Scarce Collateral, the Term Premium, and Quantitative Easing

Stephen D. Williamson
Washington University in St. Louis
Federal Reserve Banks of Richmond and St. Louis

April 7, 2013

Abstract

A model of money, credit, and banking is constructed in which the differential pledgeability of collateral and the scarcity of collateralizable wealth lead to a term premium – an upward-sloping nominal yield curve. Purchases of long-maturity government debt by the central bank are always a good idea, but for unconventional reasons. A floor system is always welfare improving, as it gives the central bank an extra degree of freedom.

1 Introduction

The Federal Reserve has recently become a world leader in the implementation of unconventional monetary policy. Conventional monetary policy typically works through open market operations in short-term government debt, with the central bank buying and selling this short-maturity debt to hit some pre-set target for the short-term nominal interest rate. This nominal interest rate target is set in a way commensurate with views about the relationships among nominal interest rates, inflation, and real economic activity. An increase in the nominal interest rate target is seen as "tightening," and a decrease is "easing." But arbitrage dictates that the short-term nominal interest rate cannot fall below zero. What if the nominal interest rate hits the zero lower bound, and the central bank thinks it has reasons to ease? Is there anything that the central bank can do?

The Fed's answer to this question is: Yes, quantitative easing (QE). Typically the nominal yield curve is upward-sloping, i.e. yield to maturity tends to increase with maturity for default-free debt. In the United States, the short-maturity nominal yield on government debt has been essentially zero since late 2008, while long-maturity yields are well above the zero lower bound. For example, on April 5, 2013, the one-month Treasury bill yield was 0.05%, while the 10-year yield on U.S. Treasury bonds was 1.71%, and the 30-year yield was 2.88%. So, Fed officials reason, if easing typically works by lowering short-term yields, why not ease by lowering long-term yields? And, if a central bank eases conventionally by purchasing short-maturity debt so as to reduce short yields, it seems it should ease unconventionally by purchasing long-maturity debt so as to reduce long yields.

But why should QE work? A central bank is a financial intermediary, and any power that it has to affect asset prices or real economic activity must stem from special advantages it has as a financial intermediary, relative to its counterparts in the private sector. For example, the reasons that conventional open market operations matter must stem from its monopoly over the issue of particular types of liquid liabilities. In particular, central banks issue currency, and they operate large-value payments systems that use outside money (reserve accounts) for clearing and settlement. If a central bank purchases short-maturity government debt by effectively issuing currency, then that should matter, as private financial intermediaries cannot do the same thing.

But QE, conducted at the zero lower bound, is different. In a situation where private financial intermediaries are holding excess reserves at the zero lower bound, QE amounts to a purchase of long-maturity government debt financed by the issue of reserves. In these circumstances, the central bank is turning long-maturity government debt into short-maturity debt, as the reserves are not serving a transactions role, at the margin. But private sector financial intermediaries can do exactly the same thing. Indeed, banks are in the business of transforming long-maturity debt into short-maturity debt. In situations like this, we would expect policy neutrality – QE should be irrelevant at the zero lower bound with a positive stock of excess reserves held by private financial intermediaries. Neutrality theorems – for example Wallace (1981) or the Ricardian equivalence theorem – work in exactly this way.

But Fed officials, including Ben Bernanke (2010), think that QE works. Bernanke in particular appeals to "preferred habitat," or "portfolio balance" theories of the term structure of interest rates, which at root seem to be based on a similar financial friction – market segmentation. If asset markets are sufficiently segmented, in that there are frictions to arbitrage across short and long-maturity debt, then central bank manipulation of the relative supplies of short and long-maturity debt will cause asset prices to change. But again, the central bank is not the only financial intermediary that can change the relative supplies of debt outstanding. Private financial institutions are perfectly capable of intermediating across maturities in response to profit opportunities, arising from market demands for assets of different maturities.

Since market segmentation seems a dubious rationale for QE, we take another approach in this paper. In the model constructed here, private financial intermediaries perform a liquidity/maturity transformation role, in line with Diamond and Dybvig (1983), and with some details that come from Williamson (2012). But these private financial intermediaries are inherently untrustworthy. Intermediary liabilities are subject to limited commitment, and the assets of the financial intermediary serve as collateral. Different assets have different degrees of "pledgeability," however, as in the work of Kiyotaki and Moore (2005) and Venkateswaran and Wright (2013) (see also Gertler and Kiyotaki 2011 and Monnet and Sanches 2013). A term premium (an upward-sloping nominal yield curve) will arise in equilibrium under two conditions: (i) short-maturity government debt has a greater degree of pledgeability than long-maturity government debt; (ii) collateral is collectively scarce, in that the total value of collateralizable wealth is too low to support efficient exchange.

The basic structure of the model comes from Lagos and Wright (2005) and Rocheteau and Wright (2005), with details of the coexistence of money, credit, and banking from Sanches and Williamson (2010), Williamson and Wright (2010), and Williamson (2012). In the model, there is a fundamental role for exchange using government-supplied currency, and exchange with secured credit, as the result of limited commitment and limited recordkeeping/memory. Banks act to efficiently allocate liquid assets – currency and collateralizable wealth – to the appropriate transactions.

In the model, the central bank holds a portfolio of short-maturity and long-maturity government debt, and issues currency and reserves as liabilities. Part of the message of Williamson (2012) was that the linkage between monetary and fiscal policy is critical in examining monetary policy issues, particularly as they relate to the recent financial crisis, and subsequent events. This is also true in the context of this model. The fiscal authority in our model is assumed to have access to lump-sum taxes, and manipulates taxes over time so that the real value of outstanding government debt (the debt held by the private sector and the central bank) is constant forever. This allows us to consider the scarcity of collateralizable wealth in a clear-cut way. To keep things simple, we assume there is no privately produced collateralizable wealth. Then, provided the value of the outstanding government debt – determined by the fiscal authority – is sufficiently small, collateralizable wealth is scarce, in a well-defined way.

Fiscal policy is treated as arbitrary in the model, and it may be suboptimal. The central bank takes fiscal policy as given, and optimizes. We consider two policy regimes: a channel system, under which no reserves are held by banks, and a floor system under which interest is paid on reserves and reserves are strictly positive in equilibrium. Under a channel system, open market purchases of either short-maturity or long-maturity bonds reduces nominal and real bond yields, in line with conventional wisdom. But these effects are permanent, which is unconventional. Further, asset purchases expand the central bank's balance sheet, in real terms, reduce bank deposits, and reduce inflation, and those effects are unconventional too. At the zero lower bound, QE indeed matters, but not in the way that Ben Bernanke imagines it does. Purchases of long-maturity government debt at the zero lower bound reduce the nominal yield on long-maturity government bonds. But real bond yields increase, and inflation falls.

QE is a good thing, as purchases of long-maturity government debt by the central bank will always increase the value of the stock of collateralizable wealth – essentially, short-maturity debt is better collateral than long-maturity debt. But a channel system limits the ability of the central bank to engage in long-maturity asset purchases, since the size of the central bank's asset portfolio is limited by how much currency the private sector will hold under a channel system. Floor systems are sometimes characterized as "big footprint" systems with inherent dangers, but in the model a floor system gives the central bank an extra degree of freedom. Central bank constraints which can bind in a channel system do not bind in a floor system. Indeed, when those constraints bind in the channel system, the nominal interest rate is too low, the inflation rate is too low, and there is too little private financial intermediation.

In the second section, we construct the model. The third and fourth sections contain analysis of a channel system and a floor system, respectively. The fifth section concludes.

2 Model

The basic structure in the model is related to Lagos and Wright (2005), or Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, ...$, and in each period there are two sub-periods – the centralized market (CM) followed by the decentralized market (DM). There is a continuum of buyers and a continuum of sellers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

where H_t is labor supply in the CM, x_t is consumption in the DM, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-x\frac{u''(x)}{u'(x)} < 1$. Each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

where X_t is consumption in the CM, and h_t is labor supply in the DM. Buyers can produce in the CM, but not in the DM, and sellers can produce in the DM, but not in the CM. One unit of labor input produces one unit of the perishable consumption good, in either the CM or the DM.

Next, we add structure that permits the coexistence of money and credit, and in which financial intermediation arises as an efficient equilibrium arrangement. For simplicity, the only underlying assets in this economy are government-issued currency, short-maturity government bonds, and long-maturity government bonds. Allowing for "privately-produced" assets would potentially be more interesting, but would complicate what we are trying to get across here. In the CM, debts are first paid off, then a Walrasian market opens on which currency, government bonds, and claims on banks are traded.

In the DM, there are random matches between buyers and sellers, with each buyer matched with a seller. All DM matches have the property that there is no memory – record-keeping is absent, so that a matched buyer and seller have no knowledge of each others' histories. A key assumption is limited commitment – no one can be forced to work – and so lack of memory implies that there can be no unsecured credit. If any seller were to extend an unsecured loan to a buyer, the buyer would default.

In a manner similar to Sanches and Williamson (2010) (except that in that paper unsecured credit is feasible), assume limitations on the information technology that imply that currency will be the means of payment in some DM transactions, and some form of credit (here it will be financial intermediary credit) will be used in other DM transactions. Suppose that, in a fraction ρ of DM transactions – denoted *currency transactions* – there is no means for verifying that the buyer possesses government debt or intermediary liabilities. In these meetings, the seller can only verify the buyer's currency holdings, and so currency is the only means of payment accepted in exchange. However, in a fraction $1-\rho$ of DM meetings – denoted *non-currency transactions* – the seller can verify the entire portfolio held by the buyer. Assume that, in any DM meeting, the buyer makes a take-it-or-leave-it offer to the seller. At the beginning of the CM, buyers do not know what type of match they will have in the subsequent DM, but they learn this at the end of the CM, after consumption and production have taken place.

Government debt takes two forms. A short-maturity government bond is a promise to pay one unit of outside money in the CM of period $t+1$, and this obligation sells in the CM of period t at a price z_t^s in units of money. A long-maturity government bond is a promise to pay one unit of money in every future period, and this obligation sells in period t at a price z_t^l. These long-maturity government bonds are Consols – indeed the British government once issued Consols, and still has some of these bonds outstanding.

In DM non-currency transactions, one option open to a buyer is to sell his or her holdings of currency and government debt in exchange for goods. But in the model (as in reality) government debt is not a physical object but an account balance with the government, and by assumption this account balance cannot be transferred directly to the seller at the time the transaction takes place. However, the seller can make a loan to the buyer, secured with the government debt. The problem is that there is limited commitment – essentially limited pledgeability of government debt as collateral. As in Kiyotaki and Moore (2005) or Venkateswaran and Wright (2013), it is assumed that the buyer is always able to abscond with some fraction of a particular asset that is pledged as collateral. We assume that the buyer can abscond with fraction θ_s of short-maturity debt, and θ_l of long-maturity government debt.

But credit transactions collateralized directly by government debt are inefficient. As in Williamson and Wright (2010) and Williamson (2012), there is a banking arrangement that arises endogenously to efficiently allocate liquid assets to the right types of transactions. This banking arrangement provides insurance, along the lines of what is captured in Diamond and Dybvig (1983). Without banks, individual buyers would acquire a portfolio of currency and government bonds in the CM, before knowing whether they will be in an currency transaction or non-currency transaction in the subsequent DM. Then, in a currency transaction, the buyer would be possess government debt which the seller would not accept in exchange. As well, in a non-currency transaction, the buyer would possess would possess some low-yield currency, and could have acquired more consumption goods from the seller with higher-yielding government debt.

A banking arrangement essentially permits currency to be allocated only to currency transactions, and government debt to non-currency transactions. Any agent could run a bank, which opens in the CM. The bank issues claims on currency at price q_t in units of money, with each of these claims paying off one unit of currency at the end of the CM, if the purchaser of the claim will be in a non-currency transaction in the next DM. The bank also issues claims to the period $t+1$ CM consumption good in the period t CM, with a claim to one unit of the period $t+1$ CM consumption good exchanging for s_t consumption goods in the the period t CM. These deposit claims are tradeable only if the buyer will be in a non-currency transaction in the subsequent DM. The deposit claims can be exchanged by the buyer with a seller in the DM (in a non-currency transaction), as the buyer and seller can communicate with the bank in such a transaction, and are then able to transfer the claim. Note that we have assumed that the buyer's type is observable, where type is the type of transaction the buyer makes in the DM. We could make type private information, and have a deposit contract with the bank under which the agent must choose, in an incentive compatible fashion, whether to withdraw currency or transfer a deposit claim in the DM. Those assumptions would give identical results.

2.1 Optimization by Buyers

Let ϕ_t denote the price of money in terms of consumption goods in the CM. Each buyer solves

$$\max_{m_t,d_t}\left[-q_t\hat{c}_t - s_t\hat{d}_t + \rho u\left(\beta\frac{\phi_{t+1}}{\phi_t}\hat{c}_t\right) + (1-\rho)u(\beta\hat{d}_t)\right],$$

where $\hat{c}_t$ denotes the contingent claims to currency (in units of the consumption good in the CM) at the end of the CM, and $\hat{d}_t$ denotes the quantity of deposit claims the buyer wants to be able to trade in the event of a non-currency transaction in the DM. The price of outside money in terms of CM goods is denoted by ϕ_t. The first-order conditions for an optimum are:

$$\rho u'\left(\beta\frac{\phi_{t+1}}{\phi_t}\hat{c}_t\right) = \frac{q_t\phi_t}{\beta\phi_{t+1}} \tag{1}$$

$$(1-\rho)u'(\beta\hat{d}_t) = \frac{s_t}{\beta} \tag{2}$$

2.2 Optimization by Banks

A bank has the same limited commitment problem that any individual agent has, in that the bank is borrowing from buyers in the CM, and making promises to deliver currency at the end of the period and consumption goods in the CM of the next period. We will assume that buyers who purchase contingent claims to currency can observe the bank's currency holdings, and that the bank cannot abscond with currency. Assume for example, that there is a commitment device – an ATM. However, the bank's deposit claims must be backed with collateral, and the only available collateral in the model is government debt. As for any individual, collateral held by the bank has limited pledgeability, in that the bank can abscond in the next CM with fraction θ_s of its holdings of short-term government debt, and θ_l fraction of its holdings of long-term government debt. As well, we will allow for the bank to hold a reserve account at the central bank. Reserve accounts can also serve as collateral for the bank, and the bank can abscond with θ_m fraction of its reserve account in the next CM. A bank solves the following problem in the CM of period t :

$$\max_{d_t, c_t, m_t, b_t^s, b_t^l} \left[\begin{array}{c} q_t c_t + s_t d_t - z_t^m m_t - z_t^s b_t^s - z_t^l b_t^l - \rho c_t - \\ (1-\rho)\beta d_t + \beta b_t^s \frac{\phi_{t+1}}{\phi_t} + \beta b_t^l \frac{\phi_{t+1}}{\phi_t} \left(1 + z_{t+1}^l\right) + \beta \frac{\phi_{t+1}}{\phi_t} m_t \end{array} \right], \tag{3}$$

subject to

$$c_t, m_t, d_t, b_t^s, b_t^l \geq 0$$

$$\begin{aligned} & -(1-\rho)d_t + b_t^s \frac{\phi_{t+1}}{\phi_t} + b_t^l \frac{\phi_{t+1}}{\phi_t} \left(1 + z_{t+1}^l\right) + \frac{\phi_{t+1}}{\phi_t} m_t \\ \geq \;& \frac{\phi_{t+1}}{\phi_t} \left[\theta_m m_t + \theta_s b_t^s + \theta_l b_t^l \left(1 + z_{t+1}^l\right)\right] \end{aligned} \tag{4}$$

All quantities in (3) and (4) are expressed in units of the CM consumption good in period t. In (3) and (4), c_t denotes contingent claims to currency at the end of the CM, while m_t denotes bank reserves purchased from the central bank at the price z_t^m, and paying off one unit of outside money in the CM in period $t+1$. Thus, if $z_t^m < 1$, the central bank pays interest on reserves. The quantities b_t^s and b_t^l are short-maturity and long-maturity government bonds, respectively, acquired by the bank in the CM of period t. The objective function in (3) is the present value of profits for the bank. Inequality (4) is an incentive constraint, which must be satisfied in order for the bank to be willing to meet its deposit obligations. The left-hand side of inequality (4) is the net revenue the bank receives from paying off its deposit obligations, and the right-hand side consists of what the bank can abscond with.

It seems sensible to assume that $\theta_m = \theta_s$, as reserve accounts and short-term government debt both pay off outside money in the CM of period $t+1$. We can then rewrite the incentive constraint (4) as

$$-(1-\rho)d_t + \left(b_t^s + m_t\right) \frac{\phi_{t+1}}{\phi_t}(1-\theta_s) + b_t^l \frac{\phi_{t+1}}{\phi_t} \left(1 + z_{t+1}^l\right)(1-\theta_l) \geq 0. \tag{5}$$

2.3 The Government: Fiscal Authority and Central Bank

Specifying the relationship between fiscal and monetary policy will be critical to how this model works. First, we will write the budget constraints of the central bank and the fiscal authority separately, so as to make clear what assumptions we are making. The central bank's budget constraints are

$$\phi_0 \left(C_0 + z_0^m M_0 - z_0^s \bar{B}_0^s - z_0^l \bar{B}_0^l \right) - \tau_0^f = 0 \tag{6}$$

$$\begin{aligned} &\phi_t \left[C_t - C_{t-1} + z_t^m M_t - M_{t-1} - z_t^s \bar{B}_t^s + \bar{B}_{t-1}^s - z_t^l \bar{B}_t^l + \left(z_t^l + 1 \right) \bar{B}_{t-1}^l \right] \\ -\tau_t^f &= 0, \ t = 1, 2, 3, ... \end{aligned} \tag{7}$$

Here, we have assumed that there are no outstanding assets at the beginning of period 0. In (6) and (7), C_t and M_t denote the nominal quantities of reserves and currency, respectively, at the beginning of period t, and $\bar{B}_t^s$ and $\bar{B}_t^l$ denote, respectively, the nominal quantities of short-term government debt and long-term government debt held by the central bank. The quantity τ_t^f is the transfer (in real terms) from the central bank to the fiscal authority in the CM of period t.

The budget constraints of the fiscal authority are

$$\phi_0 \left[z_0^s (\bar{B}_0^s + B_0^s) + z_0^l (\bar{B}_0^l + B_0^l) \right] + \tau_0^f - \tau_0 = 0, \tag{8}$$

$$\phi_t [z_t^s (\bar{B}_t^s + B_t^s) - \bar{B}_{t-1}^s - B_{t-1}^s - z_t^l (\bar{B}_t^l + B_t^l) + (1 + z_t^l)(\bar{B}_{t-1}^l + B_{t-1}^l)] + \tau_t^f - \tau_t = 0. \tag{9}$$

In equations (8) and (9), B_t^s and B_t^l denote, respectively, the nominal quantities of government debt held in the private sector, and τ_t denotes the real value of the transfer to each buyer in the CM in period t.

We can then consolidate the accounts of the central bank and the fiscal authority, and write consolidated government budget constraints, from (6)-(9), as

$$\phi_0 \left(C_0 + z_0^m M_0 + z_0^s B_0^s + z_0^l B_0^l \right) - \tau_0 = 0 \tag{10}$$

$$\begin{aligned} &\phi_t \left[C_t - C_{t-1} + z_t^m M_t - M_{t-1} + z_t^s B_t^s - B_{t-1}^s + z_t^l B_t^l - \left(z_t^l + 1 \right) B_{t-1}^l \right] \\ -\tau_t &= 0, \ t = 1, 2, 3, ... \end{aligned} \tag{11}$$

We will define equilibrium somewhat differently, depending on what type of monetary regime is under consideration in what follows. However, in any equilibrium, asset markets will clear, i.e.

$$\phi_t C_t = \rho c_t, \tag{12}$$

$$\phi_t M_t = m_t, \tag{13}$$

$$\phi_t B_t^s = b_t^s, \tag{14}$$

$$\phi_t B_t^l = b_t^l, \tag{15}$$

for $t = 0, 1, 2, ...$, so (in terms of the CM consumption good) the supply of currency, reserves, short-term government debt, and long-term government debt are equal to the respective demands, coming from banks. As well, supplies of claims to currency and deposits from banks must equal the demands from buyers, or

$$c_t = \hat{c}_t \tag{16}$$

$$d_t = \hat{d}_t \tag{17}$$

3 A Channel System

If $z_t^m < z_t^s$, then it is optimal for banks to hold no reserves. We can think of this regime as capturing how "channel systems" function. In a channel system, the central bank targets a short-term nominal interest rate, and pays interest on reserves at a rate below that target rate. In such systems, overnight reserves are essentially zero (absent reserve requirements). The Canadian monetary system is a channel system, and the European Monetary Union has elements of a channel system. As well, the monetary system in the United States before October 2008 was essentially a channel system, with $z_t^m = 1$, i.e. there was no interest paid on reserves. The bank solves

$$\max_{d_t, c_t, b_t^s, b_t^l} \left[\begin{array}{c} c_t(q_t - \rho) + s_t d_t - z_t^s b_t^s - z_t^l b_t^l - \\ (1-\rho)\beta d_t + \beta b_t^s \frac{\phi_{t+1}}{\phi_t} + \beta b_t^l \frac{\phi_{t+1}}{\phi_t} \left(1 + z_{t+1}^l\right) \end{array} \right], \tag{18}$$

subject to

$$c_t, d_t, b_t^s, b_t^l \geq 0$$

$$-(1-\rho)d_t + b_t^s \frac{\phi_{t+1}}{\phi_t}(1-\theta_s) + b_t^l \frac{\phi_{t+1}}{\phi_t}\left(1 + z_{t+1}^l\right)(1-\theta_l) \geq 0 \tag{19}$$

Let λ_t denote the multiplier associated with the incentive constraint (19). Then, assuming the incentive constraint (19) binds, the following must hold in equilibrium

$$q_t - \rho = 0 \tag{20}$$

$$s_t - (1-\rho)\beta - \lambda_t(1-\rho) = 0 \tag{21}$$

$$-z_t^s + \beta \frac{\phi_{t+1}}{\phi_t} + \lambda_t(1-\theta_s)\frac{\phi_{t+1}}{\phi_t} = 0 \tag{22}$$

$$-z_t^l + \beta \frac{\phi_{t+1}}{\phi_t}(1 + z_{t+1}^l) + \lambda_t(1-\theta_l)\left(1 + z_{t+1}^l\right)\frac{\phi_{t+1}}{\phi_t} = 0 \tag{23}$$

$$-(1-\rho)d_t + b_t^s \frac{\phi_{t+1}}{\phi_t}(1-\theta_s) + b_t^l \frac{\phi_{t+1}}{\phi_t}\left(1 + z_{t+1}^l\right)(1-\theta_l) = 0 \tag{24}$$

The binding incentive constraint is very important. If the constraint binds, then the bank must receive a payoff greater than zero in the CM of period $t+1$ (from equation 24) to keep it from absconding. But from (20)-(24) the present value payoff to the bank in the CM of period t is zero in equilibrium, so what the bank receives from selling claims to currency and deposit claims in the CM of period t exceeds the value of the assets it acquires. The difference is bank capital, i.e. the bank must acquire capital to keep itself honest. Bank capital also plays an important role in the context of limited commitment in models constructed by Gertler and Kiyotaki (2011) and Monnet and Sanches (2013).

We will confine attention to stationary equilibria, in which case $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$, for all t, where μ is the gross inflation rate. Then, from (20)-(23), the stationary equilibrium asset prices q, s, z^s, and z^l (prices of claims to currency, deposits, short-term and long-term government debt) satisfy

$$q = \rho \tag{25}$$

$$-z^s + \frac{1}{\mu}\left[\frac{s}{1-\rho}(1-\theta_s) + \beta\theta_s\right] = 0 \tag{26}$$

$$-z^l + \frac{1}{\mu}\left(1+z^l\right)\left[\frac{s}{1-\rho}(1-\theta_l) + \beta\theta_l\right] = 0 \tag{27}$$

Let ρc, m, b^s, and b^l denote the real quantities of currency, reserves, and short and long-term government debt, respectively, held in the private sector in a stationary equilibrium. Then, assume that the fiscal authority fixes exogenously the transfer at $t = 0$, i.e. from (10),

$$\tau_0 = V = \rho c + z^m m + z^s b^s + z^l b^l, \tag{28}$$

where $V > 0$ is a constant. This then implies that the total value of the outstanding consolidated government debt will be a constant, V, forever. Further, let V_l and V_s denote, respectively, the values of government long-term and short-term debt held collectively by the private sector and the central bank. This implies that, in a stationary equilibrium,

$$\tau_t = V(1 - \frac{1}{\mu}) + \frac{1}{\mu}\left[(z^m - 1)m + (z^s - 1)b^s + b^l\right],$$

for $t = 1, 2, 3, \ldots$. Thus, we are assuming that, under this fiscal policy regime, taxes respond passively after period 0 to central bank policy, in a manner that holds constant the value of the consolidated government debt outstanding.

In a stationary equilibrium, from (1), (2), (16), (17), (24), and (25)-(28), we obtain

$$z^s = \frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_s) + \theta_s\right] \tag{29}$$

$$z^l = \frac{\frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_l) + \theta_l\right]}{1 - \frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_l) + \theta_l\right]} \tag{30}$$

$$\frac{\beta}{\mu}u'\left(\frac{\beta}{\mu}c\right) - 1 = 0 \tag{31}$$

$$-(1-\rho)d + \frac{b^s(1-\theta_s)}{\mu} + \frac{b^l\left(1+z^l\right)(1-\theta_l)}{\mu} = 0 \tag{32}$$

$$\rho c + z^s b^s + z^l b^l = V \tag{33}$$

In equations (29)-(33), the variables we want to determine are z^s, z^l, d, c, μ, b^s, and b^l. What is exogenous and what is endogenous depends on how we want to think about monetary policy. The solution must satisfy

$$0 \leq z^s b^s \leq V_s, \tag{34}$$

$$0 \leq z^l b^l \leq V_l, \tag{35}$$

i.e. we are assuming that the government issues positive quantities of short and long-term government debt, and the most the central bank can do in this regime is to purchase government debt by issuing currency.

Equations (29) and (30) imply that the nominal yields on short-maturity and long-maturity bonds, respectively, are

$$R^s = \frac{1 - \frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_s) + \theta_s\right]}{\frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_s) + \theta_s\right]} \tag{36}$$

$$R^l = \frac{1 - \frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_l) + \theta_s\right]}{\frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_l) + \theta_l\right]} \tag{37}$$

Therefore, from (36) and (37), the nominal term premium is

$$R^l - R^s = \frac{\mu[u'(\beta d) - 1](\theta_l - \theta_s)}{\beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]\left[u'(\beta d)(1-\theta_s) + \theta_s\right]} \tag{38}$$

Two things are necessary for a strictly positive term premium. First, we require $\theta_l > \theta_s$, i.e. long-maturity government debt must be less pledgeable than short-maturity debt. Second, $u'(\beta d) > 1$, i.e. non-currency exchange is not efficient in the DM. Note that exchange is inefficient in this sense if and only if the bank's incentive constraint (32) binds. Thus, to observe a strictly positive term premium in this world, long-maturity government debt must perform more poorly as collateral than does short-maturity government debt, and collateral must be scarce in general. Note also that the nominal term premium increases with the gross inflation rate μ.

From (29) and (30), real bond yields are given by

$$r^s = \frac{1 - \beta\left[u'(\beta d)(1-\theta_s) + \theta_s\right]}{\beta\left[u'(\beta d)(1-\theta_s) + \theta_s\right]}, \tag{39}$$

$$r^l = \frac{1 - \beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]}{\beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]}, \tag{40}$$

so the real term premium is

$$\begin{aligned} r^l - r^s &= \frac{\left[u'(\beta d)(1-\theta_s) + \theta_s\right] - \left[u'(\beta d)(1-\theta_l) + \theta_l\right]}{\beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]\left[u'(\beta d)(1-\theta_s) + \theta_s\right]} \\ &= \frac{[u'(\beta d) - 1](\theta_l - \theta_s)}{\beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]\left[u'(\beta d)(1-\theta_s) + \theta_s\right]}. \end{aligned} \tag{41}$$

Therefore, a strictly positive real term premium, as with the nominal term premium, exists if and only if long-maturity debt is relatively poor collateral ($\theta_l > \theta_s$), and collateral is generally scarce ($u'(\beta d) > 1$). Further, the "fundamental" real bond yield is $\frac{1}{\beta} - 1$ for both short and long-maturity bonds, determined by the present value real payoffs when collateral is not scarce. Thus, real bond yields reflect liquidity premia, in that these yields are less than the fundamental real yield. We can measure real liquidity premia for short-maturity and long-maturity bonds, respectively, as

$$\frac{1}{\beta} - 1 - r^s = \frac{[u'(\beta d) - 1]\,(1 - \theta_s)}{\beta\,[u'(\beta d)(1 - \theta_s) + \theta_s]} \tag{42}$$

$$\frac{1}{\beta} - 1 - r^l = \frac{[u'(\beta d) - 1]\,(1 - \theta_l)}{\beta\,[u'(\beta d)(1 - \theta_l) + \theta_l]} \tag{43}$$

Therefore, from (42) and (43), liquidity premia increase with $u'(\beta d)$, i.e. with the scarcity of collateral. As well, the liquidity premium for short-maturity bonds is larger than the premium for long-maturity bonds.

3.1 Away From the Zero Lower Bound

We will first consider the case where $z_t^s > 1$ and $z_t^m < z_t^s$, with no bank reserves held in equilibrium. In the next section, we will examine the liquidity trap case in which the short-term nominal interest rate is at the zero lower bound, with $z_t^s = z_t^m = 1$.

Here, from equations (29), (30), (32), and (33), we obtain

$$-(1-\rho)\beta d\,[u'(\beta d)(1 - \theta_s) + \theta_s] - \frac{b^l \frac{\beta}{\mu}(\theta_l - \theta_s)}{1 - \frac{\beta}{\mu}\,[u'(\beta d)(1 - \theta_l) + \theta_l]} + (V - \rho c)(1 - \theta_s) = 0 \tag{44}$$

Then, letting $x_1 = \frac{\beta}{\mu}c$ and $x_2 = \beta d$ denote, respectively, consumption in currency transactions and in non-currency transactions in the DM, from (31) and (44),

$$-(1-\rho)x_2\,[u'(x_2)(1 - \theta_s) + \theta_s] - \left\{ \begin{array}{c} \frac{b^l(\theta_l - \theta_s)}{u'(x_1) - [u'(x_2)(1-\theta_l) + \theta_l]} \\ -[V - \rho x_1 u'(x_1)](1 - \theta_s) \end{array} \right\} = 0 \tag{45}$$

As well, from (29) and (30) we can solve for bond prices in terms of x_1 and x_2 :

$$z^s = \frac{[u'(x_2)(1 - \theta_s) + \theta_s]}{u'(x_1)} \tag{46}$$

$$z^l = \frac{u'(x_2)(1 - \theta_l) + \beta\theta_l}{u'(x_1) - [u'(x_2)(1 - \theta_l) + \theta_l]} \tag{47}$$

As well, from (31) the gross inflation rate is

$$\mu = \beta u'(x_1). \tag{48}$$

In equilibrium, $z^s < 1$, or

$$\frac{[u'(x_2)(1 - \theta_s) + \theta_s]}{u'(x_1)} < 1 \tag{49}$$

As well, from (33), (34), and (35), the following must be satisfied in equilibrium:

$$\begin{aligned} 0 &\leq \frac{b^l\left[u'(x_2)(1-\theta_l)+\theta_l\right]}{u'(x_1)-\left[u'(x_2)(1-\theta_l)+\theta_l\right]} \leq V_l \\ &\leq \rho x_1 u'(x_1) + \frac{b^l\left[u'(x_2)(1-\theta_l)+\theta_l\right]}{u'(x_1)-\left[u'(x_2)(1-\theta_l)+\theta_l\right]} \leq V \end{aligned} \tag{50}$$

3.1.1 Conventional Open Market Operations in Short-Maturity Debt

Consider an expansion in the central bank's balance sheet, in real terms, holding constant the value of the outstanding stock of long-maturity government debt, so that

$$z^l b^l = k,$$

where $k > 0$ is a constant. Then, we can write (45) as

$$(1-\rho)x_2\left[u'(x_2)(1-\theta_s)+\theta_s\right] + \frac{k(\theta_l-\theta_s)}{u'(x_2)(1-\theta_l)+\theta_l} = \left[V-\rho x_1 u'(x_1)\right](1-\theta_s) \tag{51}$$

Therefore, an increase in the real stock of currency outstanding, ρc, implies a one-for-one increase in the real quantity of short-maturity government bonds held by the central bank, and a corresponding reduction in the quantity of short-maturity government debt held in the private sector. As a result, x_1 increases and, since the left-hand side of (51) is increasing in x_2 and the right-hand side is decreasing in x_1, therefore x_2 falls. From (48), the inflation rate falls, and from (46) and (47) nominal bond prices rise and nominal bond yields fall. Then, from (39) and (40), real bond yields fall.

In one sense, what happens in response to a conventional open market purchase by the central bank is conventional, in that nominal bond yields fall - both at the short and the long end of the yield curve. Real bond yields fall as well. What is unconventional about this, in part, is that it is permanent. Further, we might think of this as a monetary expansion – indeed, the size of the central bank's balance sheet has increased, in real terms. Yet the inflation rate falls, and bank deposits fall as well, in real terms. The central bank asset purchase has reduced the quantity of collateralizable wealth, and therefore caused a contraction in the banking sector. The quantity of currency in circulation has increased, and in order to hold that higher stock of currency, buyers must be rewarded with a higher rate of return on currency, i.e. the inflation rate must fall.

But what could seem more natural? This conventional open market purchase has increased the quantity of financial intermediation done by the central bank, and as a result reduced the quantity of private financial intermediation.

3.1.2 Quantitative Easing

Next, consider unconventional open market operations, i.e. expansion of the central bank's balance sheet through purchases of long-maturity government debt. In this case, fix the real value of short-maturity government debt outstanding, or

$$z^s b^s = k,$$

In this case, we can write (45) as

$$-(1-\rho)x_2\left[u'(x_2)(1-\theta_s)+\theta_s\right]+k(1-\theta_s)+\frac{[V-k-\rho x_1 u'(x_1)](1-\theta_l)\left[u'(x_2)(1-\theta_s)+\theta_s\right]}{[u'(x_2)(1-\theta_l)+\theta_l]}=0. \tag{52}$$

Then, as with conventional open market operations, equation (52) is strictly decreasing in x_2 and strictly decreasing in x_1. Therefore, an expansion of the central bank's balance sheet – in real terms – with the expansion consisting of purchases of long-maturity government debt will increase x_1, reduce x_2, reduce the inflation rate, and reduce real and nominal bond yields.

3.2 Zero Short-Term Nominal Interest Rate

We want to consider the case in the channel system in which the target nominal interest rate is zero, and so $z_t^m = z_t^s = 1$. This implies that banks are willing to hold a positive stock of reserves. The bank's problem is

$$\max_{d_t,c_t,b_t^s,b_t^l}\left[\begin{array}{c} c_t(q_t-\rho)+s_t d_t-(m_t+b_t^s)-z_t^l b_t^l \\ -(1-\rho)\beta d_t+\beta\left(m_t+b_t^s\right)\frac{\phi_{t+1}}{\phi_t}+\beta b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right) \end{array}\right], \tag{53}$$

subject to

$$c_t, d_t, b_t^s, b_t^l, m_t \geq 0$$

$$-(1-\rho)d_t+\left(m_t+b_t^s\right)\frac{\phi_{t+1}}{\phi_t}(1-\theta_s)+b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)(1-\theta_l)\geq 0 \tag{54}$$

As in the previous section, let λ_t denote the multiplier associated with the incentive constraint (54). Then, assuming the incentive constraint (19) binds, the following must hold in equilibrium

$$q_t-\rho=0 \tag{55}$$

$$s_t-(1-\rho)\beta-\lambda_t(1-\rho)=0 \tag{56}$$

$$-1+\beta\frac{\phi_{t+1}}{\phi_t}+\lambda_t(1-\theta_s)\frac{\phi_{t+1}}{\phi_t}=0 \tag{57}$$

$$-z_t^l+\beta\frac{\phi_{t+1}}{\phi_t}(1+z_{t+1}^l)+\lambda_t(1-\theta_l)\left(1+z_{t+1}^l\right)\frac{\phi_{t+1}}{\phi_t}=0 \tag{58}$$

$$-(1-\rho)d_t+\left(m_t+b_t^s\right)\frac{\phi_{t+1}}{\phi_t}(1-\theta_s)+b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)(1-\theta_l)=0 \tag{59}$$

In a stationary equilibrium, from equations (55)-(59),

$$q=\rho \tag{60}$$

$$\frac{s}{1-\rho} = \frac{\mu - \beta\theta_s}{(1-\theta_s)}. \tag{61}$$

$$z^l = \frac{1 - \theta_l + \frac{\beta}{\mu}(\theta_l - \theta_s)}{\left(1 - \frac{\beta}{\mu}\right)(\theta_l - \theta_s)} \tag{62}$$

Then, from (**??**) and (62), short and long nominal bond yields are

$$R^s = 0$$

$$R^l = \frac{\left(1 - \frac{\beta}{\mu}\right)(\theta_l - \theta_s)}{1 - \theta_l + \frac{\beta}{\mu}(\theta_l - \theta_s)}. \tag{63}$$

Thus, from (63), there is a term premium even at the zero lower bound, provided $\mu > \beta$, which will hold in equilibrium.

Next, from (28), (1), (59) and (60)-(62), we can determine equilibrium quantities as the solution to:

$$u'\left(\frac{\beta}{\mu}c\right) = \frac{\mu}{\beta} \tag{64}$$

$$u'(\beta d) = \frac{\mu - \beta\theta_s}{\beta(1-\theta_s)} \tag{65}$$

$$-(1-\rho)d + \frac{(1-\theta_s)\left[(m+b^s)\frac{\beta}{\mu}(\theta_l - \theta_s) + (V - \rho c)(1-\theta_l)\right]}{\mu[1 - \theta_l + \frac{\beta}{\mu}(\theta_l - \theta_s)]} = 0. \tag{66}$$

For convenience, we can rewrite (64)-(66) as in the previous subsection, in terms of x_1 and x_2. We get

$$u'(x_2) = \frac{u'(x_1) - \theta_s}{1 - \theta_s} \tag{67}$$

$$(1-\rho)x_2 = \frac{(1-\theta_s)\left\{\frac{(m+b^s)(\theta_l - \theta_s)}{u'(x_1)} + [V - \rho x_1 u'(x_1)](1-\theta_l)\right\}}{u'(x_1)(1-\theta_l) + \theta_l - \theta_s}, \tag{68}$$

and from (62) the price of long-maturity bonds is

$$z^l = \frac{u'(x_1)(1-\theta_l) + (\theta_l - \theta_s)}{[u'(x_1) - 1](\theta_l - \theta_s)} \tag{69}$$

Therefore, from (68), conventional open market operations are irrelevant, since this does not change $m + b^s$ and therefore has no effect on x_1 and x_2 or on prices. There is a liquidity trap. But note that the gross inflation rate is

$$\mu = \beta u'(x_1),$$

and so there need not be a deflation in this liquidity trap equilibrium, as in general $u'(x_1) > 1$ in equilibrium due to the scarcity of collateralizable wealth.

3.2.1 Quantitative Easing

To consider quantitative easing, fix the real value of outstanding short-maturity government debt, as in the previous section, with

$$b^s = k,$$

and $k > 0$ a constant. Then, from (33), (59), and (62), we obtain

$$-(1-\rho)\beta d + \frac{\beta (V-\rho c)(1-\theta_s)}{\mu} - \frac{\beta b^l (1-\theta_s)\frac{\beta}{\mu}}{\mu\left(1-\frac{\beta}{\mu}\right)} = 0,$$

which, using (64) and (65), we can rewrite as

$$-(1-\rho)x_2[u'(x_2)(1-\theta_s)+\theta_s]+V(1-\theta_s)-\rho x_1 u'(x_1)(1-\theta_s)-\frac{\beta b^l(1-\theta_s)\frac{\beta}{\mu}}{(u'(x_1)-1)} = 0 \tag{70}$$

Then (67) and (70), solve for x_1 and x_2 given b^l, with the price of long-maturity bonds determined by (69).

Quantitative easing consists of purchases of long-maturity bonds by the central bank, which shows up as a decrease in b^l in equation (70). This increases x_1 and x_2, and reduces the gross inflation rate μ. Further, from (69), the price of long-maturity bonds rises, the nominal long bond yield falls, and real bond yields rise. Much as in the case with a strictly positive short-term nominal interest rate, quantitative easing increases the value of the stock of collateralizable wealth, bank deposits increase, and the quantity of exchange increases in both currency and non-currency transactions. But this effect does not work through a reduction in real bond yields but is reflected in an increase in real yields.

3.3 Optimal Monetary Policy Under a Channel System

If we add expected utilities across agents in a stationary equilibrium, our welfare measure is

$$W = \rho\,[u(x_1) - x_1] + (1-\rho)[u(x_2) - x_2], \tag{71}$$

or the sum of surpluses from exchange in the DM. Under the assumption that V is sufficiently small (liquid assets are sufficiently scarce), we must have $x_1 < x^*$ and $x_2 < x^*$, where $u'(x^*) = 1$. Thus, first-best efficiency in exchange in the DM is not feasible – for currency transactions or non-currency transactions. As a result, in terms of feasible allocations, W is strictly increasing in both x_1 and x_2.

As discussed in Williamson (2012), it is important in evaluating the effects of monetary policy to take account of the costs of operating a currency system. These costs include direct costs of maintaining the stock of currency, and the indirect social costs associated with illegal transactions, theft, and counterfeiting.

A simple approach to capturing some of these costs is to assume that a fraction ω of exchanges involving currency are socially useless. Then, our welfare measure becomes

$$\hat{W} = \rho(1-\omega)\left[u(x_1) - x_1\right] + (1-\rho)[u(x_2) - x_2]$$

We will first show that it is optimal for the central bank to purchase the entire stock of long-maturity debt. This simplifies the monetary policy problem by making it a problem involving the choice of one policy instrument.

Proposition 1 *Under a channel system, It is optimal for the central bank to choose* $b^l = 0$.

Proof. In the case where $z^l < 1$, from equation (45), if $(x_1, x_2) = (\tilde{x}_1, \tilde{x}_2)$ is an equilibrium with $b^l = \tilde{b}^l > 0$, and a gross inflation rate $\tilde{\mu} = \beta u'(\tilde{x}_1)$, then there is an alternative monetary policy with $b^l < \tilde{b}^l$, $\mu < \tilde{\mu}$, $x_2 = \tilde{x}_2$, and $x_1 > \tilde{x}_1$ that is also an equilibrium, and for which welfare is higher. Thus, quantitative easing is welfare improving, and quantitative easing also increases z^s. Thus, it is possible that reducing b^l increases z^s to 1 without the central bank purchasing all the long-maturity debt. However, with $z^s = 1$, quantitative easing is unambiguously welfare-improving, from (67) and (70). Thus, $b^l = 0$ is optimal. ■

Then, from (45), (67) and (70), we can write the monetary policy problem as

$$\max_{x_1, x_2} \left\{\rho\left[u(x_1) - x_1\right] + (1-\rho)[u(x_2) - x_2]\right\} \tag{72}$$

subject to

$$-(1-\rho)x_2\left[u'(x_2)(1-\theta_s) + \theta_s\right] + \left[V - \rho x_1 u'(x_1)\right](1-\theta_s) = 0, \tag{73}$$

$$\frac{\left[u'(x_2)(1-\theta_s) + \theta_s\right]}{u'(x_1)} < 1 \tag{74}$$

and

$$\rho x_1 u'(x_1) \geq V_l \tag{75}$$

or

$$\frac{\left[u'(x_2)(1-\theta_s) + \theta_s\right]}{u'(x_1)} = 1 \tag{76}$$

In this problem, (73) describes the equilibrium relationship between x_1 and x_2 when there are no long-maturity government bonds outstanding. If the zero lower bound on the short-term nominal interest rate does not bind, as in (73), then the central bank must purchase the entire stock of long-maturity government bonds with currency, which implies constraint (75). However, if the zero lower bound binds, as in (76), then we no longer have to be concerned about constraint (75), as the central bank can purchase long-maturity bonds at the zero lower bound by issuing reserves.

If we assume that $-x\frac{u''(x)}{u'(x)} = \alpha < 1$, it is straightforward to show that the zero lower bound on the nominal interest rate does not bind at the optimum if and only if

$$\omega > \frac{\alpha\theta_s}{u'(\tilde{x}_1)(1-\alpha) + \alpha\theta_s}, \tag{77}$$

and

$$\rho\tilde{x}_1 u'(\tilde{x}_1) \geq V_l \tag{78}$$

where $(x_1, x_2) = (\tilde{x}_1, \tilde{x}_2)$ is the solution to (73) and (74) with equality. Inequality (77) is satisfied so long as $\alpha\theta_s$ is sufficiently small, i.e. so long as the limited commitment problem is not too severe with respect to bank holdings of short-maturity government bonds, and buyers are not too risk averse. Note that, if $\omega = 0$ and there are no costs associated with the use of currency, then $z^s = 1$ at the optimum as (77) is not satisfied. However, for the sake of argument, assume now that

$$\omega > \frac{\alpha\theta_s}{1-\alpha(1-\theta_s)},$$

so that (77) is always satisfied. Then, we have three possibilities. First, if (78) does not hold, then it is optimal for the short-term nominal interest rate to be zero with (76) holding. Then, $(x_1, x_2) = (\tilde{x}_1, \tilde{x}_2)$ is optimal. Second, (78) holds, but (75) binds at the optimum. Let $\hat{x}_1$ denote the solution to

$$\rho\hat{x}_1 u'(\hat{x}_1) = V_l$$

Then, from (73), let $\hat{x}_2$ denote the solution to

$$(1-\rho)\hat{x}_2\left[u'(\hat{x}_2)(1-\theta_s) + \theta_s\right] = V_s(1-\theta_s). \tag{79}$$

In this second case, the optimum is $(x_1, x_2) = (\hat{x}_1, \hat{x}_2)$, with $\hat{x}_1 < \tilde{x}_1$ and $\hat{x}_2 > \tilde{x}_2$. Thus, nominal and real bond yields are higher than at the zero lower bound. In the third case, (75) does not bind at the optimum. Let $(x_1, x_2) = (\bar{x}_1, \bar{x}_2)$ denote the optimum in this case which, if $-x\frac{u''(x)}{u'(x)} = \alpha$, solves

$$\frac{u'(\bar{x}_1)(1-\alpha)(1-\theta_s)}{u'(\bar{x}_2)(1-\theta_s)(1-\alpha) + \theta_s} = \frac{(1-\omega)[u'(\bar{x}_1) - 1]}{u'(\bar{x}_2) - 1}$$

and (73) with $(x_1, x_2) = (\bar{x}_1, \bar{x}_2)$. Further, for (75) to be satisfied, we require

$$\rho\bar{x}_1 u'(\bar{x}_1) \geq V_l.$$

Thus, the optimal allocation is

$$(x_1, x_2) = (\bar{x}_1, \bar{x}_2), \text{ if } V_l \leq \rho\bar{x}_1 u'(\bar{x}_1),$$

$$(x_1, x_2) = (\hat{x}_1, \hat{x}_1), \text{ if } \rho\bar{x}_1 u'(\bar{x}_1) < V_l < \rho\tilde{x}_1 u'(\tilde{x}_1)$$
$$(x_1, x_2) = (\tilde{x}_1, \tilde{x}_1), \text{ if } \rho\tilde{x}_1 u'(\tilde{x}_1) \leq V_l$$

It is always optimal for the central bank to purchase the entire stock of long-maturity debt issued by the fiscal authority, as this maximizes the value of the stock of collateralizable wealth. However, it may be infeasible for the central bank to do this under a channel system with a strictly positive short-term nominal interest rate. This is because, away from the zero lower bound, there is a limitation on how much currency private sector economic agents will hold, and the efficiency gain comes from the issue of currency to buy long-maturity government debt. If the value of long-maturity government debt issued by the fiscal authority is sufficiently large, then it is efficient for the short-term nominal interest rate to be zero. Under this condition, banks are willing to hold reserves, which can then be used to finance purchases of long-maturity bonds by the central bank.

4 A Floor System

Under a floor system – the current monetary regime in place in the United States – interest is paid on financial intermediary reserves, and a positive stock of reserves is held. In this regime, $z_t^s = z_t^m$, i.e. for reserves and short-maturity bonds to be held, they must bear the same rate of return. When $z_t^s > 1$ in equilibrium, the bank's problem is identical to the channel system, except that we write the bank's incentive constraint as

$$-(1-\rho)d_t + (b_t^s + m_t)\frac{\phi_{t+1}}{\phi_t}(1-\theta_s) + b_t^l \frac{\phi_{t+1}}{\phi_t}\left(1 + z_{t+1}^l\right)(1-\theta_l) = 0 \tag{80}$$

Then, in a stationary equilibrium, from (80) and (28), we get

$$-(1-\rho)d + \frac{(b^s + m)}{\mu}(1-\theta_s) + \frac{[V - z^s(m+b) - \rho c](1+z^l)(1-\theta_l)}{\mu z^l} = 0. \tag{81}$$

The bond prices z^s and z^l are determined by (29) and (30), just as in the channel system with $z^s > 1$. As well, (31) must hold.

Substituting using (29) and (30) in (81), we obtain

$$-(1-\rho)\beta d\left[u'(\beta d)(1-\theta_l) + \theta_l\right] + (b^s + m)\frac{\beta}{\mu}(\theta_l - \theta_s) + [V - \rho c](1-\theta_l) = 0. \tag{82}$$

Then, an equilibrium is determined by (82), (31), (29) and (30). The solution must satisfy the zero lower bound constraint (49), but we replace (50) with

$$\begin{aligned} 0 \quad &\leq \quad \frac{b^l\left[u'(x_2)(1-\theta_l) + \theta_l\right]}{u'(x_1) - \left[u'(x_2)(1-\theta_l) + \theta_l\right]} \leq V_l \\ &\leq \quad \rho x_1 u'(x_1) + \frac{m\left[u'(x_2)(1-\theta_s) + \theta_s\right]}{u'(x_1)} + \frac{b^l\left[u'(x_2)(1-\theta_l) + \theta_l\right]}{u'(x_1) - \left[u'(x_2)(1-\theta_l) + \theta_l\right]} \leq V \end{aligned} \tag{83}$$

One way to think about policy is that the central bank effectively sets its policy variable – z_t^s, which by arbitrage is the price of reserves – and also chooses the composition of its portfolio, determined by $m + b^s$. Then (82), (31), (29) and (30) determine d, c, z^l and μ.

Note that conventional open market operations – swaps of reserves for short-term government debt – are irrelevant, since this does not change $m + b^s$. However, otherwise policy in this regime works in a similar fashion to the channel system with $z^s < 1$, but with some critical differences. If we write the equilibrium solution in terms of x_1 and x_2, we obtain (45), (46), and (47). The key change reflected in (83) is that the central bank, under a floor system, can now effectively convert long-maturity debt into short-maturity debt (reserves), and is not dependent on currency to finance the entire central bank portfolio. As a result, the optimal policy problem for the central bank is to maximize (72) subject to (73) and the zero lower bound

$$\frac{[u'(x_2)(1-\theta_s)+\theta_s]}{u'(x_1)} \leq 1. \tag{84}$$

Therefore, the solution to the policy problem is $(x_1, x_2) = (\bar{x}_1, \bar{x}_2)$, and so the nominal interest rate is always strictly positive at the optimum. The floor system in general is welfare-improving. The floor system is no worse if the value of long-maturity government debt issued is sufficiently small, and is strictly better if the quantity of long-maturity debt is large. If the floor system improves welfare, it does so with a higher nominal interest rate than in the channel system, a higher real interest rate, a larger private banking system (more deposits), and a higher inflation rate.

5 Conclusion

We can conclude that, when collateralizable wealth is scarce and asset prices reflect liquidity premia on collateralizable wealth, then term premia can arise. Such term premia represent an opportunity for monetary policy. Under these circumstances, QE is welfare improving, and a floor system allows for efficient asset purchases by the central bank. "Monetary" policy is about more than money, and could in principle encompass management of the whole structure of the outstanding consolidated government debt. But this raises fundamental questions about the independence of central banks, and the appropriate roles for the fiscal authority and the central bank.

6 References

Bernanke, B. 2010. "The Economic Outlook and Monetary Policy," speech at the Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming, August 27, 2010, http://www.federalreserve.gov/newsevents/speech/bernanke20100827a

Gertler, M. and Kiyotaki, N. 2011. "Financial Intermediation and Credit Policy in Business Cycle Analysis," in *Handbook of Monetary Economics.*

Kiyotaki, N. and Moore, J. 2005. “Liquidity and Asset Prices,” *International Economic Review* 46, 317—349.

Lagos, R. and Wright, R. 2005. “A Unified Framework for Monetary Theory and Policy Analysis.” *Journal of Political Economy* 113, 463–484.

Monnet, C. and Sanches, D. 2013. “Private Money and Banking Regulation,” working paper, Federal Reserve Bank of Phildadelphia.

Rocheteau, G. and Wright, R. 2005. “Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium.” *Econometrica* 73, 175-202.

Sanches, D. and Williamson, S. 2010. “Money and Credit with Limited Commitment and Theft.” *Journal of Economic Theory* 145, 1525-1549.

Venkateswaran, V. and Wright, R. 2013. “Pledgability and Liquidity: A New Monetarist Model of Financial and Macroeconomic Activity,” working paper.

Wallace, N. 1981. “A Modigliani-Miller Theorem for Open-Market Operations,” *American Economic Review* 71, 267-274.

Williamson, S. 2012. “Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach,” *American Economic Review* 102, 2570-2605.

Williamson, S. and Wright, R. 2010. “New Monetarist Economics: Methods.” *Federal Reserve Bank of St. Louis Review* 92(4): 265-302.