Debt Overhang and Monetary Policy

Costas Azariadis[*] **James Bullard**[†] **Aarti Singh**[‡] **Jacek Suda**[§]

This version: 8 May 2013
Preliminary and incomplete, comments welcome

Abstract

We study a theory in which households borrow during the first half of a 241-period life cycle as part of a DSGE. Households confront a persistent regime-switching process on aggregate labor productivity growth. When the economy switches to the high growth regime, there is more borrowing based on expectations of higher future income. When the economy switches back to the low growth regime, some households will have borrowed "too much" given contemporaneous income levels—the hallmark of debt overhang. A powerful central bank can intervene in private credit markets to influence real yields. If the central bank does intervene to keep real rates lower, consumption will be reallocated relative to a *laissez faire* case. The reallocation will generally be away from those households saving for retirement and possibly away from those households that are heavy users of money to smooth income fluctuations.

Keywords: Debt overhang, monetary policy, life cycle, heterogeneous households, productivity boom. *JEL codes*: E4, E5.

[*]Washington University in St. Louis, and Federal Reserve Bank of St. Louis.

[†]Federal Reserve Bank of St. Louis. Any views expressed are my own and do not necessarily reflect the views of others on the Federal Open Market Committee.

[‡]University of Sydney.

[§]Banque de France. Any views expressed do not necessarily reflect the views of the Banque de France or the European Central Bank.

1 Introduction

1.1 Broad motivation

The financial crisis and recession of 2007-2009 in the U.S. is often thought to be related to the "over-indebtedness" of households. We take "over-indebtedness" to mean that, *ex-post*, households wish they had not borrowed so much given contemporaneous income levels. This is the element of regret which is the hallmark of debt overhang. Figure 1 illustrates some of the evidence on deleveraging during the most recent recession as compared to previous recessions in the U.S. While real household debt levels increased during the time period encompassing other recessions, in 2007-2009 real household debt levels clearly decreased. This seems to indicate that the U.S. household sector wished to shed debt during this period, marking a stark difference between the most recent episode and earlier ones.

We study a key reason why life cyle households may have "borrowed too much," mostly as mortgage debt, during the run-up to the financial crisis: They rationally expected their income would be higher than it actually turned out to be. Given high income expectations, some households would have wanted to borrow substantially more (and others save substantially more) in order to smooth expected lifetime consumption. When the boom period ends, some life cycle households have "too much debt" given the now lower levels of current and expected future income. We explore this idea as a theory of debt overhang.

We then turn to ask how a powerful central bank that can intervene to influence real interest rates in asset markets may or may not be able to improve on the debt overhang situation. The typical intuition in many policy circles post-2009 has been that debt overhang rationalizes low real interest interest rates—low real rates are an elixir for the deleveraging process. Therefore, advocates argue, if the central bank can lower real returns on household debt during the overhang period they should definitely do so. However, others have simultaneously argued that low real rates are hurting savers in the economy and therefore may be a poor policy prescription during the over-

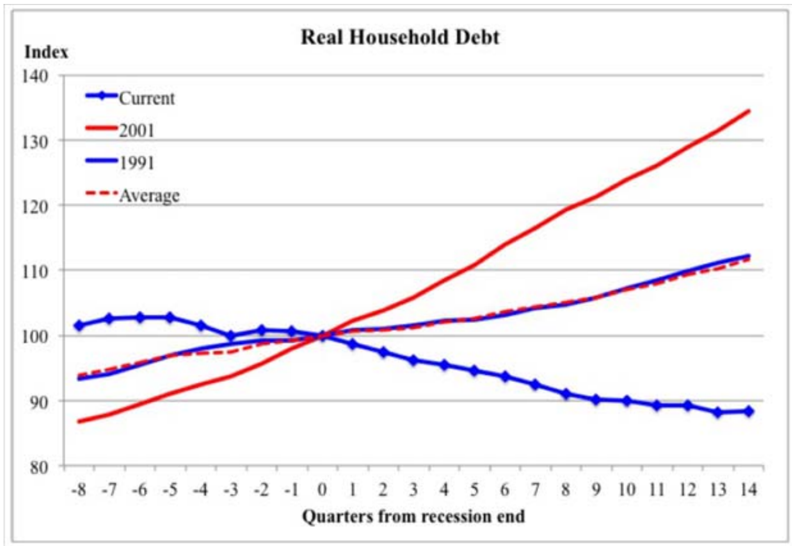

Figure 1: Real household debt levels declined during the 2007-2009 time frame, the diamonds in this Figure. The time scale indicates the quarters before and after the recession. This deleveraging contrasts with previous postwar recessions.

hang era. The model of this paper can help to address some of these issues more directly. In the economy studied here, the central bank will be able to influence real yields on household debt. However, doing so reallocates current and expected future consumption among households, and this reallocation may be viewed as undesirable.

1.2 What we do

We consider a simple and stylized 241-period general equilibrium life cycle model of medium-term movements in real private debt levels, real interest rates, and inflation.[1] This economy features real household debt as part of the stationary equilibrium. Household debt and currency are the only

[1] We think of this as a quarterly model, but many key effects involve transitions over several or many periods. We think it is important to calibrate the pace of these transitions to the quarterly frequency so that they can be appropriately compared to results from other models.

assets. We divide the population into two groups, credit market participants and credit market non-participants. The participant group trades freely in smoothly operating private credit markets.[2] Households supply one unit of labor inelastically in each period, but their productivity varies over the life cycle. We study the situation in which the households' life cycle productivity endowment is concentrated in the middle of life, so that they accumulate debt during the first portion of the life cycle and hold positive assets during the second portion. We augment this economy with a stochastic income growth process—an aggregate shock. In particular, aggregate labor productivity growth follows a two-state regime-switching process with persistent regimes. There is no idiosyncratic uncertainty, and we compute the equilibrium by keeping track of the distribution of asset holdings among the 241 cohorts. All economic actors in the model are Bayesian learners. Once some credit market participant households infer that they are in a persistent high growth regime, they will wish to borrow more in order to smooth life-cycle consumption. When the economy switches back to the low growth regime, some households in the economy suffer from a type of debt overhang. We discuss the nature of this situation in detail in the main text.

A generally smaller group of credit market non-participants are precluded from the credit market altogether. Their productivity endowment profile is smaller and intermittent, so that they can earn income only sporadically and wish to consume at times when income is unavailable. To smooth consumption in this situation, these households use currency issued by the central bank. We want to think of these households as corresponding to the unbanked or nearly-unbanked households in the U.S.—some estimates have put the size of this group at 15 percent of the U.S. population. We ensure that the real rate of return on currency is less than the real rate of return in the credit market in the equilibria we study. This means that credit market participant households will not wish to hold currency in the stationary equilibrium.

[2] There are no impediments to credit in this model. There are no borrowing constraints, and debt is always fully repaid. There is no role for collateral.

The central bank in this economy is powerful and independent. We take independence to mean that the central bank can transact as it wishes with other actors in the economy, including the government and the private sector credit market—at prevailing market prices—but that it does not give gifts to any agent in the economy. We ask what, if anything, a powerful central bank may wish to do to mitigate the effects of the debt overhang.

1.3 Main findings

The stationary equilibrium of this economy naturally generates substantial levels of household debt relative to GDP. Stationary equilibrium real rates of return are closely related to the real rate of growth in the economy—in the long run average behavior stationary equilibrium, the one period real rate of return would be exactly equal to the output growth rate which in turn is driven solely by the pace of growth in labor productivity.[3] Versions of this result are a general feature of models in this class, but the exact correspondence between the long-run pace of growth and the long-run real interest rate is due to the somewhat stylized set of assumptions we used to design the model. Periods of high growth are periods of high interest rates, however, there are transition effects and in addition, households hedge behavior because of the possibility that the economy may switch to the other regime. Persistent periods of high labor productivity growth induce some households to borrow more (and other households to save more) in order to smooth consumption over the life cycle. The period of the "debt overhang" puts heavy pressure on the households that have borrowed extensively during the high income growth regime. Generally speaking, relative to the high growth regime, these households must either consume less, save less, or default on their debt (we do not study the default possibility in this paper).

The powerful central bank supplies currency to the economy's non-participant households and targets a moderate rate of inflation. The central bank can intervene if it so desires in the private credit market, and real returns in that

[3] In this sense the credit market portion of the economy is dynamically efficient.

market can be substantially influenced by the intervention activity. We define "laissez-faire" as a case in which the powerful central bank does not intervene to influence the real interest rate on private debt.[4] Under the laissez-faire policy, the period of debt overhang is associated with a real interest rate somewhat higher than the output growth rate associated with the low labor productivity growth regime. This is the nature of the DSGE in this model, and we spend some time in the main text understanding this effect.

The monetary policymakers in this economy may have the intuition that the real interest rate should be very low during the period of debt overhang. If the powerful central bank intervenes in the private debt market, the real interest rate on private debt can in fact be kept at a lower level during the period of the debt overhang. This type of intervention re-allocates income in the economy, generally away from those are saving for retirement and possibly away from those who make heavy use of currency to smooth consumption. We do not study optimal monetary policy in this paper, but we comment on the likely direction for that at the end of the main text.

2 Facts

2.1 Labor productivity and the tech boom

Our model is one of medium term dynamics in real yields, inflation, and real debt levels. We want to think of the U.S. technology boom beginning in the 1990s and the associated increase in labor productivity growth as initially heralding a "new era" in household income prospects. We see this event as causing households to borrow more (and other households to save more) than they otherwise would in order to smooth life-cycle consumption. The increase in labor productivity growth is perceived to be quite persistent, but ultimately does not last, causing a significant "debt overhang" in the sense that after the boom contemporaneous debt levels are too high given contemporaneous and future expected household income levels.

[4] We use this term even though the central bank is still active in the part of the economy involving the non-participant households.

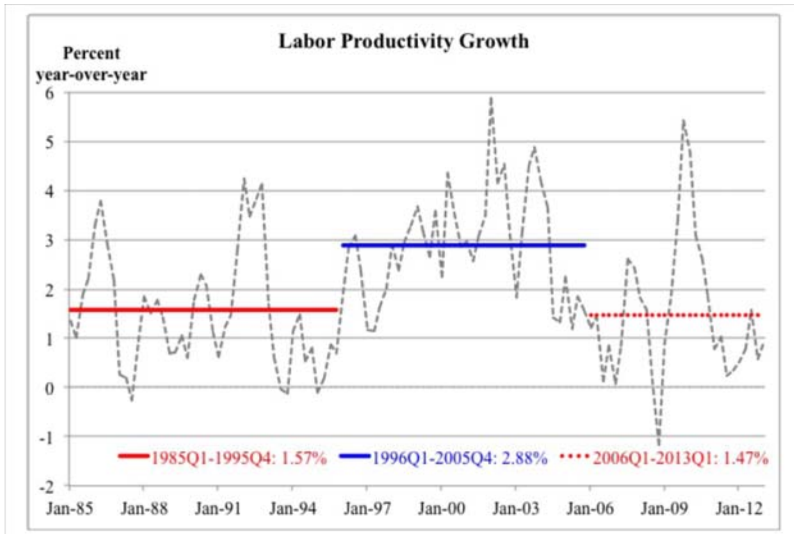

Figure 2: U.S. labor productivity growth from 1985 through 2013Q1. We will view this data as consistent with a regime-switching process with a mean of 1.5 percent in the low regime and 3.0 percent in the high regime.

Figure 2 illustrates the data on labor productivity growth in the U.S. since 1985. The increase in labor productivity growth was fairly substantial and persistent during the decade 1996-2005. In the ten years from 1986Q1 to 1995Q4, average quarterly productivity increases in the U.S. measured at an annual rate were 1.57 percent. In the ten years from 1996Q1 to 2005Q4, the pace of increase per quarter was 2.88 percent at an average annual rate. And, from 2006Q1 to 2012Q4, the pace has fallen again to 1.49 percent. We note that peak life cycle household labor income would be about 50 percent higher in an economy with 3.0 percent income growth versus 1.5 percent. We will use these data to inform the type of shock we would like to feed through our model economy. We will think of two regimes, one in which productivity growth increases at a pace of 1.5 percent per year, and another in which productivity increases at 3.0 percent per year. We will view the probability of switching between the two regimes as small, so that each regime will be quite persistent. We will view the economy as beginning in the low regime, switching to the high regime for a decade, and then switching to the low regime and remaining there.[5]

Mian and Sufi (2011) suggest that the 1995 U.S. household debt-to-income ratio was about 1.15, but that by 2005, it was approximately 1.65. This is a 43 percent increase, and we will keep this percentage increase in mind to see if we can get the amount of debt outstanding to increase by a similar magitude in our model over a ten-year time frame. However, we caution that the Mian and Sufi (2011) debt-to-income ratio is not directly comparable to the debt-income ratios produced in our model because our model is one of *net* asset-holding, whereas in the data it can be difficult to net out all assets. Still, there can be little doubt that the level of debt outstanding increased substantially during the decade 1996-2005, and we will try to assess the ability of the model to generate substantial increases.

2.2 Monetary policy during the debt overhang

The Fed policy in response to the debt overhang has been to keep real interest rates low. One benchmark for what constitutes a low real interest rate is the rate of real output growth. For instance, at the time of this writing the U.S. five-year TIPS real yield is about -1.43 percent, whereas the real output growth rate is about 2.0 percent measured from one year earlier. We will consider central bank policies that attempt to keep real yields lower than the real output growth rate during the period of debt overhang.

3 Environment

3.1 Segmented markets

Households are divided into two types, labeled "participants" and "non-participants." Both participant and non-participant household cohorts are atomistic, identical and have mass one, and we will analyze each participant cohort as if there were only one member. Households live in discrete time for

[5] We can also simulate an economy with ongoing shocks and report properties of the equilibrium.

$T+1 = 241$ periods, which we think of as corresponding to a quarterly model in which households begin economic life in their early 20*s* and continue until death. A new cohort of households enters the economy each period such that there is no population growth. The economy continues into the infinite past, so that $-\infty < t < +\infty$. The only assets in the economy are consumption loans in the credit market and currency. Labor is supplied inelastically but households have different levels of labor productivity at different stages of the life cycle. We will study the case where life cycle productivity is concentrated in the middle portion of the life cycle for credit market participants.

3.2 Labor productivity

There is an exogenous wage $w(t)$ which follows

$$w(t) = \lambda(t)w(t-1), \tag{1}$$

with $w(0) > 0$. We think of $\lambda(t) \geq 1$ and allow $\lambda(t)$ to follow a standard two-state regime-switching process. In the high state, we set the value of $\lambda(t)$ to the approximate value of the average labor productivity growth rate in the U.S. from 1996 to 2005, namely $\lambda_h = 1.03^{1/4}$. In the low state we set the mean of $\lambda(t)$ to the approximate average labor productivity growth rate from 1986 to 1995 and again from 2006 to 2013, namely $\lambda_\ell = 1.015^{1/4}$. We will assume that λ takes on either of these two values according to a standard regime-switching process. The stochastic process within each regime can have positive variance and positive serial correlation.

We study two versions of the economy differentiated by information assumptions. In the *complete information* version, the value of $\lambda(t)$ is revealed by nature at the beginning of each date t. In this version, the actors in the model do not face an inference problem. The value of $\lambda(t)$ delineates future wage expectations. In the *incomplete information* version, nature does not reveal the value of $\lambda(t)$ at the beginning of date t, and instead all actors in the model must infer the state given the most recent observations on endogenous variables. In this version we follow the approach of Bullard and Singh (2011) and allow the agents to be Bayesian learners with respect to the state of the system and hence with respect to real wage expectations.[6] As documented by Bullard and Singh (2011) for a representative agent economy, the presence of Bayesian learning will moderate the behavior of the households relative to the complete information case, provided the regimes are sufficiently close together to create and interesting inference problem for the state of productivity. However, Bayesian learning is unlikely to change the more fundamental conclusions about the role of monetary policy in this model.

3.3 The participant household problem

Let $c_h(t)$ denote the consumption of the participant cohort h at date t. The cohort born at date $h = t$ maximizes utility

$$\max_{c} E_0 \sum_{s=0}^{T} \beta^s \ln c_t(t+s) \tag{2}$$

where $\beta > 0$ is the discount factor.[7] Participants are endowed with life cycle productivity e_i, $i = 0, 1, ..., T$. The participant households will not hold currency because we will ensure that the real rate of return on currency is lower than the real rate of return on private debt. The participant households' have a sequence of budget constraints that can be expressed as

$$c_t(t) = e_0 w(t) - a_t(t) \tag{3}$$

$$c_t(t+1) = e_1 w(t+1) + R(t) a_t(t) - a_t(t+1) \tag{4}$$

$$c_t(t+2) = e_2 w(t+2) + R(t+1) a_t(t+1) - a_t(t+2) \tag{5}$$

$$\ldots$$

$$c_t(t+T) = e_T w(t+T) + R(t+T-1) a_t(t+T-1). \tag{6}$$

where $a_h(t)$ is the asset holding of cohort h at date t, $w(t)$ is the real wage at date t, and $R(t)$ is the real rate of return on loans originated at date t and

[6] In the current version of the paper, we only study the complete information case.

[7] In general equilibrium life cycle models, the discount factor need not be less than one. We may set $\beta = 1$ for some purposes in this paper.

maturing at date $t+1$ in the part of the economy where the credit market is operating. We will ensure that $R(t) > R^m(t)\ \forall t$ where $R^m(t)$ is the real rate of return on currency holdings.

3.4 Productivity endowments

The productivity endowments of the household are given by $e = \{e_s\}_{s=0}^{240}$. We use

$$f(s) = \mu_0 + \mu_1 s + \mu_2 s^2 + \mu_3 s^3 + \mu_4 s^4 \tag{7}$$

such that $f(0) = 0$, $f(60) = 57/100$, $f(120) = 1$, $f(180) = 57/100$, and $f(240) = 0$. Solving these five equations yields the values for μ_i, $i = 0, ..., 4$. This is a stylized endowment profile which emphasizes that beginning and end of life productivity is near or equal to zero. It also puts the bulk of income in the middle half of life. When the three-period model is studied, it will be convenient to talk about the $\{0, 1, 0\}$ case as an approximation to this endowment profile. It is also symmetric. This means that the agents in the economy would exactly balance the need for saving into relative old age with the need for borrowing in relative youth. This would mean that, if in addition $\beta = 1$, the long-run stationary equilibrium real interest rate would be $R = \lambda$. We think this is a good benchmark to use as it will facilitate both our own understanding and calculations as well as communication and understanding with readers. In a robustness section we use other endowment profiles $f(s)$ and show that the main ideas are the same.

3.5 The non-participant household problem

Non-participants are precluded from using the credit market. Like their participant counterparts, they live $T + 1 = 241$ periods. We will discuss these agents according to their stage of life $s = 0, 1, ..., 239, 240$. In stage of life 0, these agents are inactive. They do not consume, nor do they earn labor income. In odd-dated stages of life, these agents have a productivity endowment $\gamma \in (0, 1)$. We will think of this γ value as being fairly low—in addition, there is no life cycle aspect to the value of γ. The households born at date t then earn income $\gamma w\,(t+s)$, $s > 0$, $s = 1, 3, ..., 239$. In the even periods, the non-participant households consume. The period utility for households born at date t in these periods is $\beta^s \ln c_t\,(t+s)$, $s = 2, 4, ..., 240$. Again, we can set $\beta = 1$ if desired.

The non-participant agents evidently earn income only intermittently. They move income into periods when they need to consume by holding currency. With upward sloping wages (via $\lambda > 1$), the households will not wish to carry money beyond one period, as there is no reason to save beyond one period. Accordingly, they will solve a series of two period problems, saving everything by holding money, and then consuming everything before working (supplying labor inelastically) in the following period.[8]

Some non-participants will have labor income at a date where other non-participants will wish to consume. That is, some will be in an even stage of life $s = 2, 4, ...$ while others will be in an odd stage of life $s = 1, 3, ...$ However, we do not allow credit between these agents. Only currency can change hands.

3.6 The powerful central bank

We envision the central bank as being set up to be independent. We interpret "independent" to mean that the central bank is free to interact in the economy through trade at market prices without interference or direction from the rest of the government. The central bank does not give gifts to any agent in the economy, including the rest of the government, nor does it consume at any date.

Each period the central bank will print currency which is valued by non-participant households. The new currency will be used to buy some of the consumption good from working non-participants in the market for currency. We will view the monetary authority as conducting this policy in order to maintain an established inflation target. (We will comment on optimal policy at the end of the paper.)

[8] This form of the two-period problem eliminates any steady state in which no agent wishes to hold currency.

The monetary authority does not keep the consumption purchased in the market for currency, but instead takes it to the government. We want to think of this interaction as follows. The central bank is independent and does not give gifts to any participant in the economy. Accordingly, the central bank lends the consumption acquired in the market for currency to the government at the rate of return then prevailing in the credit market. The government then consumes the good,[9] and the central bank is left with a note from the government. This note is backed implicitly by lump-sum future taxes, but in normal times these taxes will never need to be levied. Instead, in the next period the central bank returns to the government with more of the consumption good acquired in the market for currency plus the note acquired from the government in the previous period. The government then issues a new note to the central bank promising to repay in the following period, and the new note covers the amount of consumption plus the previous period's note plus interest. In the baseline equilibria we study, this process continues indefinitely, leading government debt to pile up on the balance sheet of the central bank.

If the central bank does not intervene in the market for credit, this process would be inconsequential. The central bank is turning over real seigniorage revenue to the government in each period, and the government is handing paper back to the central bank which is implicitly backed by future taxes that would never have to be levied so long as the process continues in this fashion. The size of the central bank balance sheet would be a record of the entire history of past seigniorage revenue in the economy.[10,11]

Let's now turn to the possibility of intervention in the credit market by the central bank. Intervention in the credit market may at first blush be

[9] These resources leave the economy.

[10] The transaction is handled in this way to maintain the notion that the central bank is an independent entity which transacts with all actors in the economy at market prices, giving gifts to no agent.

[11] At the opening of the finanical crisis, the Fed had a balance sheet of about $800 billion which might be interpreted in this way. Chairman Bernanke was asked at Congressional testimony in the early phases of the crisis, How much do you have [to battle the crisis]? His response was, "I have $800 billion."

viewed as irrelevant based on the important result due to Wallace (1981). The government debt that is issued to the central bank pays the same rate of return as the privately-issued debt in the credit market, and savers in the economy would be indifferent between holding the privately-issued or the publicly-issued paper. We can deduce that a swap of the debt on the central bank's balance sheet for the debt held by the middle-aged and older households in the credit market would have no effect on market prices and therefore would be a neutral policy.

However, the central bank does have a method of intervention that will have important real consequences in this situation. Suppose that the economy has been in the "normal" situation described above for a long time so that the balance sheet of the central bank is sizeable relative to GDP. The central bank normally takes today's seigniorage revenue and the note issued yesterday by the government to the government an exchanges all of it for a new note. But, the central bank can also exchange only a portion of these resources for a new note, instead demanding repayment in the consumption good for the remainder of the government debt. Because transactions take place at arm's length, the government cannot refuse this but instead has to raise revenue through lump-sum taxes to pay back the central bank in the consumption good. The central bank then offers this consumption to the younger, borrowing households in the economy in exchange for their privately-issued paper. This type of intervention potentially has important real effects on the private credit market.

This is not to say that such an intervention is desirable. We have been deliberately vague concerning the nature of the taxes that might have to be levied in this situation, and the nature of the tax arrangements would be an important consideration in whether such a policy would ultimately be desired or not from a welfare perspective. But the outlined intervention above does suggest that (1) such an intervention could be contemplated by the central bank, (2) that it would be quasi-fiscal in nature, (3) that it could be of a scale that would have important consequences for the private credit market,[12] and,

[12]The central bank could simply take today's seigniorage revenue and not remit it to the

above all, (4) that this is a very simple process of the central bank stopping its policy of rolling over treasury securities and instead deciding to purchase privately-issued paper.

Another consideration is that the policy of the monetary authority to intervene in the private debt market is presumably temporary. This means that the central bank will eventually reverse its position, selling its privately-issued debt, taking the revenue and investing the resulting sum back into government-issued debt once more. At that point, the government could reverse its previous imposition of lump-sum taxes on households. This seems to be consistent with the idea that monetary policymakers often express the idea of holding an "all Treasuries" portfolio in the long run.

These considerations give rise to the following. We let $H(t)$ denote the currency stock at date t, and we let $P(t)$ denote the price level prevailing at date t. The real value of the currency at date t is $m(t) = H(t)/P(t)$. We let $A^{cb}(t)$ denote the holdings of one-period private sector debt acquired by the central bank at date t and maturing at date $t+1$. We let $G(t)$ be government revenue at date t, and we let $G^{b}(t)$ be the holdings of government debt acquired by the central bank at date t and maturing at date $t+1$.

The government budget constraint is

$$A^{cb}(t)+G^{b}(t) = R(t-1)\left[A^{cb}(t-1)+G^{b}(t-1)\right]+\frac{H(t)-H(t-1)}{P(t)}. \quad (8)$$

We let $N(t)$ be the nominal yield on private debt, and $R^{m}(t) = P(t)/P(t+1) = R(t)/N(t)$ be the real yield on money holdings (that is, the gross nominal yield on private debt must be defined as $N(t) \equiv R(t)/R^{m}(t)$). The gross inflation rate between date t and $t+1$ is therefore $1/R^{m}(t) = P(t+1)/P(t)$.[13]

government but instead offer that consumption to young borrowers in exchange for their paper. We have emphasized the story in the text because the scale of the intervention is likely to be much more consequential.

[13]We have not specified any taxes in this version because in normal times, the government would not have to levy these taxes. We intend to be more specific about the tax arrangements in future versions. In general, a lump-sum tax scheme would have to be age-dependent and temporary and may involve the non-participant households.

3.7 Equilibrium

3.7.1 Overview

Equilibrium is a sequence $\{R(t), R^m(t)\}_{t=-\infty}^{+\infty}$ in which households maximize utility subject to the constraints imposed and markets clear. The problem is one of heterogeneous households facing an aggregate shock. Accordingly we track the distribution of asset holdings in order to calculate the equilibrium. We calculate the equilibrium for the laissez faire case as a baseline. In this case, the central bank is not intervening in the private credit market, and so we can think of the economy of the non-participants as operating independently. When we allow central bank intervention, it will be via the government budget constraint, and so we will not have to keep track of asset-holding in the non-participant part of the economy.

3.7.2 The participants' problem

Participant households entering the economy at date t maximize subject to their sequence of budget constraints. The wage is exogenous and stochastic. We can normalize the wage $w(0) \equiv 1$, the date before this optimization problem is posed. There will be a given distribution of asset holdings across cohorts at any date t in the economy. We set the date zero distribution of asset holdings to be consistent with long-run average behavior of the system.

3.7.3 The state of the system and learning

The state of the system is the value of λ. In the Bayesian problem, λ is viewed as being dependent on a latent variable. Accordingly, nature chooses a value for λ at each date, but it is not revealed to any of the participants in the economy. Instead, households have an expectation of the value of λ labelled λ^e. The discussion in Bullard and Singh (2011) indicates that the Bayesian learning problem will tend to moderate behavior relative to a complete information case in which the state λ is observed directly. In that paper, the state variables are the expected level of technology and the level of the capital stock. The current paper has no capital stock.

What is the wisdom of including a Bayesian learning problem, as opposed to simply assuming that the level of λ is revealed at the beginning of every period? The answer is that it may moderate the transition paths from what they would otherwise be, so that the model would predict less sharp and extreme behavior in response to regime switches. The downside of a "non-learning" version is that one is in that case implicitly assuming that all households understand immediately in 1996 that the level of labor productivity had switched to the new and persistent regime, and would begin to behave consistently with the switch. This seems a little unrealistic, but perhaps it is not critically important for the story we are telling.

Accordingly, our first efforts have been to solve for the equilibrium of this economy when the level of labor productivity is simply revealed by nature at the beginning of each date. This is the "complete information" version of the model.

3.7.4 First order conditions

For the agent born at date t, the first order conditions combined with the present value version of the budget constraint imply that first period consumption can be written as

$$c_t(t) = \frac{\Xi_t(t)}{B} \tag{9}$$

where $B = 1 + \beta + \beta^2 + ... + \beta^T$, and where $B = T + 1$ if $\beta = 1$. In this expression $\Xi_t(t)$ represents the expected discounted present value of all future lifetime income, where the discounting is at the expected real rate of interest in each future period. Similar expressions are available for all other agents making decisions at date t, except that for all other agents the initial level of asset holdings (positive or negative) upon entering the period will be part of the expression. The problem of each household can be written in dynamic programming form as their current consumption problem plus the value of being in the next stage of life.

In the laissez-faire economy, total asset holding must sum to zero. This means

$$A(t) = a_{t-T}(t) + a_{t-T+1}(t) + ... + a_{t-1}(t) + a_t(t) = 0. \tag{10}$$

This is an expression in expected wages and real interest rates along with the distribution of asset holdings.

The system has a certain long-run average behavior which can be described as follows. The Markov switching means that in the long run the agents will expect the system to be equally likely to be in either state. In this 50/50 state, expected output growth will be the average growth rate of the two states, namely 2.25 percent at an annual rate. The problem has been designed so that, if $\beta = 1$, the expected long-run stationary equilibrium involves $R = \lambda$, where λ is this average value.

4 An example

4.1 Three periods

To fix ideas, let's consider a three-period life cycle economy. We let the wage $w = 1$ for all t. Participants maximize

$$V_t^p = \ln c_t(t) + \ln c_t(t+1) + \ln c_t(t+2). \tag{11}$$

The endowment pattern is given by $(e_0, e_1, e_2) = (0, 1, 0)$. Participant households may hold currency or private debt; however, they will choose to hold only private debt in equilibrium. They maximize utility by choice of consumption subject to a three-period version of the budget constraints listed above. Aggregate holdings of private sector debt at date t must be

$$A(t) = a_{t-1}(t) + a_t(t). \tag{12}$$

Since we have assumed the first period endowment is zero, $a_t(t)$ represents the amount of borrowing in the economy. Let $x_h(t)$ denote the consumption of the non-participant cohort h at date t. Lifecycle utility for cohort $h = t$ is

$$V_t^{np} = \ln x_t(t+2). \tag{13}$$

The endowment pattern is given by $\gamma(0,1,0)$. Non-participants may only hold non-negative amounts of currency. Private debt and currency are traded in two distinct competitive markets that clear at each date $t = 0, 1, 2, ...$ with real gross yields denoted by $\{R(t), R^m(t)\}$, respectively.

4.2 The steady state

For this example economy we assume an extreme form of regime switching in which the transition probabilities between states are arbitrarily small, making the states themselves perfectly persistent. The low income state in the example $(0,1,0)$ endowment economy with log preferences, $\beta = 1$, and laissez faire, and with $w = 1 \ \forall t$, has the following features:

(1) The real return on private debt $R = 1$.

(2) Consumption for participants is perfectly smooth at $1/3$.

(3) The society carries real debt equal to $1/3$ of a unit of income at all dates.

(4) An inflation target $\pi^\star$ is maintained at all dates.

(5) The non-participants experience the real return $R^m = 1/\pi^\star < R$ currency holdings.

4.3 The shock

At the date of the shock, middle-aged incomes are higher than anticipated and are expected to remain at the elevated level by all actors in the economy. The borrowing cohort now wishes to borrow more, and the middle-aged lending cohort wishes to save more. If nothing else happened, this would create a new steady state with higher income, more lending, and more debt. One period after the shock, nature reveals that the higher income situation was temporary, not permanent, and income reverts to its original steady state level. The cohort that borrowed based on expectations of higher income now experiences a "debt overhang."

Figures 3 and 4 show the path of real interest rates in the credit market in response to this very persistent income shock. In the laissez faire case,

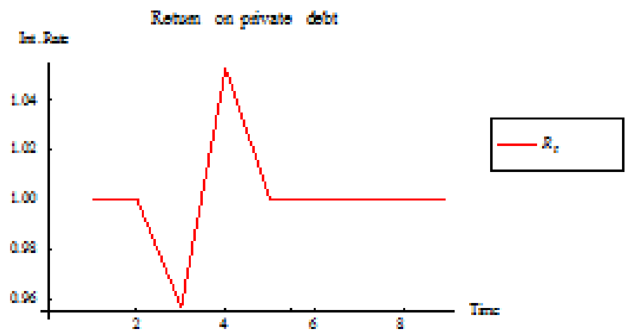

Figure 3: The return on private debt in the laissez faire economy, 3-period example. The perfectly persistent income shock causes real returns in this market to fall, then rise before returning to steady state.

Figure 4: The real return on private debt in the economy where the central bank intervenes to hold interest rates low for an additional period, instead of allowing rates to rise as in the laissez faire case. This policy is intended to be an elixir for debt overhang.

where the central bank does not intervene, the real return initially falls in the period of the income shock, which is period 3 in the simulation. Because there is more income in middle age than was expected in this period, and because this increase in middle age income is expected to be permanent, the young households wish to borrow more and the middle age households wish to save more relative to period 2. The interest rate falls in period 3 because the middle age households did not know that they would be receiving more income in middle age and so did not borrow enough in youth. Even though the new generation now wishes to borrow more, there are still extra resources for the middle age generation and this depresses real yields in the period of the shock. In period 4, the shock turns out to be temporary. This is the period of the debt overhang. The middle age of this period are under heavy pressure because they borrowed a lot in youth expecting high income in middle age. They have less resources than expected. The youth of period 4 now expect lower income in their middle age and so only want to borrow the "normal" amount, but this is more than the middle age can provide. Accordingly, the real interest rate must rise to clear the market. In Figure 4, the central bank intervenes to keep the real interest rate low during period 4, the period of debt overhang.

4.4 Welfare

A key question is what the effects of an intervention to keep the real interest rate low during the period of debt overhang might be. We calculate lifetime utility for the various cohorts of the model under laissez faire, under the policy of keeping rates lower for longer, and under a third policy of keeping the real interest rate constant at all times in the market for private debt.

Table 1 indicates that the two situations in which the central bank intervenes in the credit market reallocate consumption relative to the laissez faire case. The $T-1$ cohort is middle aged at the time of the income shock. They receive the extra income and accordingly they do well in this calculation. The T cohort is young at the date of the income shock and middle-aged at the date of the debt overhang. The $T+1$ cohort is young in the period of the

Table 1: Relative consumption

	LF P, NP	Low P, NP	Constant P, NP
T-2 cohort	$1.00, 1.17$	$1.00, 1.17$	$1.00, 1.35$
T-1 cohort	$1.08, 1.00$	$1.08, 0.65$	$1.10, 0.63$
T cohort	$1.03, 1.00$	$1.00, 1.34$	$1.00, 1.19$
T+1 cohort	$0.98, 1.00$	$1.01, 1.00$	$1.00, 1.00$
T+2 cohort	$1.00, 1.00$	$1.00, 1.00$	$1.00, 1.00$
T+3 cohort	$1.00, 1.00$	$1.00, 1.00$	$1.00, 1.00$

Table 1: Average consumption is relative to the steady state average consumption.

debt overhang. The $T+1$ cohort fairs the worst under laissez faire because they have to borrow at high rates but they do not enjoy any extra income. The T cohort is the sufferer under the normal debt overhang narrative—but this cohort is able to borrow a lot in youth at relatively low rates and so actually enjoys a portion of the income winfall that occurs with the positive income shock. The two interventionist policies "punish savers" in credit markets by reallocating consumption toward cohort $T+1$ (they get either 1.01 or 1.00 of average consumption when the central bank intervenes versus 0.98 under laissez faire) and away from cohort T, middle-aged savers (they get 1.00 under either policy versus 1.03 under laissez faire). To the extent that intervention is financed through higher inflation, the non-participant cohorts are adversely affected.

5 The full model

5.1 Aspects of the stationary equilibrium

In this version of the paper, we report results for a 25 period version of the model instead of 241 periods. This is only for computational simplicity for this draft.

A lot of the intuition for the stationary equilibrium can be obtained by recalling that the long-run expected stationary equilibrium involves $R = \lambda$, where λ is the average rate of labor productivity growth, and hence the average growth rate of the economy, where the average is taken across the two regimes. The stationary equilibrium involves real interest rates $R(t)$ that are close to the two regime values for $\lambda(t)$, that is, λ_H and λ_L, but not quite equal to these values. This is because the households hedge their behavior, putting a small weight on the possibility that the system will switch to the other regime.[14] Thus the stationary equilibrium involves real interest rates which switch between the two output growth rates, but which are somewhat lower than the output growth rate in the high state and somewhat higher than the output growth rate in the low state. On average in the long run the real rate is then equal to the output growth rate.

However, there is more to the story of the stationary equilibrium, because the heterogeneous households have existing levels of asset holding at each date t and it takes time for the distribution of asset holdings to converge to the distribution associated with either the high or the low state following a switch.

5.2 An experiment

We conduct an experiment in which we simulate a labor productivity growth shock similar to the one observed in the U.S. between 1996Q1 and 2005Q4 and discussed earlier in the text concerning Figure 2. In this experiment, we keep the system in the low state initially and we simulate the system for a sufficient amount of time to eliminate any transient dynamics associated with the initial conditions. We then allow the system to switch to the high state and remain there for 10 periods before switching back to the low state and remaining there. We expect the first switch to induce higher levels of household debt (and household saving) in the economy. We expect the second switch to induce debt overhang and deleveraging. We first discuss a laissez faire case, and then we investigate credit market intervention by the central

[14] We use a transition probability matrix that puts a probability of 0.95 on remaining the current regime and a probability of 0.05 on switching to an alternative regime.

bank.

A consideration of the problem of the non-participants indicates that the demand for currency will be given by

$$m^d(t) = \frac{\gamma T}{2} w(t) = \frac{\gamma T}{2} \lambda(t). \tag{14}$$

Since, as discussed earlier, $R^m(t) = P(t)/P(t+1) = R(t)/N(t)$, and since real money supply $H(t)/P(t)$ must be equal to the above, we can write

$$N(t) = \frac{\gamma T}{2} \frac{w(t) R(t) P(t+1)}{H(t)}. \tag{15}$$

The central bank's operations in the currency market can then be viewed as choosing a nominal interest rate $N(t)$ to achieve a target, such as a target rate of inflation. This can be accomplished by choosing a value for the currency stock at each date t. In this expression, imagine that the central bank follows a constant gross currency growth rate $\theta > 1$ to achieve and inflation target, and imagine that the system is in the long-run expected stationary equilibrium in which $R = \lambda$ and λ is the average of the two regime values. Normalize the date 0 wage to 1 and normalize the date 0 price level to 1 as well. Set the initial currency stock to $2/\gamma T$. The rate of inflation is then θ/λ. The right hand side of the expression becomes $\frac{\lambda^t \lambda (\theta/\lambda)^{t+1}}{\theta^t} = \theta$, that is, the long-run expected nominal interest rate is equal to the gross rate of currency creation.

5.3 Preliminary results

We begin with a laissez faire case in Figure 5. This figure shows the path of the real interest rate in the credit market in the stationary equilibrium of the economy subject to the regime switches described above. The horizontal line and shading at the bottom of the picture indicates the low value of $\lambda(t)$, λ_L, measured at an annual rate. The horizontal line and shading at the value $\lambda_H = 1.03$ at an annual rate indicates the high regime. The real interest rate following many periods in a row in the low state is just slightly above the output growth rate in the low state. This is depicted in the left portion of the Figure from date 30 to date 35. The system then switches to the high regime.

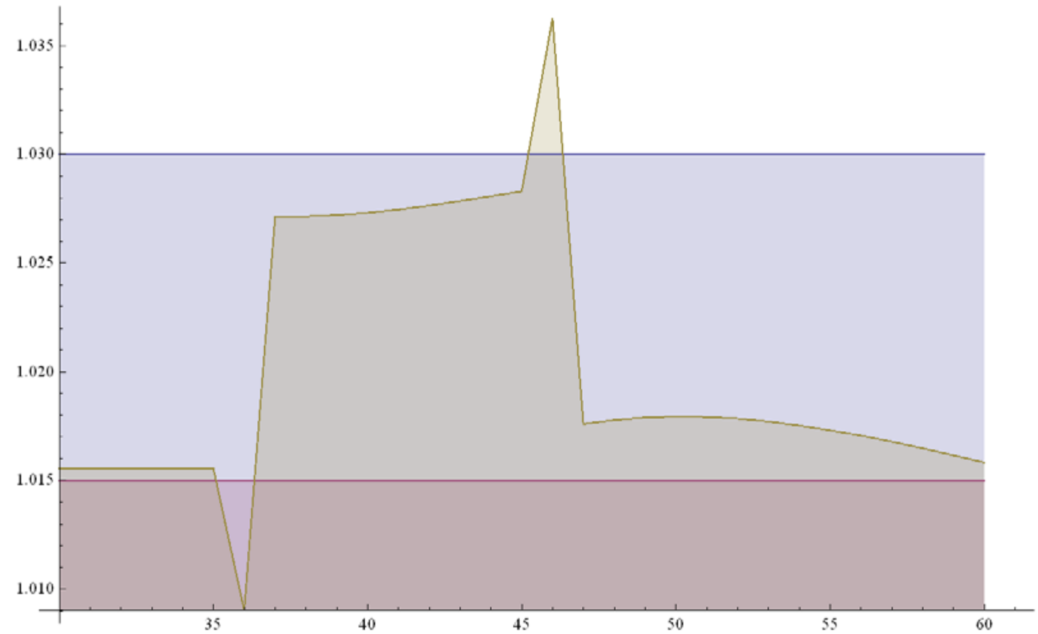

Figure 5: The equilibrium for the real interest rate for a period characterized by just two switches in regime and no central bank intervention.

The initial response is for the real interest rate to fall for the reasons outlined in the example of the previous section. The rate then rises toward the high growth rate but it takes some time to fully converge to an unchanging value consistent with the high state. The system then switches back to the low regime and the dynamics are reversed. It would take a longer simulation to get all the way back to the real interest rate observed at date 30.

A key question is whether there is an increase in the household debt-to-GDP ratio in this economy during the boom period. There is, and it is fairly substantial. The model naturally generates a household debt-to-GDP ratio in the range of 1.10 to 1.20 if the system remains in the low state for a long time, very similar to the household debt-to-GDP ratios reported by Mian and Sufi (2011) for 1995. We again caution however that the model is one of net debt held, not gross, so that a smaller debt-to-GDP ratio may be a more appropriate match to the data. The equilibrium debt-to-GDP ratio increases as much as 20 percentage points during some simulations from trough to peak. This is not as much as the increase reported by Mian and Sufi (2011) from 1995 to 2005, which they report at about 50 percentage points to a ratio of 1.65, but again, that is gross not net. The increase in the household debt level in the model is driven by the increase in expected income along with the persistence of the regime. One aspect that is more difficult to interpret is that once a regime shift occurs, the asset levels do not change that quickly while the pace of GDP growth falls or rises immediately. This means that ratios rise or fall immediately after regime shifts because of the change in the level of output relative to the expectations on which the asset holdings are based.

A related issue is whether household deleverage during the period of debt overhang. That definitely occurs as part of the equilibrium of the model. The system switches back to the low regime and the households begin to transition asset holdings toward the distribution that is consistent with the low state.

In Figure 6 we consider the same two regime switches, corresponding broadly to the 1996-2005 experience in the U.S., but we now allow the central bank to intervene in private debt markets to keep yields lower. In this Figure, the spike following the second regime switch (that is, entering the period of debt overhang) has disappeared relative to the laissez faire case, and the continuing intervention pushes yields lower than they would otherwise be. The household debt-to-GDP ratio falls somewhat faster under this policy than under the laissez faire policy.

As in the three period example, it is clear that this intervention in the credit market will reallocate consumption among cohorts. But the comparisons that will make the most sense are not as clear. This economy has moved through two regime shifts that have affected cohorts in different ways. Figure 7 illustrates two lifetime consumption profiles, those of households born in periods 45 and 46, which is around the time of the switch to the low regime. The lifetime consumption profile would increase at a constant rate λ in the long-run expected stationary equilibrium of the system. However, the system

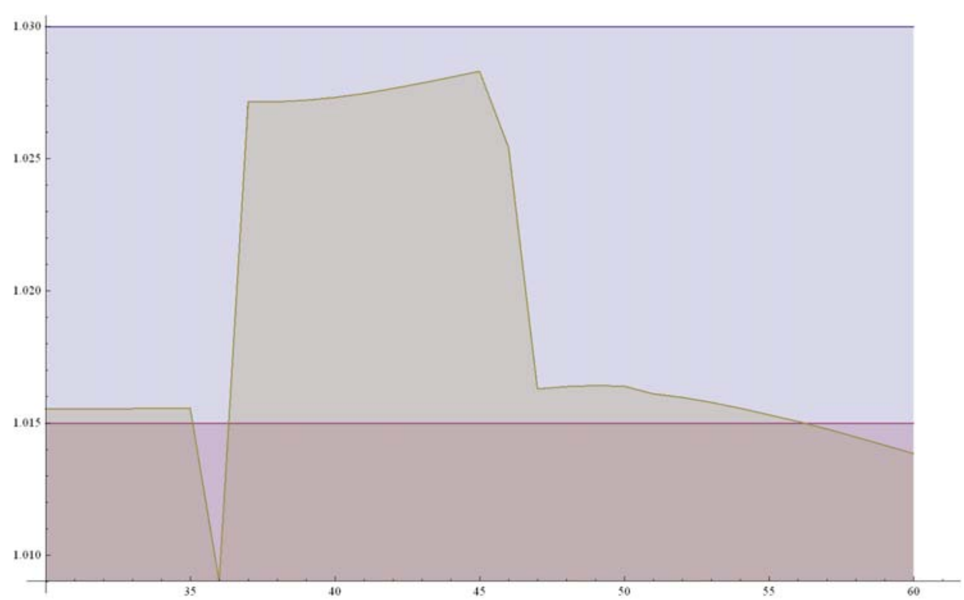

Figure 6: The equilibrium for the real interest rate in the credit market following the same two regimes switches but allowing central bank intervention to keep rates lower in the period of the debt overhang.

is never in that state, with or without central bank intervention. Therefore, while reallocation is occurring, we still need to develop good metrics to illustrate the magnitude of the reallocation.

5.4 Optimal monetary policy

We have calculated some of the effects of a central bank intervention in the market for private debt in this model. We have not tried to address the question of optimal monetary policy. Here we offer a few comments on this question. First, there are two problems in this economy that could potentially be addressed by monetary policymakers. One is the basic problem of the non-participants. They need an asset that helps them smooth consumption better than the one offered in this paper. The second problem is that the debt in the private credit market is not state-contingent. If we take this as given, it may be possible to design policies that would help the private sector mimic

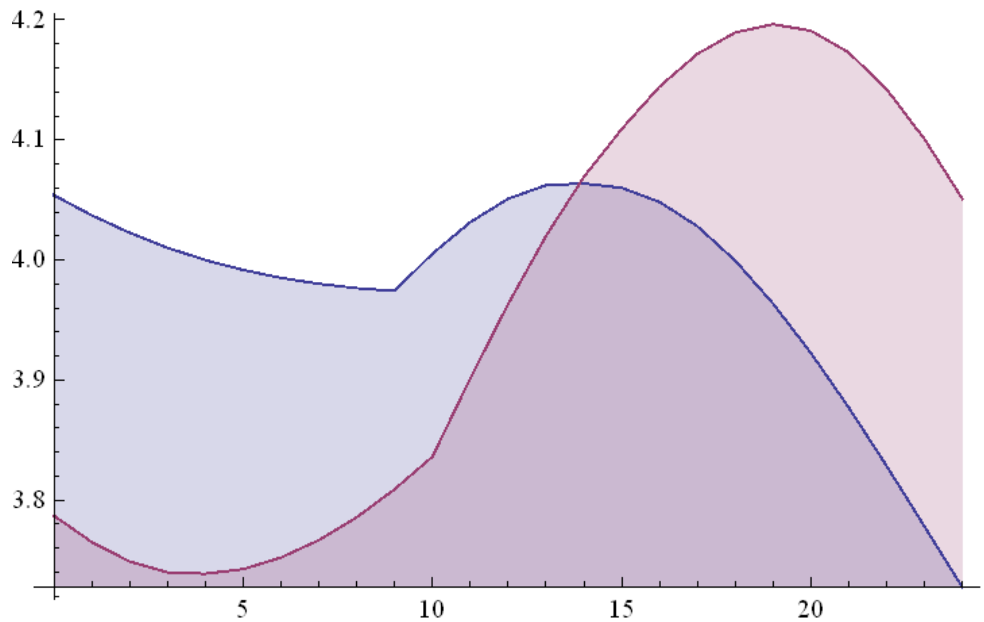

Figure 7: Lifetime consumption sequences for participant cohorts 45 and 46 in the simulation with central bank intervention in the credit market.

the outcome that would be achieved with state contingent debt. We have not attempted to design such a policy.

5.5 Catastrophe

We note that this model has a sort of catastrophe embedded. We have separated the private credit market from the market for currency, and we have assumed that credit market participants choose not to hold money because it is dominated in rate of return by private debt. If this condition breaks down, the credit market participants will begin to more seriously consider holding currency. In particular, in a state that features a sufficiently low return to private debt, savers in the economy will be indifferent between holding currency and holding the paper issued by the young borrowers. The demand for money will rise.

By itself, this is not a problem, but our conception is that the private credit market is much larger than the market for currency. The increase in money demand in this situation could be substantial. A central bank that was supplying currency only by issuing money in excess of the output growth rate—a policy that would work well in normal times—could inadvertently foster a deflationary environment in this situation. This could drive real interest rates to high levels exactly in a situation where interest rates would normally be quite low due to the low rate of growth. This sounds like a depression fueled by poor monetary policy.

6 Recent related literature

We will comment on some recent related literature in subsequent versions here.

7 Conclusions

This model has the ability to address some core issues concerning monetary policy and the problem of "debt overhang." The overhang in this economy is induced by a period of high income growth that is expected to be persistent. Life cycle households wish to borrow more (and other households wish to save more) as part of the equilibrium associated with the higher income growth. When the economy switches back to a lower growth regime, some households have "too much debt" relative to current and future expected income levels, the hallmark of debt overhang. A powerful central bank with a large balance sheet can intervene in this private credit market should it so desire. However, such an intervention will reallocate consumption generally toward borrowers and away from savers without a clear purpose.

References

[1] Azariadis, C., J. Bullard, and B. Smith. 2001. "Public and Private Circulating Liabilities." *Journal of Economic Theory* 99(1/2): 59-116.

[2] Bullard, J., and A. Singh. 2012. "Learning and the great moderation." *International Economic Review* 53(2): 375-397.

[3] Cao, D., and J-P L'Huillier. 2013. "Technological revolutions and debt hangovers: Is there a link?" Working paper #16/12, Einaudi Institute for Economics and Finance.

[4] Cecchetti, S., M.S. Mohanty, and F. Zampolli. 2011. "The real effects of debt." Working paper #352, September. Presented at the Jackson Hole Symposium, August 2011.

[5] Eggertsson, G. B. and P. Krugman. 2012. "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach." *Quarterly Journal of Economics* 127(3): 1469-1513.

[6] Glover, A., J. Heathcote, D. Krueger, and J.V. Ríos-Rull. 2011. "Intergenerational redistribution in the great recession." NBER Working Paper #16924.

[7] Guerrieri, V. and G. Lorenzoni. 2011. "Credit crises, precautionary saving, and the liquidity trap." NBER Working Paper #17583.

[8] Occhino, F., and A. Pescatori. 2012a. "Debt overhang in a business cycle model." Manuscript, Federal Reserve Bank of Cleveland.

[9] Occhino, F., and A. Pescatori. 2012b. "Leverage, investment, and optimal monetary policy." Manuscript, Federal Reserve Bank of Cleveland.

[10] Koenig, E. 2012. "Like a good neighbor: Monetary policy, financial stability, and the distribution of risk." Manuscript, Federal Reserve Bank of Dallas. Forthcoming, *International Journal of Central Banking*.

[11] Sheedy, K. 2012. "Debt and incomplete financial markets: A case for nominal GDP targeting." Manuscript, London School of Economics.

[12] Wallace, N. 1981. "A Modigliani-Miller theorem for open-market operations." *American Economic Review* 71: 267-274.

[13] Woodford, M., and V. Curdia. 2010. "Conventional and unconventional monetary policy." Federal Reserve Bank of St. Louis *Review*.

[14] Woodford, M. 2012. "Methods of policy accomodation at the interest-rate lower bound. Presented at the Jackson Hole Symposium, August.