

Collateral, liquidity and debt sustainability*

Stefan Niemann[†] Paul Pichler[‡]

May 19, 2014

Abstract

We study Markov-perfect optimal fiscal policy in an infinite-horizon economy with financial frictions and default on public debt. The government's policy instruments include a distortionary income tax, public spending, debt issuance and a fractional haircut imposed on outstanding debt. Government bonds allow agents to smooth consumption over time and provide collateral and liquidity services that mitigate financial frictions. We show that there exists a *fiscal limit*, i.e., a unique threshold level of debt such that the government fully services outstanding debt below the threshold and fractionally defaults above it. In a calibrated economy the fiscal limit corresponds to a debt-to-output ratio of around 94% and the haircut imposed at the limit is around 40%. Steady state debt amounts to 84% of output. For the model to generate these empirically plausible predictions, both reputational costs of default and contemporaneous costs due to lost collateral and liquidity are critically needed.

JEL *classification*: E44, E62, H21, H63

Keywords: sustainability; financial frictions; sovereign default; domestic debt; endogenous haircut.

*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Oesterreichische Nationalbank or the European System of Central Banks.

[†]Department of Economics, University of Essex, United Kingdom. E-mail: sniem@essex.ac.uk

[‡]Corresponding author. Economic Studies Division, Oesterreichische Nationalbank, Austria. E-mail: paul.pichler@oenb.at

1 Introduction

The sustainability of public debt has become a serious concern to investors and policy-makers during the recent financial crisis. Fears of sovereign default and the associated rising borrowing costs have forced several European countries into severe fiscal austerity measures. In the United States, concerns about the sustainability of public debt have featured prominently in the recent debate on the *fiscal cliff*. They are also prevalent in Japan, which faces the highest debt-to-GDP ratio among OECD countries.

The countries referred to above are developed economies where a substantial fraction of government debt is held domestically. The recurrent concerns about their debt sustainability indicate that the incentives of governments to default on debt held by domestic residents are not yet well understood. This is partly due to the scarcity of recent historical default episodes in advanced economies, but also due to the scarcity of relevant theoretical quantitative studies. Indeed, the literature on sovereign default has mainly studied the sustainability of *external* debt in developing economies – the empirically relevant case before the crisis.¹ The (normative) fiscal policy literature, in turn, has focused on the determination of taxes and domestic government debt in environments with and without commitment, yet abstracting from the possibility of outright government default.² The present paper addresses this shortcoming by introducing fractional default as a policy instrument into a model of optimal discretionary fiscal policy.

Our core framework is the classic closed-economy model of Lucas and Stokey (1983) augmented with endogenous government consumption spending as in Debortoli and Nunes (2013). We extend this core model in two directions. First, we allow the government to decide, in each period, on the fraction of outstanding debt it repays. Second, we introduce moral hazard frictions into the economy’s production sector. As a consequence, firms must finance their wage bill in advance using collateralized loans; and the scale of profitable investment projects is limited by entrepreneurs’ access to external finance. The existence of these frictions generates a role for government debt as *collateral* and, when it is tradable on a secondary market, as *private liquidity*. Hence, besides allowing risk-averse agents to smooth consumption over time,

¹Prominent examples include Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008).

²Important papers are Barro (1979), Lucas and Stokey (1983), Aiyagari, Marcet, Sargent, and Seppala (2002), Klein, Krusell, and Rios-Rull (2008).

government bonds mitigate the financial frictions in the economy.

Our analysis of Markov-perfect optimal fiscal policy proceeds in two steps. In a first step, we examine optimal discretionary tax, spending and debt issuance policies in the presence of financial frictions, while still assuming that the government is committed to fully honor its outstanding debt. This allows us to examine in detail how the collateral and liquidity demand for government debt shapes optimal fiscal policies and affects the determination of steady state debt. We find that, in the presence of financial frictions, the steady state level of debt is strictly positive, unlike in earlier models that predict negative or zero long-run debt (Aiyagari, Marcet, Sargent, and Seppala, 2002; Debortoli and Nunes, 2013). In a calibrated economy steady state debt amounts to 84% of output and the banking sector's demand for collateral constraint is fully satiated, which shows that, at the margin, debt accumulation is driven by the liquidity demand for government bonds.

In a second step, we introduce outright default as an admissible policy instrument. This requires careful consideration of the costs of default. Since public debt plays an essential role as collateral and as a source of liquidity, default leads to endogenous repercussions for financial intermediation; the induced default costs are proportional to the size of the imposed haircut. In line with the open-economy literature on sovereign debt (e.g. Arellano, 2008) we also postulate reputational fixed costs resulting from the government's temporary market exclusion. Unlike this literature, however, the government is not forced to run a balanced budget during an exclusion spell. The government can still issue debt, but this debt is not tradeable on secondary markets and hence not liquid. This loss in liquidity hampers entrepreneurial activity and raises the government's borrowing costs during a default episode. The government's Markov-perfect default policy balances the costs of default against the additional tax distortions under full repayment. Since the latter are increasing in debt, there is a maximum sustainable level of debt, a *fiscal limit*. In our calibrated economy this limit corresponds to more than 90% of output. For debt levels exceeding the fiscal limit the discretionary government exercises its default option. The optimal haircut at the fiscal limit is approximately 40%, which is roughly in line with empirical evidence on sovereign haircuts discussed in Cruces and Trebesch (2013).

Our work is related to a number of recent papers that study the determination of public

debt under optimal discretionary fiscal policy, though without default. In a model without capital and with exogenous government expenditure, Krusell, Martin, and Rios-Rull (2006) uncover a multiplicity of steady states that depend on initial conditions and are thus similar to those under full commitment. Considering endogenous government expenditure instead, Debortoli and Nunes (2013) establish convergence to zero long-run debt as a robust outcome driven by the government's interest rate manipulation motive.³ Our model nests their economy as a special case and inherits a generalized interest rate manipulation motive. Moreover, we consider outright default as part of the optimal policy mix and study the sustainability of public debt.

This focus on debt sustainability is also central to the vast literature on external sovereign debt and default. There, debt is held externally, fiscal policy is largely absent, governments decide about default in a discretionary fashion, and costs of default are exogenous. Notable recent exceptions include the studies by Cuadra, Sanchez, and Sapriza (2010) who examine the role of fiscal policy, Mendoza and Yue (2012) who assess business cycle implications in an environment with endogenous default costs, and Adam and Grill (2012) who analyze optimal sovereign default as the solution to a Ramsey plan.

Strategic default on domestic government debt has recently been studied by D'Erasmus and Mendoza (2012), Juessen and Schabert (2012), Sosa-Padilla (2012) and Pouzo (2013). However, different from our paper, D'Erasmus and Mendoza (2012) focus on redistributive implications. Juessen and Schabert (2012) consider a setup with risk-neutral agents and exogenous default costs. Similar to our approach, Sosa-Padilla (2012) invokes a working capital constraint to generate endogenous default costs, but the government's default decision is binary and debt is again priced by risk-neutral agents. Finally, Pouzo (2013) proceeds under the assumption that the government can commit to its tax policy but not to the repayment of outstanding debt; as in our model, default triggers a temporary breakdown of the primary bond market, but debt continues to be traded on secondary markets and hence retains a positive valuation in anticipation of a future recovery of the primary market.

³The effects of lack of commitment have also been studied in models with capital (Klein, Krusell, and Rios-Rull, 2008; Ortigueira, Pereira, and Pichler, 2012) and in monetary economies where nominal government debt is a source of dynamically inconsistent incentives (Ellison and Rankin, 2007; Diaz-Gimenez, Giovannetti, Marimon, and Teles, 2008; Martin, 2009).

Finally, the consideration of the role of public debt and endogenous default costs in the presence of financial frictions connects our paper to models with incomplete markets in the tradition of Aiyagari (1994). Woodford (1990) and Holmstrom and Tirole (1998) show how public debt can help to relax financial constraints, while Aiyagari and McGrattan (1998) and Angeletos, Collard, Dellas, and Diba (2013) explore implications for optimal policy under commitment. Brutti (2011) and Gennaioli, Martin, and Rossi (2013) study sovereign default in three-period economies where sovereign default destroys firms' ability to insure against idiosyncratic shocks or the balance sheets of domestic banks, respectively. They find that financial frictions can render sizeable government debt positions sustainable even in the absence of reputational costs of default. Our paper re-examines these findings in a fully dynamic environment which allows to analyze the determination of long-run debt and shows that reputational fixed costs of default are critical to generate sustainability of public debt under fractional default. This result obtains despite the important role of public debt for production in our model.

The rest of this paper is organized as follows. In Section 2 we lay out our model economy. In Section 3 we examine Markov-perfect optimal fiscal policy while maintaining the assumption of commitment to full debt repayment. In Section 4 we introduce the option of fractional default. In Section 5 we study the quantitative implications of optimal fiscal policies in a calibrated economy. In Section 6 we conclude.

2 The Environment

The economy is populated by households, firms and a government. There is a single non-storable output good, which is either consumed by households or transformed at a unitary rate into a public good by the government. The government can commit to fully repay outstanding debt, but it lacks inter-temporal commitment to its choices for the income tax rate, public spending and debt issuances.⁴

Households. There is a continuum of measure one of identical, infinitely-lived households.

⁴The assumption of commitment to full debt repayment will be relaxed in Section 4.

The preferences of a representative household $j \in [0, 1]$ are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^j, 1 - n_t^j, g_t), \quad (1)$$

where $\beta \in (0, 1)$ is a time discount factor, c_t^j and n_t^j denote consumption and labor effort of household j , and g_t denotes the level of public good provision. The period utility function $u(\cdot)$ is assumed to be additively separable in its three arguments and twice continuously differentiable, with partial derivatives $u_c > 0$, $u_{cc} < 0$, $u_l > 0$, $u_{ll} \leq 0$, $u_g > 0$ and $u_{gg} \leq 0$.

Each household is composed of three types of members: workers, bankers and entrepreneurs.⁵ Workers supply labor to competitive firms; the other agents either become bankers or get access to an entrepreneurial investment technology. The assignment to these two activities is stochastic; an individual agent becomes banker with probability $1 - \theta$ and entrepreneur with probability θ , respectively. Household j enters period t with a stock of b_t^j government bonds. Initially, all bonds are held by bankers and entrepreneurs, each of them holding the same amount b_t^j . Then, the household members separate, and individuals learn their type (banker or entrepreneur) before the government's policy decisions are announced.

Workers and firms. Workers supply their labor services n_t , taking the wage rate w_t as given. Firms are perfectly competitive and have access to a production technology that transforms labor into consumption goods at a unitary rate. Specifically, the technology allows the representative firm to produce

$$y_t^1 = \tilde{n}_t, \quad (2)$$

where \tilde{n}_t denotes labor hired by the firm. Production is subject to a moral hazard problem which, in the absence of monitoring, makes it impossible for firms to pledge funds to workers and outside creditors. Firms must therefore finance their wage bill in advance, and they can do so using intra-period loans from financial intermediaries (bankers).

Bankers. Bankers act as delegated monitors. In order to meet the firms' demand for working

⁵Each household comprises a continuum $[0, 1]$ of workers and a continuum $[0, 1]$ of agents who become either bankers or entrepreneurs.

capital, they issue deposits contracts, d_t , to outside creditors (i.e., to workers from households other than their own; cf. Gertler and Karadi, 2011). However, although banks have a greater capacity to pledge funds to outside creditors, they are also subject to moral hazard. They can therefore only issue deposits if they are able to post collateral to cover at least a fraction $\xi^c \in (0, 1)$ of the amount issued. Government bonds are the sole source of collateral available to bankers, such that the collateral constraint facing a representative banker from household j is given by

$$d_t^j \leq \frac{b_t^j}{\xi^c}, \quad (3)$$

where d_t^j denotes the deposits issued. Note that the banking sector is competitive, and hence working capital loans do not carry a positive interest rate unless the supply of loans is depressed by the bankers' availability of collateral. Aggregating across firms and bankers, equilibrium in the bank-intermediated market for working capital loans implies that the economy's aggregate wage bill is constrained by

$$w_t \tilde{n}_t \leq \frac{(1 - \theta)b_t}{\xi^c}. \quad (4)$$

Entrepreneurs. Entrepreneurs have access to a profitable investment technology. Specifically, they can invest in projects that deliver a gross return $R > 1$ per unit of investment (both in consumption goods). Denoting by X_t^j the investment scale of the representative entrepreneur from household j , the investment technology is characterized by

$$y_t^{j,2} = R X_t^j. \quad (5)$$

Similar to the operation of banks, there is a moral hazard problem that limits entrepreneurs' access to external finance. As a consequence, internal investment, x_t^j , is necessary to attract external funds, e_t^j . External funds take the form of intra-period loans from workers and bankers that pay zero interest as there is no discounting within the period. To raise the consumption goods required for internal investment, entrepreneurs sell their liquid assets (government bonds) on the secondary market; hence, $x_t^j = z_t b_t^j$, where z_t denotes the bond's market price. They

then augment their internal funds by acquiring external funds subject to the constraint

$$e_t^j \leq \frac{x_t^j}{\xi^l}, \quad (6)$$

where $\xi^l \in (0, 1)$. Constraint (6) is always binding when $R > 1$, resulting in an investment scale of $X_t^j = \frac{1+\xi^l}{\xi^l} z_t b_t^j$ per entrepreneur.

Aggregation. After production in the competitive and entrepreneurial sector has taken place, workers, bankers and entrepreneurs transfer their earnings back to the household. Consumption-savings decisions are then made at the household level; hence there is perfect consumption insurance within households. Aggregating over household members, the total income of household j in period t is the sum of the wage income earned by workers, overall profits between firms and bankers in the competitive sector, and entrepreneurs' net return from investment,

$$I_t^j = w_t n_t^j + (1 - w_t) \tilde{n}_t^j + \theta(R - 1) \frac{1 + \xi^l}{\xi^l} z_t b_t^j. \quad (7)$$

Note that (7) does not include income from maturing government debt b_t^j . The household's budget constraint is given by

$$c_t^j + q_t b_{t+1}^j \leq (1 - \tau_t) I_t^j + b_t^j, \quad (8)$$

where τ_t is a proportional income tax and q_t denotes the price of a newly issued government bond that promises one unit of wealth in the beginning of $t + 1$.

The government. The government is benevolent and maximizes the utility (1) of the representative household. Its policy tools are the income tax τ_t , the level of public good provision g_t , and the issuance of new debt B_{t+1} .⁶ The government's budget constraint is given by

$$g_t + B_t \leq \tau_t I_t + q_t B_{t+1}. \quad (9)$$

The government cannot commit to a fixed policy path over time. It can, however, make credible policy announcements within a given time period. This timing structure implies that

⁶In Section 4 we will introduce the haircut on outstanding debt as an additional policy tool.

the government is a Stackelberg leader vis-à-vis the private sector.

3 Markov-perfect optimal fiscal policy without default

Under lack of commitment, the government in a given time period can choose policy variables for that period but it cannot control policy variables for the future. To characterize the optimal policies we adopt a *primal approach*. Accordingly, the incumbent government directly chooses consumption c , labor n , and debt issuance b' for the current period, taking as given the policy rules $\{\hat{c}, \hat{n}, \hat{b}\}$ employed by future governments, and subject to the requirement that its choices are consistent with a private-sector equilibrium.

Private-sector equilibrium. Households in our model are atomistic and take prices $(w_t, z_t, q_t)_{t=0}^{\infty}$ and policies $(\tau_t, g_t, B_{t+1})_{t=0}^{\infty}$ as given. They choose consumption, labor supply, labor demand, and savings to maximize their objective function (1). Adopting recursive notation and dropping the superscript j , the optimization problem faced by the representative household reads

$$\begin{aligned} \tilde{V}(b; \tau, g, B) = & \max_{c, n, \tilde{n}, b'} u(c, 1 - n, g) + \beta \tilde{V}(b'; \tau', g', B') \\ & - \lambda \left(c + qb' - (1 - \tau) \left[wn + (1 - w)\tilde{n} + \theta(R - 1) \frac{1 + \xi^t}{\xi^t} zb \right] - b \right) \\ & - \mu \left(w\tilde{n} - \frac{(1 - \theta)b}{\xi^c} \right). \end{aligned}$$

From the first-order conditions associated with this problem, it is straightforward to show that the households' policies in a private-sector equilibrium satisfy the Euler equation

$$q = \beta \frac{u'_c}{u_c} (1 + \pi' + \phi') \quad (10)$$

and the budget constraint

$$u_c c + \beta u'_c (1 + \pi' + \phi') b' = \frac{u_l}{w} n + u_c (1 + \pi) b, \quad (11)$$

where

$$\pi = \theta(1 - \tau)(R - 1)\frac{1 + \xi^l}{\xi^l}, \quad (12)$$

$$\phi = (1 - \theta)(1 - \tau)\frac{(1 - w)}{w\xi^c}, \quad (13)$$

denote the liquidity and the collateral premia on government bonds, respectively.⁷

The optimal fiscal policy problem. Inspection of the private-sector equilibrium conditions shows that the aggregate state vector in our model consists of only one variable, b . The policy rules $\{\hat{c}, \hat{n}, \hat{b}\}$ are thus of the form $c = \hat{c}(b)$, $n = \hat{n}(b)$, and $b' = \hat{b}(b)$. Via households' optimal consumption-leisure choice and equations (12) and (13) these rules further imply decision rules for the tax rate, $\hat{\tau}(b)$, the liquidity premium, $\hat{\pi}(b)$, and the collateral premium, $\hat{\phi}(b)$, respectively. Plugging these functions into Euler equation (10), we can write the bond pricing function as

$$Q(u_c, b') = \beta \frac{u_c(\hat{c}(b'))}{u_c} \left(1 + \hat{\pi}(b') + \hat{\phi}(b')\right). \quad (14)$$

Note that, as households have a finite intertemporal elasticity of substitution, the bond price depends on the current and future marginal utility of consumption.⁸ Note also that the wage rate falls below labor productivity if firms' access to working capital loans is strictly constrained by the bankers' pledgeable collateral (cf. equation (13)). This allows us to write the wage rate as a function

$$\omega(b, n) = \begin{cases} 1 & \text{if } (1 - \theta)b > \xi^c n \\ \frac{(1 - \theta)b}{\xi^c n} & \text{otherwise.} \end{cases} \quad (15)$$

Using the aggregate resource constraint to substitute for public consumption in the household utility function (1), the discretionary government's optimization problem under commitment

⁷The Euler equation (10) highlights the three roles played by government bonds in our model: (i) they allow households to shift consumption over time; (ii) they provide liquidity and hence allow households to increase entrepreneurial investment; and (iii) they are a source of collateral to bankers.

⁸This property sets our model apart from related papers that determine bond prices on the basis of quasilinear utility, thus eliminating the government's interest rate manipulation motive.

to full debt repayment is then given by

$$V(b) = \max_{c,n,b'} u(c, 1 - n, n + rb - c) + \beta V(b') \quad (16)$$

$$+ \gamma \left(u_c c + u_c Q(u_c, b') b' - \frac{u_l}{\omega(b, n)} n - u_c (1 + \pi) b \right),$$

where γ is a non-negative Lagrangian multiplier and $V(b')$ is the continuation value function.

Definition 1. A Markov-perfect equilibrium is a set of policy functions $\mathcal{P} = \{\hat{c}, \hat{n}, \hat{b}\}$, a value function V and a bond pricing function Q such that:

- (i) given the value function V and the bond pricing function Q , the policy functions $\{\hat{c}, \hat{n}, \hat{b}\}$ solve the government's optimization problem (16);
- (ii) given the policy functions $\{\hat{c}, \hat{n}, \hat{b}\}$, the bond pricing function Q satisfies (14);
- (iii) given the policy functions $\{\hat{c}, \hat{n}, \hat{b}\}$, the value function satisfies the Bellman equation

$$V(b) = u(\hat{c}(b), 1 - \hat{n}(b), \hat{n}(b) + rb - \hat{c}(b)) + \beta V(\hat{b}(b)).$$

The following proposition characterizes the optimal debt policy in a Markov-perfect equilibrium.

Proposition 1. In a Markov-perfect equilibrium, the optimal debt policy satisfies the generalized Euler equation

$$\gamma' \left\{ u'_c (1 + \pi') - \frac{u'_l n'}{(w')^2} \omega'_1 \right\} - u'_g r = \gamma u'_c (1 + \pi' + \phi') \{1 + \varepsilon_b^q\}, \quad (17)$$

where $r = \theta(R - 1) \frac{1 + \xi^l}{\xi^l}$ and where ε_b^q denotes the elasticity of the bond price q with respect to changes in debt issuance b' .

The generalized Euler equation (GEE) equates the marginal cost of entering the next period with a higher stock of outstanding debt to the marginal benefit of relaxing implementability constraint (11) via issuing additional debt. For an economy without any role for government

debt as collateral or liquidity, the case studied by Debortoli and Nunes (2013), the GEE simplifies to

$$\gamma' = \gamma \left(1 + \varepsilon_{b'}^q \right). \quad (18)$$

A steady state in their model is hence characterized by either $\gamma^* = 0$ or $\varepsilon_{b'}^q{}^* = 0$. The first case corresponds to an undistorted steady state where the government holds enough assets to implement the first-best allocation. The second case corresponds to a distorted steady state where either the bond price is locally invariant to changes in debt, or $b^* = 0$ such that changes of the bond price do not have budgetary effects. Debortoli and Nunes (2013) show with a simple analytical example, as well as more general numerical examples, that steady states with $b^* = 0$ and $\hat{c}_b(b^*) > 0$, that is, a locally increasing consumption policy function, are generic in their economy.⁹ This result is rooted in the *interest rate manipulation motive* faced by the government under lack of commitment.

In our generalized model, where government bonds provide liquidity and collateral services, there is no longer convergence to zero long-run debt. This is a trivial result under a collateral role for government debt, for otherwise zero debt would imply zero production. But positive debt also emerges when the production sector is not subject to a collateral constraint, $\xi^c = 0$, as long as there is a liquidity role for debt.

Proposition 2. *If government bonds provide liquidity services, $\xi^l > 0$ and $r > 0$, the steady state features a strictly positive level of government debt, $b^* > 0$.*

The underlying intuition is best understood as follows. If public debt has a role as private liquidity, the bond price includes a liquidity premium,

$$Q(u_c, b') = \beta \frac{u_c(b')}{u_c} (1 + \hat{\pi}(b')). \quad (19)$$

The incumbent government seeks to increase current bond prices via manipulation of the future state b' . This is achieved for changes in b' which induce an increase in $u_c'(1 + \pi')$. While u_c'

⁹Key to the emergence of an increasing consumption policy function is the fact that government expenditure is endogenous.

is decreasing in $\hat{c}(b')$, the opposite is true for the liquidity premium since $\pi_c = -\pi \frac{u_{cc}}{u_c} > 0$. Accordingly, there are conflicting motives for the manipulation of $\hat{c}(b')$ because, given $\hat{c}_b(b') > 0$, a decumulation of debt increases future marginal utility u'_c but decreases the future liquidity premium π' . These conflicting motives balance each other at a positive level of debt.

The role of government debt in mitigating financial frictions also suggests that the accumulation of moderate levels of debt may have positive welfare effects. Indeed, the following proposition shows that, at low levels of debt, welfare is generically increasing in b .

Proposition 3. *If government bonds serve as collateral, $\xi^c > 0$, or provide liquidity, $\xi^l > 0$, under a sufficiently high return to investment in the entrepreneurial sector,*

$$r > \frac{u_c}{u_l} \left(\frac{1 - \frac{u_l}{u_g}}{\frac{u_l}{u_g} - \frac{u_{ll}}{u_l} n} \right), \quad (20)$$

the accumulation of moderate levels of debt has positive welfare effects. Without financial frictions, i.e., if $\xi^c = 0$ and $r = 0$, welfare is monotonically decreasing in debt.

For the government's value function to be increasing in debt, the marginal benefit from relaxing the collateral constraint and from increased liquidity must exceed the marginal cost from increased taxation. The marginal benefit is generically (weakly) decreasing in the level of debt, while the marginal cost due to increased tax distortions is strictly increasing in debt. Accordingly, the value function in our model has an inverted U-shape.

Tightly linked to the shape of the value function is the emergence of a *debt Laffer curve*, i.e., a situation where a marginal increase in the quantity of debt issued is associated with a reduction in the revenue for the government from that operation.

Proposition 4. *The optimal debt policy is subject to a Laffer curve: When marginal debt has positive (negative) welfare effects, the economy is on the 'bad' ('good') side of the debt Laffer curve.*

A debt Laffer curve emerges whenever bond prices fall strongly in the amount of debt issued, $\varepsilon_{b'}^q < -1$. But this happens precisely when debt is scarce – and therefore valuable – so that collateral and liquidity premia are both relevant and highly sensitive to the amount of debt.

4 Markov-perfect optimal fiscal policy with default

We now introduce the option of fractional default, i.e., the government can decide in a discretionary manner on the fraction $\rho \in [0, 1]$ of outstanding debt it repays. At the same time, and in line with the sovereign default literature, we introduce reputational costs of default. Following a default, the government is temporarily excluded from the primary bond market, and outstanding bonds can no longer be traded on the secondary market. The duration of the market exclusion is stochastic; with a constant probability α an excluded government can re-access the bond market in the next period. However, we assume that during the bond market exclusion the government can still issue debt in the form of loans. Loans serve as collateral but are not tradeable on secondary markets and hence not a source of liquidity.¹⁰ The loss in liquidity hampers entrepreneurial activity and raises the government's borrowing cost during a default episode. Finally, note that the costs via reduced collateral depend on the size of the implemented haircut, whereas the repercussions of market exclusion are of a fixed cost nature.

The optimal fiscal policy problem under fractional default. It is convenient to cast the incumbent government's optimal policy problem under the option to default as a two-stage decision problem. The government first decides whether or not to repay the entirety of its outstanding debt. Conditional on this decision, the government then chooses its relevant policy instruments. Define $V^o(b)$ as the value function for a government that has the option to default and starts the current period with b outstanding bonds. This value function satisfies

$$V^o(b) = \max\{V^{nd}(b), V^d(b)\}, \quad (21)$$

where $V^{nd}(b)$ is the value conditional on full repayment ($\rho = 1$) and $V^d(b)$ is the value condi-

¹⁰Consistent with this assumption is the empirical evidence presented in Bai, Julliard, and Yuan (2012). These authors analyze Eurozone sovereign bond markets in the period 2006-2012 and find that secondary market liquidity has been significantly reduced during the recent crisis, with markets basically drying up in countries that received a bailout (Greece and Portugal). Note also that the complete exclusion also from the primary market for debt considered in the literature (cf. Arellano, 2008) has the counterfactual implication of zero outstanding debt following a default. Introducing a primary market for loans is one way to address this concern.

tional on default ($\rho < 1$). The no-default value function is the solution to

$$V^{nd}(b) = \max_{c,n,b'} u(c, 1 - n, n + rb - c) + \beta V^o(b') \quad (22)$$

$$+ \gamma \left(u_c c + u_c Q^b(u_c, b') b' - \frac{u_l}{\omega(b, n)} n - u_c (1 + \pi) b \right),$$

where $\omega(b, n)$ and $Q^b(u_c, b')$ are the pricing functions for labor and newly issued bonds, respectively. The government's value function under default is given by

$$V^d(b) = \max_{\rho \in [0,1]} \tilde{V}^d(\rho b), \quad (23)$$

where

$$\tilde{V}^d(\rho b) = \max_{c,n,\ell'} u(c, 1 - n, n - c) + \beta W^o(\ell') + \gamma \left(u_c c + u_c Q^\ell(u_c, \ell') \ell' - \frac{u_l}{\omega(\rho b, n)} n - u_c \rho b \right)$$

is the value function conditional on a given repayment rate $\rho < 1$, and ℓ' and $Q^\ell(u_c, \ell')$ denote newly issued loans and the underlying pricing function, respectively. This formulation makes clear that what ultimately matters for allocations and welfare is the *effective state* ρb . Since this state can be regulated via the repayment policy $\rho \in [0, 1]$, the default value function $V^d(b)$ is necessarily *non-decreasing* over the entire state space. Specifically, $V^d(b)$ is increasing whenever the optimal policy prescribes full debt repayment, and constant whenever it prescribes partial default. Finally, $W^o(\ell)$ is the value function of a government that starts the period with ℓ outstanding loans,

$$W^o(\ell) = \alpha \max\{W_a^{nd}(\ell), W^d(\ell)\} + (1 - \alpha) \max\{W_e^{nd}(\ell), W^d(\ell)\}, \quad (24)$$

where $W_a^{nd}(\ell)$ is the value function conditional on full repayment of a government that regains access to the bond market in the beginning of the period, $W_e^{nd}(\ell)$ is the no-default value function of a government that remains excluded from the bond market, and $W^d(\ell)$ is the value function conditional on default.¹¹

¹¹It is not necessary to index $W^d(\ell)$ by a or e , since default precludes the current government's option of immediate bond market access. The value functions in (24), along with the optimal default policy for loans, are formally characterized in the Web Appendix, which also provides a formal definition of the Markov-perfect

The following proposition characterizes the optimal default policy when the government has access to the bond market.

Proposition 5. *The government's optimal default policy is of a fiscal limit type, i.e., the government optimally defaults if and only if its inherited stock of bonds exceeds a threshold level \bar{b} . The optimal repayment policy takes the form*

$$\hat{\rho}(b) = \begin{cases} 1 & \text{if } b \leq \bar{b} \\ \underline{b}/b & \text{if } b > \bar{b} \end{cases}$$

where \underline{b} is the lowest level of (effective) debt that maximizes post-default welfare, $\underline{b} = \arg \max_b \tilde{V}^d(b)$. At \underline{b} the collateral constraint is strictly binding.

Equivalent results are available for the case where the government is excluded from the bond market and outstanding debt is in the form of loans. The intuition behind Proposition 5 is readily seen (see also Figure 2 below). The value function conditional on default, $V^d(b)$, is non-decreasing over the entire state space and constant whenever the optimal policy prescribes $\rho < 1$. Denote this constant level of welfare by \bar{V}^d . The value function conditional on full repayment, $V^{nd}(b)$, has an inverted U-shape. Hence there exists a unique default threshold \bar{b} , implicitly defined via $V^{nd}(\bar{b}) = \bar{V}^d$. For higher levels of debt, $b > \bar{b}$, the optimal haircut reduces effective debt to the lowest level that maximizes post-default welfare. As a result of balancing the marginal benefits and costs of default, this post-default level of effective debt \underline{b} necessarily induces a strictly binding collateral constraint. The role of public debt in providing collateral and liquidity services is thus an important force in disciplining the discretionary government's default incentives. But a government that exercises its default option will always find it optimal to make the post-default level of debt so scarce that financial intermediation is hampered.

Based on Proposition 5 we can formally examine the sustainability of optimal fiscal policy in the face of strategic default incentives.

Definition 2. *Given a state space \mathbb{B} , a Markov-perfect optimal fiscal policy \mathcal{P} is sustainable over the region $\tilde{\mathbb{B}} \subset \mathbb{B}$ if*

equilibrium under the option to default.

- (i) the incumbent government finds it optimal to employ the policy \mathcal{P} and to fully honor inherited debt for all $b \in \tilde{\mathbb{B}}$ when it perceives that all future governments will employ the policy \mathcal{P} and fully honor inherited debt in $\tilde{\mathbb{B}}$; and
- (ii) the debt policy $\hat{b} \in \mathcal{P}$ is ergodic, i.e., it satisfies $\hat{b}(b) \in \tilde{\mathbb{B}}$ for all $b \in \tilde{\mathbb{B}}$.

Two comments are in order. First, if there exists a sustainable region $\tilde{\mathbb{B}}$, then the Markov-perfect optimal fiscal policy under the option to default coincides with the Markov-perfect optimal fiscal policy under commitment to full repayment over this region. Second, under our concept of sustainability, default does not occur in equilibrium if the initial level of debt is in a sustainable region. We now examine the sustainability of optimal fiscal policy in the complete absence of reputational fixed costs.

Proposition 6. *Absent reputational costs of default, no Markov-perfect optimal fiscal policy is sustainable over regions on the ‘good’ side of the debt Laffer curve.*

To understand the underlying intuition, notice that the value function $V^{nd}(b)$ inherits the properties discussed in Proposition 4: It is *increasing* in debt on the ‘bad’ side of the Laffer curve and *decreasing* on the ‘good’ side. Denote by \bar{V} the maximum of $V^{nd}(b)$ and by b^{**} the level of debt at which the maximum is attained, i.e., $V^{nd}(b^{**}) = \bar{V}$. It is then straightforward to see that the default value function in the absence of reputational costs satisfies $V^d(b) = V^{nd}(b)$ for all $b \leq b^{**}$ and $V^d(b) = \bar{V}$ for all $b > b^{**}$. Hence the government defaults for all debt levels on the ‘good’ side of the Laffer curve. Reputational costs of default are therefore critically needed to support debt positions that are consistent with conventional pricing of public debt ($\varepsilon_{b'}^q > -1$). Notice also that this result obtains despite our assumptions which assign an important role to public debt as a source of collateral and private liquidity.

5 A calibrated economy

We now examine the key quantitative properties of Markov-perfect optimal fiscal policies in a calibrated economy.

Calibration. We consider an instantaneous utility function u that is additively separable and allows for curvature in all its arguments,

$$u(c, 1 - n, g) = (1 - \omega_g) \left[\omega_c \frac{c^{1-\sigma_c} - 1}{1 - \sigma_c} + (1 - \omega_c) \frac{(1 - n)^{1-\sigma_l} - 1}{1 - \sigma_l} \right] + \omega_g \frac{g^{1-\sigma_g} - 1}{1 - \sigma_g}, \quad (25)$$

where ω_c and ω_g denote preference weights on private and public consumption, and σ_c , σ_l and σ_g are elasticities.¹² We target data at annual frequency and select parameter values as follows. The three elasticities σ_c , σ_l and σ_g are each set to the value 2, which is in the middle of the parameter range typically considered in the macroeconomic literature. The preference weights are chosen such that, in the model's steady state, $g^*/c^* = 0.25$ and $n^* = 0.3$; the resulting values are $\omega_c = 0.15$ and $\omega_g = 0.015$. The collateral parameter is set to $\xi^c = 0.4$, corresponding to a leverage ratio of 2.5. The parameter θ governs the relative importance of production in the competitive and entrepreneurial sectors. Beyond that, the parameters R , θ and ξ^l matter only jointly, as determinants of the return to entrepreneurial investment, $r = \theta(R - 1) \frac{1+\xi^l}{\xi^l}$. The individual parameter values are selected in line with evidence from the Survey of Consumer Finances (SCF). As discussed in Moskowitz and Vissing-Jorgensen (2002), the SCF reports a median of the distribution of capital gains in private business investment of roughly 7%. This motivates our choice of $r = 0.07$. For simplicity, we set $\theta = 0.25$ and $\xi^l = \xi^c = 0.4$, implying $R = 1 + \frac{r\xi^l}{\theta(1+\xi^l)} = 1.08$. We choose the discount factor $\beta = 0.92$ to match an annual risk-free real interest rate of about 3% in the presence of a steady state liquidity premium. Finally, we set $\alpha = 0.5$ which implies that, on average, the bond market is impaired during the default period and the two following periods. This duration of two years is consistent with the empirical evidence reported by Bai, Julliard, and Yuan (2012), and it is also broadly in line with estimates and calibrations reported in the sovereign debt literature (cf. Cruces and Trebesch, 2013; Arellano, 2008; Cuadra, Sanchez, and Sapriza, 2010).

Our parameter choices are summarized in Table 1. For given parameters, we solve the model numerically, using a combination of standard projection and dynamic programming techniques. This approach allows us to handle the non-differentiabilities and non-concavities arising due to

¹²With this utility specification, our model nests the economy of Debortoli and Nunes (2013) as a special case, which allows us to assess the effects of collateral, liquidity and default against a well-defined benchmark.

Table 1: Parameter values

Parameter	Value	Description
σ_c	2	elasticity of private consumption
σ_g	2	elasticity of public consumption
σ_l	2	elasticity of leisure
ω_c	0.15	weight of consumption (priv.+publ.) vs. leisure
ω_g	0.015	weight of public vs. private consumption
ξ^c	0.4	inverse of leverage ratio
r	0.07	return to entrepreneurial investment
θ	0.25	share of entrepreneurial investors
ξ^l	0.4	inverse of leverage ratio
R	1.08	gross return on investment projects
β	0.92	discount factor
α	0.5	market reaccess probability

financial constraints and government default.¹³

Steady state. The steady state values of key endogenous variables are presented in Table 2. Output in the competitive sector is roughly equal to $y^{1*} = 0.3$, in line with our calibration target. Value added in the entrepreneurial sector is significantly smaller, $y^{2*} = 0.019$, such that total output is given by $y^* = 0.3220$. Private and public consumption amount to 80% and 20% of total output, respectively ($c^* = 0.2578$, $g^* = 0.0642$). The steady state level of debt is positive as predicted by Proposition 2. In particular, our parameter choices imply a sizeable steady state debt position of $b^* = 0.2712$, which corresponds to a debt-to-GDP ratio equal to 84%. The steady state bond price $q^* = 0.97$ implies an annual interest rate close to our calibration target of 3%. The steady state tax rate is $\tau^* = 22.5\%$. Finally, the collateral constraint is not binding at the steady state, and the wage rate is thus equal to labor productivity, $w^* = 1$. Accordingly, while its exclusive role as collateral makes debt essential, there is endogenous accumulation of debt beyond the satiation point for collateral demand. Thus, at the margin, the driver behind the positive level of steady state debt is its liquidity role. Finally, note that the steady state is in a sustainable region where the government fully repays debt, $\rho = 1$.

Policy functions. The optimal policy functions are displayed in Figure 1.¹⁴ These functions

¹³The Web Appendix provides further details on our computational algorithm.

¹⁴We restrict attention to the case when outstanding debt is in the form of bonds. Plots of the policy functions for the case when outstanding debt is in the form of loans are available in the Web Appendix.

Table 2: Steady state values

Variable	Steady state
y^1	0.3030
y^2	0.0190
y	0.3220
c	0.2578
g	0.0642
b	0.2712
b/y	0.8422
q	0.9699
τ	0.2248
w	1.0000
ρ	1.0000

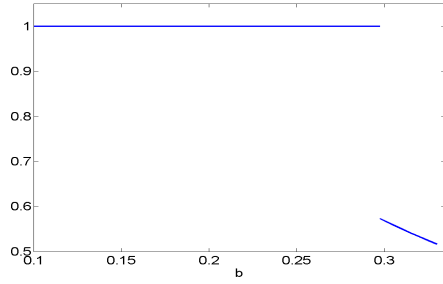
display kinks in the region of the state space where the collateral constraint kicks in as well as discontinuities at the fiscal limit. To discuss the optimal policy functions, we find it convenient to partition the state space $\mathbb{B} = [b_{min}, b_{max}]$ into four regions that differ significantly in how optimal policies react to variations in the inherited debt level. In the first region, $\mathbb{B}^1 = [b_{min}, b_1)$, debt is so scarce that the collateral constraint is strictly binding. In the second region, $\mathbb{B}^2 = [b_1, b_2)$, the collateral constraint is non-binding under optimal policies, but its existence nevertheless affects the government's optimal policy trade-offs. In the third region, $\mathbb{B}^3 = [b_2, b_3]$, the collateral constraint has no distortionary effects on optimal policies. Finally, whereas the government fully repays debt in regions \mathbb{B}^1 , \mathbb{B}^2 and \mathbb{B}^3 , it fractionally defaults in region $\mathbb{B}^4 = (b_3, b_{max}]$.¹⁵

In region \mathbb{B}^1 the collateral constraint is strictly binding. The scarcity of collateral constrains labor demand, such that the market clearing wage rate falls short of labor productivity ($w < 1$) and output, private consumption and public consumption are depressed. Since the collateral constraint is less stringent the higher the initial debt stock, the policy functions \hat{n} , \hat{w} , \hat{c} and \hat{g} are increasing in b . The same is true for the equilibrium bond price under the optimal debt policy, which prescribes an immediate transition to region \mathbb{B}^3 for all initial debt levels in \mathbb{B}^1 .

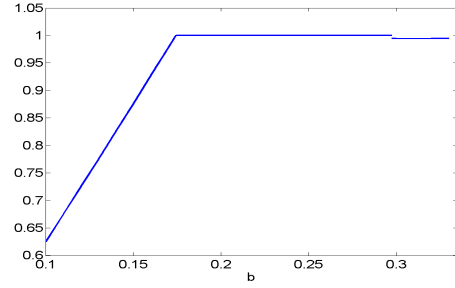
In region \mathbb{B}^2 the collateral constraint becomes non-binding under optimal policies, but its

¹⁵In our calibrated economy, $b_1 = 0.1745$, $b_2 = 0.1800$ and $b_3 = 0.2975$, which corresponds to approximately 54%, 56% and 94% of steady state output, respectively.

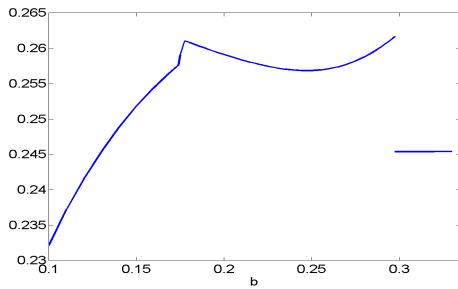
Figure 1: Policy functions



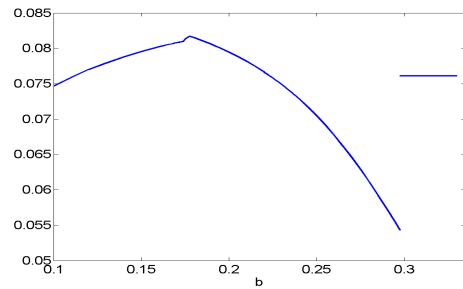
(a) Repayment rate



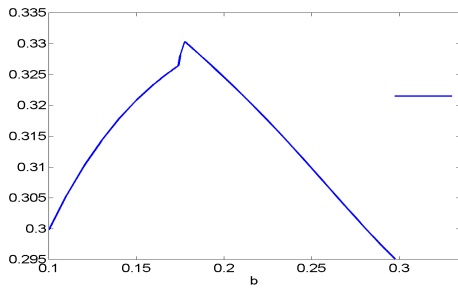
(b) Wage rate



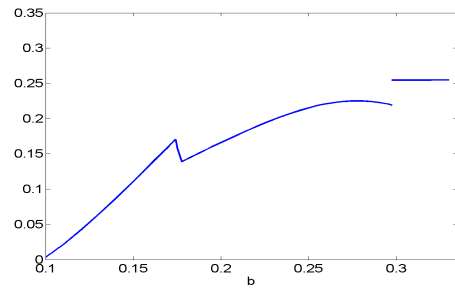
(c) Consumption



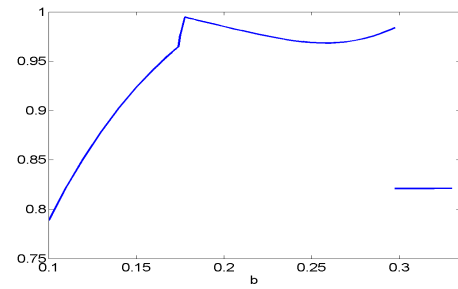
(d) Public spending



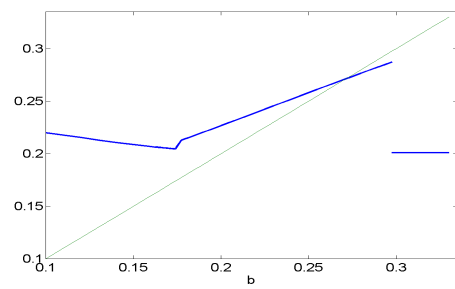
(e) Labor



(f) Tax rate



(g) Price of new debt



(h) Debt issuance

existence still distorts the optimal policy trade-off. In particular, the government relies more on taxation relative to debt issuance to finance public spending. In doing so, it depresses labor supply and prevents the collateral constraint from becoming binding. Hence, taxation is attractive relative to debt issuance because the income tax does not distort the equilibrium labor allocation. This is because, even under lower taxes, labor would still be depressed by the scarcity of pledgeable collateral.

In region \mathbb{B}^3 , which contains the steady state b^* , optimal policies are not affected by the collateral constraint. Public debt thus converges to a level that is sufficient to fully satiate the demand for collateral. The wage rate is constant and equal to labor productivity, $w = 1$. Labor supply and public spending are monotonically decreasing in debt. By contrast, private consumption and the bond price are non-monotonic, which reflects the opposite movement of the two elements $-u_c(b')$ and $\hat{\pi}(b')$ – contained in the bond pricing function (19). Consumption and bond prices are decreasing in b for relatively low levels of debt in \mathbb{B}^3 and increasing in b for high levels of debt. In particular, we have $\hat{c}_b(b^*) > 0$. A reverse pattern is found for the tax rate, which reflects the government’s effort to sustain liquidity premia $\pi = (1 - \tau)r$ and thus bond prices. The debt policy function is increasing in b with a slope below one, indicating that the steady state b^* is stable. Finally, throughout \mathbb{B}^3 , social welfare is monotonically decreasing in b . This illustrates that the adverse tax distortion effect resulting from a higher level of indebtedness dominates the positive liquidity effect.

Finally, in region \mathbb{B}^4 the government defaults on its inherited debt. Independent of the initial stock of debt in this region, the government’s optimal haircut reduces effective debt to the same optimal level \bar{b} ; at the fiscal limit $\bar{b} = b_3$, the haircut amounts to about 40%. The optimal policy functions are thus flat over \mathbb{B}^4 .¹⁶ Government default affects the value of the assets held by private agents directly via the reduced repayment and indirectly via the loss in their liquidity. Households respond to the reduction in the value of their assets by increasing labor supply and reducing private consumption,¹⁷ whereas the government responds to the reduction in its

¹⁶Note that the debt policy in region \mathbb{B}^4 actually refers to the government’s issuance of loans rather than bonds; this is because the government loses access to the bond market when defaulting.

¹⁷The wealth effect on labor supply associated with default is driven by the linear separable preferences in our calibrated economy. It may be overturned by considering GHH-preferences, as is often done in the sovereign debt literature.

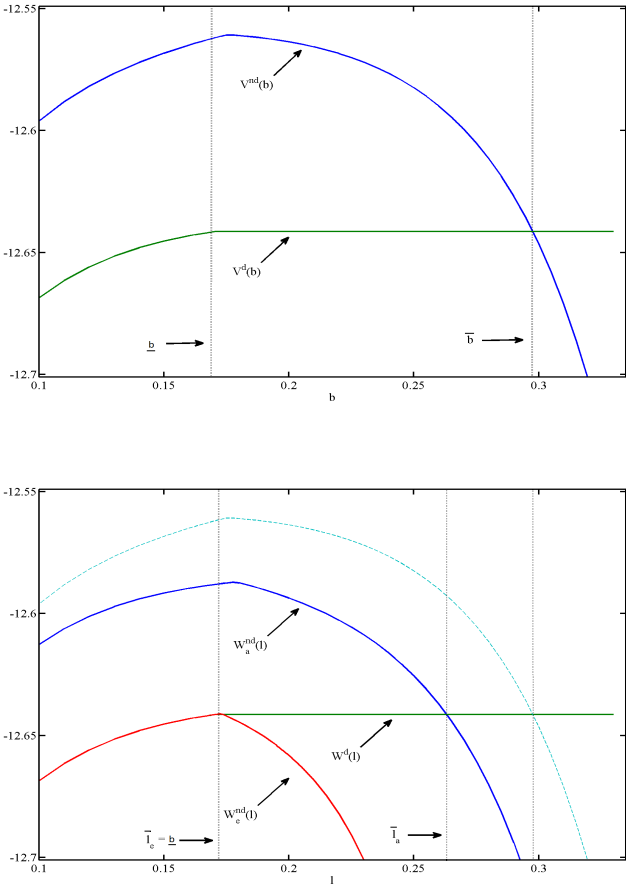
liabilities by increasing public consumption. The higher level of public spending is financed via increased taxes. This is optimal since the labor tax is not distortionary due to the now binding collateral constraint. By contrast, the government has to pay high interest rates on its newly issued debt because, following default, the government is confined to finance itself via loans, which do not carry a liquidity premium and are also subject to significant default risk (see the discussion below).

Welfare. Figure 2 shows the value functions of the government under the option to default. The top panel contrasts the government's value function conditional on no default ($V^{nd}(b)$) and on default ($V^d(b)$) when bond markets are fully operational. Under full repayment, the government's value function is of an inverted U-shape. Under partial default, it is monotonically increasing for low levels of debt and constant from the threshold level $\underline{b} = 0.1705$ onwards. The two value functions intersect at the fiscal limit $\bar{b} = b_3 = 0.2975$, which corresponds to about 94% of steady state output.

The bottom panel of Figure 2 contrasts the government's value functions when outstanding debt is in the form of loans, as formalized in equation (24). The no-default value functions $W_a^{nd}(\ell)$ and $W_e^{nd}(\ell)$ have an inverted U-shape. The value function conditional on default actually coincides with its counterpart when maturing debt is in the form of bonds, $W^d(\ell) = V^d(b)$ for $\ell = b$; this is because default hampers the liquidity of maturing bonds. In analogy to the case of fully operational bond markets, there thus emerge two further default thresholds for maturing loans. Conditional on regaining market access, the government fully honors its debt up to the point where $W_a^{nd}(\ell)$ and $W^d(\ell)$ intersect, which corresponds to $\bar{\ell}_a = 0.2630$ in our calibrated economy. On the other hand, $W_e^{nd}(\ell) \leq W^d(\ell)$ globally, and the inequality becomes strict when $\ell > \underline{b}$; when excluded from the bond market, the government thus fully honors its maturing loans up to this threshold, $\bar{\ell}_e = \underline{b} = 0.1705$.

Figure 3 further illustrates how the three default thresholds $\{\bar{b}, \bar{\ell}_a, \bar{\ell}_e\}$ depend on the market re-access probability α . We observe that \bar{b} and $\bar{\ell}_a$ are both monotonically decreasing in α . This reflects that a higher probability of market re-access lowers the expected cost of the bond market exclusion triggered by default; accordingly, the maximum sustainable level of debt is reduced. Quantitatively, however, an increase in α above our benchmark of $\alpha = 0.5$ has only relatively

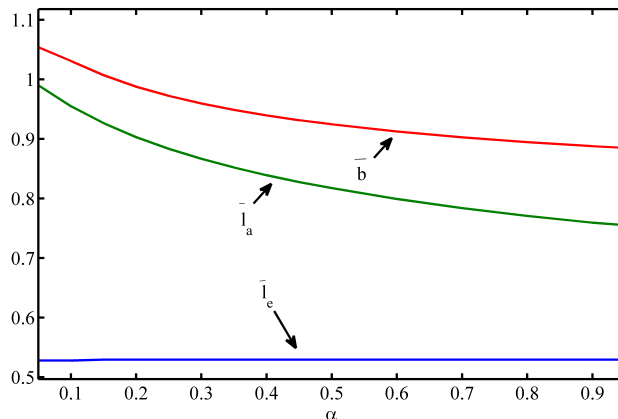
Figure 2: Value functions



minor consequences for the fiscal limit: expressed as a fraction of steady state output, it changes from 94% for $\alpha = 0.5$ to 89% for $\alpha = 0.9$. Finally, the default threshold $\bar{\ell}_e$ is independent of α because the government is already excluded from the bond market and thus incurs only the contemporaneous costs due to the reduction in pledgeable collateral. At the threshold $\bar{\ell}_e$ these costs exactly balance the benefits of default due to reduced tax distortions. As also the benefits are independent of the re-access probability, so is the default threshold $\bar{\ell}_e$.

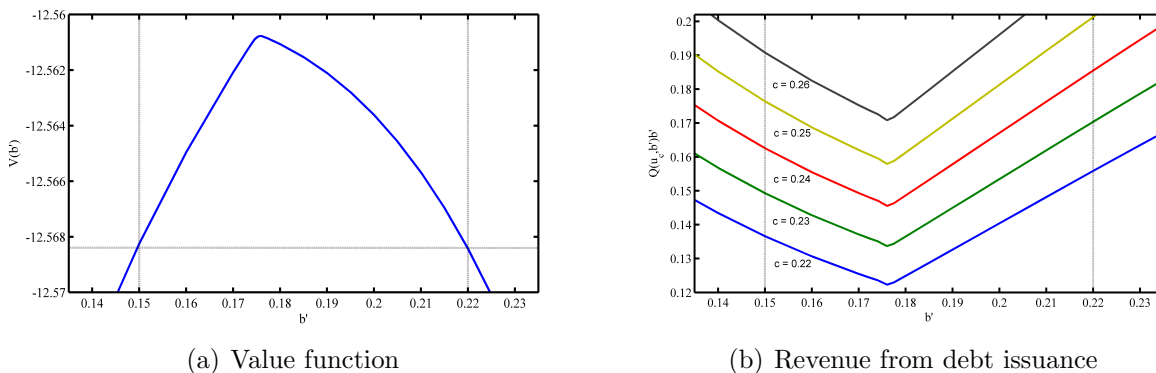
The debt Laffer curve. Inspection of the debt policy function \hat{b} displayed in Figure 1 shows that, independent of the initial debt stock, the government always issues an amount of debt that is sufficient to ensure a non-binding collateral constraint in the future. To understand the intuition behind this finding, first note that, as prescribed by Proposition 3, the social welfare

Figure 3: Default thresholds as a fraction of steady state output



function has an inverted U-shape. Specifically, the welfare function is initially upward-sloping in region \mathbb{B}^1 , where the collateral constraint is strictly binding, and later downward-sloping. Given this shape, for every relevant choice $b' \in \mathbb{B}^1$ there hence exists an alternative choice $\tilde{b}' > b'$ such that $V^{nd}(\tilde{b}') = V^{nd}(b')$. Since b' and \tilde{b}' deliver the same continuation payoff to the government, a necessary condition for b' to be optimal is to generate a higher current revenue from debt creation compared to \tilde{b}' . Formally, given the optimal choice of current consumption, $c = \hat{c}(b)$, the debt issuance b' can be an optimal choice only if $Q(u_c, b')b' > Q(u_c, \tilde{b}')\tilde{b}'$. Figure 4

Figure 4: Welfare and the debt Laffer curve



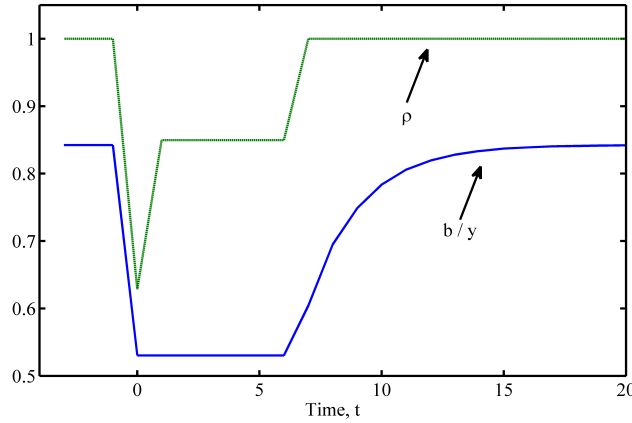
shows that this is not the case in our calibrated economy. In particular, the figure shows that debt choices $b' \in \mathbb{B}^1$ generate a *lower* current revenue than the corresponding choices $\tilde{b}' \in \mathbb{B}^3$.¹⁸

¹⁸As an example, consider an inherited level of debt $b = 0.1$ and the two alternative debt choices $b' = 0.15$

This pattern reflects the *debt Laffer curve* discussed in Proposition 4. Facing declining bond prices associated with suboptimal, low choices of $b' \in \mathbb{B}^1$, the government responds by an aggressive debt policy in order to escape the Laffer curve region. An important corollary to this observation is that, at least in the context of our quantitative results, the statement of Proposition 6 can be further generalized. Specifically, regions on the ‘bad’ side of the debt Laffer curve are not ergodic so that Markov-perfect optimal fiscal policy fails to be sustainable also here.

Post-default dynamics. Given our non-stochastic environment, default never occurs in equilibrium provided the government’s initial level of liabilities does not exceed the fiscal limit. In order to explore the dynamics of public debt following default, we now examine the following out-of-equilibrium event. We assume that in period 0, with initial debt $b_0 = b^*$, the government deviates from its optimal policy by choosing to default (more precisely, we consider the optimal deviation policy conditional on default). Figure 5 traces the subsequent dynamics, assuming optimal behavior from period 1 onwards. In period 0 the government repays only a fraction

Figure 5: Post-default dynamics



$\rho = 0.63$ of outstanding debt. This default triggers the government’s temporary exclusion from the bond market. Underlying the dynamics presented in Figure 5 is a scenario where the exclusion lasts six periods. Thus public debt has no liquidity value for an extended period. This has two consequences. First, as the costs of default remain muted, we observe a pattern of *recurrent*

and $\tilde{b}' = 0.22$. These two choices deliver the exact same continuation welfare level $V^{nd}(0.15) = V^{nd}(0.22) = -12.5685$. Yet, the current revenue from issuing $\tilde{b}' = 0.22$ exceeds the current revenue from issuing $b' = 0.15$ for all possible values of c , including the optimal one at $c \approx 0.23$.

defaults. Second, loans to the government trade only at depressed prices, reflecting the combined effect of correctly anticipated default and their failure to provide liquidity benefits. The government therefore issues less new liabilities, $\ell' < b^*$, but repays them at an increased rate of $\rho = 0.85$. In conjunction, this policy keeps the stock of effective debt $\rho\ell$ constant throughout the period of market exclusion. In detail, the initial default brings effective debt down from 84% to 53% of steady state output; it then remains constant at this level until the government gets access to the bond market again. Once this is the case, the government ‘graduates’ from default, i.e., public debt returns to its steady state level without further defaults occurring.

Interaction of financial frictions and debt sustainability. For our calibrated economy steady state debt is at approximately 84% of output, the default threshold is at 94% of output, and the haircut imposed at this threshold is about 40%. We stress again that the full set of frictions invoked in our model is necessary to generate these empirically plausible statistics. To see this note that our calibration, particularly that of the collateral parameter ξ^c , implies a demand for collateral in the order of 50% of output. For higher levels of debt, the economy’s collateral constraint is slack, which leaves the government facing a trade-off between the liquidity services of increased debt and the associated tax distortions. The liquidity role of government debt is therefore essential to generate a steady state with government liabilities in excess of the level satiating the economy’s collateral constraint.

On the other hand, for sufficiently high levels of debt the liquidity value of government bonds tends to be dominated by the associated tax distortions, resulting in a downward-sloping value function, $V_b(b) < 0$. However, since default via *fractional repayment* of maturing debt amounts to rescaling the effective level of debt, any level of debt such that $V_b(b) < 0$ is not sustainable, unless there is some additional fixed cost of defaulting. The loss in liquidity due to the government’s exclusion from the bond market is therefore critically needed in order to sustain sizeable debt positions.¹⁹

Finally, under reasonable calibrations of the entrepreneurial return r but absent a demand for collateral ($\xi^c = 0$), the value function under default is flat already for very low levels of debt. Conditional on default occurring, this (counterfactual) scenario would induce the government

¹⁹As seen in Figure 3), debt-to-GDP ratios in the order of magnitude of 100% are sustainable under quite moderate average exclusion durations.

to impose a close to 100% haircut. Hence, the costs for production and welfare due to scarce collateral are essential to generate haircuts in the empirically observed range.

6 Conclusion

This paper has provided a quantitative framework to study the joint determinants of government debt and its sustainability in a closed economy subject to financial frictions. Fiscal policy is implemented under lack of commitment, which may extend also to the repayment of maturing government debt. Since debt is held domestically, it is valued as an instrument to smooth consumption, but also as a source of collateral and liquidity. This gives rise to endogenous default costs whose magnitude varies along with the size of the haircut on outstanding debt. Our particular interest is in three statistics for government debt: the steady state level, the maximum sustainable level (fiscal limit), and the optimal haircut rescaling the effective amount of liabilities in case of default. When default triggers the government's temporary exclusion from the bond market, the calibrated economy predicts empirically plausible outcomes for these three statistics. Another interesting feature is the prediction of possibly extended periods of recurrent defaults by governments without access to the bond market.

References

- ADAM, K., AND M. GRILL (2012): "Optimal Sovereign Default," CEPR Discussion Papers 9178, C.E.P.R. Discussion Papers.
- AGUIAR, M., AND G. GOPINATH (2006): "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 69(1), 64–83.
- AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109(3), 659–84.
- AIYAGARI, S. R., A. MARCET, T. J. SARGENT, AND J. SEPPALA (2002): "Optimal Taxation without State-Contingent Debt," *Journal of Political Economy*, 110(6), 1220–1254.

- AIYAGARI, S. R., AND E. R. MCGRATTAN (1998): “The optimum quantity of debt,” *Journal of Monetary Economics*, 42(3), 447–469.
- ANGELETOS, G.-M., F. COLLARD, H. DELLAS, AND B. DIBA (2013): “Optimal Public Debt Management and Liquidity Provision,” NBER Working Papers 18800, National Bureau of Economic Research, Inc.
- ARELLANO, C. (2008): “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 98(3), 690–712.
- BAI, J., C. JULLIARD, AND K. YUAN (2012): “Eurozone Sovereign Bond Crisis: Liquidity or Fundamental Contagion,” *Working Paper*.
- BARRO, R. J. (1979): “On the Determination of the Public Debt,” *Journal of Political Economy*, 87(5), 940–71.
- BRUTTI, F. (2011): “Sovereign defaults and liquidity crises,” *Journal of International Economics*, 84(1), 65–72.
- CRUCES, J. J., AND C. TREBESCH (2013): “Sovereign Defaults: The Price of Haircuts,” *American Economic Journal: Macroeconomics*, 5(3), 85–117.
- CUADRA, G., J. SANCHEZ, AND H. SAPRIZA (2010): “Fiscal Policy and Default Risk in Emerging Markets,” *Review of Economic Dynamics*, 13(2), 452–469.
- DEBORTOLI, D., AND R. NUNES (2013): “Lack of Commitment and the Level of Debt,” *Journal of the European Economic Association*, forthcoming.
- D’ERASMO, P. N., AND E. MENDOZA (2012): “Domestic Sovereign Default as Optimal Redistributive Policy,” *Working Paper*.
- DIAZ-GIMENEZ, J., G. GIOVANNETTI, R. MARIMON, AND P. TELES (2008): “Nominal Debt as a Burden on Monetary Policy,” *Review of Economic Dynamics*, 11(3), 493–514.
- EATON, J., AND M. GERSOVITZ (1981): “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *Review of Economic Studies*, 48(2), 289–309.

- ELLISON, M., AND N. RANKIN (2007): “Optimal monetary policy when lump-sum taxes are unavailable: A reconsideration of the outcomes under commitment and discretion,” *Journal of Economic Dynamics and Control*, 31(1), 219–243.
- GENNAIOLI, N., A. MARTIN, AND S. ROSSI (2013): “Sovereign Default, Domestic Banks and Financial Institutions,” *The Journal of Finance*, forthcoming.
- GERTLER, M., AND P. KARADI (2011): “A model of unconventional monetary policy,” *Journal of Monetary Economics*, 58(1), 17–34.
- HOLMSTROM, B., AND J. TIROLE (1998): “Private and Public Supply of Liquidity,” *Journal of Political Economy*, 106(1), 1–40.
- JUESSEN, F., AND A. SCHABERT (2012): “Fiscal Policy, Sovereign Default, and Bailouts,” *Working Paper*.
- KLEIN, P., P. KRUSELL, AND J.-V. RIOS-RULL (2008): “Time-Consistent Public Policy,” *Review of Economic Studies*, 75(3), 789–808.
- KRUSELL, P., F. M. MARTIN, AND J.-V. RIOS-RULL (2006): “Time Consistent Debt,” 2006 Meeting Papers 210, Society for Economic Dynamics.
- LUCAS, R. J., AND N. L. STOKEY (1983): “Optimal fiscal and monetary policy in an economy without capital,” *Journal of Monetary Economics*, 12(1), 55–93.
- MARTIN, F. (2009): “A Positive Theory of Government Debt,” *Review of Economic Dynamics*, 12(4), 608–631.
- MENDOZA, E. G., AND V. Z. YUE (2012): “A General Equilibrium Model of Sovereign Default and Business Cycles,” *The Quarterly Journal of Economics*, 127(2), 889–946.
- MOSKOWITZ, T. J., AND A. VISSING-JORGENSEN (2002): “The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?,” *American Economic Review*, 92(4), 745–778.
- ORTIGUEIRA, S., J. PEREIRA, AND P. PICHLER (2012): “Markov-perfect optimal fiscal policy: the case of unbalanced budgets,” Economics working papers, Universidad Carlos III.

POUZO, D. (2013): “Optimal Taxation with Endogenous Default under Incomplete Markets,”
Working Paper.

SOSA-PADILLA, C. (2012): “Sovereign Defaults and Banking Crises,” MPRA Paper 41074,
University Library of Munich, Germany.

WOODFORD, M. (1990): “Public Debt as Private Liquidity,” *American Economic Review*,
80(2), 382–88.

Appendix A – Proofs

Proof of Proposition 1. Recall the definition of $Q(u_c, b')$ via the bond pricing function (14),

$$Q(u_c, b') = \beta \frac{u_c(b')}{u_c} \left(1 + \hat{\pi}(b') + \hat{\phi}(b') \right),$$

and the associated partial derivatives,

$$\begin{aligned} Q_1(u_c, b') &= -\beta \frac{u_c(b')}{(u_c)^2} \left(1 + \hat{\pi}(b') + \hat{\phi}(b') \right), \\ Q_2(u_c, b') &= \beta \frac{u_c(b')}{u_c} \left\{ \frac{u_{cc}(b')}{u_c(b')} \hat{c}_b(b') \left(1 + \hat{\pi}(b') + \hat{\phi}(b') \right) + \left(\hat{\pi}_b(b') + \hat{\phi}_b(b') \right) \right\}. \end{aligned}$$

Similarly, from the definition of $\omega(b, n)$ in (15),

$$\omega(b, n) = \begin{cases} 1 & \text{if } (1 - \theta)b > \xi^c n \\ \frac{(1 - \theta)b}{\xi^c n} & \text{otherwise,} \end{cases}$$

with $\omega_1(b, n) = \omega_2(b, n) = 0$ when $\omega(b, n) = 1$ and otherwise

$$\begin{aligned} \omega_1(b, n) &= \frac{(1 - \theta)}{\xi^c n}, \\ \omega_2(b, n) &= -\frac{(1 - \theta)b}{\xi^c n^2}. \end{aligned}$$

The first-order conditions characterizing optimal government behavior under commitment to full debt repayment are given by

$$\begin{aligned} 0 &= u_c(1 + \gamma) + \gamma u_{cc}(c - (1 + \pi)b) + \gamma(u_{cc}Q(u_c, b')b' + u_c Q_1(u_c, b')u_{cc}b') - \gamma u_c \pi_c b - u_g \\ &= u_c(1 + \gamma) + \gamma u_{cc}(c - (1 + \pi)b) - \gamma u_c \pi_c b - u_g, \\ 0 &= u_l \left(1 + \gamma \frac{1}{\omega(b, n)} \right) - \gamma u_{ll} \frac{1}{\omega(b, n)} n + \gamma u_c \pi_n b - u_g - \gamma u_l \frac{1}{\omega(b, n)^2} n \omega_2(b, n), \\ 0 &= \beta V_b(b') + \gamma(u_c Q_2(u_c, b')b' + u_c Q(u_c, b')) \\ &= \beta V_b(b') + \gamma \beta u_c(b') \left(1 + \hat{\pi}(b') + \hat{\phi}(b') \right) \{ 1 + \varepsilon_b^q \}, \end{aligned}$$

where $\varepsilon_{b'}^q = \frac{Q_2(u_c, b')b'}{Q(u_c, b')} = \frac{u_{cc}(b')}{u_c(b')} \hat{c}_b(b')b' + \frac{(\hat{\pi}_b(b') + \hat{\phi}_b(b'))b'}{(1 + \hat{\pi}(b') + \hat{\phi}(b'))}$. The envelope condition for b is

$$V_b(b) = -\gamma \left\{ u_c(1 + \pi) - \frac{u_l n}{\omega(b, n)^2} \omega_1(b, n) \right\} + u_g r.$$

Substitution into the first-order condition with respect to b' yields the GEE (17),

$$\gamma' \left\{ u'_c(1 + \pi') - \frac{u'_l n'}{\omega(b', n')^2} \omega_1(b', n') \right\} - u'_g r = \gamma u'_c(1 + \pi' + \phi') \{1 + \varepsilon_{b'}^q\}.$$

□

Proof of Proposition 2. Under a collateral role of government debt, $\xi^c > 0$, zero debt is equivalent to zero production in our model economy, such that a positive steady state debt level emerges by construction. When there is only a liquidity role for government debt, $\xi^c = 0$ and $\xi^l > 0$, a positive steady state debt level emerges, too. This follows directly from the generalized Euler equation (17). When $\xi^c = 0$, the GEE, evaluated at the steady state, reads

$$-u_g^* r = \gamma^* u_c^* (1 + \pi^*) \varepsilon_{b'}^{q*}.$$

Since $u_g^* > 0$, $u_c^* > 0$, $r > 0$, $\pi^* > 0$ and $\gamma^* > 0$, we have that $\varepsilon_{b'}^{q*} < 0$. Since private agents cannot go short in government bonds, it follows that $b^* > 0$ and $Q_2^* < 0$. □

Proof of Proposition 3. When $\xi^c > 0$, government debt is essential for production due to its collateral role. Thus, by construction, in the neighborhood of $b = 0$ welfare is increasing in debt. When $\xi^c = 0$ but $r > 0$, $\omega_1(b, n) = 0$ and the envelope condition for b is

$$V_b(b) = -\gamma u_c(1 + \pi) + u_g r = -\gamma u_c \left(1 + \frac{u_l}{u_c} r \right) + u_g r,$$

where the second equality follows from $\pi = \frac{u_l}{u_c} r$. The first-order condition with respect to n implies

$$u_g - u_l = \gamma [u_l - u_{ll} n + u_c \pi_n b] = \gamma [u_l - u_{ll} (n + r b)],$$

where the second equality follows from $\pi_n = -\frac{u_{ll}}{u_l}\pi = -\frac{u_{ll}}{u_c}r$. Solving for γ yields

$$\gamma = \frac{u_g - u_l}{u_l - u_{ll}(n + rb)} > 0.$$

Substituting into the envelope condition and evaluating at $b = 0$,

$$V_b(0) = -\frac{u_g - u_l}{u_l - u_{ll}n}u_c \left(1 + \frac{u_l}{u_c}r\right) + u_g r.$$

It follows that $V_b(0) > 0$ if and only if

$$\left(\frac{(u_l)^2 - u_g u_{ll}n}{u_l - u_{ll}n}\right) r > \frac{u_g - u_l}{u_l - u_{ll}n}u_c,$$

or equivalently,

$$r > \frac{u_g u_c - u_l u_c}{(u_l)^2 - u_g u_{ll}n} = \frac{\frac{u_c}{u_l} - \frac{u_c}{u_g}}{\frac{u_l}{u_g} - \frac{u_{ll}}{u_l}n} = \frac{u_c}{u_l} \frac{1 - \frac{u_l}{u_g}}{\frac{u_l}{u_g} - \frac{u_{ll}}{u_l}n}.$$

Finally, absent financial frictions, that is, when $\xi^c = 0$ and $r = 0$, the envelope condition for b is unambiguously negative, $V_b(b) = -\gamma u_c < 0$. \square

Proof of Proposition 4. The marginal revenue from issuing additional debt b' is given by $\frac{dQ(u_c, b')b'}{db'} = Q(u_c, b') \left\{1 + \frac{Q_2(u_c, b')b'}{Q(u_c, b')}\right\} = Q(u_c, b') \{1 + \varepsilon_{b'}^q\}$. Accordingly, there is a debt Laffer curve whenever $\varepsilon_{b'}^q < -1$. From the GEE (17), $-V_b(b') = \gamma u_c'(1 + \pi' + \phi') \{1 + \varepsilon_{b'}^q\}$. Since $\gamma u_c'(1 + \pi' + \phi') > 0$, it follows that $V_b(b') > 0$ if and only if $\varepsilon_{b'}^q < -1$. \square

Proof of Proposition 5. Problem (23) shows that, by choosing ρ , the government can effectively regulate the state ρb in the value function $\tilde{V}^d(\rho b)$, subject to the constraint $\rho \in [0, 1]$. Accordingly, as ρ is chosen optimally, the value function $V^d(b)$ is *non-decreasing* over the entire state space. To see this formally, note that the first-order condition for ρ associated with problem (23) implies

$$\gamma \left[\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n) b - u_c b \right] \geq 0, \quad (26)$$

with equality in case of an interior solution. But then the envelope condition associated with $\tilde{V}^d(\rho b)$ implies

$$\tilde{V}_b^d(\rho b) = \gamma \left[\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n) \rho - u_c \rho \right] = \gamma \frac{\rho}{b} \left[\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n) b - u_c b \right] \geq 0, \quad (27)$$

where the weak inequality follows from (26), that is, under the optimal repayment policy associated with problem (23). It thus follows that $V^d(b)$ is non-decreasing.

Returning to (26), since $u_c b > 0$, it follows that an interior solution can only arise when $\omega_1(\rho b, n) > 0$. The same argument also implies that $\omega_1(\rho b, n) > 0$ is a necessary condition for a corner solution at $\rho = 1$.²⁰ Hence, the optimal repayment policy conditional on default, $\tilde{\rho}^d(b)$, ensures that the collateral constraint is strictly binding. Given $\omega_1(\rho b, n) > 0$, the envelope condition (27) implies

$$\tilde{V}_b^d(b) = \gamma \left[\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n) \rho - u_c \rho \right] = \gamma \left[\frac{u_l n}{\omega(\rho b, n)} \frac{1}{b} - u_c \rho \right].$$

This expression is monotonically decreasing in b and ρ . Given some ρ , there is thus a unique b such that $\tilde{V}_b^d(b) = 0$. Let \underline{b} denote the level of debt such that $\tilde{V}_b^d(\underline{b}) = 0$ when $\rho = 1$. When $\rho = 1$ and $b < \underline{b}$, $\tilde{V}^d(b)$ is increasing; a corner solution at full repayment, $\tilde{\rho}^d(b) = 1$, is thus indeed an optimizing choice, and $V^d(b)$ is increasing. Conversely, when $\rho = 1$ and $b > \underline{b}$, $\tilde{V}^d(b)$ is decreasing, which contradicts (27); the optimal repayment policy conditional on default is thus adjusted to an interior solution $\tilde{\rho}^d(b) < 1$, and $V^d(b)$ is flat. Finally, when $b = \underline{b}$, full repayment, $\tilde{\rho}^d(b) = 1$, is optimal.

Taking stock, when $b < \underline{b}$, $V^d(b)$ is strictly increasing. Moreover, at \underline{b} the collateral constraint is strictly binding. When $b < \underline{b}$, the government always finds it optimal to fully repay its maturing bonds, $\tilde{\rho}^d(b) = 1$. However, due to the market exclusion costs of default, it follows that $V^{nd}(b) > V^d(b) = \tilde{V}^d(b)$ for all $b < \underline{b}$. By contrast, for any level of debt $b > \underline{b}$ such that the government finds it optimal to default, $V^d(b)$ is constant, i.e., the value conditional on default is independent from initial debt. Denote this value by \bar{V}^d . Moreover, under the premise that the no-default value function $V^{nd}(b)$ is monotonically decreasing for large levels of debt and

²⁰A corner solution at $\rho = 0$ can never occur because debt is essential for production.

hence of an inverse U-shape, there exists a unique level of debt, $\bar{b} > \underline{b}$, such that $V^{nd}(\bar{b}) = \bar{V}^d$. By the same argument, $V^{nd}(b) \geq \bar{V}^d$ for $b \leq \bar{b}$, and $V^{nd}(b) < \bar{V}^d$ for $b > \bar{b}$. Accordingly, the government fully repays its outstanding bonds up to the threshold level \bar{b} and partially defaults if inherited debt exceeds this threshold. This is the optimal (unconditional) repayment policy associated with problem (21); denote it by $\hat{\rho}(b)$.

In order to explicitly characterize the optimal (unconditional) repayment policy $\hat{\rho}(b)$, recall first that $V^{nd}(b) \geq V^d(b)$ when $b \leq \bar{b}$; hence, $\hat{\rho}(b) = 1$ for all $b \leq \bar{b}$. Conversely, when $b > \bar{b}$, $V^{nd}(b) < V^d(b)$ and, since $\bar{b} > \underline{b}$, $\hat{\rho}(b) = \tilde{\rho}^d(b) < 1$. But this implies that, for $b \geq \underline{b}$, condition (26) holds at equality and $\omega_1(\rho b, n) > 0$. By the household's optimal consumption-leisure choice $\frac{u_l}{u_c} = (1 - \tau)\omega(\rho b, n)$, so that condition (26) implies

$$\rho b = \frac{\frac{u_l}{u_c} n}{\omega(\rho b, n)} = (1 - \tau)n. \quad (28)$$

But for interior solutions $\hat{\rho}(b) = \tilde{\rho}^d(b) < 1$, $V^d(b) = \bar{V}^d$ is constant; that is, b does not matter for allocations and welfare, and $(1 - \tau(b))n(b)$ is constant. It thus follows that the right-hand side in (28) is constant, implying that $\hat{\rho}(b)b$ must be constant and equal to \underline{b} for all b that induce an interior solution for ρ . Since $\rho \leq 1$, we thus have $\hat{\rho}(b) = \underline{b}/b$ for $b > \bar{b}$. \square

Proof of Proposition 6. For the policy \mathcal{P} to be sustainable over some region $\tilde{\mathbb{B}} \subset \mathbb{B}$, the incumbent government must find it optimal to employ the policy \mathcal{P} and to fully honor inherited debt for all $b \in \tilde{\mathbb{B}}$ when it perceives all future governments to employ the policy \mathcal{P} and to fully honor inherited debt in $\tilde{\mathbb{B}}$. Absent reputational costs, the incumbent government's optimal policy problem is given by

$$\max_{c, n, b', \rho} u(c, 1 - n, n + r\rho b - c) + \beta V(b') + \gamma \left(u_c c + u_c Q(u_c, b') b' - \frac{u_l}{\omega(\rho b, n)} n - u_c (1 + \pi) \rho b \right).$$

This problem can be decomposed into two stages: First, the government decides on the haircut on outstanding debt; second, given the haircut, it chooses the remaining policy instruments. By construction, the solution to the government's second stage problem is given by the policy functions \mathcal{P} and the corresponding value function $V(b)$. The first stage choice of the optimal

haircut $\hat{\rho}(b)$, in turn, is the solution to

$$\max_{\rho \in [0,1]} V(\rho b).$$

Thus, for any region $\tilde{\mathbb{B}}$, it is optimal to fully repay debt over the entire region $\tilde{\mathbb{B}}$ if and only if the value function is non-decreasing over the entire region $\tilde{\mathbb{B}}$. However, from Proposition 4, if the value function is non-decreasing in initial debt, this implies that the economy cannot be on the ‘good’ side of the debt Laffer curve. \square

Web Appendix – Not for publication

Characteristics of the value functions when debt is in the form of loans

The value functions $W_a^{nd}(\ell)$, $W_e^{nd}(\ell)$ and $W^d(\ell)$ satisfy

$$W_a^{nd}(\ell) = \max_{c,n,b'} u(c, 1-n, n-c) + \beta V^o(b') \quad (29)$$

$$+ \gamma \left(u_c c + u_c Q^b(u_c, b') b' - \frac{u_l}{\omega(\ell, n)} n - u_c \ell \right),$$

$$W_e^{nd}(\ell) = \max_{c,n,\ell'} u(c, 1-n, n-c) + \beta W^o(\ell') \quad (30)$$

$$+ \gamma \left(u_c c + u_c Q^\ell(u_c, \ell') \ell' - \frac{u_l}{\omega(\ell, n)} n - u_c \ell \right),$$

$$W^d(\ell) = \max_{\rho \in [0,1]} \tilde{W}^d(\rho \ell), \quad (31)$$

where $\tilde{W}^d(\rho \ell)$ denotes the value function conditional on a given repayment rate ρ on loans,

$$\tilde{W}^d(\rho \ell) = \max_{c,n,\ell'} u(c, 1-n, n-c) + \beta W^o(\ell') + \gamma \left(u_c c + u_c Q^\ell(u_c, \ell') \ell' - \frac{u_l}{\omega(\rho \ell, n)} n - u_c \rho \ell \right).$$

It is not necessary to index $W^d(\ell)$ by a or e , since default precludes the current government's option of immediate bond market access. Moreover, because default hampers the liquidity of maturing bonds, the value of defaulting is independent of whether outstanding liabilities are in the form of bonds or loans, that is, $V^d(x) = W^d(x)$ and $\tilde{V}^d(x) = \tilde{W}^d(x)$, where x denotes the (effective) amount of outstanding liabilities. Finally, note also that $W_e^{nd}(\ell) = \tilde{W}^d(\rho \ell)$ for $\rho = 1$. Accordingly, $W_e^{nd}(\ell) = W^d(\ell)$ whenever the optimal policy prescribes full debt repayment.

Optimal default policy when debt is in the form of loans

Similar arguments to those presented in the proof of Proposition 5 are readily available for the case when the government's liabilities are in the form of loans. In the region where $W^d(\ell)$ is increasing in ℓ , the optimal repayment policy conditional on default is given by $\tilde{\rho}^d(\ell) = 1$. $W^d(\ell)$ is constant for all loan levels exceeding a threshold $\underline{\ell}$. Comparison of problems (23) and

(31) shows that $W^d(x) = V^d(x)$ for $x \in \{b, \ell\}$; hence, $\underline{\ell} = \underline{b}$. Moreover, when W_a^{nd} and W_e^{nd} have an inverted U-shape, there exist unique thresholds $\bar{\ell}_a$ and $\bar{\ell}_e$ such that the government in state $s \in \{a, e\}$ fully repays its outstanding loans if and only if ℓ is below the threshold $\bar{\ell}_s$; otherwise, it partially defaults.

Finally, note that the no-default value functions satisfy $V^{nd}(x) > W_a^{nd}(x) > W_e^{nd}(x)$ globally. The first inequality follows because the liquidity services of maturing bonds are valuable, $u_g r - \gamma u_c \pi = r[u_g - \gamma u_i] > 0$; the second inequality follows because, relative to loans, there is a liquidity premium on newly issued bonds, $Q^b(u_c, x') > Q^\ell(u_c, x')$ for all $x' \in \{b', \ell'\}$. Hence, the government's default thresholds satisfy $\bar{b} > \bar{\ell}_a > \bar{\ell}_e$. Accordingly, the economy's maximum sustainable level of debt is given by \bar{b} . Moreover, the default threshold under market exclusion is given by $\bar{\ell}_e = \underline{\ell}$, which follows from the property that $W_e^{nd}(\ell) < \bar{V}^d$ if and only if $\ell > \underline{\ell}$. The optimal (unconditional) repayment policies when the government's liabilities are in the form of loans, $\hat{\rho}_s(\ell)$ for $s \in \{a, e\}$, can be constructed analogously to their counterpart $\hat{\rho}(b)$ when the maturing liabilities are bonds. When the government defaults, the optimal haircut reduces its effective liabilities (ρb or $\rho \ell$) to $\underline{b} = \underline{\ell}$, i.e., to the lowest level that is consistent with the maximum default value \bar{V}^d .

Markov-perfect equilibrium under the option to default

A Markov-perfect equilibrium under the option to default is defined as follows.²¹

Definition 3. *A Markov-perfect equilibrium under the option to default is a collection of consumption functions $\{\hat{c}_s^o, \hat{c}_s^d, \hat{c}_s^{nd}\}_{s \in \{f, a, e\}}$, labor supply functions $\{\hat{n}_s^o, \hat{n}_s^d, \hat{n}_s^{nd}\}_{s \in \{f, a, e\}}$, debt policy functions $\{\hat{b}_f^o, \hat{b}_a^o, \hat{\ell}_e^d, \hat{\ell}_e^d, \hat{b}_f^{nd}, \hat{b}_a^{nd}, \hat{\ell}_e^{nd}\}$, repayment policy functions $\{\hat{\rho}_f, \hat{\rho}_a, \hat{\rho}_e\}$, value functions $\{V_f^o, V_f^{nd}, V_f^d, \bar{V}^d, W^o, W_a^{nd}, W_e^{nd}, W^d, \bar{W}^d\}$, and pricing functions $\{Q^b, Q^\ell\}$ such that:*

- (i) given $V_f^o, W^o, Q^b, Q^\ell, \hat{\rho}_f, \hat{\rho}_a$ and $\hat{\rho}_e$, the policy functions $\{\hat{c}_f^{nd}, \hat{n}_f^{nd}, \hat{b}_f^{nd}\}$ solve problem (22); the policy functions $\{\hat{c}_f^d, \hat{n}_f^d, \hat{\ell}_f^d\}$ solve problem (23); the policy functions $\{\hat{c}_a^{nd}, \hat{n}_a^{nd}, \hat{b}_a^{nd}\}$ solve problem (29); the policy functions $\{\hat{c}_e^{nd}, \hat{n}_e^{nd}, \hat{\ell}_e^{nd}\}$ solve problem (30); the policy functions $\{\hat{c}_a^d, \hat{n}_a^d, \hat{\ell}_a^d\}$ and $\{\hat{c}_e^d, \hat{n}_e^d, \hat{\ell}_e^d\}$ solve problem (31);

²¹For clarity and ease of notation we introduce the subindex f for the case when bond markets are fully operational.

(ii) given $\{\hat{\rho}_f, \hat{\rho}_a, \hat{\rho}_e\}$, the consumption policy functions satisfy $\hat{c}_s^o = \hat{c}_s^{nd}$ when the government fully repays debt and $\hat{c}_s^o = \hat{c}_s^d$ otherwise; the labor and debt policies are constructed in the same way;

(iii) given the value functions, the repayment policy functions $\{\hat{\rho}_f, \hat{\rho}_a, \hat{\rho}_e\}$ solve (21), (23), (24) and (31);

(iv) given V^{nd} and V^d , the value function V^o satisfies (21); given $\{W_a^{nd}, W_e^{nd}, W^d\}$, the value function W^o satisfies (24);

(v) given the policy functions, the pricing functions Q^b and Q^ℓ satisfy

$$\begin{aligned} Q^b(u_c, b') &= \beta \frac{u_c(\hat{c}_f^o(b'))}{u_c} \hat{\rho}_f(b') \left(1 + \hat{\pi}_f^o(b') + \hat{\phi}_f^o(b')\right), \\ Q^\ell(u_c, \ell') &= \alpha \beta \frac{u_c(\hat{c}_a^o(\ell'))}{u_c} \hat{\rho}_a(\ell') \left(1 + \hat{\phi}_a^o(\ell')\right) + (1 - \alpha) \beta \frac{u_c(\hat{c}_e^o(\ell'))}{u_c} \hat{\rho}_e(\ell') \left(1 + \hat{\phi}_e^o(\ell')\right). \end{aligned}$$

(vi) given the policy functions and $\{V_f^o, W^o\}$, the value functions $\{V_f^{nd}, V_f^d, \tilde{V}^d, W_a^{nd}, W_e^{nd}, W^d, \tilde{W}^d\}$ satisfy (22), (23), (29), (30) and (31).

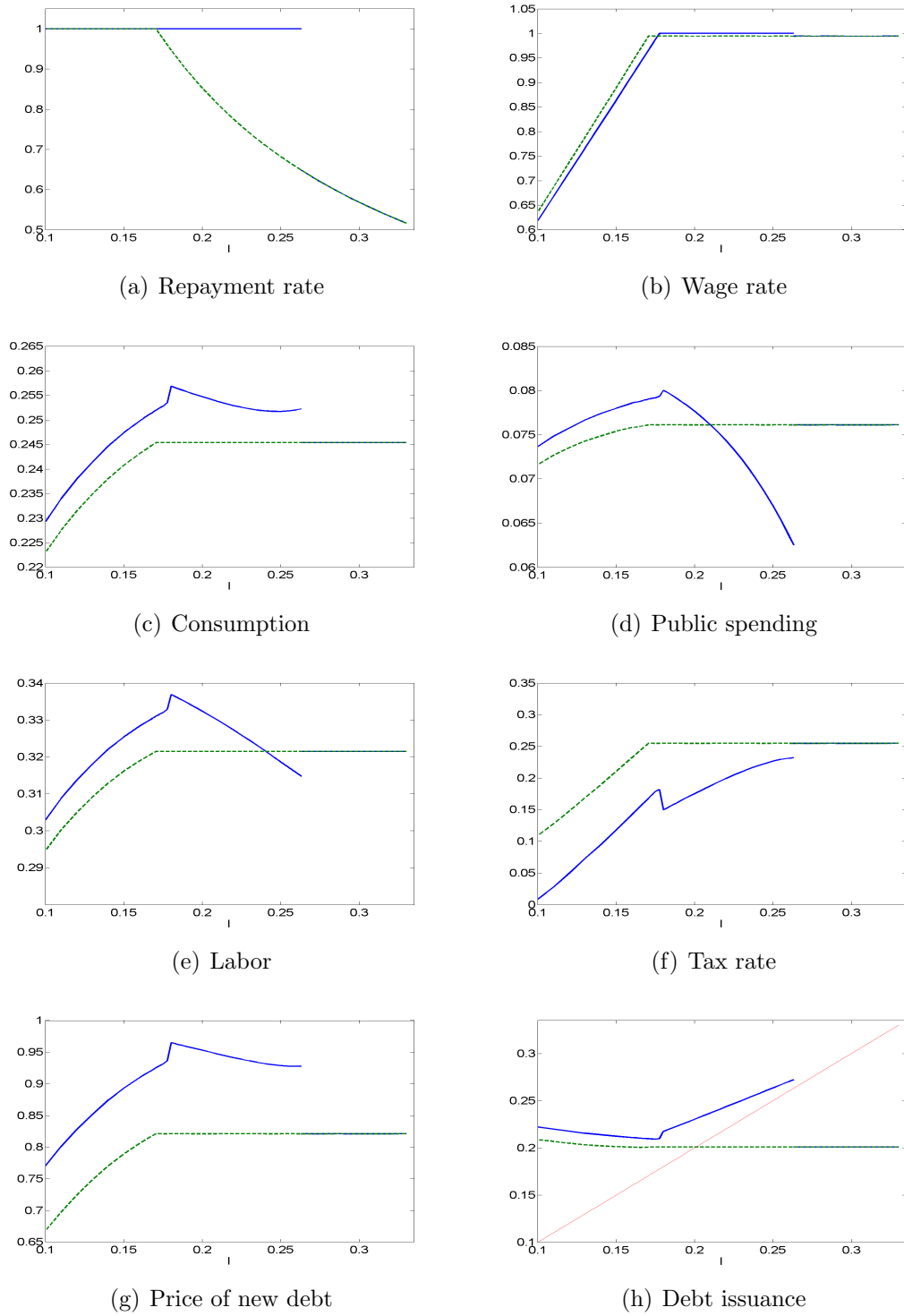
Policy functions when debt is in the form of loans

Figure 6 displays the policy functions when outstanding debt takes the form of loans, distinguishing between the situation when the government can re-access the bond market (the blue solid line) and when it cannot (the green dashed line).

Computational algorithm

This appendix provides details of our computational strategy. In $\mathbb{B}^{\#}$, the region of the state space where both the collateral constraint and the default option are irrelevant for optimal policies, the equilibrium policy and welfare functions are smooth and differentiable. They can hence easily be computed with standard projection methods. In the regions of the state space where either the collateral constraint or the default option are relevant, the policy functions

Figure 6: Policy functions under the option to default – loans



Note: The blue solid line corresponds to the optimal policy functions when the government regains access to the primary bond market ($s = a$). The green dotted line corresponds to the optimal policy functions when the government remains excluded ($s = e$).

display strong non-linearities and non-differentiabilities. Dynamic programming tools are then used to construct the equilibrium functions. Our algorithm involves the following steps:

1. Solve the model without collateral constraint ($\xi^c = 0$) and with commitment to full debt repayment over a grid of points in the state space $\mathbb{B} = [b_{min}, b_{max}]$ using standard projection methods. Denote the solution by $\hat{c}^o(b)$, $\hat{n}^o(b)$, $\hat{g}^o(b)$, $\hat{w}^o(b)$, $\hat{b}^o(b)$ and $\hat{V}^o(b)$.
2. **Find the solution for intermediate debt levels (\mathbb{B}^3).** Guess a region of the state space $[b_2, b_3] \subset \mathbb{B}$, covering the steady state b^* from Step 1, such that the collateral constraint and default are irrelevant over $[b_2, b_3]$. Construct the policy functions $\hat{c}(b)$, $\hat{n}(b)$, $\hat{g}(b)$, $\hat{w}(b)$, $\hat{b}(b)$ and the welfare function $\hat{V}(b)$ using the solution from Step 1, i.e., set $\hat{c}(b) = \hat{c}^o(b)$, $\hat{n}(b) = \hat{n}^o(b)$, etc. for all grid-points in $[b_2, b_3]$.
3. **Extend the solution to low debt levels (\mathbb{B}^1 and \mathbb{B}^2).** Set $b = b_2$. Let the next-period consumption policy and the continuation value function be given by \hat{c} and \hat{V} . Solve the current government's problem by dynamic programming to derive the optimal choices c^{opt} , n^{opt} , g^{opt} , w^{opt} , b'^{opt} . Verify that these optimal choices coincide with $\hat{c}(b)$, $\hat{n}(b)$, $\hat{g}(b)$, $\hat{w}(b)$, $\hat{b}(b)$. If not, return to Step 2 with a higher guess for b^{min} . Otherwise set $b = b_2 - \varepsilon$ and proceed to Step 4.
4. Let the next-period consumption policy and the continuation value function be given by \hat{c} and \hat{V} . Solve the current government's problem by dynamic programming to derive the optimal choices c^{opt} , n^{opt} , g^{opt} , w^{opt} , b'^{opt} and the associated welfare level V^{opt} . Update the policy and value functions at state b by setting $\hat{c}(b) = c^{opt}$, $\hat{n}(b) = n^{opt}$, $\hat{g}(b) = g^{opt}$, $\hat{w}(b) = w^{opt}$, $\hat{b}(b) = b'^{opt}$ and $\hat{V}(b) = V^{opt}$. As long as $b > b_{min}$, reduce the current b by a small amount and repeat Step 4. If $b = b_{min}$, proceed to Step 5.
5. In a similar vein, construct the value functions when outstanding debt is the form of loans, \hat{W}_a^{nd} and \hat{W}_e^{nd} , at a relevant grid of points in the state space. Set $\bar{V}^d = \max(\hat{W}_e^{nd})$ and set \underline{b} equal to the debt level where $\max(\hat{W}_e^{nd})$ is attained.
6. **Extend the solution to large debt levels (\mathbb{B}^4).** Set $b = b_3$ and verify that $\hat{V}^o(b) \geq \bar{V}^d$. If this is not the case, choose a lower value for b_3 and go back to Step 2. Otherwise set $b = b_3 + \varepsilon$, set $\hat{\rho}(b) = 1$ for all $b \in [b_{min}, b_3]$, and proceed to Step 7.

7. Check whether $\hat{V}^o(b) \geq \bar{V}^d$. If this is true, the government has no incentive to default; set $\hat{\rho}(b) = 1$, $\hat{V}(b) = \hat{V}^o(b)$, $\hat{c}(b) = \hat{c}^o(b)$, $\hat{n}(b) = \hat{n}^o(b)$, etc. Conversely, if $\hat{V}^o(b) < \bar{V}^d$, the government does have an incentive to default. In this case set $\hat{\rho}(b) = \underline{b}/b$, $\hat{V}(b) = \bar{V}^d$, etc. As long as $b < b_{max}$, increase the current b by a small amount and repeat Step 7. When $b = b_{max}$ stop.