induced by the gross position in bonds.

## 6 Quantitative analysis with incomplete markets