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ABSTRACT

Is the UK triple-A?*

The immediate background to this paper is the downgrade of the U.K.'s credit rating in February 2013, the market's view that this should have occurred earlier, and the emphasis in fiscal policy on reducing debt rather than recovery from recession. We propose a measure of the U.K. sovereign credit rating based on an open economy macroeconomic model that is simple to compute and easily automated. Whether based on an ad hoc debt-GDP limit or a DSGE model of an open economy, our measure downgrades the U.K.'s sovereign credit rating from the middle of 2008. From 2010 the rating improves and is nearly restored to triple-A by 2012.

JEL Classification: E62, H30 and H60
Keywords: credit ratings and debt default

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"When a government spends, its citizens must eventually pay, either today or tomorrow, either through explicit taxes or implicit ones like inflation and defaults on debt"

Thomas J. Sargent's speech at the Nobel Banquet in the Stockholm City Hall, 10 December 2011 (www.nobelprize.org).

1 Introduction

In February 2013 the credit rating agency (CRA) Moody's downgraded the U.K.'s credit rating by one notch, from triple-A to Aa1. The main reasons given were the unusually weak performance of the U.K. economy over the previous five years, the expectation of several more years of poor growth, the failure to reduce the debt-GDP ratio which is expected to peak in 2016, later than originally planned. This downgrade of the UK follows downgrades in 2011 to the credit ratings of France and the U.S.. Like the U.K., the sovereign debt of France and the U.S. had always previously been rated triple-A.

While the announcement did not surprise the markets, its timing did as, like the U.S., the U.K. retained its triple-A rating throughout the financial crisis; see Polito and Wickens (2012) for an analysis of the U.S. credit rating. The failure of the CRAs to anticipate the 2008 crisis or to either downgrade or revise sovereign credit outlooks in the aftermath of the financial crisis led to much criticism of them. Evaluating - and even interpreting - a country's sovereign credit rating is, however, neither straightforward nor uncontroversial. It does not necessarily imply, for example, that a country is more likely to default on its debt, even though the probability of default is a key factor in determining a credit rating. The IMF (2012) referred to U.K. policymakers as having always been committed to "preserve Britain's proverbial creditworthiness". Further, historically, the U.K. has never defaulted on its sovereign liabilities and has always received the highest rating quality by all (CRAs) since they began rating U.K. sovereign securities. Nor does a downgrade necessarily imply that the cost of borrowing will increase. The cost of borrowing for France and the U.S. fell after their downgrades, possibly reflecting that the market had already priced in the downgrade.

In December 2011, the U.K. Treasury Committee launched an inquiry on the accountability, methodology and impact of the work of CRAs. Similar enquiries launched in the U.S. and Europe have raised concerns about the transparency of the procedures adopted by CRAs and the timing of downgrades; and suggested that one possible way forward is for investors to produce their own independent measure of the credit rating.[1] One reason often advanced for the dependence on CRAs is the expense and complexity of providing a credit rating.

This paper describes a simple, transparent and easily automated methodology for deriving the sovereign credit ratings of advanced open economies with a floating exchange rate. Our measure of the credit rating is entirely data-driven

[1] See Security and Exchange Commission (2011), European Commission (2011) and U.K. Treasury Committe (2011).

and can be readily calculated by governments and market participants. We argue that it provides a clear benchmark against which to evaluate the analyst-driven sovereign credit ratings issued by CRAs.

The credit rating is computed by adapting to sovereign debt Merton (1974)'s model of credit risk and default. Thus, calculating a credit rating is similar to pricing an American option. It entails estimating the probability that the debt-GDP ratio will exceed a given limit or threshold over a given time horizon and mapping this probability into a credit rating. Uncertainty about the credit rating can be accounted for by incorporating uncertainty about the likely trajectory of the debt-GDP forecast and about the appropriate value of the debt limit.

We form the forecast of the debt-GDP ratio using an open economy reduced-form model that allows for time variation in both parameters and volatility. This is based on rolling-window estimation of a VAR, and so can be easily estimated and updated. We consider two types of debt limit. The first is an ad hoc limit and is interpreted as a threshold beyond which a government is either unable or unwilling to repay its sovereign liabilities. The longer the time horizon and the lower the value of the debt limit, the more likely it is that the debt-GDP ratio will exceed the debt threshold and a credit rating downgrade will occur. Further, uncertainty about the future trajectory of the debt-GDP ratio and the debt limit make downgrades more likely and longer lasting.

The second type of debt limit is derived from dynamic general equilibrium theory and measures the maximum borrowing capacity of an economy. It therefore measures exclusively the ability of a government to alter fiscal policy in the future to meet its outstanding financial obligations. An open economy real business cycle model with distortionary taxation on income from consumption, labor and capital is used to derive four alternative versions of this type of debt limit. These depend on whether fiscal policy changes are anticipated or unanticipated by market participants and, if unanticipated, whether they could stem from changes in government expenditure policy, government tax policy or both. The model is solved using a nonlinear algorithm which, if applied to time-varying data, delivers a time series of state-dependent distributions of each version of the debt limit. This shows how the maximum borrowing capacity of an economy changes over time and hence reflects exclusively a government's ability to use its fiscal instruments to repay its financial obligations.

This methodology is used to derive quarterly time-series of the U.K. sovereign credit rating for both types of debt limit.[2] We find that the U.K. sovereign credit rating is triple-A for most of this period. There are, however, two instances where the U.K. is downgraded. This is by one notch for 1 or 2 quarters following the sterling's exit from the Exchange Rate Mechanism in 1992 and, more substantially, for several quarters from the middle of 2008, in the aftermath of the run on Northern Rock in September 2007. From the end of 2010 onwards, the credit rating obtained from the model begins to recover towards triple-A. There is still, however, a large degree of uncertainty about its value by

[2] These range from 1979:2 to 2012:2 when using ad hoc limits and, due to limited data availability, from 1995:2 to 2012:2 when using the theory-based measures of the debt limit.

the end of 2012.

The paper is set out as follows. Section 2 describes our analytical framework for measuring the credit rating. We explain in detail how we define and measure the default probability, how we map this into a credit rating and the model used for forecasting the trajectory of the U.K. debt-GDP ratio. We also identify a number of features of the U.K. data that are found subsequently to be important in determining its credit rating. In section 3 we derive a U.K. credit rating for the period 1979:2-2012:2 based on ad hoc debt limits. We emphasize the effect of uncertainty about the debt-GDP limit and the forecasts on our measure of the credit rating. Section 4 develops the open economy real business cycle model and derives four alternative measures of the maximum borrowing capacity of the U.K. economy that are based on its repayment ability. These are the intertemporal government budget constraint limit (IGBCL), the natural debt limit (NDL), the fiscal limit (FL) and the maximum debt limit (MDL). We then solve the model using a nonlinear simulation algorithm that delivers state-dependent distributions of the debt limits. This is used to compute time-varying measures of each debt-GDP limits for the U.K. from 1995:2 to 2012:2. Credit ratings are then calculated for each measure of debt limit. Section 5 summarises our results. The paper also contains three appendices. Appendix A provides details of the data. Appendix B describes the derivation of the stationary equilibrium solution of the structural model used for the derivation of the debt limits. Appendix C describes the algorithm used for computing the state-dependent distribution of the debt limits in stationary equilibrium.

2 Measuring the sovereign credit rating

A sovereign credit rating, broadly defined, is an opinion about the likelihood of default of a government, based on its perceived ability and willingness to service its debt. This interpretation originates from the early 1900s and has been adopted by the three largest CRAs - Fitch Ratings, Moody's and Standard & Poor's, see Gaillard (2012). Following Merton's (1974) model of credit risk, the likelihood of defaulting on either corporate or sovereign debt is measured by the probability that the value of liabilities at a future horizon will exceed some threshold level or debt limit above which it is deemed that the debtor will either be unwilling or unable to service its financial obligations. A default event is a situation in which a debtor misses, delays or changes the terms of a contractually-obligated interest or principal payment.

Our methodology for measuring the sovereign credit rating involves three steps. First, we adapt Merton's model of credit risk to sovereign debt to compute the probability that the forecast of the debt-GDP ratio will exceed a debt limit within a given time horizon. To the extent that the debt limit is a default threshold, this identifies a default probability. Second, we specify the macroeconomic model we use to forecast the debt-GDP ratio and we identify a number of features of the U.K. data that have a strong bearing on the credit ratings we obtain. Third, we map our measure of the default probability into a credit rating using historical records provided by CRAs.

This measure of the sovereign credit rating is based solely on the probability of default, and hence on the ability and willingness of a government to meet its financial obligations. It may not give the same sovereign credit ratings as those issued by CRAs. These are based on subjective evaluations of a number of quantitative and qualitative factors.[3] Rather than trying to replicate the credit ratings issued by the CRAs, instead we seek a benchmark measure of the sovereign credit rating, with a clear economic content, that can be easily replicated by any investor and/or market analyst. Discrepancies between our model-based measure of the credit rating and the credit ratings issued by the CRAs indicate that the CRAs may be influenced by additional factors to those incorporated in our measure. For example, in 2012 the three largest CRAs revised the outlook on the U.K. sovereign credit rating from stable to negative and announced that a downgrade could be imminent. All three CRAs raised concerns about the U.K. government's ability to put the debt burden on a downward trajectory by fiscal year 2015-16. As this increased uncertainty about the pace of fiscal consolidation, they said that it may lead them to downgrade the U.K.[4] These assessments appear to reflect an opinion about the timing of fiscal policy improvements rather than default in the strict sense. Even when explaining the downgrade in February 2013 Moody's do not stress the probability of eventual default.[5] Nonetheless, in this paper we reflect the original interpretation of the credit rating by basing our measure on the probability of default.

2.1 The probability of sovereign default

The determination of the probability that the debt-GDP ratio will exceed a given threshold at some point over a given time horizon is a similar problem to that of pricing an American option. Our conceptual framework is an adaptation of Merton's (1974) model of credit risk. The debt-GDP ratio satisfies the one-period government budget constraint (GBC). Expressed as a proportion of nominal GDP, the GBC can be written as:

$$\frac{d_t}{y_t} + (1+\rho_t)\,\frac{b_{t-1}}{y_{t-1}} = \frac{b_t}{y_t},$$

where y_t is real GDP, $\frac{d_t}{y_t}$ is the primary deficit-GDP ratio, ρ_t is the output-adjusted real interest rate on government debt and $\frac{b_t}{y_t}$ is the debt-GDP ratio. The primary deficit $\frac{d_t}{y_t}$ is defined as the difference between government expenditures on goods and services ($\frac{g_t}{y_t}$) plus transfers ($\frac{z_t}{y_t}$), both expressed as a proportion of GDP, and the ratio of government revenues to GDP ($\frac{v_t}{y_t}$) which includes seigniorage revenues. The discount rate ρ_t is the nominal interest rate on government bonds (i_t^b) less the inflation rate (π_t) and the growth rate of GDP (γ_t).

[3] See Gaillard (2012) for a detailed description of the methodologies used by CRAs.
[4] See, for example, Moody's (2012).
[5] See Moody's (2013).

The debt-GDP ratio in period $t+h$ is therefore

$$\frac{b_{t+h}}{y_{t+h}} = -\sum_{j=1}^{h}\left[\Pi_{s=1}^{j}\left(1+\rho_{t+s}\right)\frac{d_{t+j}}{y_{t+j}}\right] + \Pi_{s=1}^{h}\left(1+\rho_{t+s}\right)\frac{b_t}{y_t},$$

where the right-hand side is the cumulative saving generated by current and future primary surpluses from t to $t+h$ plus the interest cost of rolling-over the current debt-GDP ratio until period $t+h$.

Default occurs between periods t and $t+h$ if the expected value of the debt-GDP ratio conditional on information available in period t exceeds the threshold (debt limit) $\overline{\frac{b_{t+h}}{y_{t+h}}}$. The probability of sovereign default by period $t+h$ (hazard rate) is the probability of not defaulting prior to year $t+h$ but defaulting in year $t+h$, and hence is given by

$$p_{t,t+h} = p_{t+h}\left(1-p_{t+h-1}\right)\left(1-p_{t+h-2}\right)...\left(1-p_{t+1}\right).$$

p_{t+h} denotes the probability of defaulting in period $t+h$ given the information available in period t, and is measured by

$$p_{t+h} = \Pr\left(\frac{b_{t+h}}{y_{t+h}} \geq \overline{\frac{b_{t+h}}{y_{t+h}}}|\Phi_t\right),$$

where $\Pr(.)$ is assumed to be the normal probability density function and (Φ_t) denotes information available at time t.[6]

The default threshold $\overline{\frac{b_{t+h}}{y_{t+h}}}$ represents the amount of debt that a country will be either willing or able to repay at a specific time in the future. In practice, market analysts and investors may have in mind a debt-GDP threshold of their own, which may depend upon considerations both about a government's ability to meet its financial obligations using fiscal policy and its willingness to service its debt.

The debt-GDP ratio at time $t+1$ may be decomposed into

$$\frac{b_{t+1}}{y_{t+1}} = E_t\frac{b_{t+1}}{y_{t+1}} + \xi_{t+1}$$

where $E_t\frac{b_{t+1}}{y_{t+1}}$ is the expectation of the debt-GDP ratio by the end of period $t+1$ conditional on information available in t, and ξ_{t+1} is the corresponding innovation in period $t+1$. The latter may be written as

$$\xi_t = \sigma_t\varepsilon_t,$$

where $\varepsilon_t \sim i.i.d.\,(0,1)$. It then follows that the debt-GDP ratio for period $t+h$ may be written as

$$\begin{aligned}\frac{b_{t+h}}{y_{t+h}} &= E_t\frac{b_{t+h}}{y_{t+h}} + \eta_{t+h} \\ \eta_{t+h} &= \Sigma_{s=1}^{h}\xi_{t+s}\end{aligned}$$

[6] When the evolution of the debt-GDP ratio follows a geometric Brownian motion, the sovereign default probability can be expressed in a form equivalent to the option pricing model of Merton (1974).

where $V_t(\eta_{t+h}) = \sigma^2_{\eta,t+h} = \Sigma^h_{s=1}\sigma^2_{t+s}$ is the conditional variance of the debt-GDP ratio.

Defining

$$DD_{t+h} = \frac{E_t \frac{b_{t+h}}{y_{t+h}} - \overline{\frac{b_{t+h}}{y_{t+h}}}}{\sigma_{\eta,t+h}} \quad (1)$$

as the distance-to-default of sovereign debt, the probability of sovereign default in period $t+h$, given information in period t is

$$p_{t+h} = \Pr\left(-DD_{t+h} \leq \zeta_{t+h}|\Phi_t\right), \quad (2)$$

where

$$\zeta_{t+h} = \frac{\eta_{t+h}}{\sigma_{\eta,t+h}}.$$

The probability of default therefore increases as $E_t \frac{b_{t+h}}{y_{t+h}} - \overline{\frac{b_{t+h}}{y_{t+h}}}$ widens and the uncertainty surrounding the forecasts of the debt-GDP ratio (σ_{t+h}) increases. This probability will change over time as changes in the base year and the information will alter the forecast of the debt-GDP ratio and its uncertainty, and the debt threshold.

The probability of default in any period between t and $t+h$ (the cumulative default probability) is

$$p^c_{t,t+h} = \sum_{j=1}^{h} p_{t,t+j}, \quad (3)$$

which is calculated assuming a standard cumulative normal distribution.

Equation (1) measures the distance-to-default for given values of the debt-GDP limit, the point forecast and the standard deviation of the debt-GDP ratio at a specific time horizon. Uncertainty about these three components can be accounted for by constructing distributions of the debt-limit, debt-GDP forecast and its volatility at each time horizon. These can then be used to derive the distribution of the distance-to-default, which can be translated into a distribution of the default probability using equations (2) and (3).

2.2 Mapping the probability of default into a credit rating

Having obtained the required measure of the probability of default, we map this into a credit rating using the historical record on credit ratings and default probability provided by Moody's (2011). This provides the annual cumulative default probability for 7 types of sovereign bond ratings over a 10-year horizon.[7] We construct a quarterly mapping based on 19 grades over the whole 10-year cumulative default probabilities using linear interpolation on the existing data.[8] The resulting quarterly mapping is reported in Table 1. Several features about these data are worth observing. The rating scale is similar to that employed

[7] This is based on data about credit rating and sovereing default history for the period 1983-2011. Similar tables are provided by most of CRAs.

[8] Full details are provided in Polito and Wickens (2012).

by the three main CRAs. Long and short–term rating refer to bonds with maturity beyond and before 1-year respectively. Aaa denotes the highest credit quality while C identifies junk bonds. Investment grade (low risk bonds) includes all bonds rated between Aaa and Baa3, while bonds rated from Ba1 below are referred as speculative grade (high risk bonds). The cumulative default probability is measured by the end of year 1, 5 and 10. For robustness we also include the average cumulative default probability over the whole 10-year horizon. In the empirical application in sections 3 and 4, we will use the 1-year horizon to calculate the model-based credit rating over the short-term, while the 5-year, the 10-year and the average horizons are used as alternative time periods for the computation of the long term rating.

	Rating		Cumulative default probability			
Category	Long-term	Short-term	1-year	5-year	10-year	average
Investment grade	Aaa	Prime - 1	0.000	0.000	0.000	0.000
	Aa1	Prime - 1	0.026	0.245	0.317	0.208
	Aa2	Prime - 1	0.053	0.490	0.634	0.415
	Aa3	Prime - 1	0.079	0.736	0.952	0.623
	A1	Prime - 1	0.106	0.981	1.269	0.830
	A2	Prime - 1/2	0.132	1.226	1.586	1.038
	A3	Prime - 1/2	0.159	1.471	1.903	1.245
	Baa1	Prime - 2	0.185	1.717	2.221	1.453
	Baa2	Prime- 2 or 3	0.212	1.962	2.538	1.660
	Baa3	Prime-3	0.238	2.207	2.855	1.868
Speculative grade	Ba1	Not Prime	0.415	3.950	8.197	3.942
	Ba2	Not Prime	0.592	5.692	13.540	6.017
	Ba3	Not Prime	0.769	7.435	18.882	8.092
	B1	Not Prime	1.643	9.989	20.785	10.196
	B2	Not Prime	2.517	12.542	22.687	12.299
	B3	Not Prime	3.391	15.096	24.590	14.403
	Caa	Not Prime	10.139	21.005	27.334	19.607
	Ca	Not Prime	16.888	26.914	30.079	24.812
	C	Not Prime	23.636	32.823	32.823	30.016

Source: www.moodys.com (Rating) and authors' calculations

Table 1: Sovereign credit rating scale and cumulative default probabilities

Our measure of the U.K. sovereign credit rating is obtained by comparing our model-based ex-ante default probabilities over a 10-year forecast period with the ex-post values in Table 1. This provides a precise mapping between sovereign credit rating and default probability. Ratings issued by CRAs represent an ordinal rank-ordering of countries creditworthiness and do not target specific probabilities of default. Thus discrepancies between the model-based and the actual credit rating will highlight factors beyond the default probability that may contribute to the determination of the analyst-driven measure of the credit rating.

2.3 Forecasting the sovereign debt-GDP ratio and its volatility

We forecast the debt-GDP ratio and obtain a measure of the uncertainty of this forecast using a rolling-window VAR (ROVAR) whose specification reflects the variables in the GBC. Other forecasting methods could, of course, be used instead. There are several reasons for our choice of a VAR. Although we could use a structural macroeconomic model such as a DSGE model, Wickens (2012) has shown that there is little difference in the forecasts of DSGE and VAR models. Both tend to miss turning points. They have similar dynamic structures with both mean-reverting following a turning point more strongly than economic data show. The main differences between a VAR and the solution to a DSGE model is that the DSGE model typically includes expected future exogenous variables and its internal dynamics are a restricted rather than an unrestricted VAR. The reasons that the forecasts from a DSGE model are similar to that of a VAR is, first, that being exogenous, and hence having no theory to guide them, forecasts of the exogenous variables are usually based on a pure time series model, as they would be if the exogenous variables were included in an unrestricted VAR. Second, if the restrictions are correct, then the forecasts from a DSGE model will have smaller forecast error variances than those from an unrestricted VAR but, if the restrictions are incorrect, then an unrestricted VAR will give less biased forecasts. Further advantages to using an unrestricted VAR are that it is theory-free and it is easy to implement empirically, both of which are attractive features for market participants seeking to determine their own credit-rating measure.

In order to improve the forecasting performance we take account of the uncertainty arising from changes in the processes generating the exogenous variables and any variations in the parameters by estimating the VAR using a rolling-window data period. It is then easy to up-date the mean and variance of the probability distribution of the debt-GDP ratio on the latest data. Rolling analyses of time-series models are often used in finance to assess a model's stability over time. Stock and Watson (2007, 2008) have used rolling-window estimation to forecast U.S. inflation, while Orphanides and Wei (2010) have demonstrated the effectiveness of rolling-window estimation in capturing time-variation in the parameters and (the volatility of) the residuals in macro-finance models. Canova and Ferroni (2012) employ rolling samples to evaluate the impact of policy shocks for a medium-scale DSGE model. Rolling-window estimation allows changes in the parameters and volatility of a model to be data-driven. For this reason, Kapetanios et al. (2012) find that ROVAR models produce forecasts of the U.K. economy that are more accurate than those from a VAR with time-varying parameters and stochastic volatility, Primiceri (2005), and similar to those obtained from a VAR model with Markow-switching parameters and volatility, Sims and Zha (2006).

The ROVAR includes nine variables; each equation is specified with a constant and two lags of each variable; the estimation is carried out using a window size of 40 quarters from 1970:1 to 2012:2. The variables are summarized in Table 2: the debt-GDP ratio (DEBT) and the primary deficit-GDP ratio (DEF) represent the aggregate fiscal sector; the growth rate of real GDP (GDP), the inflation rate (INF), the short-term nominal interest rate (IRS) and the long-term nominal interest rate (IRL) capture the dynamics of the aggregate domestic private sector; while the last three variables, the real exchange rate depreciation (EXC), the current account-GDP ratio (CAC) and the oil inflation rate (OIL), refer to the aggregate external sector.[9] The external sector variables are included in the ROVAR to capture the impact of global factors on the domestic economy. In particular, Reinhart and Rogoff (2008) document that "peaks and troughs in commodity price cycles appear to be leading indicators of peaks and troughs in the capital flow cycle, with troughs typically resulting in multiple defaults".

Symbol	Description	Acronyms
$\frac{b_t}{y_t}$	Debt-GDP ratio	DEBT
$\frac{d_t}{y_t}$	Primary deficit-GDP ratio	DEF
γ_t	Growth rate real GDP	GDP
π_t	Inflation rate	INF
r_t^s	Short-term interest rate	IRS
r_t^l	Long-term interest rate	IRL
e_t	Real exchange rate depreciation	EXC
$\frac{x_t}{y_t}$	Current account-GDP	CAC
π_t^o	Oil inflation rate	OIL

Table 2: ROVAR: variables description

Given the 40 quarter rolling window, the forecasts start from 1979:4. Figure 1 therefore plots the data from this point. Four features of these data have a large effect on the credit ratings we obtain. First, economic growth was strong in the aftermath of the Great Inflation of the 1970s and early 1980s but this came to an abrupt end in 2007/8. Second, the deficit-GDP ratio has fluctuated considerably. It was high in the early 1990s, and rose steadily between 1999 and 2010 before turning down thereafter. Third, the debt-GDP ratio has increased continuously since 2001. Fourth, despite large fluctuations in the fiscal variables, nominal interest rates have declined steadily over the sample period, largely reflecting the fall in inflation.

[9] Full description of data sources and the derivation of each variable is provided in appendix A.

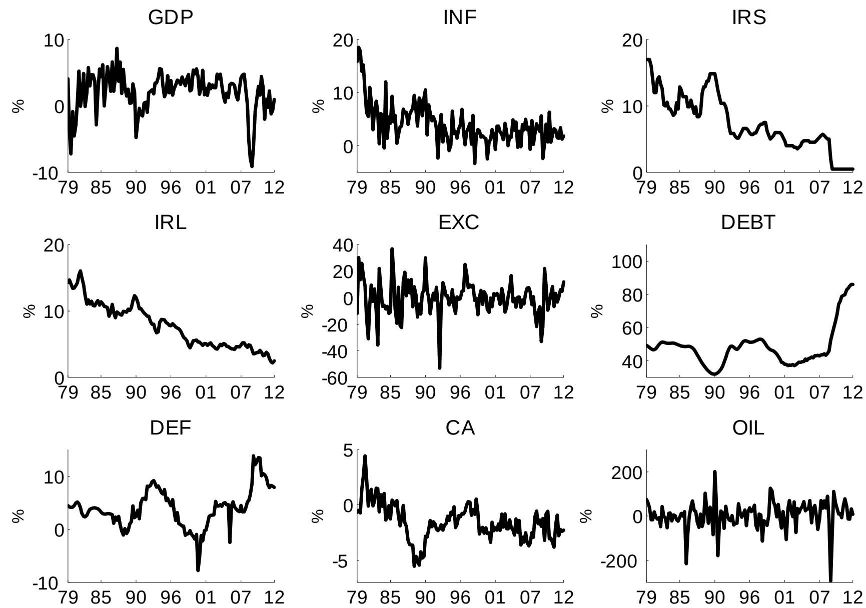

Figure 1: U.K. aggregate macroeconomic data, 1979:4-2012:2. See table 2 for variables description.

The credit rating is affected not only by the point forecast of the debt-GDP ratio but also by its uncertainty as a higher forecast error variance increases the chance of exceeding the debt limit. The great moderation of the 1990s and early 2000s is reflected in Figure 2 in the fall in the one-period ahead standard deviations of the forecast errors of economic growth, inflation and the interest rates. In sharp contrast, the standard deviations of the ratios of the deficit and debt to GDP have steadily increased.

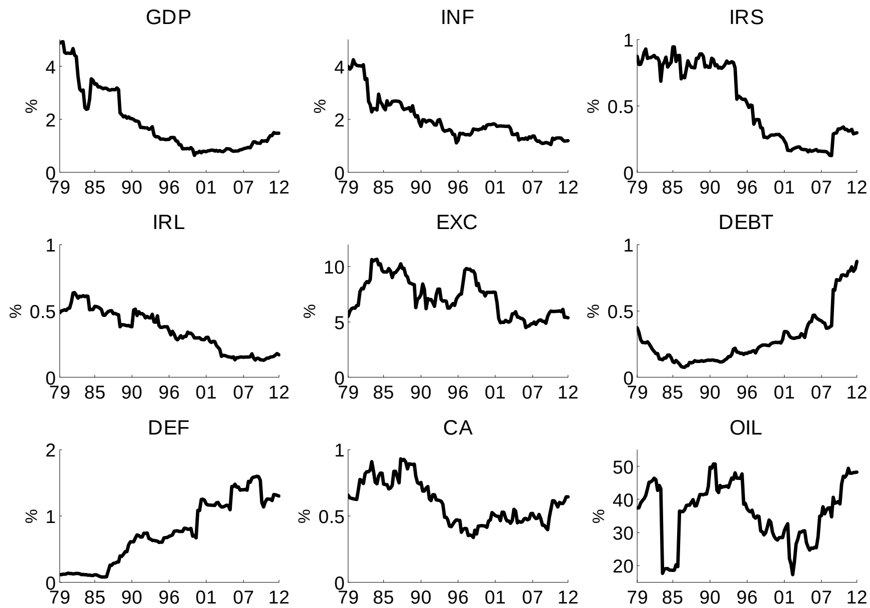

Figure 2: Standard deviation of the 1-period ahead forecast error of the U.K. macroeconomic data. 1979:4-2012:2. See table 2 for variables description.

3 The UK credit rating based on ad-hoc debt-GDP limits

First we calculate sovereign credit ratings for the U.K. data using ad hoc values of the debt limit. We are agnostic about whether this limit reflects a threshold beyond which the U.K. government is unwilling or unable to meet its financial obligations. Ad hoc limits may be used to illustrate several features of our model-based measure of the U.K. sovereign credit rating and how the data affects this.[10]

We find that the longer the time horizon and the lower the debt threshold, the more likely it is that the debt-GDP ratio will exceed the debt threshold and a credit rating downgrade will occur. This is illustrated in Figure 3, in which we plot the historic U.K. sovereign credit rating together with the credit ratings

[10]The measure of the model credit rating obtained from the application of the methodology appear to be very sensitive to large swings in debt-GDP forecasts. For this reason, the measure that we report in the paper is obtained by smoothing the initial credit rating as follows: in the first period of the sample the reported credit rating is set equal to the initial credit rating; if the new initial credit rating (from the second period onwards) is the same as the previous quarter's initial rating, the new reported rating is set equal to the rating reported in the previous quarter; if the new initial credit rating is higher (lower) than the previous period's initial rating then the reported credit rating is upgraded (downgraded) by one notch.

obtained from the model based on ad hoc debt-GDP limits of 1.5, 2 and 3 times GDP and default probabilities at three specific time horizons (1-year, 5-year and 10-year ahead).[11] Whereas the historic credit rating is triple-A throughout for all time horizons, the model-based credit rating at the five-year horizon indicates a downgrade of one notch in 1993:3 and a number of more severe, but still temporary, downgrades during the second half of the 2000s; the duration and extent of these downgrades increase as the debt-GDP target reduces.[12] At the 10-year horizon downgrades in the 1990s occur more frequently though they are still mild, whereas downgrades in the late 2000s are more severe and longer lasting.[13]

[11] The debt-GDP limits chosen reflect the history of the U.K. economy. Debt-GDP ratios higher than 150% occurred for prolonged periods of times over the past 300 years. For example, the debt-GDP ratio was above 150 per cent from 1830 to 1850, with a peak of 170%; and it has averaged approximately 200% in the 40 years between 1920 and 1960, with a peak of approximately 270% just after the end of WWII. Long data series on U.K. debt can be found in Mitchell (1988) or on the IMF data mapper (www.imf.org).

[12] Based on the lowest debt-GDP limit the downgrades start in 2008:3. The credit rating then fluctuates between Aa1 and A1 until 2012:2 when it is 2 notches below the triple-A mark. Using the 200 per cent debt-GDP limit, a less stringent criterion, the downgrades start in 2008:4. The credit rating then fluctuates between triple-A and Aa2, being only one notch below triple-A in 2012:2. With the highest debt-GDP limit, the downgrades again start in 2008:4, but the triple-A mark is restored from 2010:3 onwards.

[13] With the lowest debt limit, downgradings of one notch occur in the first two quarters of 1992, in 1993:2 and in 1997:3. The credit rating is downgraded without interruption from 2008:3 onwards. Using a 200 per cent debt-GDP limit gives a downgrade of one notch in 1992:3 and 1993:3. After 2008:3 the lowest rating attained occurs in 2011:2 and is eight notches below triple-A. The rating then recovers to A3 in 2012:2 which is five notches below triple-A. Using the highest debt limit there is a downgrade of one notch in 1993:3. After 2008:3 the lowest rating is A3 in 2010:2. It then recovers gradually, attaining triple-A by 2012:2.

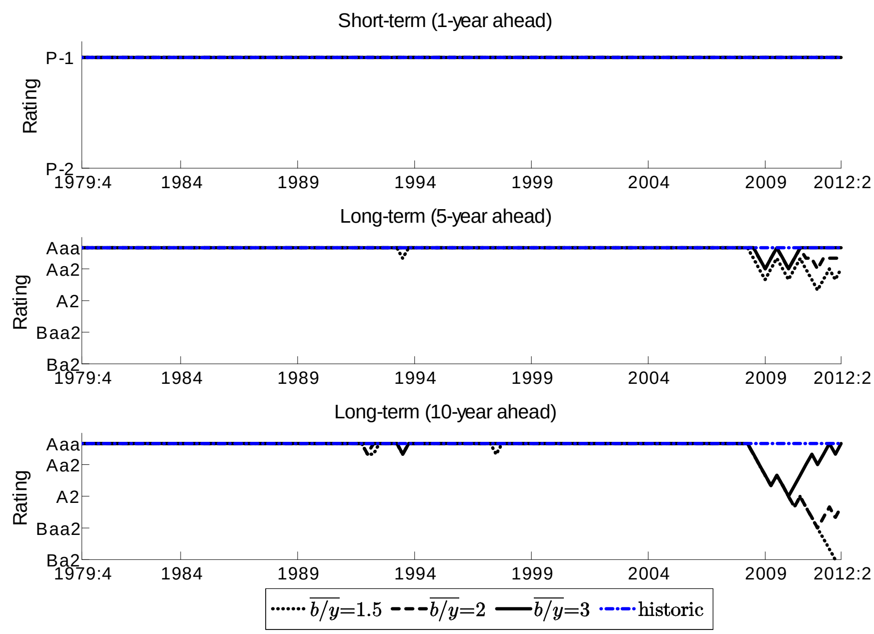

Figure 3: Model-based U.K. sovereign credit rating with ad-hoc debt-GDP limit, 1979:4-2012:2. Historic credit rating is from Moody's (2012).

We can examine the robustness of these results to (i) the choice of time-period for the determination of the default probability, (ii) the relative contribution of the trajectory of the debt-GDP forecast and the forecast error, (iii) uncertainty surrounding the debt-GDP limit and (iv) uncertainty about the distribution of the debt-GDP forecast.

To discover the effect of the choice of time period, we calculate the credit ratings based on the average probability of default over a 10-year horizon (solid line) rather than the default probability at a specific forecasting horizon as in Figure 3. The results are very similar to those in the bottom panel of Figure 3: downgrades are more frequent and longer lasting for lower debt-GDP limits, with more severe changes occurring from the second half of the 2000s.[14] To evaluate the relative contribution of the debt-GDP forecast and the uncertainty surrounding this forecast we keep constant the volatility of the debt-GDP forecast at its value for the 1-year horizon. This gives the dashed-black line in Figure 4. Keeping uncertainty constant changes the results in two respects: the mild downgrades in the 1990's are eliminated and the downgrades during the

[14] With the lowest debt limit, downgrades of one notch occur in 1992:1 and 1993:3 and again from 2008:3. With the mid-range debt limit there is a downgrade of one notch in 1993:3 and more severe downgrades from 2008:3. These reach their lowest point in 2011:3 (A3, six notches below triple-A) before rising to A1 in 2012:2 (four notches below triple-A). Using the highest debt limit there is no downgrade in the 1990s but the rating is downgraded from 2008:3, reaching a minimum of A3 (six notches below triple-A) before gradually recovering to triple-A by 2012:2.

second half of the 2000s are less severe in terms of both their magnitude and their duration.[15] These results suggest that the uncertainty surrounding the trajectory of the U.K. debt-GDP ratio plays a significant role in determining the U.K. sovereign credit rating during the latest global financial crisis.

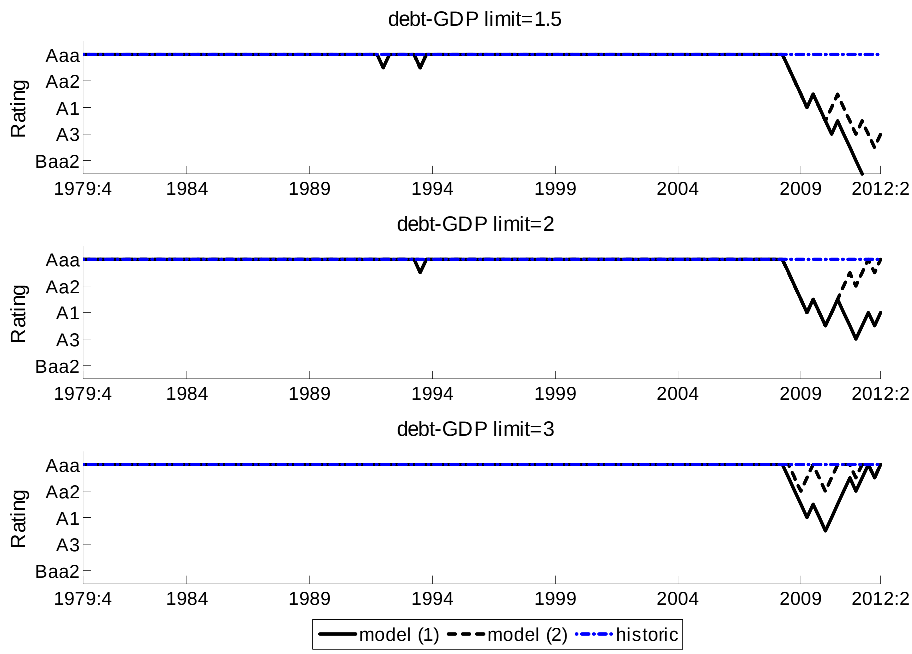

Figure 4: U.K sovereign credit rating with ad hoc debt-GDP limits based on average default probability over 10 years with either changing, model (1), or constant, model (2), volatility of the debt-GDP forecasts. Historic rating is from Moody's (2012).

To investigate the impact of uncertainty about the debt-GDP limit, we specify a grid of values for it that range from 100 to 300 per cent in 1 per cent increments. We then compute the credit rating for each of these 201 values of the debt limit. This provides for each quarter a distribution of the sovereign credit rating defined over this interval. The median value of these distributions is shown in Figure 8 together with their 16th and 84th percentiles (as in Primiceri, 2005). These percentiles provide confidence bands which can be used to

[15] With the lowest debt limit there is a downgrade of one notch in 1993:3. The credit rating is gradually downgraded from 2008:3 before stabilizing at A3 (six notches below triple-A) by 2012:2. There are no downgrades in the 1990s for the other two debt limits. With the 200 per cent debt-GDP limit the downgrade from triple-A starts from 2008:3 reaching its lowest point in 2010:2 of A2 (six notches below triple-A) before gradually recovering the triple-A mark by 2012:2. With the highest debt limit the triple-A rating is lowered from 2008:4 and then fluctuates between the top three notches. By 2012:2 this is again triple-A.

evaluate the statistical significance of the difference between the credit rating measure from the model and the historic credit rating. For short time horizons the model produces a Prime-1 credit rating. For longer time horizons the model downgrades the U.K. credit rating for short periods in the 1990s. In all cases, except for the one-notch downgrade of 1993:3, the difference between the model sovereign credit rating and the triple-A mark is not statistically significant. In contrast, the downgrades towards the end of the 2000s are statistically significantly different from triple-A.[16]

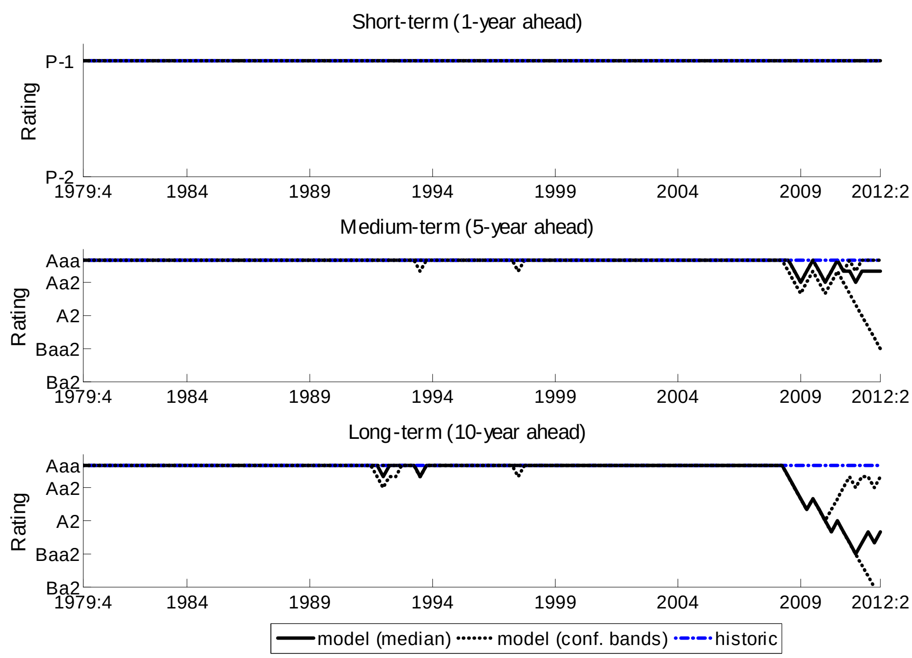

Figure 5: Median, 16th and 84th percentiles of the model-based U.K. credit rating based on an uniform distribution of the debt-GDP limit over the 100-300 per cent range. Historic rating is from Moody's (2012).

To examine the effect of uncertainty about the distribution of the forecasts on the credit rating we bootstrap the forecasts. If Y_t denotes the vector of observations from 1970:1 to 2012:2 on the nine variables included in the ROVAR (see table 2) then we bootstrap these by re-sampling the residuals from the full sample VAR 2000 times. The ROVAR is then estimated for each sample and

[16] For a 5-year horizon, the U.K. credit rating is estimated to be triple-A until the third quarter of 2008. After that there are three instances of a statistically significant downgrade from triple-A by either one or two notches. These are between 2008:4-2009:2, 2009:4-2010:2 and 2010:4-2011:2. For a 10-year horizon, there are statistically significant downgrades in 1993:3 (by one notch) and from 2008:3. Credit ratings based on using the 10-year average cumulative default probability are very similar to those for the 10-year horizon.

a bootstrapped distribution of the debt-GDP forecasts is computed for a 40-period horizon. The median value of the debt-GDP forecast and the 16th and 84th percentiles are then obtained for each point in the sample. The results are shown in Figure 6. We find that the U.K. credit rating is statistically significantly different from triple-A only from the onset of the global financial crisis. The behaviour of the median credit rating depends on the debt limit; with the lowest debt limit it falls from 2009; with the intermediate debt-GDP it stabilizes from 2010; with the highest debt GDP limit it begins to recover from 2010.[17]

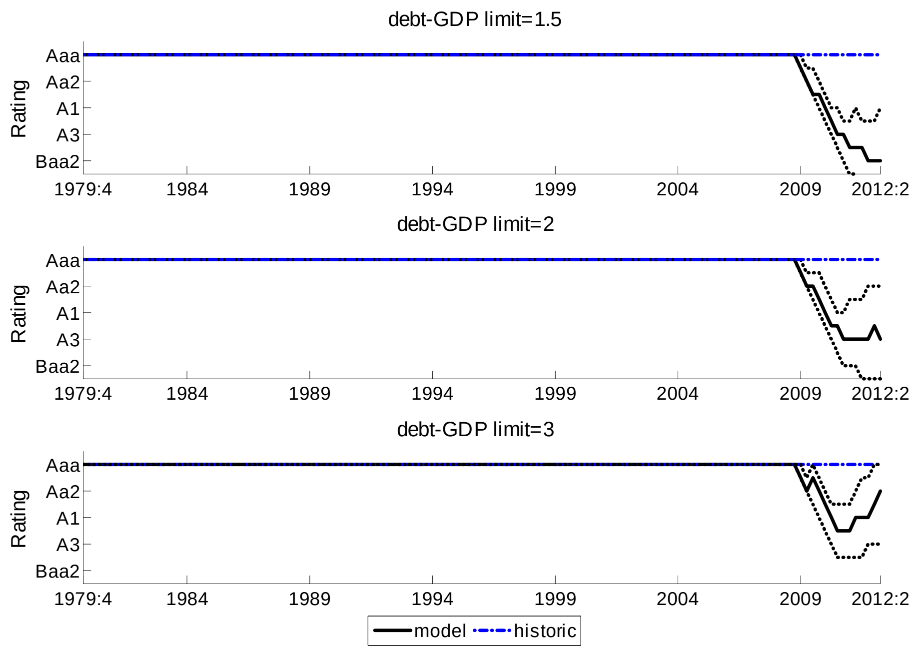

Figure 6: U.K. model-based credit rating, uncertainty about the distribution of the debt-GDP forecast.

[17] The credit rating is statistically significantly different from triple-A for all three debt limits from 2009:2. Using the lowest debt limit the credit rating falls to Baa2 (eight notches below triple-A). For the 200 per cent debt limit it stabilizes at A3 (six notches below triple-A). With the highest debt limit the credit rating falls to A2 (five notches below triple-A) by 2010:3 and then by 2012:2 recovers to Aa2 (two notches below triple-A). From 2012:1 the difference between the estimated and actual credit ratings is no longer statistically significant.

4 The UK credit rating using theory-based debt-GDP limits

We now derive a measure of the credit rating based exclusively on a government's ability to use its fiscal policy to repay its debt that is derived from a stylized DSGE model of an open economy. This is in contrast to the use of ad hoc limits when it is not clear whether a downgrade reflects a threshold beyond which the U.K. government is unwilling or is unable to meet its financial obligations. A narrower view of the sovereign credit rating may be more appropriate for advanced countries like the U.K. that are deemed to be fully committed to repaying their sovereign debt. It provides a measure of the credit rating based purely on a country's ability to use its fiscal policy to enable it to repay its sovereign debt. Any discrepancies with the credit ratings issued by CRAs can then be attributed to non-fiscal factors in the judgements of the CRAs.

The DSGE model can be used to determine the maximum borrowing capacity of the U.K. economy. Four versions of the debt limit may be derived that differ due to the assumptions made about future fiscal policy: any changes in future fiscal policy may be either anticipated or unanticipated by market participants and, if unanticipated, they may stem from changes in expenditure policy, tax policy or both. The debt limits are derived from an open economy real business cycle model with a floating exchange rate and distortionary taxation. The model does not include inflation. Consequently, the debt limits do not take account of the possibility that the government could inflate away its debt obligations. This would be considered de-facto default by the CRAs.

4.1 The model

The economy has four sectors: households, firms, the government and the rest of the world. There is imperfect substitutability between home and foreign goods. Households derive utility from total consumption c_t and leisure $1-n_t$, and seek to maximize

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-n_t\right)$$

where E_0 denotes a mathematical expectation conditioned on time 0 information, $\beta \in (0,1)$ is the household discount factor, $u(.)$ is a twice continuously differentiable, increasing, strictly concave utility function and n_t denotes labor supply. Households face a sequence of budget constraints

$$\begin{aligned}(1+\tau_t^c)c_t + k_t + b_t^D + s_t f_t &= (1-\tau_t^n)w_t n_t + \left(r_t^k - \delta\right)\left(1-\tau_t^k\right)k_{t-1} + \\ &\quad + k_{t-1} + (1+r_t)\, b_{t-1}^D + z_t + (1+r_t^*)\, s_t f_{t-1}\end{aligned}$$

where k_t, b_t^D, f_t, w_t, s_t, r_t^k, r_t, r_t^*, z_t, δ, τ_t^c, τ_t^n and τ_t^k respectively denote physical capital, government bonds held by domestic households, real net foreign assets and denominated in foreign currency, wages, the real exchange rate (defined as the home currency per unit of foreign currency), the real rate of return from capital, the domestic real rate of return on bonds, the real rate of return on foreign assets, government transfers, the rate of physical depreciation, the tax rate on consumption, the tax rate on labour income and the tax rate on net income from capital, $r_t^k - \delta$. Total consumption is assumed to satisfy the CES function

$$c_t = \left[\phi \left(c_t^H\right)^{1-\frac{1}{\eta}} + (1-\phi)\left(c_t^F\right)^{1-\frac{1}{\eta}}\right]^{\frac{1}{1-\frac{1}{\eta}}},$$

with c_t^H, c_t^F, ϕ and η denoting goods purchased domestically, goods purchased from abroad, the relative expenditure weight on domestic and foreign goods, and the elasticity of substitution between domestic and foreign goods respectively. Output is generated by a labor-augmenting Cobb-Douglas production function

$$y_t = k_t^\alpha \left(A_t n_t\right)^{1-\alpha} \tag{4}$$

where A_t denotes technological progress and α is the income share of capital. The government budget constraint is

$$g_t + (1+r_t)\, b_{t-1}^D + (1+r_t)\, b_{t-1}^F + z_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k \left(r_t^k - \delta\right) k_{t-1} + b_t^D + b_t^F$$

where g_t is government expenditure in goods and services, and b_t^F is government debt held abroad and denominated in domestic currency. In order to allow the reconciliation of total revenue and tax revenue in the data, z_t is measured as gross transfers net of any source of government revenue other than taxation.

The balance of payments and the national income identity are:

$$\begin{aligned} s_t f_t - b_t^F &= x_t + (1+r_t^*)\, s_t f_{t-1} - (1+r_t)\, b_{t-1}^F \\ y_t &= c_t + g_t + k_t - (1-\delta)\, k_{t-1} + x_t \end{aligned}$$

where x_t denotes net foreign trade expressed in domestic currency. Appendix B shows that for the utility function

$$u(c_t, 1-n_t) = \log c_t + \psi \log (1-n_t) \tag{5}$$

the stationary equilibrium debt-GDP ratio is

$$\frac{b^D + b^F}{y} = \frac{b}{y} = \frac{1}{r^*}\left\{ \begin{array}{c} \tau^c \chi \left(\frac{1}{\varphi k} - 1\right) + \tau^n (1-\alpha) \\ +\tau^k \alpha \left[1 - \delta \left(\frac{\beta^{-1}-1}{1-\tau^k} + \delta\right)^{-1}\right] - \frac{g}{y} - \frac{z}{y} \end{array} \right\} \tag{6}$$

where

$$\chi = \frac{(1-\tau^N)}{\psi(1+\tau^C)}(1-\alpha) \tag{7}$$

$$\varphi = \left[\frac{\beta^{-1} - 1 + \delta\left(1-\tau^k\right)}{\alpha A^{1-\alpha}\left(1-\tau^k\right)}\right]^{\frac{1}{1-\alpha}} \tag{8}$$

$$k = \frac{\mu + (1+\tau^c)(g+x)}{[(1+\tau^c)\Omega + \mu\varphi]} \tag{9}$$

with

$$\mu = \frac{1}{\psi}(1-\tau^n)(1-\alpha)A^{1-\alpha}\varphi^{-\alpha}, \tag{10}$$

$$\Omega = (A\varphi)^{1-\alpha} - \delta \tag{11}$$

which is non-linear in the three tax rates τ_t^c, τ_t^n and τ_t^k.

The stationary equilibrium solution for this open-economy model differs in three respects from that for a closed economy. First, the debt-GDP ratio includes government debt held abroad in addition to that held by domestic residents. Second, it requires the real world rate of interest instead of the domestic real rate of interest. Third, the revenue generated by the consumption tax depends on the equilibrium value of capital which, in an open economy, is affected by net trade.

4.2 Stationary equilibrium debt-GDP limits

The stationary equilibrium solution for the debt-GDP ratio in equation (6) can be used to determine alternative measures of the debt-GDP limit in an open economy. The existence of an equilibrium solution implies that the intertemporal GBC is satisfied and that a government cannot roll over its liabilities forever (the No-Ponzi game condition). It also implies that governments can borrow at a rate that allows an equilibrium to exist. The resulting stationary equilibrium debt-GDP ratio must be equal to the market expectation of discounted stationary equilibrium future primary surpluses. In this respect equation (6) is a debt-GDP limit identifying a government's borrowing capacity based on the market's anticipation of the future evolution of fiscal and monetary policy. We will refer to this measure of the debt-GDP limit as IGBCL.

Further measures of the debt-GDP limit can be derived by considering the potential maximum impact of unanticipated changes in fiscal policy which are, by definition,unpredictable. Nonetheless, to the extent that government revenues and expenditures are bounded (from above and below respectively) market participants would be able to determine the maximum potential impact of unexpected changes in fiscal policy on the stationary equilibrium debt-GDP ratio. For example, as government expenditure is bounded from below, it is non-negative. Hence, imposing the additional constraints that $\frac{g}{y} = \frac{z}{y} = 0$ in equation (6), the stationary equilibrium debt-GDP ratio becomes

$$\frac{b}{y}^{NDL} = \frac{1}{r^*}\left\{ \begin{array}{c} \tau^c \chi \left(\frac{1}{\varphi k} - 1\right) + \tau^n (1-\alpha) \\ +\tau^k \alpha \left[1 - \delta \left(\frac{\beta^{-1}-1}{1-\tau^k} + \delta\right)^{-1}\right] \end{array} \right\}. \tag{12}$$

Equation (12) is an adaptation to government policy of Aiyagari (1994)'s natural debt limit. We therefore refer to this as the NDL. In a DSGE model, the NDL corresponds to the representative household's wealth and follows from the requirement that, by eliminating its expenditures, it must be feasible for the household to repay its state contingent debt in every possible state. When applied to fiscal policy, the NDL limit precludes a government from being able to finance higher debt levels from unanticipated reductions in expenditure; in order to increase the stationary equilibrium debt-GDP limit there must be unanticipated increases in tax or monetary policy.[18]

In an economy with distortionary taxation, government revenue is bounded from above due to the Laffer effect. A further stationary equilibrium value of the debt-GDP limit may then be obtained when government revenue is at the peak of the Laffer hill:

$$\frac{b}{y}^{FL} = \frac{1}{r^*}\left\{ \begin{array}{c} \tau^c \chi \left(\frac{1}{\varphi k} - 1\right) + \tau^{n,\max}(1-\alpha) \\ +\tau^{k,\max}\alpha \left[1 - \delta\left(\frac{\beta^{-1}-1}{1-\tau^k} + \delta\right)^{-1}\right] - \frac{g}{y} - \frac{z}{y} \end{array} \right\} \tag{13}$$

where we replace τ^n and τ^k in equation (6) with the tax rates $\tau^{n,\max}$ and $\tau^{k,\max}$ that maximize tax revenues from labor and capital respectively. Equation (13) is, in effect, an adaptation to an open economy (with distortionary taxation on income from labor, capital and consumption) of the fiscal limit derived for closed economy with only income taxation by Davig, Leeper and Walker (2010). We refer to this debt-GDP limit as the FL. The FL identifies the point where the government no longer has the ability to increase its borrowing capacity by unanticipated changes in tax policy. Nonetheless, it could still either change its expenditure policy or use monetary policy, or both. We note that as there is no Laffer effect in real business cycle models with distortionary taxation for consumption taxation - see Trabandt and Uhlig (2011) - the FL is computed by setting the tax rate on consumption at its equilibrium value.

The maximum stationary equilibrium value of the debt-GDP ratio is obtained by imposing on equation (6) the conditions for both the NDL and the FL:

$$\frac{b}{y}^{MDL} = \frac{1}{r^*}\left\{ \begin{array}{c} \tau^c \chi \left(\frac{1}{\varphi k} - 1\right) + \tau^{n,\max}(1-\alpha) \\ +\tau^{k,\max}\alpha \left[1 - \delta\left(\frac{\beta^{-1}-1}{1-\tau^k} + \delta\right)^{-1}\right] \end{array} \right\}, \tag{14}$$

We refer to this as the MDL. When the stationary equilibrium debt-GDP value reaches the MDL, a government can no longer use unanticipated changes in fiscal policy to finance additional debt and so would then need to resort to monetary policy.

In principle the open economy model could be solved using a standard perturbation approach, for example, by taking a local approximation based on a Taylor expansion. The IGBCL can then be computed directly from the stationary equilibrium solution of the debt-GDP in equation (6), while the NDL, the FL and the MDL can be computed by imposing the additional restrictions described

[18] By extending the model to include prices of domestic consumption goods and imposing a cash-in-advance constraint on consumption, via its supply of money the government would be able to use monetary policy to repay its liabilities. This would be equivalent to imposing an inflation tax on households.

above. As perturbation methods are local approximations, they are reliable only when the shocks represent small deviations from the steady state. They are not, therefore, suitable for large temporary deviations of the debt-GDP ratio from its stationary equilibrium. Furthermore, the solution of a rational expectation model obtained from a perturbation method can only be implemented on stationary data. We therefore derive the stationary equilibrium solution of the debt-GDP ratio in equation (6) using a nonlinear Monte Carlo Markov Chain algorithm based on the simulation method proposed by Judd (1998) for solving rational expectations models. The algorithm is consistent with Coleman's (1991) nonlinear solution method, which was recently employed by Bi (2011) for computing the FL for a number of advanced countries. Appendix C describes the algorithm in detail. We note that the algorithm provides state-dependent distributions of each of the four debt-GDP limits and is based on using rolling window means of (i) the ratio of government expenditures to GDP, (ii) the ratio of transfers to GDP, (iii) the shocks to technological progress and (iv) the actual tax rates.

4.3 Empirical results

The four theoretical measures of the debt-GDP limit depend mainly on the evolution of four variables, all expressed as a proportion to GDP: government expenditures on goods and services; government transfers; actual government tax revenues; and the tax revenues that could be generated in each period at the peak of the Laffer hill.[19] These four components are shown for the period 1995:2-2012:2 in Figure 7. Several features are worth commenting on. First, tax revenues (actual and maximum) do not display any evident upward or downward trend. Second, government spending on goods and services displays a mild downward trend until about 1997 and an evident upward trend afterwards. Third, transfers show a gradual upward trend until the first half of 1990s, decline in the second half of the 1990s, and are then stable until the global financial crisis when they start to increase. There is a striking difference between the behaviour of the components of the U.K. budget and those for the U.S.: U.S. government spending on goods and services as a proportion to GDP is trendless over the past 40 years, while transfers have a positive trend, see Polito and Wickens (2012).

[19] The actual ratio of tax revenues to GDP is calculated by totalling the revenues from taxes on production and imports, direct taxes and social security contributions. Net transfers are obtained by summing social security contributions and capital transfers paid by the government and subtracting non-tax revenues. Expenditures on goods and services are obtained by subtracting social security payments, capital transfers and interest payments from total government expenditures. All data are taken from the OECD Economic Outlook (Datastream, October 2012). Actual government revenue depends on the average tax rates on consumption, capital and labor. These tax rates are taken from the Eurostat (2012), but are available only from 1995 to 2012 (see appendix C, point 3 for details). Consequently, all our results are now from 1995. The tax rates that maximize tax revenues are calculated numerically in step 4 of the solution algorithm described in appendix C.

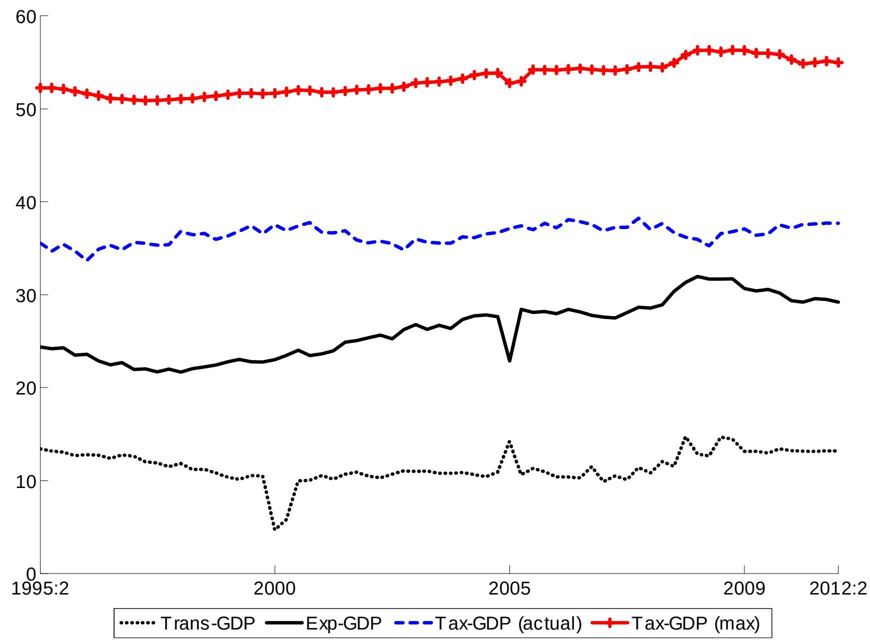

Figure 7: U.K. government budget components, 1995-2012.

The four debt limits for the period 1995.2-2012.2 are shown in Figure 8. MDL and NDL are relatively stable over the whole sample period. This reflects the assumption that total government spending is zero in equilibrium ($\frac{g}{y} = \frac{z}{y} = 0$) and that the ratio of tax revenues to GDP is relatively stable. In contrast, the behaviour FL and IGBCL are both affected by fluctuations in government expenditures. They are stable until about 1997 but afterwards start to trend downwards due to increases in government expenditures and transfers. The values of MDL and NDL are implausibly high, being respectively on average about 9 and 8 times U.K. GDP. In contrast, the values of FL and IGBCL are well within the range of U.K. debt-GDP ratios observed historically: in the 1990s FL is about 3 times U.K. GDP, and falls to about 2.6 by 2012; in 1995 IGBCL is 1.9 times U.K. GDP and declines to about 1.3 by 2012.

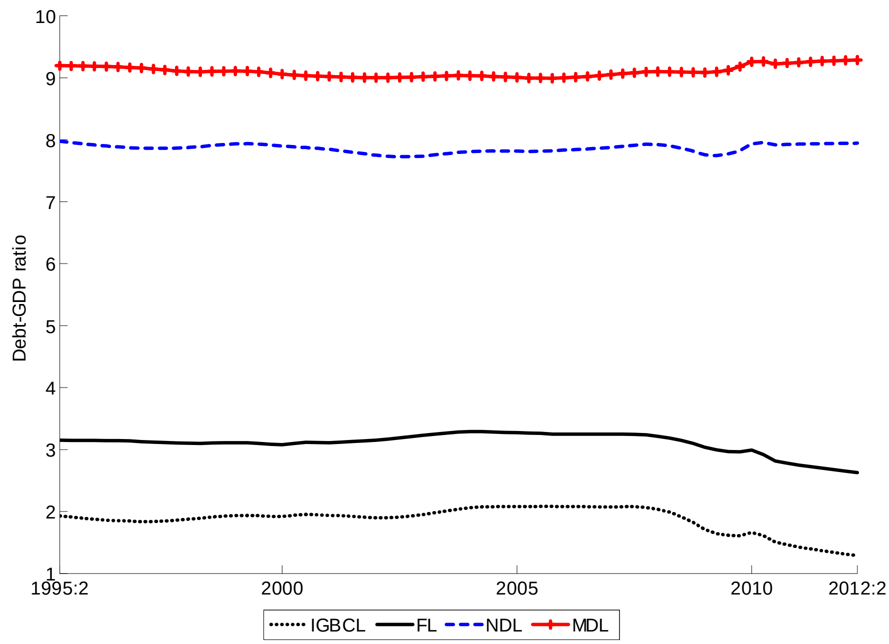

Figure 8: U.K. theoretical debt-GDP limits, 1995-2012.

These values for the debt-GDP limits are subject to uncertainty and this uncertainty varies over time. This is shown in Figures 9 and 10 where we show numerically generated state-dependent probability density functions for the debt-limits at various dates. Figure 9 shows the distributions for the two highest debt-GDP limits, MDL and NDL. They are relatively stable for the whole sample period. The distributions of FL and the IGBCL shown in Figure 10 display a noticeable shift to the left from the start of the financial crisis implying a more stringent limit.

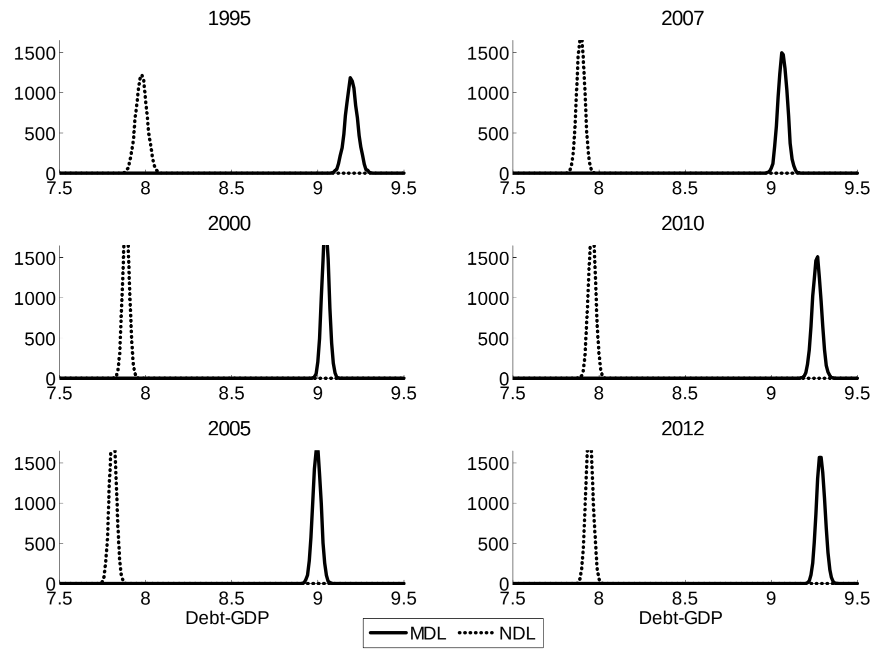

Figure 9: Changes in the state-dependent probability density functions of the U.K. natural debt limit (NDL) and maximum debt limit (MDL), 1995-2012.

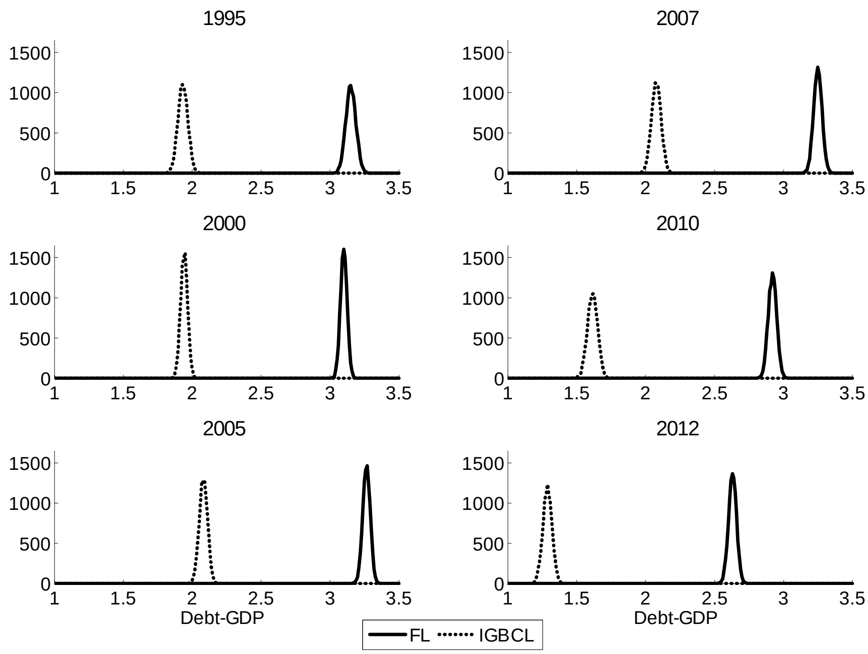

Figure 10: Changes in the state-dependent probability density functions of the U.K. intertemporal GBC limit (IGBCL) and fiscal limit (FL), 1995-2012.

The credit ratings associated with the four debt-GDP limits (MDL, NDL, FL and IGBCL) are shown in Figure 11 for 5 and 10 year horizons over the period 1995:2 - 2012:2 .[20] As already observed in the analysis with ad-hoc limits, downgrades are more likely the lower is the debt-GDP threshold and the longer is the time horizon. Using the two highest debt-GDP limits a temporary downgrade occurs around 2009-2010, but the triple-A rating is restored by the end of the sample period. For MDL and a 5-year horizon, the U.K.'s credit rating is downgraded from triple-A by one notch in 2008:4 and by two notches in 2009:1 before recovering to triple-A in 2009:3. For MDL and a 10-year horizon, the credit rating is further downgraded by one notch in 2009:1, two notches in 2010:2 before gradually recovering to triple-A in 2010:3. NDL gives similar results except at the 5-year horizon as a further 1 notch downgrade occurs in 2009:4.

For FL and IGBCL the downgrades are more severe and longer lasting. For IGBCL, once lost, a triple-A rating is never restored regardless of the time horizon. For FL and a 5-year horizon, the credit rating is downgraded from triple-A in 2008:4 and then fluctuates between the top three notches until 2012:2. Over a 10-year horizon, the credit rating is downgraded from triple-A in 2008:3

[20] The results for the 1-year horizon are not reported as they are triple-A for each debt-GDP limit throughout the whole sample.

and continues to fall, reaching a lowest grade of A2 (five notches below triple-A) in 2012:2 before rising to Aa1 (one notch below triple-A) by 2012:2. For IGBCL the credit rating is downgraded from triple-A in 2008:3 and continues falling thereafter.

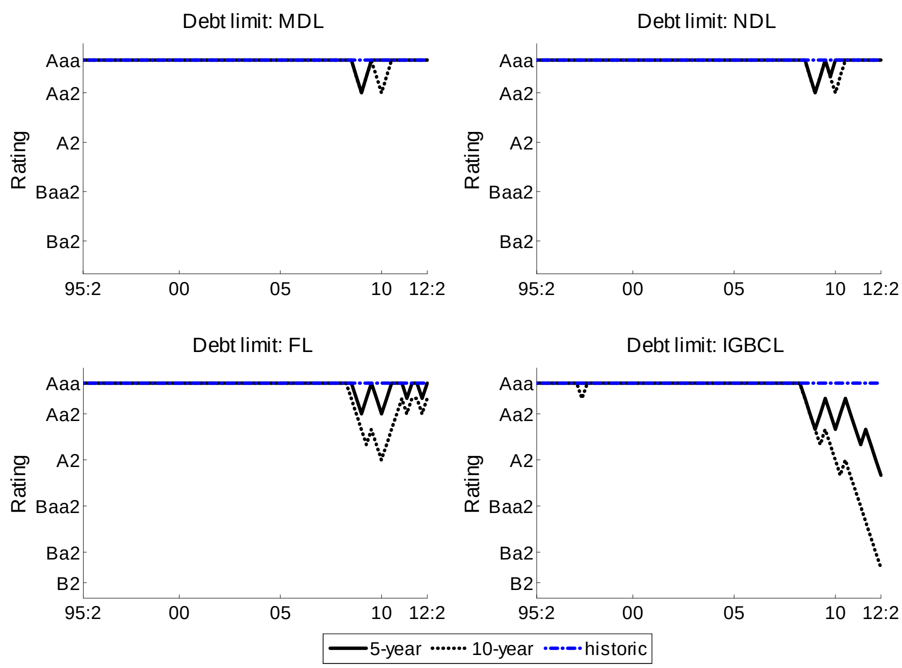

Figure 11: U.K. model-based credit rating under alternative theoretical debt-GDP limits, 1995-2012.

The effect of uncertainty about the debt-GDP limits on the credit rating is shown in Figure 11. MDL and IGBCL provide upper and lower bounds for the debt-GDP limit over the period 1995:2-2012:2. The distribution of the credit rating may be constructed at each point of time by computing the credit rating at each point in a grid formed from one per cent increments in the interval obtained from the difference between the credit ratings for MDL and IGBCL. The median values and the 16th and 84th percentiles of the distributions are shown in Figure 12. We observe a significant difference between the model-based credit rating and the actual triple-A rating for a 5-year horizon during the period 2008:4-2009:2 and in 2009:4. For a 10-year horizon the difference is significant for seven quarters starting in 2008:4.

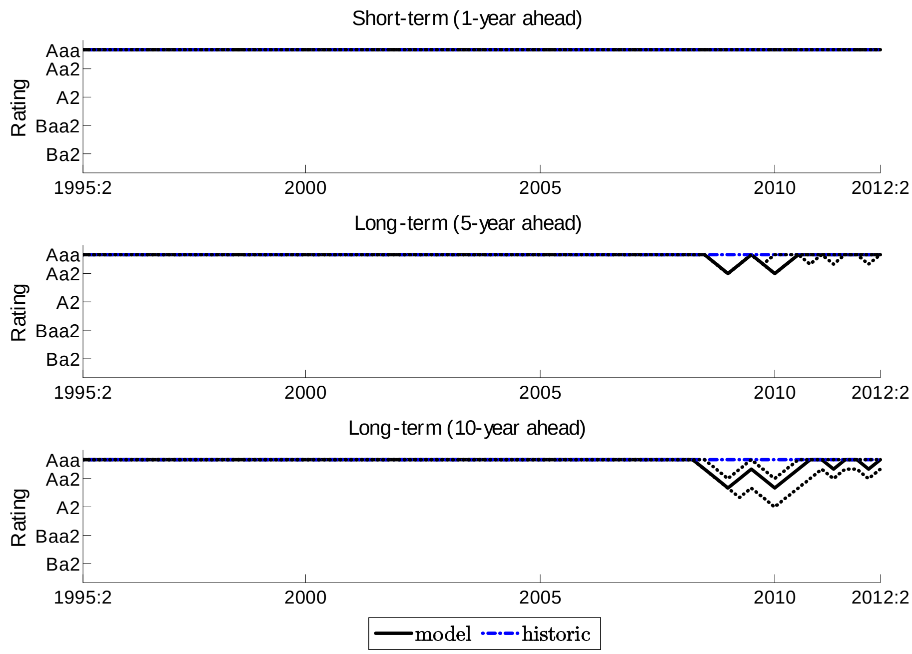

Figure 12: U.K. sovereign credit rating, uncertainty about the debt-GDP limit.

The effect of uncertainty about the distribution of the ROVAR forecast of the debt-GDP ratio can be obtained by bootstrapping the ROVAR. 1000 sets of debt-GDP forecasts and standard errors are generated over a 40-quarter horizon from 1995:2-2012:2 and used to construct numerical distributions for the credit ratings over the grid of values described above. The median and the 16th and 84th percentiles of these distributions are shown in Figure 13 for the period 1995:2-2012:2. This provides a measure of the model credit rating that takes account both of uncertainty about the trajectory of the debt-GDP forecast and the maximum borrowing capacity of the U.K. economy. For a 1-year horizon, the credit rating is triple-A. For a 5-year horizon the credit rating is statistically significantly different from triple-A from 2009:2-2010:3, and for a 10-year horizon it is significantly different over the period 2009:1-2012:1.[21]

[21] For a 5-year horizon the credit rating is downgraded by up to Aa2 (two notches below triple-A) in the first three quarters of 2010 before gradually recovering to triple-A. For a 10-year horizon, it is downgraded to A1 (four notches below triple-A) in the last three quarters of 2010, before gradually recovering to triple-A.

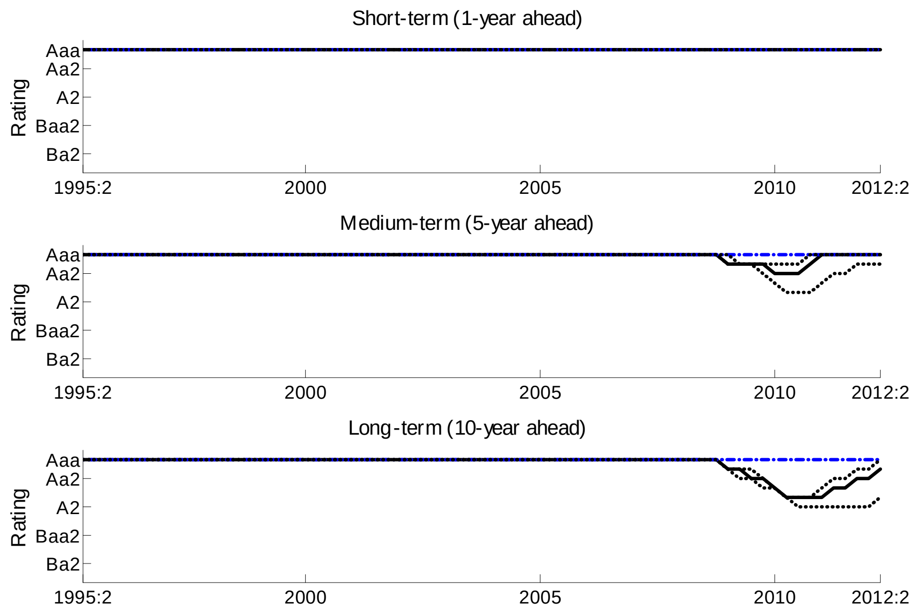

Figure 13: U.K. sovereign credit rating, uncertainty about the distribution of the debt-GDP forecasts and the debt-GDP limit.

5 Conclusions

In this paper we have shown that it is feasible to produce a benchmark sovereign credit rating that can be easily automated and used by governments to assess the quality of their fiscal stance. It can also be used by financial markets in pricing government debt.

Our measure is based on evaluating the probability that a forecast of the debt-GDP ratio will exceed a certain limit or threshold and mapping this probability into a credit rating. We use a rolling-window VAR to form the forecasts, but any other forecasting model could be used instead, and we compare ad hoc debt-GDP limits with limits obtained from a calibrated DSGE model of the economy. The paper extends the theory of fiscal limits of Polito and Wickens (2012) from a closed to an open economy.

Our results suggest that there was a good case for downgrading U.K. sovereign debt from the middle of 2008. This is in contrast to the credit ratings issued by the three main CRAs which gave U.K. sovereign debt a triple-A rating throughout the financial crisis. It was only in 2012 that the CRAs put the U.K. on a negative outlook and threatened a downgrade, and only in February 2013 that Moody's downgraded the U.K.'s credit rating by one notch. Although our results indicate that the U.K. should have been downgraded from triple-A, they also suggest an improved rating from 2010.

Clearly, our theory-based measure of sovereign credit ratings fails to consider many factors that the CRAs take into account such as the willingness of a government to default and a country's historical record of default. For example, the U.K. has never defaulted and is unlikely to do so in the foreseeable future. Nonetheless, the U.K. government has clearly been concerned to avoid a credit downgrade as this could make government borrowing much more expensive.

We observed at the start of this paper that macroeconomic policy in many countries, including the U.K., appeared to be driven more by concerns about the size of its debt, and the cost of servicing this if it lost its triple-A credit rating, than recession. This has proved to be very controversial politically. Our results provide some justification for this policy stance.

6 References

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A Data

The table below lists the data used for the construction of the variables included in the ROVAR model. The series for the variable denoted as DEBT(2) is taken from the Eurostat Government Finance Statistics of the Eurostat, while all other series are taken from Datastream (accessed on 25 July 2012). We note that IRS is the base rate of the Bank of England; EXC is a trade-weighted multilateral real exchange rate; OIL refers to crude oil spot price of Brent; and the series for the fiscal variables refer to the general government.

Variable	Description	Availability	Source	Mnemonic	Format
NGDP	Nominal GDP	1960:1 - 2012:3	OECD EO	UKOCFGPNB	Millions, s.a.
RGDP	Real GDP	1960:1 - 2012:3	OECD EO	UKOCFGVOD	Millions, s.a.
DGDP	GDP deflator	1960:1 - 2012:3	OECD EO	UKOCFDGDE	Index, s.a.
IRS	Short rate	1950:1 - 2012:3	BoE	UKPRATE.	%, s.a.
IRL	Long rate	1960:1 - 2013:3	OECD EO	UKOCFILTR	%, s.a.
EXC	Real exch. rate	1970:1 - 2012:3	JP Morgan	UKXTW..RF	Index, n.s.a.
OIL	Crude oil price (U.S. $)	1960:1 - 2012:3	OECD EO	OCOCBRNTB	value, s.a.
CAC	Current account-GDP	1960:1 - 2012:3	OECD EO	UKOCFC%GQ	%, s.a.
DEBT(1)	Gross debt-GDP	1966 -2013	OECD EO	UKOCFGL%	% GDP, s.a.
DEBT(2)	Gross debt-GDP	2000:1-2012:2	Eurostat	n.a.	% GDP
TOTREC	Gov. total receipts	1970:1 - 2012:3	OECD EO	UKOCFYRQQ	% GDP, s.a.
TOTEXP	Gov. total disbursements	1970:1 - 2012:3	OECD EO	UKOCFGU%Q	% GDP, s.a.
INTEXP	Int. paid	1960:1 - 2012:3	OECD EO	UKOCFIPYB	Millions, s.a.
INTREC	Int. receipts	1970:1 - 2012:3	OECD EO	UKOCFIRCB	Millions, s.a.
GFCF	Gov. fixed cap. format.	1960:1 - 2012:3	OECD EO	UKOCFGFXB	Millions, s.a.
GFCC	Gov. fixed cap. cons.	1960:1 - 2012:3	OECD EO	UKOCFGCCB	Millions, s.a.

Table 3: Data: sources and description

These series are used to create a quarterly dataset of the nine variables included in the ROVAR model that covers the period 1970:1-2012:2. The table below summarizes how each variable has been constructed. We note that all data are annualized and are percentage; the derivation of the primary deficit is based on the government accounts definition in the OECD Economic Outlook Database inventory 91, Table A2-4 on page 31. Quarterly data on the debt-GDP ratio are only available from 2000:1; consequently the data for the period 1970:1-1999:4 are derived by linearly interpolating the available annual data.

Variable	Acronyms	Derivation
$\frac{b_t}{y_t}$	DEBT	From 1970:1-1999:4: quarterly interpolation of $DEBT(1)$
		From 2000:1 to 2012:2: $DEBT(2)$
$\frac{d_t}{y_t}$	DEF	$TOTEXP + \frac{GFCF}{NGDP} \times 100 - \frac{INTEXP}{NGDP} \times 100$
		$-\left[TOTREC - \frac{INTREC}{NGDP} \times 100\right]$
γ_t	GDP	$\Delta ln(RGDP) \times 400$
π_t	INF	$\Delta ln(DGDP) \times 400$
r_t^s	IRS	IRS
r_t^l	IRL	IRL
e_t	EXC	$\Delta ln(EXC) \times 400$
$\frac{x_t}{y_t}$	CAC	CAC
π_t^o	OIL	$\Delta ln(OIL) \times 400$

Table 4: Derivation of the variables included in the ROVAR

B Stationary equilibrium debt-GDP ratio

The first-order conditions (FOCs) for consumption of domestic goods, consumption of foreign goods, labor, capital, domestic and net foreign assets are

$$
\begin{aligned}
1 &: \frac{\partial \mathcal{L}}{\partial c_t^H} = \beta^t u_{c,t} p_t^H - \lambda_t (1+\tau_t^c) p_t^H = 0, \\
2 &: \frac{\partial \mathcal{L}}{\partial c_t^F} = \beta^t u_{c,t} p_t^F - \lambda_t (1+\tau_t^c) p_t^F = 0, \\
3 &: \frac{\partial \mathcal{L}}{\partial n_t} = \beta^t u_{n,t} + \lambda_t (1-\tau_t^n) w_t = 0, \\
4 &: \frac{\partial \mathcal{L}}{\partial k_t} = \lambda_{t+1} \left[1 + \left(r_{t+1}^k - \delta\right)\left(1-\tau_{t+1}^k\right)\right] - \lambda_t = 0 \\
5 &: \frac{\partial \mathcal{L}}{\partial b_t^D} = \lambda_{t+1}\left(1 + r_{t+1}\right) - \lambda_t = 0 \\
6 &: \frac{\partial \mathcal{L}}{\partial f_t} = \lambda_{t+1} s_{t+1} \left(1 + r_{t+1}^*\right) - s_t \lambda_t = 0.
\end{aligned}
$$

Given the assumptions for the utility function (5) and the production function (4), the Euler equations for the intratemporal equilibrium between labor and consumption; the income identity and the no-arbitrage equilibrium conditions are:

$$
\begin{aligned}
\frac{(1+\tau_{t+1}^c) c_{t+1}}{(1+\tau_t^c) c_t} &= \beta \left\{1 + \left[\alpha k_{t+1}^{\alpha-1} \left(A_{t+1} n_{t+1}\right)^{1-\alpha} - \delta\right] \left(1 - \tau_{t+1}^k\right)\right\} \\
\psi \frac{c_t}{1-n_t} &= \frac{(1-\tau_t^n)}{(1+\tau_t^c)} (1-\alpha) A_t k_t^\alpha \left(A_t n_t\right)^{-\alpha} \\
k_t^\alpha \left(A_t n_t\right)^{1-\alpha} &= c_t + g_t + k_t - (1-\delta) k_{t-1} + x_t \\
1 + \left(r_{t+1}^k - \delta\right)\left(1 - \tau_{t+1}^k\right) &= 1 + r_{t+1} = \frac{s_{t+1}}{s_t}\left(1 + r_{t+1}^*\right).
\end{aligned}
$$

The stationary equilibrium solution for capital is given by equation (9), while those for consumption and labor are:

$$c = \Omega k - g - x \tag{15}$$

$$n = \varphi k \tag{16}$$

where Ω and φ are defined in (11) and (8) respectively. The stationary equilibrium solutions for the other variables are:

$$\begin{aligned}
Y &= k^{\alpha}(An)^{1-\alpha} \\
r^{k} &= \alpha k^{\alpha-1}(An)^{1-\alpha} \\
w &= A(1-\alpha)k^{\alpha}(An)^{-\alpha} \\
r^{*} &= r = \left[\alpha k^{\alpha-1}(An)^{1-\alpha} - \delta\right]\left(1-\tau^{k}\right) \\
x &= r^{*}\left(b^{F} - sf\right)
\end{aligned}$$

These results show that in an open economy different values for the steady states of capital, consumption and labour are obtained than for a closed economy. The ratio between labor and capital is however unaffected in the steady state.

These results can be combined to obtain the stationary equilibrium values for the capital-output ratio, the labour-output ratio, the consumption-output and the real wage as:

$$\begin{aligned}
\frac{k}{y} &= \left[\frac{\beta^{-1}-1}{\alpha\left(1-\tau^{k}\right)} + \frac{\delta}{\alpha}\right]^{-1}. \\
\frac{y}{n} &= A\left[\frac{\beta^{-1}-1}{\alpha\left(1-\tau^{k}\right)} + \frac{\delta}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}} \\
\frac{c}{y} &= \chi\left(\frac{1}{\varphi k} - 1\right) \\
w &= (1-\alpha)A\left[\frac{\beta^{-1}-1}{\alpha\left(1-\tau^{k}\right)} + \frac{\delta}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}
\end{aligned}$$

with χ defined in (7). The equilibrium values of the capital-GDP ratio, labour-GDP ratio and real wage are the same as in the closed economy. The equilibrium value of the consumption-GDP ratio is different because it depends on the steady-state value of capital which in turn is affected by net trade.

Finally, the stationary equilibrium debt-GDP ratio is derived from the equilibrium solution to the GBC

$$\frac{b}{y} = \frac{1}{r^{*}}\left(\frac{v}{y} - \frac{g}{y} - \frac{z}{y}\right),$$

where

$$\begin{aligned}\frac{v}{y} &= \tau^c \frac{c}{y} + \tau^n w \frac{n}{y} + \tau^k \left(r^k - \delta\right) \frac{k}{y} + \frac{q}{y} \\ b &= b^F = b^D \\ r^* &= r.\end{aligned}$$

The tax-GDP ratio can therefore be formulated as

$$\frac{v}{y} = \tau^c \chi \left(\frac{1}{\varphi k} - 1 \right) + \tau^n (1 - \alpha) + \tau^k \left\{ \alpha - \delta \left[\frac{\beta^{-1} - 1}{\alpha (1 - \tau^k)} + \frac{\delta}{\alpha} \right]^{-1} \right\} + \frac{q}{y},$$

From this we obtain the stationary equilibrium debt-GDP ratio in equation (6).

C Markov Chain Monte Carlo algorithm

The Markov Chain Monte Carlo simulation includes the following steps:

1. Estimate the time-varying volatility of technology shocks. From the log transformation of the labour-augmenting Cobb-Douglas production, we derive a time-series for the logarithm of technological progress using

$$\ln A_t = \frac{1}{1 - \alpha} \left[\ln y_t - \alpha \ln k_t - (1 - \alpha) \ln n_t \right]$$

   over the period 1970:1-2012:2. This uses real GDP data described in appendix A, together with data on private sector employment and private nonresidential gross fixed capital formation from the OECD Economic Outlook (Datastream, October 2012) and assumes a capital share of output of 0.3. We then measure the rolling-window (40 quarters) standard deviation of the derived series for $\ln A_t$ which is used as proxy for the time-varying volatility of technological progress. Given the size of the rolling window, this series ranges from 1979:4 to 2012:2.

2. Estimate the time-varying mean and volatility of $\frac{g_t}{y_t}$ and $\frac{z_t}{y_t}$. These are derived by calculating rolling window (40 periods) means and standard deviations for the time series for government expenditure-GDP and transfers-GDP described in Appendix A. As data are available from 1970:1 onward, these time-varying means and standard deviations range from 1979:4 to 2012:2.

3. Estimate time-varying average tax rates on consumption, capital and labour. We use the data provided by Eurostat (2012). They are annual data ranging from 1995 to 2010 for consumption and labour and until 2009 for capital. These data are interpolated to create quarterly time series from 1995:1 to 2012:2.

4. Estimate Laffer hills. We simulate numerically the stationary equilibrium solution to the model over the period 1979:4-2012:2 using rolling-window mean values of $\frac{g_t}{y_t}$ (from 2) and τ^c (from 3) and allowing in each quarter τ^n and τ^k to range from 0.01 to 0.99 (step = 0.01). We then use grid search to find the combination of τ^n and τ^k that maximizes the revenue-GDP ratio in each quarter. This yields a series of $\tau^{n,\max}$ and $\tau^{k,\max}$ that correspond with the pick of the Laffer hill at each quarter of the sample period. The simulation is carried out using β=0.95, α=0.3224, δ=0.012, ψ=0.6, and $\overline{A} = 1$. The exchange rate is normalized to 1, $s_t = 1$.

5. Stochastic simulation of the shocks. We assume that the natural logarithm of $\frac{g_t}{y_t}$, $\frac{z_t}{y_t}$ and A_t follows an AR(1) process with time-varying volatility (taken from 1 and 2) and time-varying mean (for $\frac{g_t}{y_t}$ and $\frac{z_t}{y_t}$) taken from 2. The mean of the technological progress is normalized to 1. Thus we specify

$$\ln s_t = (1 - \rho^s) \ln \overline{s}_t + \rho^s \ln s_{t-1} + \epsilon_t^s,\ \epsilon_t^s \sim N\left(0, \sigma_s^2\right)$$

with $s = \frac{g_t}{y_t}$, $\frac{z_t}{y_t}$ and A_t. We simulate these AR(1) process 200 times at each quarter over the 1979:4-2011:2 period using a constant mean reversion coefficient $\rho^s = 0.553$.

6. Compute time-varying stationary equilibrium. Using the tax rates from either 3 or 4, we calculate the steady-state solution to the model and the implied consumption path, for each of the 200 values of $\frac{g_t}{y_t}$ and A_t simulated from 5.

7. Compute time-varying debt-GDP limits. Using the simulated values of $\frac{v_t^{\max}}{y_t}$, $\frac{v_t}{y_t}$, $\frac{g_t}{y_t}$ and $\frac{z_t}{y_t}$ we calculate the present value of the maximum deficit obtained by setting expenditure to zero and generating the maximum tax revenue. Discounting is carried out using the stochastic discount factor generated by the consumption series simulated in 6.

8. Posterior distribution of the debt-GDP limits. We repeat steps 5-7 10000 times to get the posterior mean and standard deviation of each of the debt-GDP limits.