Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk*

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Abstract

Households face large income uncertainty that varies substantially over the business cycle. We examine the macroeconomic consequences of these variations in a model with incomplete markets, liquid and illiquid assets, and a nominal rigidity. Heightened uncertainty depresses aggregate demand as households respond by hoarding liquid "paper" assets for precautionary motives, thereby reducing both illiquid physical investment and consumption demand. This translates into output losses, which the central bank can only prevent by providing sufficient liquidity. We show that the welfare consequences of uncertainty shocks crucially depend on a household's asset position. Households with little human capital but high illiquid wealth lose the most from an uncertainty shock and gain the most from stabilization policy.

Keywords: Incomplete Markets, Nominal Rigidities, Uncertainty Shocks.

JEL-Codes: E22, E12, E32,

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1 Introduction

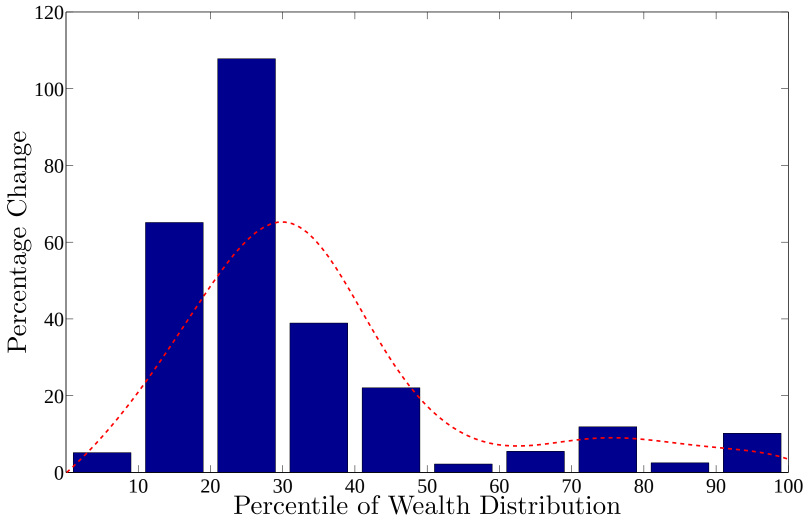
The Great Recession has brought about a reconsideration of the role of uncertainty in business cycles. Increased uncertainty has been documented and studied in various markets, but uncertainty with respect to household income stands out in its size and importance. Shocks to household income are persistent and their variance changes substantially over the business cycle. The seminal work by Storesletten et al. (2001) estimates that during an average NBER recession, income uncertainty faced by U.S. households, interpreted as the variance of persistent income shocks, is more than twice as large as in expansions.

These sizable swings in household income uncertainty lead to variations in the propensity to consume if asset markets are incomplete so that households use precautionary savings to smooth consumption. This paper quantifies the aggregate consequences of this precautionary savings channel of uncertainty shocks by means of a dynamic stochastic general equilibrium model. Since increases in precautionary savings will affect output negatively only if output depends on demand and if not all additional savings translate into investment, we model households to have access to two types of assets to smooth consumption. They can either hold money or invest in illiquid but dividend paying capital. We augment this incomplete markets framework in the tradition of Bewley (1980) by sticky prices à la Calvo (1983).

In this economy, when idiosyncratic income uncertainty increases, individually optimal asset holdings rise and consumption demand declines. Importantly, households also rebalance their portfolios toward the liquid asset because it provides better consumption smoothing. These effects are reminiscent of the observed patterns of the share of liquid assets in the portfolios of U.S. households during the Great Recession (see Figure 1). According to the 2010 *Survey of Consumer Finances*, the share of liquid assets in the portfolios increased relative to 2004 across all wealth percentiles, with the strongest relative increase for the lower middle-class. In our model, this portfolio rebalancing towards money implies that the decline in consumption will not be offset one-to-one by demand for goods through investment; on the contrary, it is reinforced by a decline in investment demand. Consequently, aggregate demand declines even more strongly than consumption demand.

This decline in total aggregate demand leads to falling prices. If prices were fully flexible, the drop in consumption demand could be offset by an increase in real balances through falling goods prices. This Pigou (1943) effect would suffice to bring the economy back to equilibrium. With sticky prices, however, not all firms are able to adjust their

Figure 1: Portfolio share of liquid assets by percentiles of wealth, 2010 vs. 2004



Notes: Portfolio share: Net liquid assets/Net total assets. Net liquid assets: cash, money market, checking, savings and call accounts, as well as government bonds and T-Bills net of credit card debt. Cash holdings are estimated by making use of the *Survey of Consumer Payment Choice* for 2008, as in Kaplan and Violante (2011). Households with negative net liquid or net illiquid wealth, as well as the top 5% by net worth, are excluded from the sample. The bar chart displays the average change in each wealth decile, and the dotted line an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

prices downward. As a result, the decline in demand leads to a decline in output.

Quantitatively, we find that following a two standard deviation increase in household income uncertainty, aggregate activity decreases by roughly 1.63% on impact and 1.44% over the first year under the assumption of a monetary policy that follows a constant nominal money growth rule (Friedman's "k% rule"). This is about the effect size that Fernández-Villaverde et al. (2011) report for a two standard deviation shock to fiscal policy uncertainty at the zero lower bound. The economy recovers only sluggishly after 21 quarters.

We argue that the liquidity of money relative to capital is key for the decline in aggregate demand. This result is independent of the degree of liquidity. When physical capital is more liquid, money and capital become more homogenous assets, and households hold less money. At the same time money demand becomes more elastic with respect to uncertainty. As a result, the disinflation needed to satisfy the excess demand for money remains largely unaffected.

We model the illiquidity of capital as infrequent participation in the capital market, where capital can only be traded from time to time. This can be considered as an approximation to a more complex trading friction as in Kaplan and Violante (2011), who follow the tradition of Baumol (1952) and Tobin (1956) in modeling the portfolio choice between liquid and illiquid assets.

Since the relative price of capital falls but the value of money increases upon an uncertainty shock, such a shock has rich distributional consequences. Our welfare calculations imply that households rich in physical or human capital lose the most, because factor returns fall in times of high uncertainty. In contrast, welfare losses decline in money holdings as their value appreciates. To understand the welfare consequences of systematic policy responses to uncertainty shocks, we compare a regime where monetary policy follows Friedman's k%-rule to one where monetary policy provides additional money to stabilize inflation. Since an uncertainty shock effectively works like a demand shock in our model, monetary policy is able to reduce the negative effects on output and alleviate welfare consequences. On average, households would be willing to forgo 0.65% of their consumption over the first 24 quarters to eliminate the uncertainty shock, but this number is reduced to 0.14% with stabilization. In the latter regime, households rich in human capital pay the cost of the stabilization policy, because they save (partly in money) and thereby finance the seigniorage. Moreover, without stabilization, these households profit from low prices of the illiquid asset in which they accumulate their long-term savings.

The remainder of the paper is organized as follows. Section 2 starts off with a review of the related literature. Section 3 develops our model, and Section 4 discusses the solution method. Section 5 introduces our estimation strategy for the income process and explains the calibration of the model. Section 6 presents the numerical results. Section 7 concludes. An Appendix follows that provides details on the numerics, the robustness checks, and the estimation of the uncertainty process from income data.

2 Related Literature

Our paper contributes to the recent literature that explores empirically and theoretically the aggregate effects of time-varying uncertainty. The seminal paper by Bloom (2009) discusses the effects of time-varying (idiosyncratic) productivity uncertainty on firms' factor demand, exploring the idea and effects of time-varying real option values of investment. This paper has triggered a stream of research that explores under which conditions such variations have aggregate effects.[1]

A more recent branch of this literature investigates the aggregate impact of uncertainty shocks beyond their transmission through investment and has also broadened the sources of uncertainty studied. The first papers in this vein highlight non-linearities in the New Keynesian model, in particular the role of precautionary price setting.[2] Fernández-Villaverde et al. (2011), for example, look at a medium-scale DSGE model à la Smets and Wouters (2007). They find that at the zero lower bound output drops by 1.7% on impact after a joint two standard deviation shock to the volatility of taxes on capital, labor, and consumption if countervailing fiscal policy response is ruled out.[3] In a similar framework, Basu and Bundick (2011) highlight the labor market response to uncertainty about aggregate TFP and time preferences. They argue that, if uncertainty increases, the representative household will want to save more and consume less. Then, with King et al. (1988) preferences, the representative household will also supply more labor, which in a New Keynesian model depresses output through the "paradox of toil." When labor supply increases, wages and hence marginal costs for firms fall. This increases markups when prices are sticky, which finally depresses demand for consumption and investment, and a recession follows. Overall, they find similar aggregate effects to Fernández-Villaverde et al. (2011), in particular at the zero-lower bound.

While our paper also focuses on precautionary savings, it differs substantially in the transmission channel. We are agnostic about the importance of the "paradox of toil," because it crucially relies on a wealth effect in labor supply. We therefore assume Greenwood et al. (1988) preferences to eliminate any direct impact of uncertainty on labor supply to isolate the demand channel of precautionary savings instead.[4] Moreover, since we focus on *idiosyncratic* income uncertainty, we can identify the uncertainty process outside the model from the Panel Study of Income Dynamics (PSID).

This focus on idiosyncratic uncertainty and the response of precautionary savings links our paper to Ravn and Sterk (2013) and Rendahl (2013). Both highlight the

[1] To name a few: Arellano et al. (2012), Bachmann and Bayer (2013), Christiano et al. (2010), Chugh (2012), Gilchrist et al. (2010), Narita (2011), Panousi and Papanikolaou (2012), Schaal (2011), and Vavra (2014) have studied the business cycle implications of a time-varying dispersion of firm-specific variables, often interpreted as and used to calibrate shocks to firm risk, propagated through various frictions: wait-and-see effects from capital adjustment frictions, financial frictions, search frictions in the labor market, nominal rigidities, and agency problems.

[2] With sticky prices, firms will target a higher markup the more uncertain future demand is.

[3] Born and Pfeifer (2011) report an output drop of 0.025% for a similar model and a similar joint policy risk shock under a slightly different calibration. Regarding TFP risk they hardly find any aggregate effect.

[4] Similarly, in a search model, higher uncertainty about match quality might translate into longer search and more endogenous separation. Thus it is not clear a priori whether labor supply would increase or decrease on impact.

importance of idiosyncratic unemployment risk. In their setups, households face unemployment risk in an incomplete markets model with labor market search and nominal frictions. Both papers differ in their asset market setup and the shocks considered. Ravn and Sterk (2013) look at a setup with government bonds as a means of savings. They then study a joint shock to job separations and the share of long-term unemployed. This increases income risk and hence depresses aggregate demand because of higher precautionary savings. They find that such first moment shocks to the labor market can be significantly propagated and amplified through this mechanism.

Rendahl (2013) consider a model with money and equity instead, where equity is not physical capital as in our model, but is equated with vacancy-ownership. In addition, they assume wage rigidity. When wages are sticky, precautionary money demand leads to deflation, pushing up real wages. Because the labor intensity of production cannot be adjusted, this immediately decreases the equity yield on existing and newly formed vacancies. This effect on equity returns induces portfolio adjustments by households. It increases the relative return of money thereby inducing a shift toward it, which amplifies the output drop. Our transmission mechanism shares this feature, but additionally highlights the importance of liquidity. Households increase their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset, because its services in consumption smoothing become more valuable to households. We find that the liquidity effect is more important than the relative return effect in our model where the labor intensity of production can be adjusted.

Finally, our work relates to Gornemann et al. (2012). We discuss the distributional consequences of uncertainty shocks and of systematic monetary policy response. We find that both differently affect households that differ in their portfolios due to differential price movements. This portfolio composition aspect is new in comparison to Gornemann et al., because we introduce decisions regarding nominal versus real asset holdings to the household's problem.

3 Model

We model an economy inhabited by two types of agents: (worker-)households and entrepreneurs. Households supply capital and labor and are subject to idiosyncratic shocks to their labor productivity. These shocks are persistent and have a time-varying variance. Households self-insure in a liquid nominal asset (money) and a less liquid physical asset (capital). Liquidity of money is understood in the spirit of Kaplan and Violante's (2011) model of wealthy hand-to-mouth consumers, where households hold capital, but trading capital is subject to a friction. We model this trading friction as limited participation in the asset market. Every period, a fraction of households is randomly selected to trade physical capital. All other households may only adjust their money holdings.[5] While money is subject to an inflation tax and pays no dividend, capital can be rented out to the intermediate-good-producing sector on a perfectly competitive rental market. This sector combines labor and capital services into intermediate goods and sells them to the entrepreneurs.

Entrepreneurs capture all pure rents in the economy. For simplicity, we assume that entrepreneurs are risk neutral. They obtain rents from adjusting the aggregate capital stock due to convex capital adjustment costs and, more importantly, from differentiating the intermediate good. Facing monopolistic competition, they set prices above marginal costs for these differentiated goods. Price setting, however, is subject to a pricing friction à la Calvo (1983) so that entrepreneurs may only adjust their prices with some positive probability each period. The differentiated goods are finally bundled again to the composite final good used for consumption and investment.

The model is closed by a monetary authority that provides money in positive net supply and adjusts money growth according to the prescriptions of a Taylor type rule, which reacts to inflation deviations from target. All seigniorage is wasted.

3.1 Households

There is a continuum of ex-ante identical households of measure one indexed by i. Households are infinitely lived, have time-separable preferences with time-discount factor β, and derive felicity from consumption c_{it} and leisure. They obtain income from supplying labor and from renting out capital. A household's labor income $w_t h_{it} n_{it}$ is composed of the wage rate, w_t, hours worked, n_{it}, and idiosyncratic labor productivity, h_{it}, which evolves according to the following AR(1)-process:

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N\left(0, \sigma_{ht}\right). \tag{1}$$

Households have Greenwood-Hercowitz-Huffman (GHH) preferences and maximize the discounted sum of felicity:

$$V = E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u\left(c_{it} - h_{it} G(n_{it})\right). \tag{2}$$

[5] We choose to exclude trading as a choice, and hence we use a simplified framework relative to Kaplan and Violante (2011) for numerical tractability. Random participation keeps the households' value function concave, thus making first-order conditions sufficient, and therefore allows us to use a variant of the endogenous grid method as an algorithm for our numerical calculations. See Appendix A for details.

The felicity function takes constant relative risk aversion (CRRA) form with risk aversion ξ:

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \quad \xi > 0,$$

where $x_{it} = c_{it} - h_{it}G(n_{it})$ is household i's composite demand for the bundled physical consumption good c_{it} and leisure. The former is obtained from bundling varieties j of differentiated consumption goods according to a Dixit-Stiglitz aggregator:

$$c_{it} = \left(\int c_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}.$$

Each of these differentiated goods is offered at price p_{jt} so that the demand for each of the varieties is given by

$$c_{ijt} = \left(\frac{p_{jt}}{P_t} \right)^{-\eta} c_{it},$$

where $P_t = \left(\int p_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$ is the average price level.

The disutility of work, $h_{it}G(n_{it})$, determines a household's labor supply given the aggregate wage rate through the first-order condition:

$$h_{it}G'(n_{it}) = w_t h_{it}. \tag{3}$$

We weight the disutility of work by h_{it} to eliminate any Hartman-Abel effects of uncertainty on labor supply. Under the above assumption, a household's labor decision does not respond to idiosyncratic productivity h_{it}, but only to the aggregate wage w_t. Thus we can drop the household-specific index i, and set $n_{it} = N_t$. Scaling the disutilty of working by h_{it} effectively sets the micro elasticity of labor supply to zero. Therefore, it simplifies the calibration as we can calibrate the model to the income risk that households face without the need to back out the actual productivity shocks. What is more, without this assumption, higher *realized* uncertainty leads to higher productivity inequality and hence increases aggregate labor supply.[6]

We assume a constant Frisch elasticity of aggregate labor supply with γ being the inverse elasticity:

$$G(N_t) = \frac{1}{1+\gamma} N_t^{1+\gamma}, \quad \gamma > 0,$$

and use this to simplify the expression for the composite consumption good x_{it}. Exploit-

[6]Without the assumption, n_{it} would be increasing in h_{it} and hence the aggregate effective labor supply, $\int h_{it}n_{it}di$, would increase when the dispersion of h_{it} increases. While it would not change the household's problem in its asset choices and the choice of x_{it}, it would complicate aggregation.

ing the first-order condition on labor supply, the disutility of working can be expressed in terms of the wage rate:

$$h_{it}G(N_t) = h_{it}\frac{N_t^{1+\gamma}}{1+\gamma} = \frac{h_{it}G'(N_t)N_t}{1+\gamma} = \frac{w_t h_{it} N_t}{1+\gamma}.$$

In this way the demand for x_{it} can be rewritten as:

$$x_{it} = c_{it} - h_{it}G(N_t) = c_{it} - \frac{w_t h_{it} N_t}{1+\gamma}.$$

Total labor input supplied is given by

$$\tilde{N}_t = N_t \int h_{it} di.$$

Following the literature on idiosyncratic income risk, we assume that asset markets are incomplete. Households can only trade in nominal money, $\tilde{m}_{it}$, that does not bear any interest and in capital, k_{it}, to smooth their consumption. Holdings of both assets have to be non-negative. Moreover, trading capital is subject to a friction.

This trading friction allows only a randomly selected fraction of households, ν, to participate in the asset market for capital every period. Only these households can freely rebalance their portfolios. All other households obtain dividends, but may only adjust their money holdings. For those households participating in the capital market, the budget constraint reads:

$$c_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + w_t h_{it} N_t, \quad m_{it+1}, k_{it+1} \geq 0,$$

where m_{it} is real money holdings, k_{it} is capital holdings, q_t is the price of capital, r_t is the rental rate or "dividend," and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. We denote real money holdings of household i at the end of period t by $m_{it+1} := \frac{\tilde{m}_{it+1}}{P_t}$.
Substituting the expression $c_{it} = x_{it} + \frac{w_t h_{it} N_t}{1+\gamma}$ for consumption, we obtain:

$$x_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + \frac{\gamma}{1+\gamma} w_t h_{it} N_t, \quad m_{it+1}, k_{it+1} \geq 0. \tag{4}$$

For those households that cannot trade in the market for capital the budget constraint simplifies to:

$$x_{it} + m_{it+1} = \frac{m_{it}}{\pi_t} + r_t k_{it} + \frac{\gamma}{1+\gamma} w_t h_{it} N_t, \quad m_{it} \geq 0. \tag{5}$$

Note that we assume that depreciation of capital is replaced through maintenance such that the dividend, r_t, is the net return on capital.

Since a household's saving decision will be some non-linear function of that household's wealth and productivity, the price level, P_t, and accordingly aggregate real money, $M_{t+1} = \frac{\tilde{M}_{t+1}}{P_t}$, will be functions of the joint distribution Θ_t of (m_t, k_t, h_t). This makes Θ_t a state variable of the household's planning problem. This distribution evolves as a result of the economy's reaction to shocks to uncertainty that we model as time variations in the variance of idiosyncratic income shocks, σ^2_{ht}. This variance follows a stochastic volatility process, which allows us to separate shocks to the variance from shocks to the level of household income.

$$\sigma^2_{ht} = \bar{\sigma}^2 \exp(s_t), \quad s_t = \rho_s s_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N\left(-\frac{\sigma^2_s}{2(1-\rho^2_s)}, \sigma_s\right), \tag{6}$$

where $\bar{\sigma}^2$ is the steady state labor risk that households face, and s shifts this risk. Shocks ε_t to income risk are the only aggregate shocks in our model.

With this setup, the dynamic planning problem of a household is then characterized by two Bellman equations: V_a in the case where the household can adjust its capital holdings and V_n otherwise:

$$\begin{aligned}
V_a(m,k,h;\Theta,s) =& max_{k',m'_a} u[x(m,m'_a,k,k',h)] \\
&+ \beta\left[\nu EV^a(m'_a,k',h',\Theta',s') + (1-\nu)EV^n(m'_a,k',h',\Theta',s')\right] \\
V_n(m,k,h;\Theta,s) =& max_{m'_n} u[x(m,m'_n,k,k,h)] \\
&+ \beta\left[\nu EV^a(m'_n,k,h',\Theta',s') + (1-\nu)EV^n(m'_n,k,h',\Theta',s')\right]
\end{aligned} \tag{7}$$

In line with this notation, we define the optimal consumption policies for the adjustment and non-adjustment cases as x^*_a and x^*_n, the money holding policies as m^*_a and m^*_n, and the capital investment policy as k^*. Details on the properties of the value functions (smooth and concave) and policy functions (differentiable and increasing in total resources), the first-order conditions, and the algorithm we employ to calculate the policy functions can be found in Appendix A.

3.2 Intermediate Goods Producers

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = \tilde{N}^\alpha_t K^{(1-\alpha)}_t.$$

Let MC_t be the relative price at which the intermediate good is sold to entrepreneurs.

The intermediate-good producer maximizes profits,

$$MC_tY_t = MC_t\tilde{N}_t^{\alpha}K_t^{(1-\alpha)} - w_t\tilde{N}_t - (r_t+\delta)K_t,$$

but it operates in perfectly competitive markets, such that the real wage and the user costs of capital are given by the marginal products of labor and capital:

$$w_t = \alpha MC_t\left(K_t/\tilde{N}_t\right)^{1-\alpha} \tag{8}$$

$$r_t+\delta = (1-\alpha)MC_t\left(\tilde{N}_t/K_t\right)^{\alpha} \tag{9}$$

3.3 Entrepreneurs

Entrepreneurs differentiate the intermediate good and set prices. They are risk neutral and have the same discount factor as households. We assume that only the central bank can issue money so that entrepreneurs participate in neither the money nor the capital market. This assumption gives us tractability in the sense that it separates the entrepreneurs' price setting problem from the households' saving problem. It enables us to determine the price setting of entrepreneurs without having to take into account households' intertemporal decision making. Under these assumptions, the consumption of entrepreneur j equals her current profits, Π_{jt}. By setting the prices of final goods, entrepreneurs maximize expected discounted future profits:

$$E_0\sum_{t=0}^{\infty}\beta^t\Pi_{jt}. \tag{10}$$

Entrepreneurs buy the intermediate good at a price equalling the nominal marginal costs, MC_tP_t, where MC_t is the real marginal costs at which the intermediate good is traded due to perfect competition, and then differentiate them without the need of additional input factors. The goods that entrepreneurs produce come in varieties uniformly distributed on the unit interval and each indexed by $j\in[0,1]$. Entrepreneurs are monopolistic competitors, and hence charge a markup over their marginal costs. They are, however, subject to a Calvo (1983) price setting friction, and can only update their prices with probability θ. They maximize the expected value of future discounted profits by setting today's price, p_{jt}, taking into account the price setting friction:

$$\max_{\{p_{jt}\}}\sum_{s=0}^{\infty}(\theta\beta)^sE\Pi_{jt,t+s} = \sum_{s=0}^{\infty}(\theta\beta)^sEY_{jt,t+s}(p_{jt}-MC_{t+s}P_{t+s}) \tag{11}$$

$$s.t.: Y_{jt,t+s} = \left(\frac{p_{jt}}{P_{t+s}}\right)^{-\eta} Y_{t+s},$$

where $\Pi_{jt,t+s}$ is the profits and $Y_{jt,t+s}$ is the production level in $t+s$ of a firm j that set prices in t.

We obtain the following first-order condition with respect to p_{jt}:

$$\sum_{s=0}^{\infty} (\theta\beta)^s E Y_{jt,t+s} \left(\frac{p^*_{jt}}{P_{t-1}} - \underbrace{\frac{\eta}{\eta-1}}_{\mu} MC_{t+s} \frac{P_{t+s}}{P_{t-1}} \right) = 0, \tag{12}$$

where μ is the static optimal markup.

Recall that entrepreneurs are risk neutral and that they do not interact with households in any intertemporal trades. Moreover, *aggregate* shocks to the economy are small and homoscedastic, since the only aggregate shock we consider is the shock to the variance of housdehold income shocks. Therefore, we can solve the entrepreneurs' planning problem locally by log-linearizing around the zero inflation steady state, without having to know the solution of the households' problem. This yields, after some tedious algebra (see, e.g., Galí (2008)), the New Keynesian Phillips curve:

$$\log \pi_t = \beta E_t(\log \pi_{t+1}) + \kappa(\log MC_t + \mu), \tag{13}$$

where

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

We assume that besides differentiating goods and obtaining a rent from the markup they charge, entrepreneurs also obtain and consume rents from adjusting the aggregate capital stock. Since the dividend yield is below their time-preference rate, in equilibrium entrepreneurs never hold capital. The cost of adjusting the stock of capital is $\frac{\phi}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t + \Delta K_{t+1}$. Hence, entrepreneurs will adjust the stock of capital until the following first-order condition holds:[7]

$$q_t = 1 + \phi \frac{\Delta K_{t+1}}{K_t}. \tag{14}$$

[7] Note that we assume capital adjustment costs only on new capital (or on the active destruction of old capital) but not on the replacement of depreciation. Depreciated capital is assumed to be replaced at the cost of one-to-one in consumption goods, and replacement is forced before the capital stock is adjusted at a cost. This differential treatment of depreciation and net investment simplifies the equilibrium conditions substantially, because the user cost of capital and hence the dividend paid to households do not depend on the next period's stock of capital, and the decisions of non-adjusters are not influenced by the price of capital q_t.

3.4 Goods, Money, Asset, and Labor Market Clearing

The labor market clears at the competitive wage given in (8); so does the market for capital services if (9) holds. We assume that the money supply is given by a monetary policy rule that adjusts the growth rate of money in order to stabilize inflation:

$$\frac{M_{t+1}}{M_t} = (\theta_1/\pi_t)^{1+\theta_2}. \tag{15}$$

Here M_{t+1} is the real balances at the end of period t (with the timing aligned to our notation for the households' budget constraint). The coefficient $\theta_1 \geq 1$ determines steady-state inflation, and $\theta_2 \geq 0$ the extent to which the central bank attempts to stabilize inflation around its steady-state value: the larger θ_2 the stronger is the reaction of the central bank to deviations from the inflation target. When $\theta_2 \to \infty$ inflation is perfectly stabilized at its steady-state value. We assume that the central bank wastes any seigniorage buying final goods and choose the above functional form for its simplicity.[8]

The money market clears whenever the following equation holds:

$$(\theta_1/\pi_t)^{1+\theta_2} M_t = \int [\nu m_a^*(m,k,h;q_t,\pi_t) + (1-\nu) m_n^*(m,k,h;q_t,\pi_t)]\, \Theta_t(m,k,h) dmdkdh, \tag{16}$$

with last end-of-period real money holdings given by

$$M_t := \int m_t \Theta_t(m,h) dmdh.$$

Last, the market for capital has to clear:

$$\begin{aligned} q_t &= 1 + \phi \frac{K_{t+1} - K_t}{K_t} = 1 + \nu\phi \frac{K_{t+1}^* - K_t}{K_t}, \qquad (17) \\ K_{t+1}^* &:= \int k^*(m,k,h;q_t,\pi_t) \Theta_t(m,k,h) dmdkdh, \\ K_{t+1} &= K_t + \nu(K_{t+1}^* - K_t), \end{aligned}$$

where the first equation stems from competition in the production of capital goods, the

[8] For the baseline calibration this is an innocuous assumption. With constant nominal money growth, the changes in seigniorage are negligible in absolute terms. Steady-state seigniorage is 1% of annual output, since money growth is 2% and the money-to-output ratio is 50%. When inflation drops, say, from 2% to 0, the real value of seigniorage increases, but only from .98% to 1% of output. As $\theta_2 \to \infty$, seigniorage occasionally turns negative. It is numerically very expensive to put a constraint on M_t, and hence we abstain from doing so to keep the dynamic problem tractable. This unboundedness of seigniorage only affects the effectiveness of the stabilization policy. The central bank can commit to decrease seigniorage more in the future without the requirement of (weakly) positive seigniorage.

second equation defines the aggregate supply of funds from households trading capital, and the third equation defines the law of motion of aggregate capital. The goods market then clears due to Walras' law, whenever both money and capital markets clear.

3.5 Recursive Equilibrium

A *recursive equilibrium* in our model is a set of policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r, w, \pi, q\}$, aggregate capital and labor supply functions $\{N, K\}$, distributions Θ_t over individual asset holdings and productivity, and a perceived law of motion Γ, such that

1. Given $\{V_a, V_n\}$, Γ, prices, and distributions, the policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$ solve the households' planning problem, and given the policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equations (7).

2. The labor, the final-goods, the money, the capital, and the intermediate-good markets clear, i.e., (8), (13), (16), and (17) hold.

3. The actual law of motion and the perceived law of motion Γ coincide, i.e., $\Theta' = \Gamma(\Theta, s')$.

4 Numerical Implementation

The dynamic program (7) and hence the recursive equilibrium is, of course, not computable, because it involves the infinite dimensional object Θ_t.

4.1 Krusell-Smith Equilibrium

To turn this problem into a computable one, we assume that households predict future prices only on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998). Specifically, we make the assumption that households condition their expectations only on last period's aggregate real money holdings, M_t, the aggregate stock of capital, K_t, and the uncertainty state, s_t. The reasoning behind this choice goes as follows: (16) determines inflation, which in turn depends on the current money stock. Once inflation is fixed, the Phillips curve (13) determines markups and hence wages and dividends. These will pin down asset prices by making the marginal investor indifferent between money and physical capital. If asset-demand functions, $m_{a,n}^*$ and k^*, are sufficiently close to linear in human capital, h, and in non-human wealth, m, k, at the mass of Θ_t, we can expect approximate aggregation to hold. For our exercise, the three aggregate states – s_t, M_t, and K_t – are sufficient to describe the evolution of the aggregate economy.

While the law of motion for s_t is pinned down by (6), households use the following log-linear forecasting rules for current inflation and the price of capital, where the coefficients depend on the uncertainty state:

$$\log \pi_t = \beta_\pi^1(s_t) + \beta_\pi^2(s_t)\log M_t + \beta_\pi^3(s_t)\log K_t \tag{18}$$

$$\log q_t = \beta_q^1(s_t) + \beta_q^2(s_t)\log M_t + \beta_q^3(s_t)\log K_t. \tag{19}$$

The law of motion for real money holdings, M_t, then follows from the monetary policy rule and is given by:

$$\log M_{t+1} = \log M_t + (1+\theta_2)(\log\theta_1 - \log\pi_t).$$

The law of motion for K_t results from (17).

Fluctuations in q and π happen for two reasons: As uncertainty goes up, the self-insurance service that households receive from the illiquid capital good decreases. In addition, the rental rate of capital falls as firms' markups increase. When making their investment decisions, households need to predict the next period's capital price q' to determine the expected return on their investment. Since all other prices are known functions of the markup, only π' and q' need to be predicted.

Technically, finding the equilibrium is similar to Krusell and Smith (1997), as we need to find market clearing prices within each period. Concretely, this means the posited rules, (18) and (19), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate n independent sequences of economies for $t = 1, \ldots, T$ periods, keeping track of the actual distribution Θ_t. In each simulation the sequence of distributions starts from the stationary distribution implied by our model without aggregate risk. We then calculate in each period t the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rule (18) and (19) from period $t+1$ onward. Having determined the market clearing prices, we obtain the next period's distribution Θ_{t+1}. In doing so, we obtain n sequences of equilibria. The first 500 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (18) and (19) from the simulated data and update the parameters accordingly. By using $n = 10$ and $T = 1500$, it is possible to make use of parallel computing resources and obtain 10.000 equilibrium observations. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (18) and (19) approximate the aggregate behavior of the economy fairly well. The minimal within sample R^2 is above 99%. Also the out-of-sample performance (see Den Haan (2010)) of the forecasting rules is good. See Appendix E.

4.2 Solving the Household Planning Problem

In solving for the households' policy functions we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. We approximate the idiosyncratic productivity process by a discrete Markov chain with 11 states and time-varying transition probabilities, using the method proposed by Tauchen (1986). The stochastic volatility process is approximated in the same vein using 5 states.[9] Details on the algorithm can be found in Appendix A.4.

5 Calibration

We calibrate the model to the U.S. economy. Where possible we identify parameters from the behavior of the model in steady state without fluctuations in uncertainty. We check whether the time-averages of the simulated variables in the model with uncertainty shocks are close to their steady-state values and find only negligible differences. The aggregate data used for calibration spans 1980 to 2006. One period in the model refers to a quarter of a year. The choice of parameters as summarized in Tables 1 and 2 is explained next. We present the parameters as if they were individually changed in order to match a specific data moment, but all calibrated parameters are determined jointly of course.

5.1 Income Process

We estimate the income process and hence uncertainty faced by households from income data in the Cross-National Equivalent File (CNEF) of the Panel Study of Income Dynamics (PSID), excluding the low-income sample. We construct household income as pre-tax labor income plus private and public transfers minus all taxes (based on TAXSIM), and control for observable household characteristics in a first stage regression. We use the residual income to estimate the parameters governing the idiosyncratic income process

[9] We solve the household policies for 50 points on the grid for money and 80 points on the grid for capital using equi-distant grids on log scale. For aggregate money and capital holdings we use a relatively coarse grid of 5 points each. We experimented with changing the number of gridpoints without a noticeable impact on results.

Table 1: Estimated parameters of the income process

Parameter	Value	Description
ρ_h	0.94	Persistence of income
$\bar{\sigma}$	0.24	Average STD of innovations to income
ρ_s	0.93	Persistence of the income-innovation variance, σ_h^2
σ_s	0.19	Conditional STD (log scale) of σ_h^2

Notes: All values are adapted to the quarterly frequency of the model. For details on the estimation see Appendix B.

– $\rho_s, \rho_h, \bar{\sigma}$, and σ_s – with a Kalman filter. Details on data selection and the estimation procedure can be found in Appendix B.

In a first stage regression for log-income, we control for the effects of age (non-parametrically), household size, and educational attainment interacted with up to squared-order terms in age. We then generate income growth variances by age groups for the years 1970-2009 from these filtered data. Based on these age-year variances, the parameters of interests are estimated by Bayesian estimation using a Kalman filter. The priors for this estimation correspond to the estimates by Storesletten et al. (2004) for $\rho_h, \bar{\sigma}$, and σ_s, but are flat for the remaining parameters for which the literature does not provide any guidance. We find the autocorrelation of the persistent component of quarterly earnings, ρ_h, to be around 0.94 and an average standard deviation of quarterly persistent earnings shocks of $\bar{\sigma} = 0.24$. The persistence of shocks to income risk, ρ_s, is relatively high with an quarterly autocorrelation of 0.93. The annual coefficient of variation for income risk, $\frac{\sigma_s}{\bar{\sigma}}$, is 0.47 and hence comes close to a doubling of the variance in recessions, as estimated in Storesletten et al.[10] Table 1 summarizes the parameter estimates, where the values are adapted to the quarterly frequency of our model.

[10] Storesletten et al. estimate the variance of persistent shocks to annual income to be 126% higher in times of below average GDP growth than in times of above average GDP growth. This implies that the unconditional *annual* coefficient of variation of s is roughly 0.5.

5.2 Preferences and Technology

While we can estimate the income process directly from the data, all other parameters are calibrated within the model. Table 2 summarizes our calibration. In detail, we choose the parameter values as follows.

5.2.1 Households

For the felicity function, $u = \frac{1}{1-\xi}x^{1-\xi}$, we set the coefficient of relative risk aversion $\xi = 4$, as in Kaplan and Violante (2011). The time-discount factor, β, and the asset market participation frequency, ν, are jointly calibrated to match the ratios of liquid and illiquid assets to output. We equate illiquid assets to all capital goods at current replacement values relative to nominal GDP, and thus take a standard annual capital-to-output ratio of 283%. In our baseline calibration, this implies an annual real return for illiquid assets of 3.9%. We equate liquid assets to claims of the private sector against the government and not to inside money, because the net value of inside claims does not change with inflation. Specifically, we look at average U.S. federal debt for the years 1980 to 2006 held by the private sector. This yields an annual money-to-output ratio of 33%. For details on the steady-state asset distribution, see Appendix C. The calibrated participation frequency $\nu = 5.5\%$ is close to Kaplan and Violante's estimate for working households in their state-dependent participation framework. We take a standard value in the New Keynesian literature for the Frisch elasticity of labor supply, $\gamma = 0.5$. We provide a robustness check with a more conservative estimate of the Frisch elasticity of labor supply, $\gamma = 2$, which follows the estimates by microeconometric studies.

5.2.2 Intermediate, Final, and Capital Goods Producers

We parameterize the production function of the intermediate good producer according to the U.S. National Income and Product Accounts (NIPA). In the U.S. economy the income share of labor is about 2/3. Accounting for profits we hence set $\alpha = 0.715$.

To calibrate the parameters of the entrepreneurs' problem, we use standard values for markup and price stickiness that are widely employed in the New Keynesian literature. The Phillips curve parameter κ implies an average price duration of 4 quarters, assuming flexible capital at the firm level. The steady-state marginal costs, $exp(-\mu) = 0.91$, imply a markup of 10%. For simplicity, we set the entrepreneurs' discount factor equal to the households' discount factor.

We calibrate the adjustment cost of capital, $\phi = 25$, to match an investment to output volatility of 3.

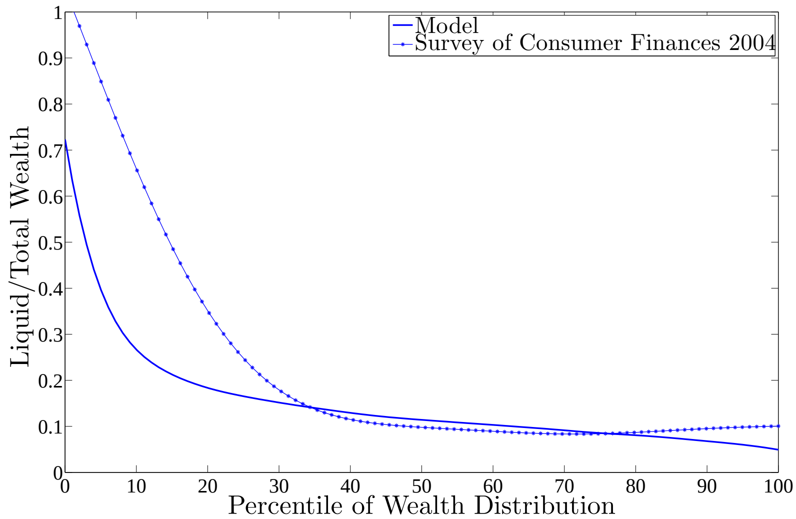
Table 2: Calibrated parameters

Parameter	Value	Description	Target
Households			
β	0.981	Discount factor	$K/Y = 283\%$ (annual)
ν	5.5%	Participation frequency	$M/Y = 33\%$ (annual)
ξ	4	Coefficient of rel. risk av.	Standard value
γ	0.5	Inverse of Frisch elasticity	Standard value
Intermediate Goods			
α	0.715	Share of labor	Income share of labor of 0.65
δ	1.35%	Depreciation rate	NIPA: Fixed assets
Final Goods			
κ	0.09	Price stickiness	Mean price duration of 4 quarters
μ	0.1	Markup	10% markup (standard value)
Capital Goods			
ϕ	25	Capital adjustment costs	Relative investment volatility of 3
Monetary Policy			
θ_1	1.005	Money growth	2% p.a.
θ_2	0	Inflation stabilization	No stabilization
or:	10^6		Perfect stabilization

5.2.3 Central Bank

We set the average growth rate of money, θ_1, such that our model produces an average annual inflation rate of 2%, in line with the usual inflation targets of central banks and roughly equal to average inflation in the U.S. between 1980 and 2012. We do not have a good estimate for the reaction of the money supply to inflation, θ_2, and hence set it either to zero, i.e., the central bank follows Friedman's $k\%$ rule, for inactive policy or to 10^6 to capture a central bank policy with strong inflation stabilization.

Figure 2: Share of liquid assets in total net worth against percentiles of wealth



Notes: For graphical illustration we make use of an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15. For the definition of net liquid assets see Figure 1.

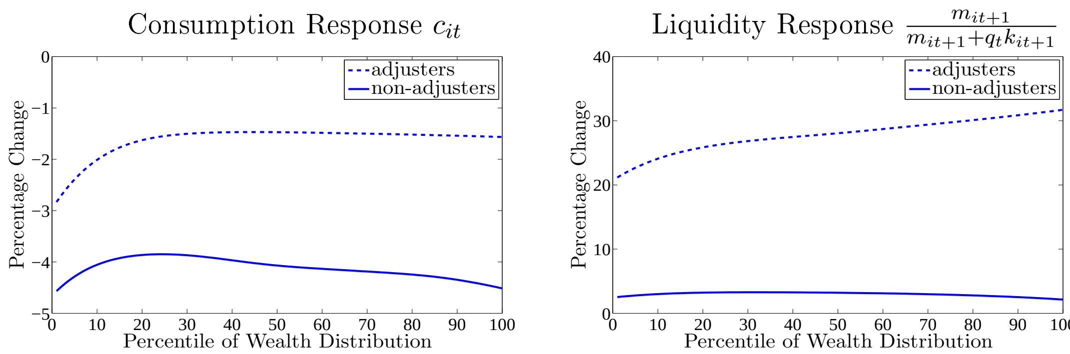
6 Quantitative Results

6.1 Household Portfolios and the Individual Response to Uncertainty

In our model, households hold money because it provides better short-term consumption smoothing than capital, as the latter can only be traded infrequently. Of course, this value of liquidity decreases in the amount of money a household holds, because a household rich in liquid assets will likely be able to tap into its illiquid wealth before running down all liquid wealth. For this reason, richer households, who typically hold both more money and more capital, hold less liquid portfolios. The poorest households, on the contrary, hold almost all their wealth in the liquid asset. This holds true in the actual data as well as in our model. While our model matches relatively well the shape of the actual liquidity share of household portfolios at all wealth percentiles, it underestimates the share of liquid assets for the lowest deciles; see Figure 2, which compares our model to the *Survey of Consumer Finances* 2004.

So what happens to total savings and its composition when uncertainty increases? In response to the increase in income uncertainty, households aim for higher precautionary savings to be in a better position to smooth their consumption. Since the liquid asset

Figure 3: Partial equilibrium response – Change in individual policy upon an uncertainty shock keeping prices and expectations constant at steady-state values

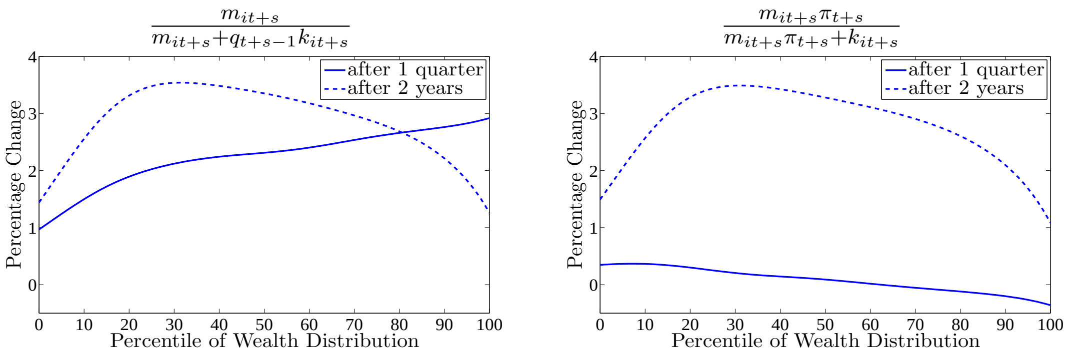


Notes: Reaction of individual consumption demand and portfolio choice of adjusters and non-adjusters at constant prices and price expectations relative to the policy at average uncertainty. The policies are averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. High uncertainty corresponds to the highest uncertainty gridpoint we consider, which is equal to an 85% increase in uncertainty. As with the data, we use an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

is better suited to this purpose, households first increase their demand for this asset – in fact, they even reduce holdings of the illiquid asset to increase the liquidity of their portfolio. Figure 3 shows how households' portfolio composition and consumption policy react to an increase in uncertainty without imposing any market clearing. The figure displays the relative change in the respective policy compared to the average uncertainty state. For this exercise, we evaluate households' consumption policies and the portfolio choice of adjusters and non-adjusters moving to the highest uncertainty state. We compute the policies under the expectation that all prices are at their steady-state values. Hence, we alter only income uncertainty. Across all wealth levels, households wish to increase their savings (i.e., decrease their consumption) as well as the liquidity of their portfolios when uncertainty goes up. Adjusters can do so by tipping into their capital account and thus their consumption falls less. This flight to liquidity leads to falling demand for capital even though total savings increase.

The change in the liquidity of household portfolios in general equilibrium is displayed in Figure 4; the left-hand panel shows the change in value terms; the right-hand panel shows the change in quantities, i.e., at constant prices. Portfolio liquidity initially increases at all wealth levels – in particular in value terms because the price of illiquid assets

Figure 4: General equilibrium response – Change in the liquidity of household portfolios



Notes: Change in the distribution of liquidity at all percentiles of the wealth distribution at equilibrium prices and price expectations $s = 1$ and 8 quarters after a two standard deviation shock to income uncertainty. The liquidity of the portfolios is averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. The left-hand panel shows the change including changes in prices; the right-hand panel shows the pure quantity responses. As with the data, we use an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

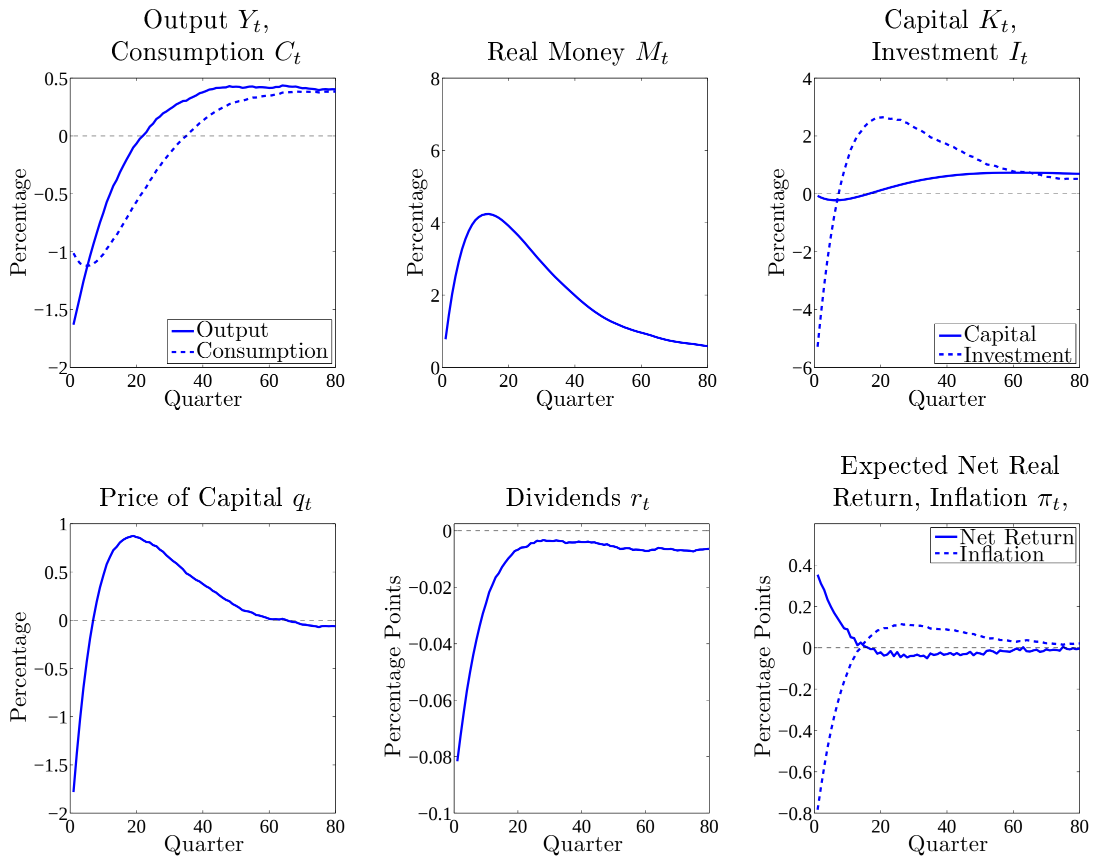
drops sharply as we will see in the next section. The increase in the share of liquid assets is overall smaller than in Figure 3 and least pronounced for the poorest, because of the negative income effect. After two years, the increase in liquidity is concentrated at households somewhat below median wealth. By then, rich households aiming at lower liquidity shares have had enough time to save in the illiquid asset, exploiting their lower prices. Interestingly, this picture is exactly what we found in Figure 1, where the strongest increase in the liquidity of the portfolios is for the lower middle class. Only the magnitude of changes in the liquidity of household portfolios during the Great Recession is much more dramatic.

6.2 Aggregate Consequences of Uncertainty Shocks

6.2.1 Main Findings

This simultaneous decrease in the demand for consumption and capital upon an increase in uncertainty leads to a decline in output. Figure 5 displays the impulse responses of output and its components, real balances and the capital stock as well as asset prices and returns for our baseline calibration. The assumed monetary policy follows a strict money growth rule, i.e., it is not responsive to inflation. After a two standard deviation

Figure 5: Uncertainty shock under constant money growth



Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$
Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over the response of 10.000 runs of the model. All rates (inflation, dividends, etc.) are *not* annualized.

increase in the variance of idiosyncratic productivity shocks, output drops on impact by 1.63% and only returns to the normal growth path after roughly 21 quarters. Over the first year the output drop is 1.44% on average.

The output drop in our model results from households increasing their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset. In times of high uncertainty, households dislike illiquid assets because of their limited use for short-run consumption smoothing. Conversely, the price of capital decreases on impact by more than 1.5%. Since the demand for the liquid asset is a demand for paper and not for (investment) goods, demand for both consumption and investment goods falls.

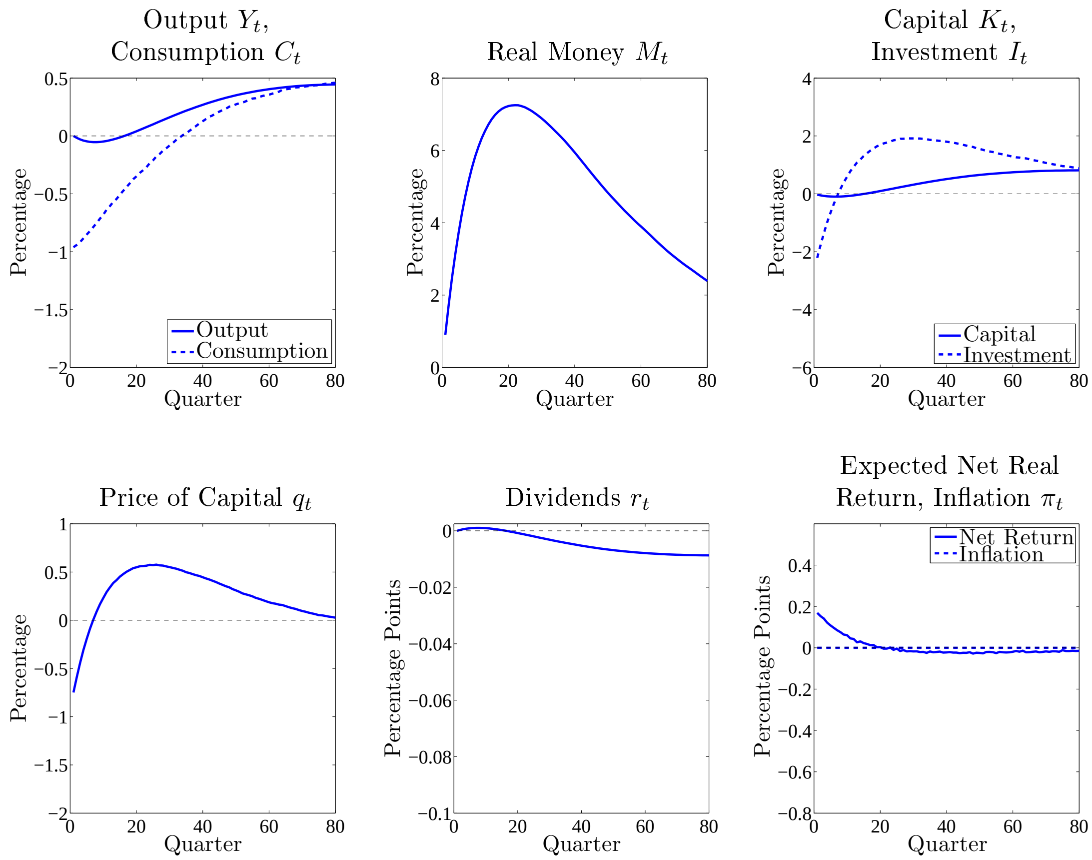
This decrease in demand puts pressure on prices. Inflation falls by about 80 basis points on impact, increasing the average markup in the economy. Thus, the marginal return on capital, r_t, and consequently investment demand decline. What is more, lower inflation also reduces the tax on money. The disinflation ensuing from the flight to liquidity therefore increases the relative return of money, further amplifying the portfolio adjustment. Interestingly, uncertainty shocks move capital prices and expected returns much more (and in the opposite direction) than they move dividends (35 vs. -8 basis points, quarterly).

6.2.2 Stabilization Policy

How much of this is driven by the increased value of liquidity, and how much by the differential impact of disinflation on the return of money and on dividends? We can isolate the liquidity effect from the relative-return effect when we look at a monetary policy that is stabilizing the economy. Under this policy inflation is fixed and output barely moves. Also dividends are virtually constant. Thus, the relative-return effect vanishes in the case of strict inflation targeting. In other words, this setup identifies the partial equilibrium response in the model as all prices (except for capital prices) are basically kept fixed. The corresponding impulse responses are displayed in Figure 6. As a consequence of the stabilization, the price of capital falls less, but it still falls by about 0.75%. The expected return on capital increases by about 20 basis points. The total income of households almost stays constant in the first 5 years and hence money demand peaks at an even higher level than without stabilization.

In other words, the portfolio adjustment is to a large extent driven by the liquidity effect. After roughly 5 years, real balances have increased to a point where households are well insured and want to increase their holdings of the illiquid asset again. Moreover, as liquid wealth has become abundant, households expect higher inflation in the future.

Figure 6: Uncertainty shock under inflation targeting



Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$
Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over the response of 10.000 runs of the model. All rates (inflation, dividends, etc.) are *not* annualized.

Hence, money becomes an unappealing asset, and the portfolio adjustment reverses.

How does the central bank achieve this increase in household wealth in real terms? Of course, one element in this is that money per se is more valuable for consumption smoothing in times of high uncertainty, but under the stabilization policy, it is the additional commitment of the central bank to *lower* seigniorage in the future, when uncertainty decreases again. This increases the real value of the money stock in the model. In this sense, monetary policy in our model has a strong fiscal policy dimension.

6.2.3 How Important Is the (Il)liquidity of Capital?

Our calibration suggests that households can adjust their capital holdings on average every 18 quarters. This restricted access to savings in capital limits its use for short-run consumption smoothing considerably. If capital were easier to access, it would become more and more of a substitute for money in terms of its use for consumption smoothing. Hence, aggregate money holdings decline as ν increases. Figure 7 plots the impulse responses for an average adjustment frequency of once a year (i.e., $\nu = 20\%$). In this case money holdings are only 15% of annual output in the steady state, which is close to a calibration to the monetary base.[11]

Figure 7 (a) shows that the output drop is larger with a higher portfolio adjustment frequency, although the share of money in the economy is significantly smaller and capital is very liquid in comparison to the baseline calibration. Households increase their money holdings slightly faster and average holdings peak at a somewhat higher level, while investment in capital falls twice as much. Hence, the drop in inflation is more pronounced. Money demand reacts more elastically to uncertainty as more households are able to adjust their portfolio. Consequently, the flight to liquidity is stronger and happens faster than before – in the build-up and in the reverse.

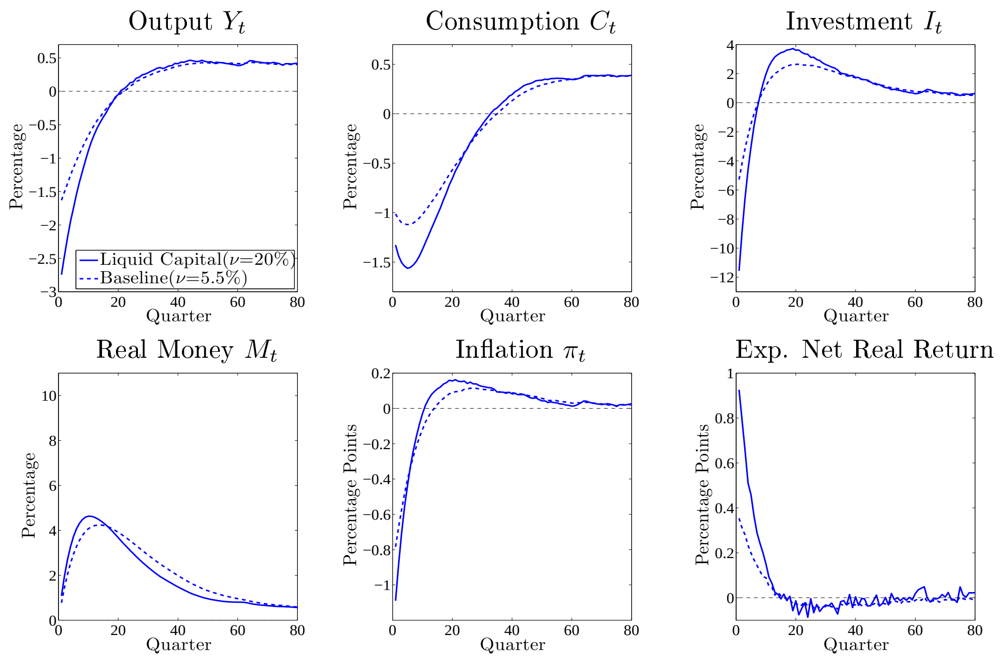
The economy with a stabilizing central bank, where we can more clearly observe the partial equilibrium demand effects, supports this interpretation. In Figure 7 (b), the real money stock jumps up by almost 2% on impact and quickly reaches 10%.

In summary, the macroeconomic effects of uncertainty shocks are robust to changes in ν. While in the limit with perfectly liquid capital money is driven out of the economy, the economy seems to not converge toward the "Aiyagari" economy without money. In the "Aiyagari" case, investment replaces consumption demand one-for-one when uncertainty hits. As long as households hold even tiny amounts of money for liquidity-consumption

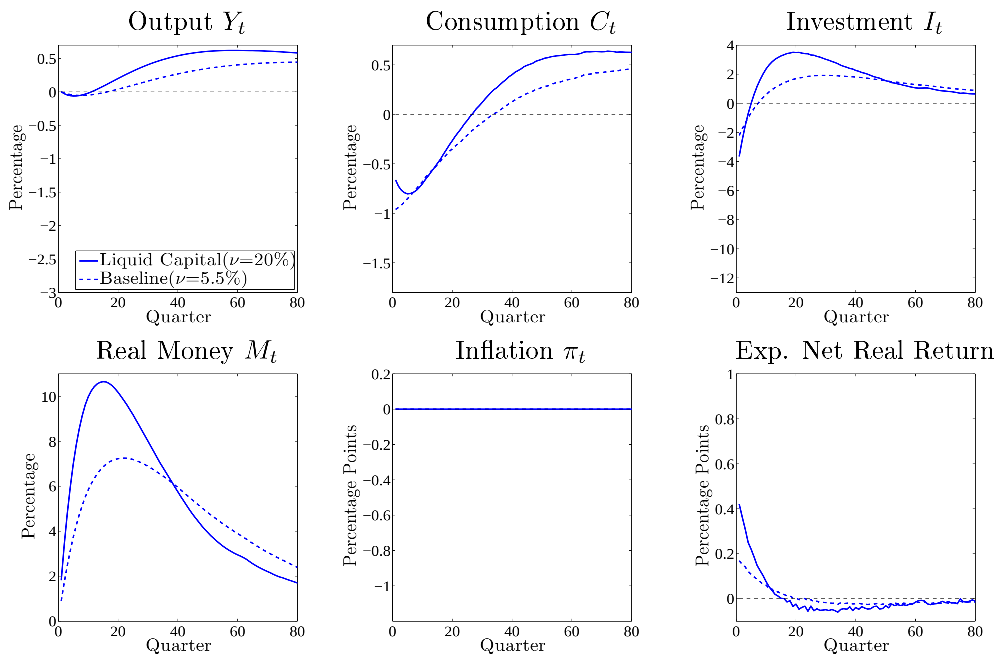
[11] The annual monetary base (i.e., the St. Louis Fed adjusted monetary base) to output ratio from 1980 to 2006 is 7% on average.

Figure 7: Uncertainty shock with liquid capital ($\nu = 20\%$)

7 (a): Constant money growth



7 (b): Inflation targeting



Notes: See Figure 5 or 6.

smoothing reasons, the value of money increases with income uncertainty and money demand is higher in uncertain times, which creates deflationary pressures.

In other words, and more generally speaking, uncertainty shocks will affect aggregate demand negatively only if they trigger precautionary savings in paper and not in real assets. In our model, it is the increased value of liquidity that is responsible for the portfolio adjustment toward money.

6.3 Redistributive and Welfare Effects

So far we have described the aggregate dimension of an uncertainty shock and its repercussions. Since such shocks affect the price level, asset prices, dividends, and wages differently, our model predicts that not all agents (equally) lose from the decline in consumption upon an uncertainty shock. For example, if capital prices fall, those agents that are rich in human capital but hold little physical capital could actually gain from the uncertainty shock. These agents are net savers. They increase their holdings of physical capital and can do so now more cheaply.

To quantify and understand the relative welfare consequences of the uncertainty shock and of systematic policy response, one would normally just look at the change in a household's value function. However, since solving directly for the value function is prohibitively time consuming in our model, we instead simulate and compare two sets of economies: one where the uncertainty state simply evolves according to its Markov chain properties and another set where, at time T, we exogenously increase income uncertainty, σ^2_{ht}, by setting the shock to uncertainty to $\epsilon_T = 2\sigma_s$, a 2 standard deviation increase. We then let the economies evolve stochastically. We trace agents over the next S periods for both sets of economies, and track their period-felicity u_{iT+t} to calculate for each agent with individual state (h, m, k) in period T the discounted expected felicity stream over the next S periods as:

$$v_S(h, m, k) = E\left[\sum_{t=0}^{S} \beta^t u_{T+t} \middle| (h_T, m_T, k_T) = (h, m, k)\right],$$

where u_{T+t} is the felicity stream in period $T + t$ under the household's optimal saving policy. For large S, v_S approximates the actual household's value function.

We then determine an equivalent consumption tax that households would be willing to face over the next S quarters in order to eliminate the uncertainty shock at time T

as:

$$\texttt{CE} = -\left(\frac{v_S^{\texttt{shock}}}{v_S^{\texttt{no shock}}}\right)^{1/\xi} + 1. \tag{20}$$

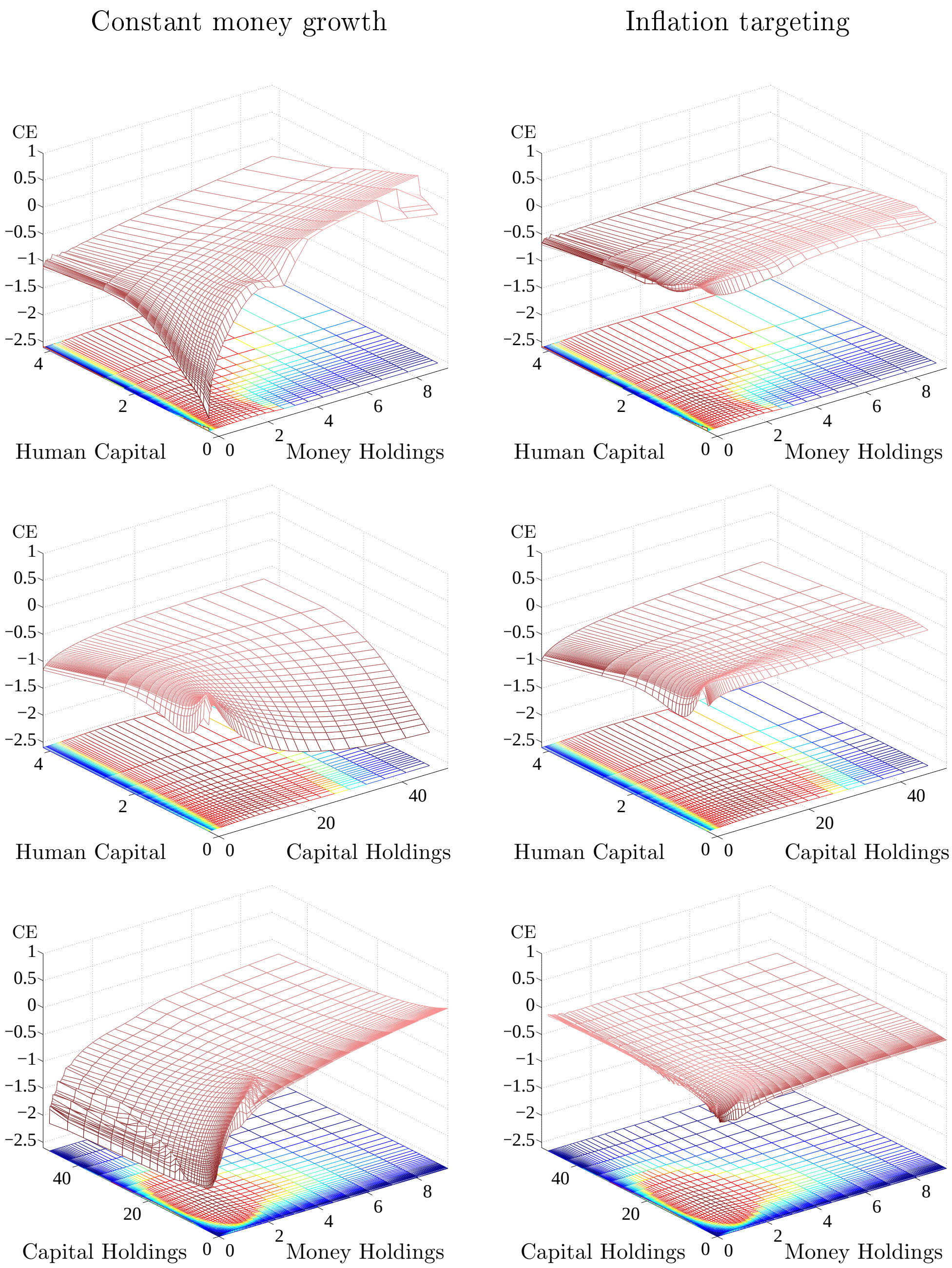
Figure 8 displays the relative differences in v_S for $S = 24$ quarters in terms of consumption equivalents, CE, between the two sets of simulations of the economy. This time horizon captures the welfare consequences of the recession following the uncertainty shock. See Appendix F for an assessment of welfare after more than 65 years, when the initial position, (h_T, m_T, k_T), has washed out in the sense that the conditional and the unconditional distributions are almost identical. Of course, in the long run there are no differences between the two sets of economies.

On average, households would be willing to forgo roughly 0.65% of their consumption over 6 years to eliminate the uncertainty shock. This average loss masks heterogeneous effects across households with different asset positions and human capital. While monetary policy can reduce the cost to roughly 0.14% on average, it also shifts the burden of the shock between households. Figure 8 displays the expected welfare costs of households conditioning on two of the three dimensions of the (h, m, k)-space – integrating out the missing dimension. On the bottom of each of the graphs, we also display the conditional distributions.

Without stabilization, money rich and physical asset poor households lose the least. These are households that typically acquire physical capital in exchange for their money holdings, and they can do so at favorable capital prices after the uncertainty shock. For a similar reason, the gradient in human capital is relatively flat. After the shock, human capital rich households suffer from lower wages, but as savers they are partly compensated, because they can acquire physical capital at lower prices. Table 3 summarizes the figures numerically. In this table we condition on just one dimension of the households' portfolio, and display the average relative welfare gains. We do so in two ways: First, we calculate welfare conditional on one asset taking the conditional distribution of the other two assets into account. Second, we also report welfare effects at median asset holdings of the respective other assets. The latter isolates the direct effect in the dimension of interest.

Table 3 shows that the intervention of the central bank helps households with physical assets. In particular wealthy agents with low human capital profit the most from stabilization (see Figure 8). Conversely, the capital poor but human-capital rich households profit the least from stabilization, because it is them who finance the increased seigniorage by accumulating money.

Figure 8: Welfare after 6 years



Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (20). The graphs refer to the conditional expectations of CE with respect to the two displayed dimensions, respectively. The missing dimension has been integrated out. Capital and money are reported in terms of quarterly income. On the bottom of each graph, the conditional density of the $\Theta(h, m, k)$-distribution is displayed.

Table 3: Welfare after 6 years

	Policy regime: Money growth rule									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-1.42	-0.79	-0.54	-0.40	-0.27	-0.44	-0.54	-0.64	-0.74	-0.88
Median	-1.28	-0.75	-0.49	-0.32	-0.02	-0.44	-0.51	-0.60	-0.71	-0.90
	Quintiles of Human Capital									
Conditional	-0.90	-0.73	-0.61	-0.50	-0.55					
Median	-0.79	-0.62	-0.56	-0.52	-0.68					

	Policy regime: Inflation targeting									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.16	-0.17	-0.13	-0.10	-0.13	-0.19	-0.13	-0.13	-0.13	-0.13
Median	-0.34	-0.18	-0.10	-0.05	0.03	-0.19	-0.13	-0.12	-0.11	-0.08
	Quintiles of human capital									
Conditional	-0.01	-0.05	-0.13	-0.15	-0.31					
Median	-0.01	-0.04	-0.13	-0.17	-0.36					

Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (20). Conditional refers to integrating out the missing dimensions, whereas Median refers to median asset holdings of the respective other assets. We track households over 24 quarters and average over 200 independent model simulations.

6.4 Robustness

To be added

7 Conclusion

This paper examines how variations in uncertainty about household income affect the macroeconomy through precautionary savings. We build a model with a nominal friction in which households may save in a liquid and an illiquid asset – merging incomplete markets with New Keynesian modeling. In this model, higher uncertainty about income triggers a flight to liquidity because it is superior for short-run consumption smoothing. This reduces not only consumption but also investment and hence depresses economic activity.

Calibrating the model to match the evolution of uncertainty about household income in the U.S., we find that a spike in income uncertainty can lead to substantive output, consumption, and investment losses. This may help us to understand the slow recovery of the U.S. economy during the Great Recession, for which we document a shift toward liquid assets across all percentiles of the U.S. wealth distribution. We find that a two standard deviation increase in household income uncertainty generates output losses in the environment we study that are as large as the ones Fernández-Villaverde et al. (2011) report for aggregate policy uncertainty at the zero lower bound.

The welfare effects of such uncertainty shocks crucially depend on a household's asset position and the stance of monetary policy. Monetary policy that drastically increases the money supply in times of increased uncertainty limits the negative welfare effects of uncertainty shocks but redistributes from the asset poor to the asset rich.

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A Dynamic Planning Problem with Two Assets

The dynamic planning problem of a household in the model is characterized by two Bellman equations, V_a in the case where the household can adjust its capital holdings and V_n otherwise

$$\begin{aligned} V_a(m,k,h;\Theta,s) =& max_{k',m'_a \in \Gamma_a} u[x(m,m'_a,k,k',h)] \\ &+ \beta \left[\nu EV^a(m'_a,k',h',\Theta',s') + (1-\nu)EV^n(m'_a,k',h',\Theta',s')\right] \\ V_n(m,k,h;\Theta,s) =& max_{m'_n \in \Gamma_n} u[x(m,m'_n,k,k,h)] \\ &+ \beta \left[\nu EV^a(m'_n,k,h',\Theta',s') + (1-\nu)EV^n(m'_n,k,h',\Theta',s')\right] \end{aligned} \tag{21}$$

where the budget sets are given by

$$\Gamma_a(m,k,h;\Theta,s) = \{m',k' \geq 0 | q(\Theta,s)(k'-k) + m' \leq \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)}\} \tag{22}$$

$$\Gamma_n(m,k,h;\Theta,s) = \{m' \geq 0 | m' \leq \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)}\} \tag{23}$$

$$x(m,m',k,k',h) = \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} - q(\Theta,s)(k'-k) - m' \tag{24}$$

To save on notation, let Ω be the set of possible idiosyncratic state variables controlled by the household, let Z be the set of potential aggregate states, let $\Gamma_i : \Omega \to \Omega$ be the correspondence describing the feasibility constraints, and let $A_i(z) = \{(\omega, y) \in \Omega \times \Omega : y \in \Gamma_i(\omega, z)\}$ be the graph of Γ_i. Hence the states and controls of the household problem can be defined as

$$\Omega = \{\omega = (m,k) \in R^2_+ : m,k \leq \infty\} \tag{25}$$

$$z = \{h, \Theta, s\} \tag{26}$$

and the return function $F : A \to R$ reads

$$F(\Gamma_i(\omega,z),\omega;z) = \frac{x_i^{1-\gamma}}{1-\gamma} \tag{27}$$

Define the value before the adjustment/no-adjustment shock realizes as

$$v(\omega,z) := \nu V_a(\omega,z) + (1-\nu)V_n(\omega,z).$$

Now we can rewrite the optimization problem of the household in terms of the definitions above in a compact form:

$$V_a(\omega, z) = \max_{y \in \Gamma_a(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')] \tag{28}$$

$$V_n(\omega, z) = \max_{y \in \Gamma_n(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')]. \tag{29}$$

Finally we define the mapping $T : C(\Omega) \to C(\Omega)$, where $C(\Omega)$ is the space of bounded, continuous and weakly concave functions.

$$\begin{aligned} (Tv)(\omega, z) &= \nu V_a(\omega, z) + (1-\nu) V_n(\omega, z) \\ V_a(\omega, z) &= \max_{y \in \Gamma_a(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')] \\ V_n(\omega, z) &= \max_{y \in \Gamma_n(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')]. \end{aligned} \tag{30}$$

A.1 Properties of Primitives

The following properties of the primitives of the problem obviously hold:

P 1. *Properties of sets* $\Omega, \Gamma_a(\omega, z), \Gamma_n(\omega, z)$

1. Ω *is a convex subset of* R^3.

2. $\Gamma_i(\cdot, z) : \Omega \to \Omega$ *is non-empty, compact-valued, continuous, monotone and convex for all* z.

P 2. *Properties of return function* F

F is bounded, continuous, strongly concave, C^2 differentiable on the interior of A, and strictly increasing in each of its first two arguments.

A.2 Properties of the Value and Policy Functions

Lemma 1. *The mapping T defined by the Bellman equation for v fulfills Blackwell's sufficient conditions for a contraction on the set of bounded, continuous and weakly concave functions $C(\Omega)$.*

a) It satisfies discounting.

b) It is monotonic.

c) It preserves boundedness (assuming an arbitrary maximum consumption level).

d) It preserves strict concavity.

Hence, the solution to the Bellman equation is strictly concave. The policy is a single-valued function in m, k, and so is optimal consumption.

Proof. The proof proceeds item by item and closely follows Nancy L. Stokey (1989) taking into account that the household problem in the extended model consists of two Bellman equations.

a) Discounting
Let $a \in R_+$ and the rest be defined as above. Then it holds that

$$\begin{aligned}(T(v+a))(\omega, z) =& \nu \max_{y\in\Gamma_a(\omega,z)} [F(\omega, y, z) + \beta_w Ev(y, z') + a] \\ &+ (1-\nu) \max_{y\in\Gamma_n(\omega,z)} [F(\omega, y, z) + \beta_w Ev(y, z') + a] \\ =&(Tv)(\omega, z) + \beta_w a\end{aligned}$$

Accordingly, T fulfills discounting.

b) Monotonicity
Let $g : \Omega \times Z \to R^2$, $f : \Omega \times Z \to R^2$ and $g(\omega, z) \geq f(\omega, z)$ $\forall \omega, z \in \Omega \times Z$, then it follows that

$$\begin{aligned}(Tg)(\omega, z) =& \nu \max_{y\in\Gamma_a(\omega,z)} [F(\omega, y, z) + \beta_w Eg(y, z')] \\ &+ (1-\nu) \max_{y\in\Gamma_n(\omega,z)} [F(\omega, y, z) + \beta_w Eg(y, z')] \\ \geq& \nu \max_{y\in\Gamma_a(\omega,z)} [F(\omega, y, z) + \beta_w Ef(y, z')] \\ &+ (1-\nu) \max_{y\in\Gamma_n(\omega,z)} [F(\omega, y, z) + \beta_w Ef(y, z')] \\ =& Tf(\omega, z)\end{aligned}$$

The objective function for which Tg is the maximized value is uniformly higher than the function for which Tf is the maximized value. Therefore, T preserves monotonicity.

c) Boundedness
From properties **P1** it follows that the mapping T defines a maximization problem over the continuous and bounded function $[F(\omega, y) + \beta_w Ev(y, z'))]$ over the compact

sets $\Gamma_i(\omega, z)$ for $i = (a, n)$. Hence the maximum is attained. Since F and v are bounded, Tv is also bounded.

d) Strict Concavity
Let $f \in C''(\Omega)$, where $C^{''}$ is the set of bounded, continuous, strictly concave functions on Ω. Since the convex combination of two strictly concave functions is strictly concave, it is sufficient to show that $T_i[C''(\Omega)] \subseteq C''(\Omega)$, where T_i is defined by

$$T_i v = \max_{y \in \Gamma_i(\omega,z)} [F(\omega, y, z) + \beta_w E v(y, z')], i \in a, n$$

Let $\omega_0 \neq \omega_1, \theta \in (0,1), \omega_\theta = \theta\omega_0 + (1-\theta)\omega_1$.
Let $y_j \in \Gamma_i(\omega_j, z)$ be the maximizer of $(T_i f)(\omega_j)$ for $j = 0, 1$ and $i = a, n$, $y_\theta = \theta y_0 + (1-\theta) y_1$.

$$\begin{aligned}(T_i f)(\omega_\theta, z) \geq & [F(\omega_\theta, y_\theta, z) + \beta_w E f(y_\theta, z')] \\ > & \theta[F(\omega_0, y_0) + \beta_w E f(y_0, z'))] + (1-\theta)[F(\omega_1, y_1) + \beta_w E f(y_0, z')] \\ = & \theta(Tf)(\omega_0, z) + (1-\theta)(Tf)(\omega_1, z)\end{aligned}$$

The first inequality follows from y_θ being feasible because of convex budget sets. The second inequality follows from the strict concavity of f. Since ω_0, ω_1 were arbitrary, it follows that $T_i f$ is strictly concave, and since f was arbitrary that $T[C''(\Omega)] \subseteq C''(\Omega)$.

□

Lemma 2. *The value function is C^2 and the policy function C^1 differentiable.*

Proof. The properties of the choice set **P1**, of the return function **P2**, and the properties of the value function proven in (1) fulfill the assumptions of Santos's (1991) theorem on the differentiability of the policy function. According to the theorem, the value function is C^2 and the policy function C^1 differentiable.
Note that strong concavity of the return function holds for CRRA utility, because of the arbitrary maximum we set for consumption. □

Lemma 3. *The total savings $S_i^* := m_i^*(\omega, z) + q(z)k_i^*(\omega, z)$ and consumption $c_i^*, i \in a, n$ are increasing in ω if $r(z)$ is positive. In the adjustment case total savings and consumption are increasing in total resources $R_a = [q(z) + r(z)]k + m/\pi(z)$ for any r(z).*

Proof. Define $\tilde{v}(S,z) := \max_{\{m,k|m+q(z)k\leq S\}} Ev(m,k;z')$ and resources in the case of no adjustment $R_n = r(z)k + m/\pi(z)$. Since v is strictly concave and increasing, so is $\tilde{v}$ by the line of the proof of Lemma 1.d). Now we can (re)write the planning problem as

$$V_a(m,k;z) = \max_{S\leq \frac{\gamma}{1+\gamma}w(z)hN+R_a} [u(\frac{\gamma}{1+\gamma}w(z)hN + [q(z)+r(z)]k + m/\pi(z) - S) + \beta_W \tilde{v}(S,z)]$$
$$V_n(m,k;z) = \max_{m'\leq \frac{\gamma}{1+\gamma}w(z)hN+R_n} [u(\frac{\gamma}{1+\gamma}w(z)hN + r(z)k + m/\pi(z) - m') + \beta_W Ev(m',k;z')].$$

Due to differentiability we obtain the following (sufficient) first-order conditions

$$\frac{\partial u\left(\frac{\gamma}{1+\gamma}w(z)hN + [q(z)+r(z)]k + m/\pi(z) - S\right)}{\partial c} = \beta_W \frac{\partial \tilde{v}(S,z)}{\partial S}$$
$$\frac{\partial u\left(\frac{\gamma}{1+\gamma}w(z)hN + r(z)k + m/\pi(z) - m'\right)}{\partial c} = \beta_W \frac{\partial v(m',k;z)}{\partial m'}. \tag{31}$$

Since the left-hand sides are decreasing in $\omega = (m,k)$, and increasing in S (respectively m'), and the right-hand side is decreasing in S (respectively m'), $S_i^* = \begin{cases} qk' + m' \text{ if } i = a \\ qk + m' \text{ if } i = n \end{cases}$ must be increasing in ω.

Since the right-hand side of (31) is hence decreasing in ω, so must be the left-hand side of (31). Hence consumption must be increasing in ω.

The last statement follows directly from the same proof. □

A.3 Euler Equations

Denote the optimal policies for consumption, for money holdings and capital as $x_i^*, m_i^*, k^*, i \in \{a,n\}$ respectively. The first-order conditions for an inner solution in the (no-)adjustment case read

$$k^* : \frac{\partial u(x_a^*)}{\partial x} q = \beta E\left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial k} + (1-\nu)\frac{\partial V_n(m_a', k'; z')}{\partial k}\right] \tag{32}$$
$$m_a^* : \frac{\partial u(x_a^*)}{\partial x} = \beta E\left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial m} + (1-\nu)\frac{\partial V_n(m_a^*, k^*; z')}{\partial m}\right] \tag{33}$$
$$m_n^* : \frac{\partial u(x_n^*)}{\partial x} = \beta E\left[\nu \frac{\partial V_a(m_n^*, k; z')}{\partial m} + (1-\nu)\frac{\partial V_n(m_n^*, k; z')}{\partial m}\right] \tag{34}$$

Note the subtle difference between (33) and (34), which lies in the different capital stocks k' vs. k in the right-hand side expressions.

Differentiating the value functions with respect to k and m, we obtain

$$\frac{\partial V_a(m,k;z)}{\partial k} = \frac{\partial u[x_a^*(m,k;z)]}{\partial x}(q(z)+r(z)) \tag{35}$$

$$\frac{\partial V_a(m,k;z)}{\partial m} = \frac{\partial u[x_a^*(m,k;z)]}{\partial x}\pi(z)^{-1} \tag{36}$$

$$\frac{\partial V_n(m,k;z)}{\partial m} = \frac{\partial u[x_n^*(m,k;z)]}{\partial x}\pi(z)^{-1} \tag{37}$$

$$\begin{aligned}\frac{\partial V_n(m,k;z)}{\partial k} &= r(z)\frac{\partial u[x_n^*(m,k;z)]}{\partial x} \\ &+ \beta E\left[\nu\frac{\partial V_a[m_n^*(m,k;z),k;z']}{\partial k} + (1-\nu)\frac{\partial V^n[m_n^*(m,k;z),k;z']}{\partial k}\right] \\ &= r(z)\frac{\partial u[x_n^*(m,k;z)]}{\partial x} + \beta\nu E\frac{\partial u\{x_a^*[m_n^*(m,k;z),k;z],k;z'\}}{\partial x}(q(z')+r(z')) \\ &+ \beta(1-\nu)E\frac{\partial V_n\{[m_n^*(m,k;z),k;z],k;z'\}}{\partial k}\end{aligned} \tag{38}$$

Such that the marginal value of capital in non-adjustment is defined recursively.

Now we can plug the second set of equations into the first set of equations and obtain the following Euler equations (in slightly shortened notation)

$$\frac{\partial u[x_a^*(m,k;z)]}{\partial x}q(z) = \beta E\left[\nu\frac{\partial u[x_a^*(m_a^*,k^*;z')]}{\partial x}[q(z')+r(z')] + (1-\nu)\frac{\partial V^n(m_a^*,k';z')}{\partial k'}\right] \tag{39}$$

$$\frac{\partial u[x_a^*(m,k;z)]}{\partial x} = \beta E\pi'(z')^{-1}\left[\nu\frac{\partial u[x_a^*(m_a^*,k^*;z')]}{\partial x} + (1-\nu)\frac{\partial u[x_n^*(m_a^*,k';z')]}{\partial x}\right] \tag{40}$$

$$\frac{\partial u[x_n^*(m,k,;z)]}{\partial x} = \beta E\pi'(z')^{-1}\left[\nu\frac{\partial u[x_a^*(m_n',k;z')]}{\partial x} + (1-\nu)\frac{\partial u[x_n^*(m_n^*,k;z')]}{\partial x}\right] \tag{41}$$

A.4 Algorithm

The algorithm we use to solve for optimal policies given the Krusell-Smith forecasting rules is a version of Hintermaier and Koeniger's (2010) extension of the endogenous grid method, originally developed by Carroll (2006).

It works iteratively (until convergence of policies) as follows: Start with some guess for the policy functions x_a^* and x_n^* on a given grid $(m,k) \in M \times K$. Define the shadow value of capital

$$\begin{aligned}\beta^{-1}\psi(m,k;z) :=& \nu E\left\{\frac{\partial u\{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\ &+(1-\nu)E\frac{\partial V_n[m_n^*(m,k,z),k;z']}{\partial k} \\ &= \nu E\left\{\frac{\partial u\{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\ &+(1-\nu)E\left\{\frac{\partial u\{x_n^*(m_n^*(m,k,z),k;z')]}{\partial x}r(z')\right\} \\ &+(1-\nu)E\left\{\psi[m_n^*(m,k,z),k;z']\right\}.\end{aligned} \tag{42}$$

Guess initially $\psi = 0$. Then

1. Solve for an update for x_n^* using standard endogenous grid methods using equation (41), and denote $m_n^*(m,k;z)$ as the optimal money holdings without capital adjustment.

2. Find for every k' on-grid some (off-grid) value of $\tilde{m}_a^*(k';z)$ such that - combining (40) and (39) -

$$\begin{aligned}0 = \nu E&\left\{\frac{\partial u[x_a^*(\tilde{m}_a^*(k',z),k';z')]}{\partial x}\left[\frac{q(z')+r(z')}{q(z)}-\pi(z')^{-1}\right]\right\} \\ +(1-\nu)E&\left\{\frac{\partial u[x_n^*(\tilde{m}_a^*(k',z),k';z')]}{\partial x}\left[\frac{r(z')}{q(z)}-\pi(z')^{-1}\right]\right\}+(1-\nu)E\left[\frac{\psi(\tilde{m}_a^*(k',z),k';z')}{q(z)}\right]\end{aligned} \tag{43}$$

   N.B. that $E\psi$ takes the stochastic transitions in h' into account and does not replace the expectations operator in the definition of ψ. If no solution exists, set $\tilde{m}_a^* = 0$. Uniqueness (conditional on existence) of $\tilde{m}_a^*$ follows from the strict concavity of v.

3. Solve for total initial resources, by solving the Euler equation (40) for $\tilde{x}^*(k',z)$, such that

$$\begin{aligned}&\tilde{x}^*(k',z) \\ &= \frac{\partial u}{\partial x}^{-1}\left\{\beta E\pi(z')^{-1}\left[\nu\frac{\partial u\{x_a^*[m_a^*(k',z),k';z']\}}{\partial x}+(1-\nu)\frac{\partial u\{x_n^*[m_a^*(k',z),k';z']}{\partial x}\right]\right\}\end{aligned} \tag{44}$$

   where the right-hand side expressions are obtained by interpolating $x_a^*(m_a^*(k',z),k',z')$ from the on-grid guesses $x_a^*(m,k;z)$ and taking expected values with respect to z'.

This way we obtain total non-human resources $\tilde{R}_a(k', z)$ that are compatible with plans $(m^*(k'), k')$ and a consumption policy $\tilde{\tilde{x}}_a^*(\tilde{R}_a(k', z), z)$ in total resources.

4. Since (consumption) policies are increasing in resources, we can obtain consumption policy updates as follows: Calculate total resources for each (m, k) pair $R_a(m, k) = (q+r)k+m/\pi$ and use the consumption policy obtained before to update $x_a^*(m, k, z)$ by interpolating at $R_a(m, k)$ from the set $\left\{(\tilde{\tilde{x}}_a^*(\tilde{R}_a(k', z), z), R_a(k', z))\middle| k' \in K\right\}$.[12]

5. Update ψ: Calculate a new value of ψ using (38), such that

$$\begin{aligned}\psi^{new}(m, k, z) =& \beta\nu E\left\{\frac{\partial u\{x_a^*[m_n^*(m, k, z), k; z']\}}{\partial x}[q(z') + r(z')]\right\}\\ &+ \beta(1-\nu)E\left\{\frac{\partial u\{x_n^*(m_n^*(m, k, z), k; z')]}{\partial x}r(z')\right\}\\ &+ \beta(1-\nu)E\left\{\psi^{old}[m_n^*(m, k, z), k; z']\right\}. \end{aligned} \tag{45}$$

making use of the **updated** consumption policies.

B Estimation of the Income Process

B.1 Income Process

We assume that the observed log-income of a household, y_{it}, is composed of four components: a deterministic part $f(o_{it})$, a transitory part $\tau_{i,a,t}$, a persistent part $h_{i,a,t}$, and a permanent part μ_i such that

$$y_{i,a,t} = f(o_{it}) + y^*_{i,a,t} \tag{46}$$

$$y^*_{i,a,t} = \tau_{i,a,t} + h_{i,a,t} + \mu_i, \tag{47}$$

$$h_{i,a,t} = \rho_h h_{i,a-1,t-1} + \epsilon_{i,a,t}, \tag{48}$$

$$\log\sigma_t^\epsilon = (1-\rho_s)\mu_s + \rho_s \log\sigma_{t-1}^\epsilon + \varepsilon_t, \tag{49}$$

$$\varepsilon_t \sim N(0, \sigma_s), \quad \epsilon_{i,a,t} \sim N(0, \sigma_t^\epsilon), \quad \tau_{i,a,t} \sim N(0, \sigma_\tau), \quad \mu_i \sim N(0, \sigma_\mu).$$

where o_{it} is observable characteristics of the household (head), $y^*_{i,a,t}$ is the stochastic component of a household's income ("residual income"), t is calendar time, and $a + 25$ is the household's age, where we assume a is years of labor market experience. We assume that all households start with $h_{i,0,t} = 0$ when they enter the labor market.

[12] If a boundary solution $\tilde{m}^*(0) > 0$ is found, we use the "n" problem to obtain consumption policies for resources below $\tilde{m}^*(0)$.

B.2 Income Variances

Under the above assumptions, the variance of residual income, $y^*_{i,a,t}$, is given by

$$\begin{aligned} {\sigma^y_{a,t}}^2 &= \sigma^2_\tau + \sigma^2_\mu + {\sigma^h_{a,t}}^2 && (50)\\ {\sigma^h_{a,t}}^2 &= \rho^2_h {\sigma^h_{a-1,t-1}}^2 + {\sigma^\epsilon_t}^2; \quad {\sigma^h_{0,t}}^2 = 0 && (51)\\ \log \sigma^\epsilon_t &= (1-\rho_s)\mu_s + \rho_s \log \sigma^\epsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_s). && (52) \end{aligned}$$

We use the above equations to identify the parameters of interest $\{\rho_h, \rho_s, \sigma_s, \mu_s\}$ from the data. $\sigma^2_\tau + \sigma^2_\mu$ will be only identified in sum by our estimation procedure.

B.3 Data

We use the 1970-2009 CNEF files of the PSID and drop the low income sample. We keep all households in the sample that have at least two (but no more than 10) household members and a male household head no younger than 25 and not older than 60 who works at least 52 hours per year. We construct household income as the sum of labor income, and private and public transfers minus taxes.

B.4 Estimation

Our estimation procedure proceeds in two steps. First we estimate the deterministic component, $f(o_{it})$, running an OLS regression of log household income on time dummies, age dummies, schooling dummies interacted with up to a quadratic age trend, and household size dummies. We eliminate any observation where the residual of this regression exceeds 3 standard deviations in absolute value. The results are comparable to existing studies, implying a concave earnings function in age and education. The inclusion of age-schooling interactions considerably raises the R^2.

From the residuals of this regression, we then calculate the sample variance within an age-year cell, $s^2_{a,t}$, across ages, $a = 25, \ldots, 60$, from $t = 1970, \ldots, 2009$. This yields 1224 sample-variance observations, where each variance is constructed from on average 56 observations on the log-income residual.

We assume that these sample variances reflect the theoretical ones with a (sampling) measurement error, which also captures time variations in the variances of transitory shocks, such that we obtain

$$s^2_{a,t} = {\sigma^y_{a,t}}^2 + \vartheta_{a,t},$$

where ${\sigma^y_{a,t}}^2$ is given by (50).

We can now write $s_{a,t}^2$ recursively as:

$$\begin{aligned}
s_{a,t}^2 &= \rho_h^2 L s_{a,t}^2 + \sigma_t^{\epsilon\ 2} + (1-\rho_h^2 L)(u_{a,t} + \bar{u}) \text{ with } u_{a,t} \sim \mathrm{N}(0, \sigma_u) &\quad (53)\\
\log \sigma_t^{\epsilon} &= (1-\rho_{s)}\mu_s + \rho_s \log \sigma_{t-1}^{\epsilon} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_s) &\quad (54)
\end{aligned}$$

where L is the lag-operator, ρ_s is the autocorrelation coefficient of the variance of the persistent income component, and $u_{a,t} = \sigma_\tau^2 + \sigma_\mu^2 + \vartheta_{a,t}$.

We estimate the parameters using a Kalman filter. What somewhat complicates the estimation is that equation (53) holds for levels but equation (54) for logs. We overcome this problem by iterating over both equations simultaneously, where we first identify the joint innovations, $\sigma_t^{\epsilon\ 2} + (1-\rho_h^2 L)u_{a,t}$, to equation (53), and then separate it into an age-group-year specific, $(1-\rho_h^2 L)u_{a,t}$, and a common part, $\sigma_t^{\epsilon\ 2}$, for time t. The latter provides the measurement updates for equation (54).

Stacking all sample variances at date t for the different age groups, $a = 1, \ldots, A$, in vector X_t, we obtain the state space of the Kalman filter as follows – abstracting from constants for the ease of notation:

$$X_{1t} = F_1 X_{1t-1} + G_1 \begin{pmatrix} \nu_t \\ \sigma_t^{\epsilon\ 2} \end{pmatrix}, \tag{55}$$

where we have split $X_t = \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix}$ into two sets of variables:

$$\begin{aligned}
X_{1t} &= \begin{pmatrix} \mathtt{S}_t^2 \\ \mathtt{U}_t \end{pmatrix}, & X_{2t} &= \sigma_t^{\epsilon\ 2} \\
\mathtt{S}_t^2 &= \begin{pmatrix} s_{1,t}^2 \\ \ldots \\ s_{A,t}^2 \end{pmatrix}, & \mathtt{U}_t &= \begin{pmatrix} u_{1,t} \\ \ldots \\ u_{A,t} \end{pmatrix} &\quad (56)
\end{aligned}$$

and where F_1, G_1 are given by

$$F_1 = \begin{pmatrix} \rho_h^2 I_A & -\rho_h^2 I_A \\ 0_{A\times A} & 0_{A\times A} \end{pmatrix}, \quad G_1 = \begin{pmatrix} I_A & \iota_A \\ I_A & 0_{A\times 1} \end{pmatrix} \tag{57}$$

with I_A being the $A \times A$ unit matrix, ι_A a $A \times 1$ vector of ones, and $0_{A\times B}$ being the $A \times B$ zero matrix.

We can use equation (55) to solve for the transitory shocks and the common shock by bringing all predetermined states to the left-hand side of the equation and then taking the generalized inverse of G_1, i.e. by identifying it from the residuals as the common component over all age groups.

The estimate so obtained for σ_t^ϵ provides us, after log-transformation, with a measurement update for the second state equation, i.e., the assumed AR(1) process for the log of the persistent component of the variance, as stated in equation (54). After this step we have estimates for both shocks, ν_t and ε_t, so that we can evaluate the likelihood function. The measurement equation belonging to 55 simply states that $s_{a,t}^2$ is observed each period t for all ages a.

B.5 Initialization and Priors

We assume that the accumulation of shocks in the labor market starts at age 26. To abstract from the effects of household formation and retirement, we measure income variances not before age 25 nor after age 60. This gives us 31 observable variances to be initialized for the Kalman filter. We do so according to the expected accumulated shocks at labor market experience $a = 5, ..., 35$ (ages 25-60) given the assumed parameter values.

$$s_{a,1}^2 = \left[\sum_{c=1}^{a-25} \rho_h^{2c} \exp\left(2\mu_s(1-\rho_s) + \frac{\sigma_s^2}{(1-\rho_s^2)} \right) \right] + \bar{u} \tag{58}$$

where μ_s reflects the steady-state value of Equation 54, and $\bar{u}$ is the mean of the transitory shocks. All shocks are initialized at their expected value.

The variables to be estimated therefore are:

$$\mu = \begin{pmatrix} \rho_h & \rho_s & \bar{u} & \mu_s & \sigma_u^2 & \sigma_s^2 \end{pmatrix} \tag{59}$$

We choose priors according to the existing literature where possible. The priors for $\{\rho, \bar{\sigma_z}, \sigma_s\}$ are based on Storesletten et al. (2004). There is no guidance for the remaining variables, and hence we do not impose tight priors.

B.6 Results

Table 4 summarizes the parameter values that maximize the log-likelihood function given our priors.

Figure 9 plots the posterior distribution of the log-likelihood function against the prior distribution for each of the parameters. We simulate the posterior by Markov chain

Table 4: Parameter Estimates and Standard Errors

ρ	ρ_s	$\bar{\sigma_z}$	$\bar{u}$	σ_ε	σ_u
0.747	0.726	-0.481	0.026	0.205	0.019
(0.015)	(0.059)	(0.101)	(0.014)	(0.022)	(0.001)

Monte Carlo methods, in particular using the Metropolis-Hastings algorithm. We start 10 Markov chains that each draw 2500 candidate parameter vectors, and adjust the variance of the candidate generating multivariate-normal density to achieve an acceptance rate of about 25%.

C Asset Distribution

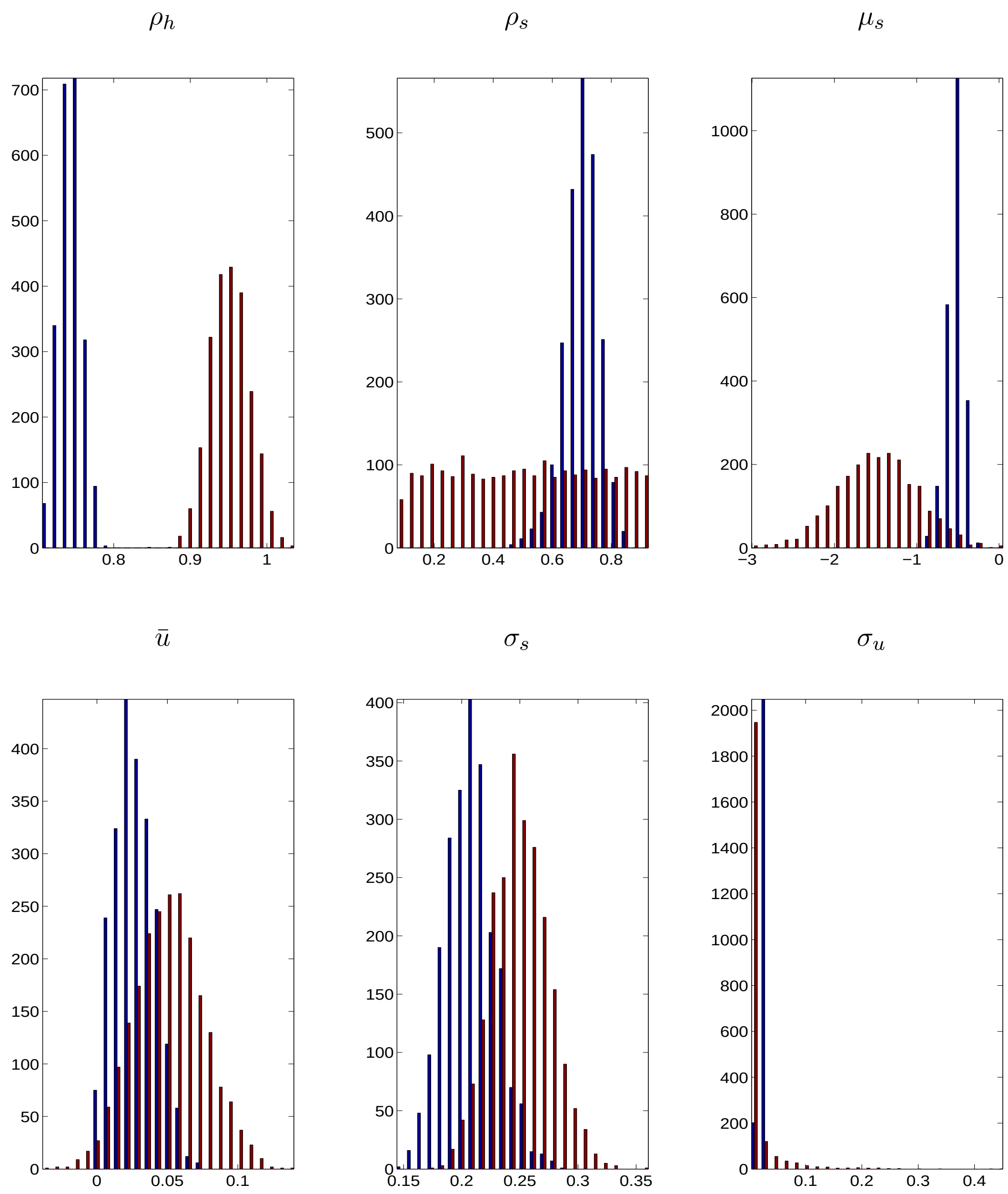
Table 5 summarizes the wealth distribution implied by our model (i.e., for the baseline calibration without fluctuations in uncertainty). As with any incomplete markets model that does not resort to heterogeneity in preferences or extremely skewed processes for idiosyncratic productivity, we fail to match the skewness in wealth documented for the U.S. Whereas the fraction of wealth held by the richest quintile is about 80% in the U.S., the top quintile in our model holds only 48% of total wealth. The same discrepancy holds for the Gini coefficient, where our model falls short as well – 0.46 versus 0.79.

Table 5: Asset Distribution

	Quintiles					Gini-Coeff.
	1st	2nd	3rd	4th	5th	
Fraction of Total Wealth	6.46	12.96	18.68	24.65	37.25	0.31
...held in Money	23.93	15.09	11.43	9.03	6.59	
...held in Capital	76.07	84.91	88.57	90.97	93.41	

These shortcomings are, however, not of great importance for our transmission mechanism. The top quintile is well insured, because they hold a sizable amount of liquid assets. Hence, they are least affected by ups and downs in uncertainty. The lower quintiles, to the contrary, are the ones building up precautionary savings and thus the ones

Figure 9: Posterior vs. Prior Distributions



Notes: The distributions in red are the priors and in blue the posteriors. The posteriors are generated with Markov chain Monte Carlo methods. We use 10 Markov chains with 5000 draws each. The acceptance rate is 25%.

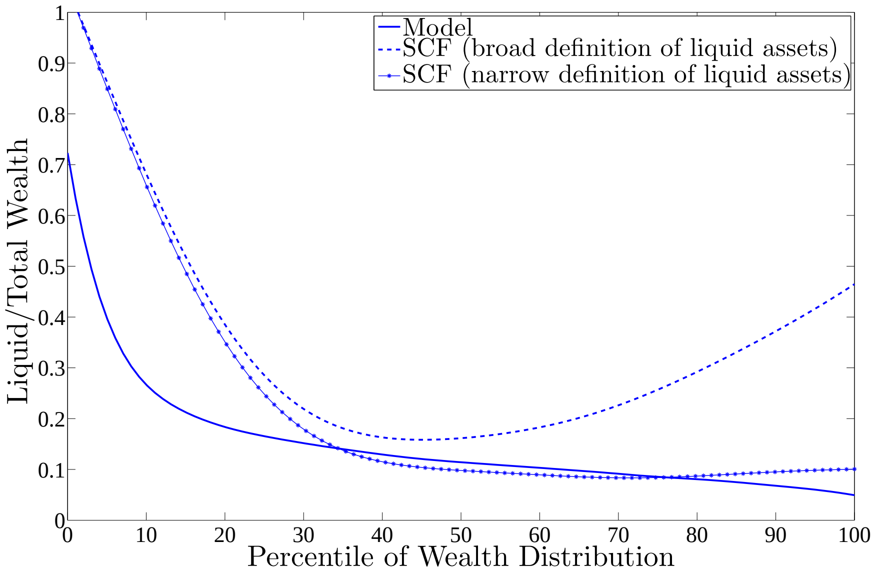
that react strongest to changes in uncertainty. Precisely in this dimension our model replicates the data fairly well. The poorest quintile in the U.S. has about zero wealth on average – including indebted households. The poorest households in our model hold close to no assets as well – only 2.6% of total wealth. The second and third quintiles also hold little wealth – 7.7% and 15.2% of total wealth.

Besides matching the net worth of the poorest quintiles fairly well, our model has implications for the ratio of liquid to illiquid assets conditional on how rich households are in total. By assumption, households start accumulating wealth in our model by hoarding money until they are able to put a fraction of their savings into their capital account. Moreover, households first save in money because of its value in consumption smoothing. Hence, our model implies that the share of liquid assets in the portfolio declines in total wealth. Figure 10 plots the prediction of our model and the data equivalent taken from the Survey of Consumer Finances 2004 (SCF) according to the definitions by Kaplan and Violante (2011). The poorest households in the U.S. and in our model do not hold any illiquid assets at all. The share of liquid assets then rapidly falls below 40% in both graphs, but rises again in the SCF for the richest households. This is because stocks, mutual funds, and non-governmental bond holdings are concentrated at the top quintiles as can be seen by comparing the broad liquidity measure, which includes all of those, to the narrow definition. If we also exclude those assets that usually induce some transaction cost (e.g., a commission) when acquiring them from a bank or broker, the share of liquid assets is substantially reduced for the asset rich.

D Equilibrium Forecasting Rules

Tables 6 and 7 display the equilibrium laws of motion for the Krusell-Smith equilibrium.

Figure 10: Share of liquid assets of total networth against percentiles of total wealth in 2004



Notes: We compare our measure of liquid net worth (see Figure 1) to a broader definition of liquid assets that includes mutual funds, stocks, and non-governmental bonds as in Kaplan and Violante (2011). For graphical illustration we make use of an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15

Table 6: Laws of Motion for the Price of Capital

Baseline Calibration	$\beta_1(s_1)$	$\beta_1(s_2)$	$\beta_1(s_3)$	$\beta_1(s_4)$	$\beta_1(s_5)$	$\beta_2(s_1)$	$\beta_2(s_2)$	$\beta_2(s_3)$	$\beta_2(s_4)$	$\beta_2(s_5)$	$\beta_3(s_1)$	$\beta_3(s_2)$	$\beta_3(s_3)$	$\beta_3(s_4)$	$\beta_3(s_5)$	R^2
Const. Money Growth	0.81	0.79	0.78	0.78	0.77	0.31	0.32	0.32	0.33	0.33	-0.39	-0.38	-0.39	-0.40	-0.41	99.38
Inflation Targeting	0.53	0.54	0.54	0.55	0.53	0.08	0.09	0.10	0.10	0.11	-0.29	-0.30	-0.30	-0.30	-0.30	98.88
Alternative Calibrations (with Constant Money Growth)																

Table 7: Laws of Motion for Inflation

Baseline Calibration	$\beta_1(s_1)$	$\beta_1(s_2)$	$\beta_1(s_3)$	$\beta_1(s_4)$	$\beta_1(s_5)$	$\beta_2(s_1)$	$\beta_2(s_2)$	$\beta_2(s_3)$	$\beta_2(s_4)$	$\beta_2(s_5)$	$\beta_3(s_1)$	$\beta_3(s_2)$	$\beta_3(s_3)$	$\beta_3(s_4)$	$\beta_3(s_5)$	R^2
Const. Money Growth	11.17	9.92	9.09	8.43	9.03	6.22	6.19	6.23	6.47	7.15	-3.78	-3.46	-3.39	-3.40	-4.12	99.79
Inflation Targeting	0.50	0.50	0.50	0.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	83.12
Alternative Calibrations (with Constant Money Growth)																

Notes: For readability all values are mutiplied by 100.

E Quality of the Numerical Solution

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on 10.000 observations. We generate the observations by simulating the model in parallel on 10 machines, letting each economy run for 1500 periods and discarding the first 500 periods. The R^2 is generally above 99% for all calibrations; see Tables 6 and 7. In the case of perfect stabilization, π_t is virtually constant, such that the R^2 of the π-forecasting is a nonsensical statistic.

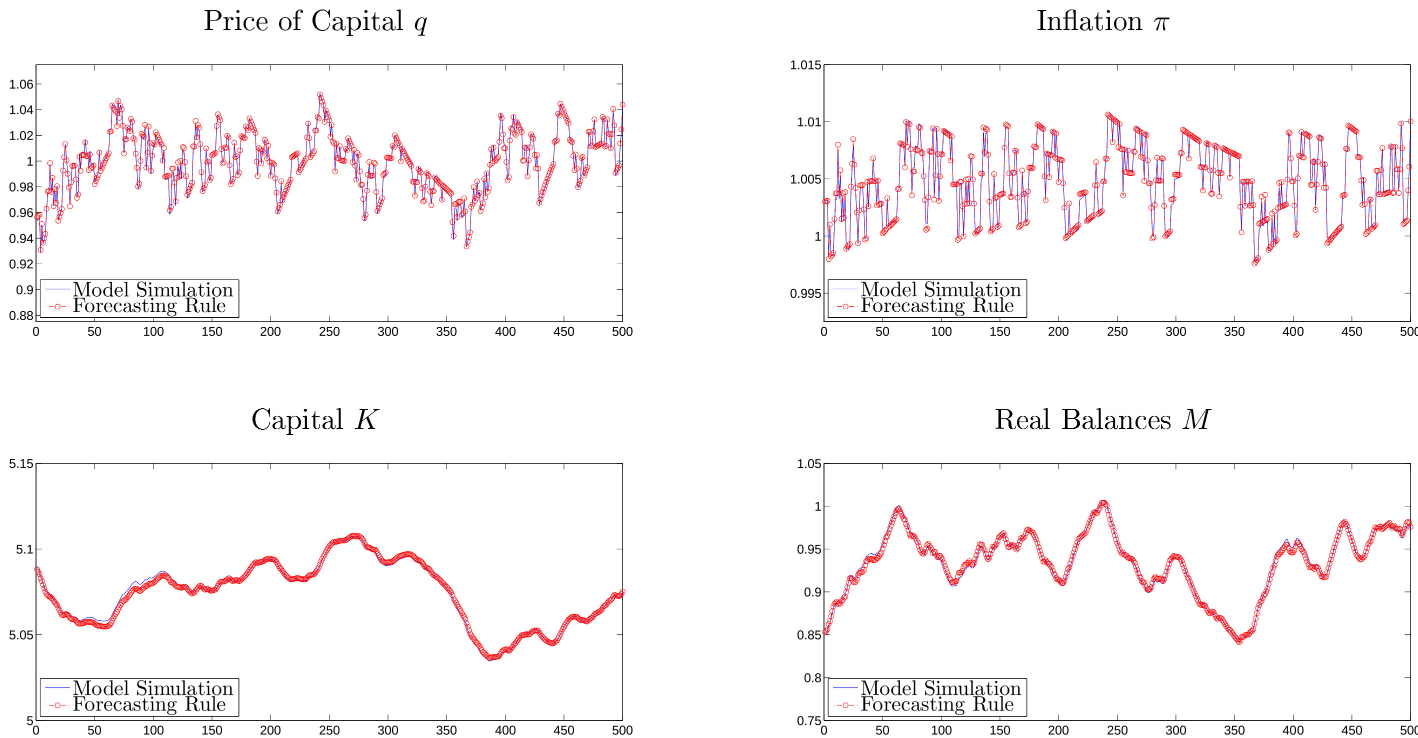
Following Den Haan (2010), we also test the out-of-sample performance of the forecasting rules. For this we initialize the model and the forecasting rules at steady state values, feed in the same shock sequence, but otherwise let them run independently. Figure 11 plots time series of the prices q and π as well as the states K and M taken from the simulation of the model and the forecasting rules. The equilibrium forecasting rules track the evolution of the underlying model without any tendency to divergence. Table 8 summarizes the mean and maximum difference between the series generated by the model and the forecasting rules. The mean error for all 4 time series is less than 0.3%. The maximum errors are small, too.

Table 8: Forecasting errors

	Price of capital q_t	Capital K_t	Inflation π_t	Real balances M_t
Mean Error	0.13	0.04	0.02	0.27
Max Error	0.43	0.11	0.06	0.91

Notes: Differences in out-of-sample forecasts between forecasting rules and model; see Den Haan (2010).

Figure 11: Out-of-sample performance of the forecasting rules



F Welfare

Table 9 provides the long run welfare effects with and without stabilization after 68 years.

Table 9: Welfare after 68 years

	Policy regime: Money growth rule									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.51	-0.37	-0.30	-0.27	-0.24	-0.15	-0.25	-0.33	-0.40	-0.50
Median	-0.54	-0.36	-0.27	-0.22	-0.08	-0.16	-0.25	-0.32	-0.38	-0.48
	Quintiles of Human Capital									
Conditional	-0.23	-0.30	-0.32	-0.34	-0.44					
Median	-0.18	-0.27	-0.30	-0.35	-0.54					
	Policy regime: Inflation targeting									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.15	-0.11	-0.10	-0.10	-0.16	-0.03	-0.09	-0.13	-0.16	-0.20
Median	-0.11	-0.07	-0.05	-0.05	-0.05	0.00	-0.03	-0.06	-0.09	-0.13
	Quintiles of human capital									
Conditional	-0.14	-0.07	-0.07	-0.10	-0.22					
Median	-0.14	-0.06	-0.06	-0.09	-0.24					

Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (20). Conditional refers to integrating out the missing dimensions, whereas Median refers to median asset holdings of the respective other assets. We track households over 275 quarters and average over 200 independent model simulations.

G Robustness Checks

Table **??** provides the calibrated parameters for our robustness checks.