A Model of Secular Stagnation

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Abstract

In this paper we propose a simple overlapping generations New Keynesian model in which a permanent (or very persistent) slump is possible without any self-correcting force to full employment. The trigger for the slump is a deleveraging shock which can create an oversupply of savings. Other forces that work in the same direction and can both create or exacerbate the problem are a drop in population growth and an increase in income inequality. High savings, in turn, may require a permanently negative real interest rate. In contrast to earlier work on deleveraging, our model does not feature a strong self-correcting force back to full employment in the long-run, absent policy actions. Successful policy actions include, among others, a permanent increase in inflation and a permanent increase in government spending. We also establish conditions under which an income redistribution can increase demand. Policies such as committing to keep nominal interest rates low or temporary government spending, however, are less powerful than in models with temporary slumps. Our model sheds light on the long persistence of the Japanese crisis, the Great Depression, and the slow recovery out of the Great Recession.

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1 Introduction

During the closing phase of the Great Depression in 1938, the President of the American Economic Association, Alvin Hansen, delivered a disturbing message in his Presidential Address to the Association (see Hansen (1939)). He suggested that the Great Depression might just be the start of a new era of ongoing unemployment and economic stagnation without any natural force towards full employment. This idea was termed the "secular stagnation" hypothesis. One of the main driving forces of secular stagnation, according to Hansen, was a decline in the population birth rate and an oversupply of savings that was suppressing aggregate demand. Soon after Hansen's address, the Second World War led to a massive increase in government spending effectively ending any concern of insufficient demand. Moreover, the baby boom following WWII drastically changed the population dynamics in the US, thus effectively erasing the problem of excess savings of an aging population that was of principal importance in his secular stagnation hypothesis.

Recently Hansen's secular stagnation hypothesis has gained increased attention. One obvious motivation is the Japanese malaise that has by now lasted two decades and has many of the same symptoms as the U.S. Great Depression - namely dwindling population growth, a nominal interest rate at zero, and subpar GDP growth. Another reason for renewed interest is that even if the financial panic of 2008 was contained, growth remains weak in the United States and unemployment high. Most prominently, Lawrence Summers raised the prospect that the crisis of 2008 may have ushered in the beginning of secular stagnation in the United States in much the same way as suggested by Alvin Hansen in 1938. Summers suggests that this episode of low demand may even have started well before 2008 but was masked by the housing bubble before the onset of the crisis of 2008. In Summers' words, we may have found ourselves in a situation in which the natural rate of interest - the short-term real interest rate consistent with full employment - is permanently negative (see Summers (2013)). And this, according to Summers, has profound implications for the conduct of monetary, fiscal and financial stability policy today.

Despite the prominence of Summers' discussion of the secular stagnation hypothesis and a flurry of commentary that followed it (see e.g. Krugman (2013), Taylor (2014), Delong (2014) for a few examples), there has not, to the best of our knowledge, been any attempt to formally model this idea, i.e., to write down an explicit model in which unemployment is high for an indefinite amount of time due to a permanent drop in the natural rate of interest. The goal of this paper is to fill this gap.

It may seem somewhat surprising that the idea of secular stagnation has not already been studied in detail in the recent literature on the liquidity trap, which does indeed already invite the possibility that the zero bound on the nominal interest rate is binding for some period of time due to a drop in the natural rate of interest. The reason for this, we suspect, is that secular stagnation does not emerge naturally from the current vintage of models in use in the literature. This,

however - and perhaps unfortunately - has less to do with economic reality than the limitations of these models. Most analyses of the current crisis takes place within representative agent models (see e.g. Krugman (1998), Eggertsson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2011) and Werning (2012) for a few well known examples). In these models, the long run real interest rate is directly determined by the discount factor of the representative agent, which is fixed. The natural rate of interest can then only temporarily deviate from this fixed state of affairs due to preference shocks. And changing the discount rate permanently (or assuming a permanent preference shock) is of no help either since this leads the intertemporal budget constraint of the representative household to "blow up" and the maximization problem of the household is no longer well defined.

Meanwhile, in models in which there is some heterogeneity in borrowing and lending (such as Curdia and Woodford (2010), Eggertsson and Krugman (2012) and Mehrotra (2012)), it remains the case that there is a representative saver whose discount factor pins down a positive steady state interest rate. But as we shall see, moving away from a representative savers framework, to one in which people transition from being borrowers to becoming savers over time due to lifecycle dynamics will have a major effect on the steady state interest rate and can open up the possibility of a secular stagnation[1].

In what follows, we begin by outlining a simple endowment economy (in the spirit of Samuelson (1958)) with overlapping generations where people go through three stages of life: young, middle aged and old. The endowment income is concentrated within the middle generation. This gives rise to demand for borrowing by the young, and gives the middle aged an incentive to save part of their endowment for old age by lending it to the young. This lending, however, is constrained by a debt limit faced by young agents. In this environment, we will see that the steady state real interest rate is no longer constrained to be only a function of an agents' discount factor. Instead, it depends on the relative supply of savings and demand for loans, and the equilibrium real interest rate may easily be permanently negative. Forces that work in this direction include a population growth slowdown, which increases the relative supply of savings, along with a tighter debt limit, which directly reduces the demand for loans. Under some conditions, income inequality, either across generations or within generations, may also generate a negative real interest rate. Interestingly enough, all three factors - increases in inequality, a slowdown in population growth, and a tightening of borrowing limits – have been at work in several economies in recent years.

One interesting result emerges when we consider a debt deleveraging shock of the kind common in the literature (see e.g. Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2011), Philippon and Midrigin (2011), Hall (2011), Mian and Sufi (2011) and Mian and Sufi (2012) for

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[1]Two closely related papers, Guerrieri and Lorenzoni (2011) and Philippon and Midrigin (2011) feature richer heterogeneity than the papers cited above, although not lifecycle dynamics which are critical to our results. These papers too, however, only deliver a temporary liquidity trap in response to a negative deleveraging shock.

both theoretical and empirical analysis). In that work, typically, this shock leads to a temporary reduction in the real interest rate as debtors pay down their debt and savers need to compensate for the drop in overall spending by increasing their spending. Once the deleveraging process is completed (debt is back to a new debt limit), the economy returns to its steady state with a positive interest rate. In our framework, however, no such return to normal occurs. Instead, a period of deleveraging puts even further downward pressure on the real interest rate so that it becomes permanently negative. The key here is that people shift from being borrowers to savers over their lifecycle. If a borrower today takes on less debt (due to the deleveraging shock, which is modeled as a tightening of the borrowing limit) then tomorrow, he has greater savings capacity since there is less debt to repay. This implies that the act of deleveraging - rather than bringing about a new steady state with a positive interest rate - will instead reduce the real rate even further by increasing the supply of savings in the future.

We then augment our simple endowment economy with a nominal price level. A key result that emerges is that under flexible prices there is a lower bound on steady state inflation, which can be no lower than the negative of the natural rate of interest. Thus, for example, if the natural rate of interest is -4%, then there is no equilibrium that is consistent with inflation below 4% in steady state. The secular stagnation hypothesis, thus, implies that long-run price stability is impossible if prices are flexible. This has profound implications for an economy with realistic pricing frictions. If a central bank can indeed force inflation below this "natural" upper bound, it does so at the expense of generating a permanent recession.

We next turn to a model in which wages are downwardly rigid as originally suggested by Keynes (1936) as an explanation for the persistent unemployment in the Great Depression. In this economy, we show that if the central bank is unwilling to tolerate high enough inflation, there is a permanent slump in output. In line with the literature that emphasizes deleveraging shocks that have short-term effects, we find that, in this economy, a long slump is one in which usual economic rules are stood on their head. The old Keynesian paradox of thrift is in full force, as well as the more recent "paradox of toil" (Eggertsson (2010)), - where an increase in potential output decreases actual output - as well as the proposition that increasing wage flexibility makes things worse rather than better (Eggertsson and Krugman (2012)).

Finally, we consider the role of monetary and fiscal policy. We find that a high enough inflation target can always do away with the slump all together, as it accommodates a negative natural interest rate. Importantly, however, an inflation target which is below what is needed has no effect in this context. This formalizes what Krugman (2014) has referred to as the "timidity trap" - an inflation target that is too low will simply do nothing in an economy experiencing a secular stagnation. We show this trap explicitly in the context of our model which only arises if the shock is permanent. Similarly, we illustrate that, in a secular stagnation environment, there are strong limitations of forward guidance with nominal interest rates. Forward guidance relies on manipu-

lating expectations after the zero lower bound shock has subsided, but as the shock in our model is permanent, manipulating these types of expectations is of much more limited value. Moving to fiscal policy, we show that either a permanent increase in government spending can eliminate the output gap, or a redistribution of income from savers to borrowers, although this latter result depends on the details of the distribution of income (we provide examples in which income redistribution is ineffective).

We have already noted that our work is related to the relatively large literature on the zero bound, in which the natural rate of interest is temporarily negative due to preference or deleveraging shocks. Our paper also relates to a different strand of the literature on the zero bound which argues that the zero bound may be binding due to self-fulfilling expectations in the absence of any shock (see e.g. Benhabib, Schmitt-Grohé and Uribe (2001) and Schmitt-Grohé and Uribe (2013)). As in that literature, our model describes an economy that may be in a permanent liquidity trap. A key difference is that the trigger for the crisis in our model is a real shock, rather than self-fulfilling expectations. This implies both different policy options, but also that in our model the liquidity trap is the unique stable equilibria, which facilitates policy experiments. Moreover, it also ties our paper to the hypothesis of secular stagnation, which relies on the natural rate of interest being permanently negative rather than self-fulfilling expectations.

2 Endowment Economy

Imagine a simple overlapping generations economy. Households live for three periods. They are born in period 1 (young), they become middle age in period 2 (middle age), and retire in period 3 (old). Consider the case in which no aggregate savings is possible (i.e. there is no capital), but that generations can borrow and lend to one another. Moreover, imagine that it is only the middle generation that receives any income, which is received (for now) in the form of an endowment Y_t. In this case, the young will borrow from the middle aged households, which in turn will save for retirement when old. We assume, however, that there is a limit on the amount of debt the young can borrow. Generally, we would like to think of this as reflecting some sort of incentive constraint, but for the purposes of this paper, it will just be an exogenous constant D_t (as the "debtors" in Eggertsson and Krugman (2012)).

More concretely, consider a representative household of a generation born at time t. It has the following utility function:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} \mathbb{E}_t \left\{ \log \left(C_t^y \right) + \beta \log \left(C_{t+1}^m \right) + \beta^2 \log \left(C_{t+2}^o \right) \right\}$$

where $0 < \beta < 1$, C_t^y is the consumption of the household when young, C_{t+1}^m its consumption when middle aged, and C_{t+2}^o its consumption while old[2]. We assume that borrowing and lending

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[2]The assumption of log preferences is not crucial to the results and has little effect on the derivations that follow.

take place via a one period risk-free bond denoted B_t^i where $i = y, m, o$ at an interest rate r_t. Given this structure we can write the budget constraints facing household of the generation born at time t at different times as:

$$C_t^y = B_t^y \tag{1}$$

$$C_{t+1}^m = Y_{t+1} - (1 + r_t)B_t^y + B_{t+1}^m \tag{2}$$

$$C_{t+2}^o = -(1 + r_{t+1})B_{t+1}^m \tag{3}$$

$$(1 + r_t)B_t^i \leq \bar{D}_t \tag{4}$$

where equation (1) corresponds to the budget constraint for the young where the household consumption is constrained by what it can borrow. Equation (2) corresponds to the budget constraint of the middle aged household who receives the endowment Y_t and repays what was borrowed when young as well as accumulating some savings B_{t+1}^m for retirement. Finally, equation (3) corresponds to the budget constraint when the household is old and consumes all its savings.

The inequality (4) corresponds to the exogenous borrowing limit (as in Eggertsson and Krugman (2012), which we assume will binding for young so that:

$$C_t^y = B_t^y = \frac{D_t}{1 + r_t} \tag{5}$$

The old at any time t will consume all their savings so that:

$$C_t^o = -(1 + r_{t-1})B_{t-1}^m \tag{6}$$

The middle aged, however, are at an interior solution and satisfy a consumption Euler equation given by:

$$\frac{1}{C_t^m} = \mathbb{E}_t \frac{1 + r_t}{C_{t+1}^o} \tag{7}$$

We assume that the size of each generation is given by N_t. Let us define the growth rate of births by $1 + g_t = \frac{N_t}{N_{t-1}}$. For an equilibrium in the bond market, we require that borrowing of the young is equal to the savings of the middle aged so that $N_t B_t^y = -N_{t-1} B_t^m$ or:

$$(1 + g_t)B_t^y = -B_t^m \tag{8}$$

An equilibrium is now defined as set of stochastic processes $\{C_t^y, C_t^o, C_t^m, r_t, B_t^y, B_t^m\}$ that solve $(1), (2), (5), (6), (7),$ and (8) given an exogenous process for $\{D_t, g_t\}$.

To analyze equilibrium determination, let us explore the equilibrium in the market for savings and loans given by equation (8) using the notation L_t^s and L_t^d; the left hand side of (8) denotes the demand for loans, L_t^d, and the right hand side its supply, L_t^s. Hence the demand for loans (using (5)) can be written as:

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t \tag{9}$$

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Figure 1: Equilibrium in the asset market
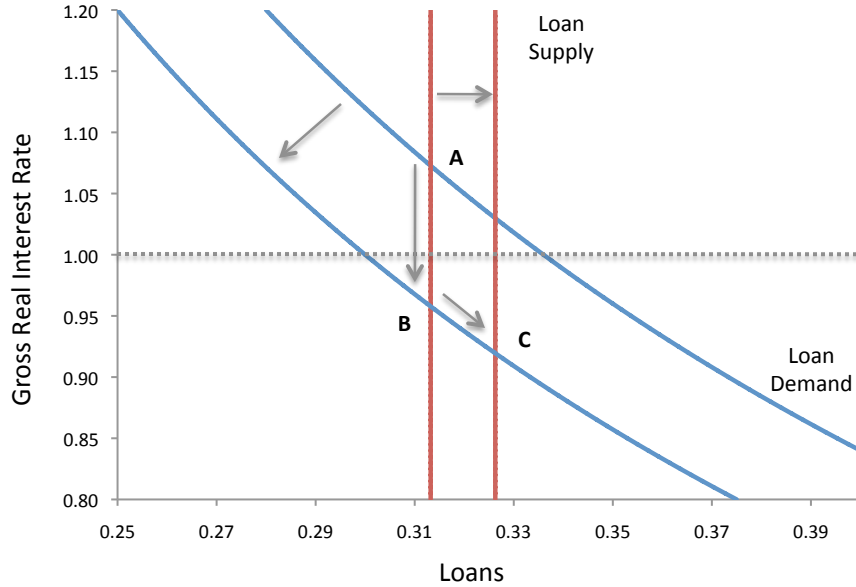
while the supply for savings - assuming perfect foresight for now - can be solved for by substituting out for C_t^m in (2), using (7) and (3), and for B_t^y by using (5). Then solving for B_t^m we obtain the supply of loans given by:

$$L_t^s = \frac{\beta}{1+\beta}\left(Y_t - D_{t-1}\right)$$ (10)

An equilibrium, depicted in Figure 1, is then determined by the intersection of the demand (L_t^d) and supply (L_t^s) for loans at the equilibrium level of real interest rates given by:

$$1 + r_t = \frac{1+\beta}{\beta}\frac{(1+g_t)D_t}{Y_t - D_{t-1}}$$ (11)

2.1 Deleveraging and Slowdown in Population Growth

The contribution of this paper is not to develop a theory of the debt limit D_t and we will simply follow the existing literature in assuming that there is a sudden and permanent reduction in its value from some high level, D^H, to a new low level D^L. Eggertsson and Krugman (2012), for example, suggest that a shock of this kind can be thought of as a "Minsky moment", a sudden realization that households' collateral constraints were too lax and their focus is on the adjustment of the economy to this new state of affairs.

A key contribution of this paper, however, is that rather than resulting in only a temporary reduction in the real interest rate, as the economy adjusts to the "new normal," this shock can instead lead to a permanent reduction in the real rate of interest under flexible prices. As we shall

see, this has fundamental implications for the nature of the slump in the model once it is extended to include other realistic frictions.

Point B in Figure 1 shows the equilibrium level of the real interest rate when there is a deleveraging shock in our model in the first period of the shock. As we can see, the shock leads directly to a reduction in the demand for loans, since the demand curve shifts from $\frac{1+g_t}{1+r_t}D^H$ to $\frac{1+g_t}{1+r_t}D^L$. There is no change in the supply of loanable funds, however, as we see in (10) since the debt payment of the middle aged agents depend upon D_{t-1}[3].

Let us compare the equilibrium in point B to point A. Relative to the previous equilibrium A, the young are now spending less, while the old are spending the same (given by D_{t-1}). Since all the endowment is consumed in our economy, this fall in spending by the young then needs to be made up by an increase in spending by the middle generation (since the old are spending all they have saved). As we see in equation (11), this means that in order to be induced to spend more today, the real interest rate has to fall for the savers to make up for the shortfall in spending.

So far, the story here is exactly the same as in Eggertsson and Krugman (2012) and the literature on deleveraging. There is a deleveraging shock which triggers a drop in spending by borrowers in the economy. To make up for it, the savers need to spend more. And the only way they can be induced to spend more is through a drop in the real interest rate. It is only in what happens next, however, that the path followed by these two models diverge.

In Eggertsson and Krugman (2012), the model reaches a new steady state next period in which once again the real interest rate is determined by the discount factor of the representative savers in the economy which implies a positive real interest rate. And here is the key difference in our model – there is no representative saver; instead, people are both borrowers and savers but at different stages in their lives. The fall in the borrowing of the young in period t, then implies that in the next period - when that person moves from becoming a borrower to a saver - the middle aged agent has more money to save (since he needs to pay less due to the reduction in D_t). This means that at time $t + 1$ there is an outward shift in the supply of savings L_t^s, as shown in Figure 1. Hence, in sharp contrast to Eggertsson and Krugman (2012) where the economy settles back on the old steady state after a brief transition with a negative interest rate during the deleveraging phase, here we see that the economy settles down at a new steady state with a permanently lower real rate of interest. Moreover, this interest rate can easily be negative depending on the size of the shock. This, as we shall see, is a condition for a permanent liquidity trap. Moreover, in a more realistic lifecycle model, this process would serve as a powerful and persistent propagation mechanism for the original deleveraging shock. More generally, even if the drop in D_t is not permanent, the point is that the natural rate of interest will then inherit the drop in D_t which may be of arbitrary duration. Other factors may matter too and as we shall see the changes in these

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[3]The fact that the there is no income or substitution effect of interest rates on L^s is not a general result but depends on our assumption of log preferences which simplifies things a bit here.

variables may also be arbitrarily persistent, providing alternative foundations of permanent drop in the natural rate of interest.

We see that D_t is not the only factor that can shift the demand for loanable funds. As revealed in equation (9), a reduction in the birth rate of the young will also reduce the demand for loans. This has the consequence of shifting back the demand for loans, and thus triggering a reduction in the real interest rate in just the same way as a reduction in D_t. The intuition here is straightforward: if population growth is slower, i.e. a lower g_t, then there are less young people. Since the young are the source of borrowing in this economy, the reduction in population growth diminishes the demand for loans. The supply for loanable funds, however, remains unchanged, since we have normalized both L_t^d and L_t^s by the size of the middle generation.

There are other forces, however, that may influence the supply of loanable funds - one of which may be inequality. This is what we turn to next.

2.2   Income Inequality

There is no general result about how an increase in inequality affects the real rate of interest. In general, this will depend on how changes in income affect the relative supply and demand for loans. As we shall see, however, there are relatively plausible conditions under which higher inequality will in fact reduce the natural rate of interest.

Consider one measure of inequality, which measures the degree of income inequality across generations. In our case, we considered the somewhat extreme form of inequality where the middle generation received all the income, while the young and old received none. If we assume instead that all generations receive the same endowment $Y_t^y = Y_t^m = Y_t^o$, then it is easy to see that there is no incentive to borrow or lend, and accordingly the real interest rate is equal to the discount factor $1 + r_t = \beta^{-1}$. It is thus inequality of income across generations that is responsible for our results and what triggers possibly negative real interest rates.

Generational inequality, however, is typically not what people have in mind when considering inequality; instead commentators often focus on unequal income of individuals within the same age cohort. Inequality of that kind can also have a negative effect on the real interest rate. Before getting there, however, let us point out that this need not be true in all cases. Consider, for example an endowment distribution $Y_t(z)$ where z denotes the type of an individual that is known once he is born. Suppose further more that once again $Y_t^y(z) = Y_t^m(z) = Y_t^o(z)$ for all z. Here once again, income is perfectly smoothed across ages and the real interest rate is given by β^{-1}. The point is that we can pick any distribution of income $Y(z)$ to support that equilibrium so that in this case income distribution is irrelevant.

Let us now consider the case when inequality in a given cohort can in fact generate negative pressure on real interest rates. When authors attribute demand slumps to a rise in inequality they

typically have in mind - in the language of old Keynesian models - that income gets redistributed from those with a high propensity for consumption to those that instead wish to save their income. We have already seen how this mechanism works in the case of inequality across generations. But we can also imagine that a similar mechanism applies if income gets redistributed within a cohort - as long as some of the agents in that cohort are credit constrained.

Suppose that all households receive no endowment while young and a strictly positive endowment in the middle and last periods of their lives. However, assume that some fraction of households receive a large endowment in their middle generation (i.e. high-income households), while the remaining households (i.e. low-income households) receive a very small endowment in the middle period of life. For simplicity, all households receive the same income endowment in old age (this could be thought of as some sort of state-provided pension like Social Security). For sufficiently low levels of the middle-period endowment and a sufficiently tight credit constraint, low-income households will remain credit constrained in the middle period of life (or alternatively we can simply assume they are "hand-to-mouth" consumers for whatever reason). These households will rollover their debt in the middle generation and only repay their debts in old age, consuming any remaining endowment. In this situation, only the high-income households will save in the middle period and will, therefore, supply savings to both other middle generation households and the youngest generations of both types.

As before, we can derive an explicit expression for real interest rate in this richer setting with multiple types of households. Under the conditions described above, the only operative Euler equation is for the high-income households who supply loans in equilibrium. The demand for loans is obtained by adding together the demand from young households and the credit constrained low-income households. The expression we obtain is a generalization of the case obtained in equation (11):

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1 + g_t + \eta_s) D_t}{(1 - \eta_s)\left(Y_t^{m,h} - D_{t-1}\right)} + \frac{1}{\beta} \frac{Y_{t+1}^o}{\left(Y_t^{m,h} - D_{t-1}\right)} \tag{12}$$

where η_s is the fraction of low-income households, $Y_t^{m,h}$ is the income of the high-income middle-generation and Y_{t+1}^o is the income of these households in the next period (i.e the pension income received by all households). If $\eta_s = 0$ and $Y_{t+1}^m = 0$, we recover the expression for the real interest rate derived in (11).

Total income for the middle-generation is weighted average of high and low income workers (i.e. $Y_t^m = \eta_s Y_t^{m,l} + (1 - \eta_s) Y_t^{m,h}$). Let us then define an increase in inequality is a redistribution of middle-generation income from low to high-income workers, without any change in Y_t^m. While this redistribution keeps total income for the middle-generation constant (by definition) it must necessarily lower the real interest rate by increasing the supply of savings which is only determined by the income of the wealthy. This can be seen by equation (12) where the real interest rate

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is decreasing in $Y_t^{m,h}$ without any offsetting effect via $Y_t^{m,l}$[4].

As this extension of our model suggests, the secular rise in wage inequality in recent decades in the US and other developed nations may have been one factor in exerting downward pressure on the real interest rate. Labor market polarization - the steady elimination of blue-collar occupations and the consequent downward pressure on wages for a large segment of the labor force - could show up as increase in income inequality among the working-age population, lowering the real interest rate in the manner described here (for evidence on labor market polarization, see e.g. Autor and Dorn (2013) and Goos, Manning and Salomons (2009)). Several other theories have been suggested for the rise in inequality. To the extent that they imply an increase in savings, they could fit into our story as well.

3   Price Level Determination

To this point, we have not described the behavior of the price level or inflation in this economy. We now introduce nominal price determination and first consider the case in which prices are perfectly flexible. In this case, we will see that the economy "needs" permanent inflation, and there is no equilibrium that is consistent with price stability. This will turn out to be critical for the analysis when we consider realistic pricing frictions, as it implies that full employment can only be obtained at permanently higher inflation rates.

As is by now standard in the literature, we introduce a nominal price level by assuming that there is traded one period nominal debt denominated in money, and that the government controls the rate of return of this asset (the nominal interest rate)[5]. The saver in our previous economy (middle generation household) now has access to risk-free nominal debt which is indexed in dollars in addition to one period risk-free real debt[6]. This assumption gives rise to the consumption Euler equation which is the nominal analog of equation (7):

$$\frac{1}{C_t^m} = \beta E_t \frac{1}{C_{t+1}^o}(1 + i_t)\frac{P_t}{P_{t+1}} \tag{13}$$

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[4]As emphasized earlier, not all forms of income inequality should be expected to have a negative effect on the real interest rate. If middle-generation income is drawn from a continuous distribution, a mean-preserving spread that raises the standard deviation of income could be expected to have effects on both the intensive and extensive margin. That is the average income among savers would rise, but this effect would be somewhat offset by an increase in the fraction of credit-constrained households (i.e an increase in η_s). The extensive margin boosts the demand for loans and would tend to increase the real interest rate. Whether inequality raises or lower rates would depend on the relative strength of these effects.

[5]There are various apporaches to microfound this, such as money in the utility function or cash-in-advance constraints.

[6]For simplicity, this asset trades in zero net supply, so that in equilibrium the budget constraints already analyzed are unchanged.

where i_t is the nominal rate and P_t is the price level. We impose a nonnegativity constraint on nominal rates. Implicitly, we assume that the existence of money precludes the possibility of a negative nominal rate. At all times:

$$i_t \geq 0 \tag{14}$$

Equation (13) and (7) imply (assuming perfect foresight) the standard Fisher equation:

$$1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} \tag{15}$$

where again recall that r_t is exogenously determined as before by equation (11) (or equation (12) in the model with income inequality). The Fisher equation simply states that the real interest rate should be equal to the nominal rate deflated by the growth rate of the price level (inflation)[7].

From (14) and (15) it should be clear that if the real rate of interest is permanently negative, there is no equilibrium consistent with stable prices. To see this, assume there is such an equilibrium so that $P_{t+1} = P_t = P^*$. Then the the Fisher equation implies that $i_t = r_t < 0$ which violates the zero bound. Hence a constant price level – price stability – is inconsistent with our model when r_t is permanently negative. The next question we ask, then, is what constant growth rates of the price level are consistent with the equilibrium?

Let us denote the growth rate of the price level – inflation – by $\Pi_t = \frac{P_{t+1}}{P_t} = \bar{\Pi}$. The zero bound and the Fisher equation then implies that for an equilibrium with constant inflation to exist, then there is a bound on the inflation rate given by $\Pi(1 + r) = 1 + i \geq 1$ or:

$$\bar{\Pi} \geq \frac{1}{1 + r} \tag{16}$$

which implies that steady state inflation is bounded from below by the real interest rate due to the zero bound.

Observe that at a positive real interest rate this bound may seem of little relevance. If, as is common in the literature using representative agent economies, for example, the real interest rate in steady state is equal to the discount factor then the bound says that $\Pi \geq \beta$. In typical calibration this implies a bound on steady state inflation of about -2% to -4%, a level of deflation most central bank target to be well above so the bound is of little empirical relevance.

In a situation such as the one we described in previous sections, however, this bound takes on a greater practical significance. If the real interest rate is permanently negative, it implies that under flexible prices steady state inflation needs to be permanently above zero and possibly well above zero depending on the value of the steady state real interest rate. This insight will be critical once we move away from the assumption that prices are perfectly flexible. In that case, if the economy

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[7]Again, we can define an equilibrium as a collection of stochastic processes $\{C_t^y, C_t^o, C_t^m, r_t, i_t, B_t^y, B_t^m, P_t\}$ that solve (1), (2), (5), (6), (7) and (8) and now in addition (13) and (14) given the exogenous process for $\{D_t, g_t\}$ and some policy reaction function for the government (e.g. an interest rate rule).

calls for a positive inflation rate – and cannot get it due to policy (e.g. a central bank committed to low inflation) – the consequence will be a permanent drop in output instead.

4   Endogenous Output

We now extend our model to allow output to be endogenously determined. Again, we will assume that the only generation that gets any income is the middle generation. Now, however, they will need to work to generate the income.

The budget constraint of this agent is again given by equations $(1) - (4)$, except we replace the budget constraint of the middle aged (2) with:

$$C_{t+1}^m = \frac{W_{t+1}}{P_{t+1}}L_{t+1} + \frac{Z_{t+1}}{P_{t+1}} - (1 + r_t)B_t^y + B_{t+1}^m \tag{17}$$

where W_{t+1} is the nominal wage rate and P_{t+1} the aggregate price level, L_{t+1} the labor supply of the middle generation, and Z_{t+1} represents the profits of the firms.

For simplicity we assume that the middle age will supply its labor inelastically at $\bar{L}$. Note that if the firms do not hire all available labor supplied, then L_t may be lower than $\bar{L}$ as we further outline below. Note that under this assumption, each of the generations' optimization conditions remain exactly the same as before.

On the firm side, we assume that firms are perfectly competitive and take prices as given. They hire labor to maximize period-by-period profits. Their problem is given by:

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t \tag{18}$$

$$\text{s.t.} \quad Y_t = L_t^\alpha \tag{19}$$

The firms' labor demand condition is then given by:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \tag{20}$$

So far we have described a perfectly frictionless production side and if this were the end of the story, our model would essentially be analogous to what we have already studied in the endowment economy. Output would now be given by $Y_t = L_t^\alpha = \bar{L}^\alpha$ and equation (20) would back out the real wage (and $\frac{W_t}{P_t}L_t + \frac{Z_t}{P_t} = Y_t$ so the middle generation's budget constraint would remain the same as in the endowment economy).

We will, however, now deviate from this frictionless world. The friction with longest history in Keynesian thought is that of downward nominal wage rigidities. Keynes put this friction at the center of his hypothesis of the slump of the Great Depression, and we follow his example here. More recently these type of frictions have been gaining increasing attention due to the weak recovery of employment from the Great Recession (see e.g. Shimer (2012) and Schmitt-Grohé

and Uribe (2013) - the latter has a similar specification for wage setting as we adopt here)[8]. A key implication of the assumption of downwardly rigid wages is that output can be permanently below steady state under that assumption. As we will see, if a central bank is unwilling to let inflation be what it "needs to be" (given by the inequality (16)), it will have to tolerate some level of permanent unemployment.

To formalize this consider a world in which the household will never accept to work for wages that are lower than in the previous period (albeit they will be happy to work for higher ones) so that nominal wages at time t cannot be lower than what they were time $t - 1$. Or to be slightly more general, imagine that the household would never accept lower wages than $\tilde{W}_t = \gamma W_{t-1} + (1 - \gamma)\alpha\bar{L}^{\alpha-1}$. If $\gamma = 1$ this corresponds to a perfectly downwardly rigid wage, but, if $\gamma = 0$, we obtain the flexible wage case already studied in which our model simplifies to the endowment economy. Formally, if we take this assumption as given, it implies that we replace the presumption that $L_t = \bar{L}$ with:

$$W_t = \max\left\{\tilde{W}_t, P_t\alpha\bar{L}^{\alpha-1}\right\} \quad \text{where} \quad \tilde{W}_t = \gamma W_{t-1} + (1 - \gamma)P_t\alpha\bar{L}^{\alpha-1} \tag{21}$$

Here we see that nominal wages can never be below $\tilde{W}_t$, i.e, they are rigid downwards to the extent given by the value of γ. If market clearing requires higher wages than the past nominal wage rate, however, this specification implies that demand equals supply and the real wage is given by (20) evaluated at $L_t = \bar{L}$.

Moving to monetary policy, now suppose that the central bank sets the nominal rate according to a standard Taylor rule:

$$1 + i_t = \max\left(1, (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \tag{22}$$

where $\phi_\pi \geq 1$ and Π^* and i^* are parameters of the policy rule that we keep constant. This rule says that the central bank tries to stabilize inflation around an inflation target (determined by i^* and Π^*) unless it is constrained by the zero bound, in which case the nominal interest rate is set at its lower bound. We define an equilibrium as a collection of quantities $\{C_t^y, C_{t,}^o, C_t^m, B_t^y, B_t^m\}$ and prices $\{P_t, W_t, r_t, i_t\}$ that solve $(1), (2), (5), (6), (7), (8), (13), (14), (20), (21)$ and (22) given an exogenous process for $\{D_t, g_t\}$.

We will now analyze the steady state of this economy and show that it may very well imply a permanent contraction at zero interest rates. We will defer to later sections discussion of transition dynamics in response to exogenous shocks and an analysis of the stability of the steady state (in particular we will establish that condions under which the steady state we analyze here is stable).

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[8]The presence of substantial nominal wage rigidity has been established empirically recently in US administrative data by Fallick, Lettau and Wascher (2011), in worker surveys by Barattieri, Basu and Gottschalk (2010), and in cross-country data by Schmitt-Grohé and Uribe (2011).

Figure 2: Steady state aggregate demand and aggregate supply curves
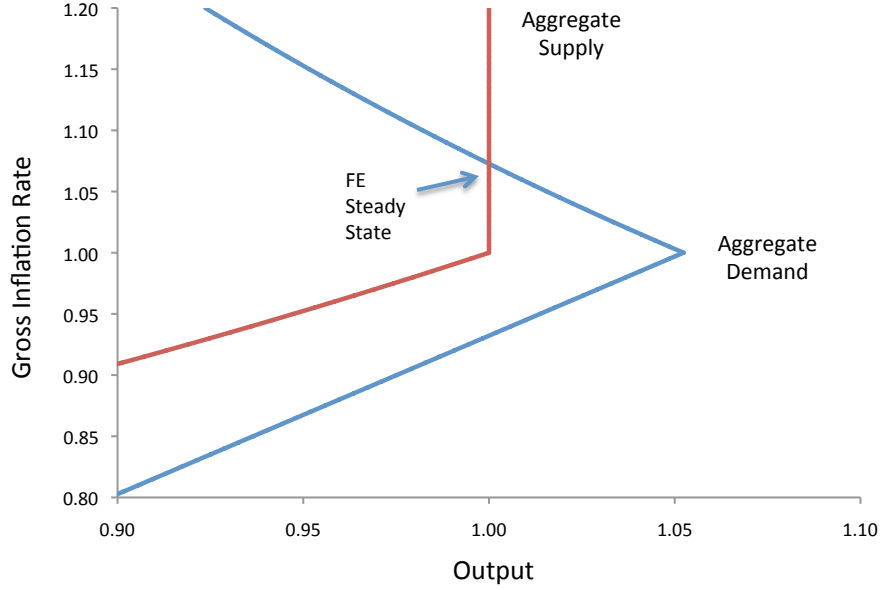
It is fairly easy to analyze the steady state of the model. It can be done in two steps, first by tracing out the aggregate supply of the economy and then aggregate demand. Consider first aggregate supply.

The aggregate supply specification of the model consists of two regimes: one in which real wages are equal to their market clearing ones (if $\Pi \geq 1$), and the other when the bound on nominal wages is binding ($\Pi < 1$). Intuitively, positive inflation in steady state means that wages behave as if they are flexible, since inflation is positive over time and the downward bound on nominal wages is not binding[9]. If wages are not downwardly rigid ($\Pi > 1$) then supply and demand for labor are equated, defining the full employment level of output:

$$Y = \bar{L}^\alpha = Y^f \ \text{ for } \ \Pi \geq 1 \tag{23}$$

where we use Y^f to denote full employment. This is shown as a solid vertical part in the AS curve in Figure 2[10].

Let us now turn to the case in which $W < \tilde{W}$ which will be the case when $\Pi < 1$. The real wage is then given by $w = \frac{(1-\gamma)\alpha\bar{L}^{\alpha-1}}{(1-\gamma\Pi^{-1})}$ and substituting this into equation (20) and using the production function (19) to express the result in terms of output we obtain:

$$Y = \left(\frac{1 - \gamma\Pi^{-1}}{1 - \gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f \ \text{ for } \ \Pi < 1 \tag{24}$$

---

[9]First observe that for $W > \tilde{W}$ in steady state we require that the real wage (denoted $w = \frac{W}{P}$) is $w \geq \gamma w \Pi^{-1} + (1-\gamma)\alpha\bar{L}^{\alpha-1}$. This is satisfied as long as $\Pi > 1$.

[10]We normalize $Y^f = 1$.

Figure 3: Steady state aggregate demand and aggregate supply curves
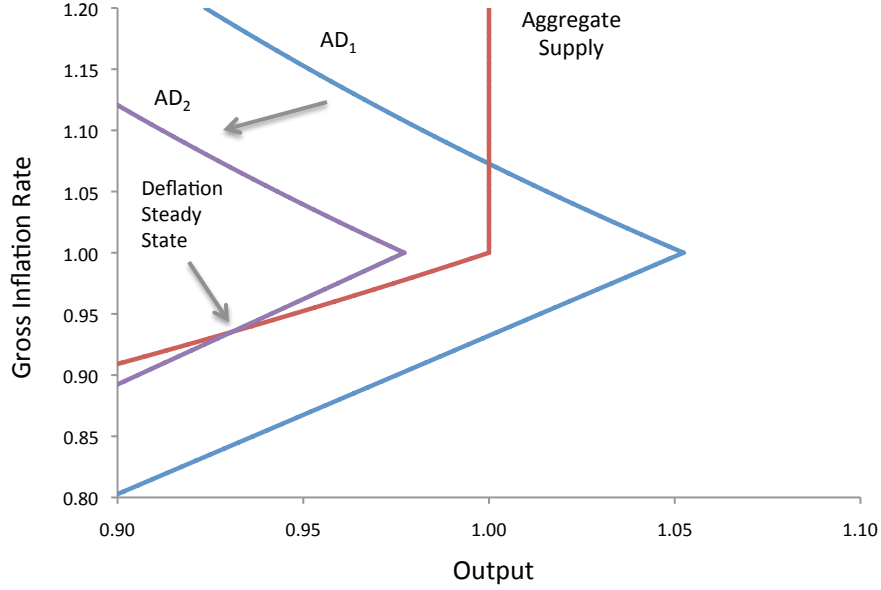
which shows that output now increases with inflation. To obtain a bit more intuition for this function, consider how it behaves to a first-order approximation local to $\Pi = 1$. We obtain:

$$d\Pi = \kappa \frac{dY}{Y^f}$$

where $\kappa = \frac{1-\alpha}{\alpha} \frac{1-\gamma}{\gamma}$. This illustrates a positive relationship between inflation and output - a classic Phillips curve relationship. The intuition is straightforward: as inflation increases, real wages decrease (as wages are rigid) and hence the firms hire more labor. Note that the degree of rigidity is indexed by the parameter γ. As γ gets closer to 1, the Phillips curve gets flatter and as it approaches zero, the Phillips curve becomes vertical in equation (24). Note that this is not a short-run relationship; instead, it describes the behavior of steady state inflation and output. This is because we assume that wages are downwardly rigid - even in the long run. The aggregate supply curve denoted AS is plotted up in Figure 2, with the vertical part given by (23) and the flatter part with (24) with the kink at $\Pi = 1$.

Let us now turn to the aggregate demand side. As in the case of aggregate supply, the demand side consists of two regimes: one when the zero bound is not binding, the other when it is. Let us start with deriving aggregate demand at positive nominal rates. Demand is obtained by summing up the consumption of the three generations and substituting out for the nominal interest rate with the policy reaction function (22). Combining equations $(11), (15), (22)$ and assuming $i > 0$, we get:

$$\Gamma^* \Pi^{\phi_\pi - 1} = \frac{1 + \beta (1 + g) D}{\beta} \frac{D}{Y - D} \quad \text{for} \ i > 0 \tag{25}$$

where $\Gamma^* \equiv (1 + i^*)(\Pi^*)^{-\phi_\pi}$. To gain some intuition, it is again helpful to consider a first order approximation of this schedule local to $\Pi = 1$. We obtain:

$$(\phi_\pi - 1)d\Pi = -\frac{1}{\bar{Y} - \bar{D}}dY$$

where $dY \equiv Y - \bar{Y}$ and bar denotes a variable evaluated in steady state. This relationship is the top portion of the AD curve in Figure 2. It shows that as inflation increases output decreases. The logic for this is familiar: if inflation increases, then the central bank raises the nominal interest rate by more than one for one (since $\phi_\pi > 1$), which in turn increases the real interest rate and reduces demand.

Consider now the situation in which $i = 0$. In this case, again combining equations $(11), (15), (22)$ and assuming $i = 0$, we get:

$$Y = D + \psi\Pi \ \text{ for } \ i = 0$$

where and $\psi = \frac{1+\beta}{\beta}(1 + g)D$ which now leads to an upward sloping relationship between inflation and output. The logic should again be relatively familiar to the literatur short-run liquidity trap: as inflation increases, the nominal interest rate remains constant, thus reducing the real interest rate. This increases demand as the middle generation will now want to consume more. This is represented by the downward sloping segment of the AD curve in Figure 2.

In order to ensure that an equilibrium exists at a zero nominal interest rate, we make the assumption that:

$$\kappa < \psi^{-1} \tag{26}$$

or in terms of the fundamental parameters $\left(\frac{1+\beta}{\beta}(1 + g)D\right)^{-1} > \frac{1-\alpha}{\alpha}\frac{1-\gamma}{\gamma}$. What this condition states is that we require that the AS curve in Figure 2 is flatter than the AD curve. For this to be the case, we simply require that wages not be overly flexible, i.e we need large enough γ. If wages are completely downwardly rigid, then the AS curve is horizontal and an equilibrium always exists. This requirement is tied to the result we obtained in Section 3, that under flexible prices no equilibrium exists for an inflation target that is below the negative of the natural rate of interest. We will develop this point more towards the end of Section 5, but let us now focus on what the equilibrium looks like taking this restriction as given so that we know the two lines must intersect once the zero bound is binding.

4.1 A Long Slump

As Figure 2 shows, in times characterized by a positive natural rate, the aggregate demand curve intersects the vertical component of the aggregate supply curve. Therefore, nominal rates are positive, inflation equals the inflation target, and output is at the full employment level of output

Table 1: Parameter values used in AD and AS figures

Description	Parameter	Value
Population growth	$\bar{g}$	0.2
Collateral constraint	$\bar{D}$	0.28
Discount rate	β	0.77
Wage adjustment	γ	0.5
Taylor coefficient	ϕ_π	2
Labor supply	$\bar{L}$	1

Y_f. If the inflation target is $\Pi^* = 1$, then monetary policy stabilizes the price level and the nominal rate equals the real rate.

The making of a long-run output slump is shown in Figure 3, where we have illustrated the effect of an 8% tightening of the collateral constraint D_t. Figures 2 and 3 display the nonlinear aggregate demand and supply curves in steady state where the parameters chosen are given in Table 1. It should be emphasized that the AD and AS curves in the figures are not linearized; they are the nonlinear expressions given by equations (23) - (25).

As Figure 3 illustrates, the collateral shock shifts the AD curve inward from AD_1 to AD_2. As can be seen from equation (25), a reduction in D reduces output for any given inflation rate since demand from younger households is reduced and is not offset by any reduction in the real interest rate. This shock moves the economy off the full-employment line to the deflation steady state. In this particular numerical example, output falls some 6%. So long as monetary policy is given by equation (22), demand will remain too low to attain full employment. The reduction is obtained by wage rigidity; steady state deflation raises steady state real wages depressing demand for labor. This lands the economy into the strange land of liquidity trap economics.

5   Keynesian Paradoxes

As with standard New Keynesian analyses of the zero lower bound, our OLG environment delivers many of the same Keynesian paradoxes discussed in Eggertsson (2010) and Eggertsson and Krugman (2012). These paradoxes can be illustrated graphically using the aggregate demand and aggregate supply framework developed in the previous section.

The paradox of thrift in the old Keynesian literature is the proposition that if everybody tries to save more, there will be less aggregate demand – and accordingly less income from which to save. Aggregate savings falls as a consequence of all households trying to save more. We do not have any aggregate saving in this economy as there is no investment. However, precisely the same mechanism is operative in the sense that an increase in the desire to save reduces demand

Figure 4: Paradox of thrift
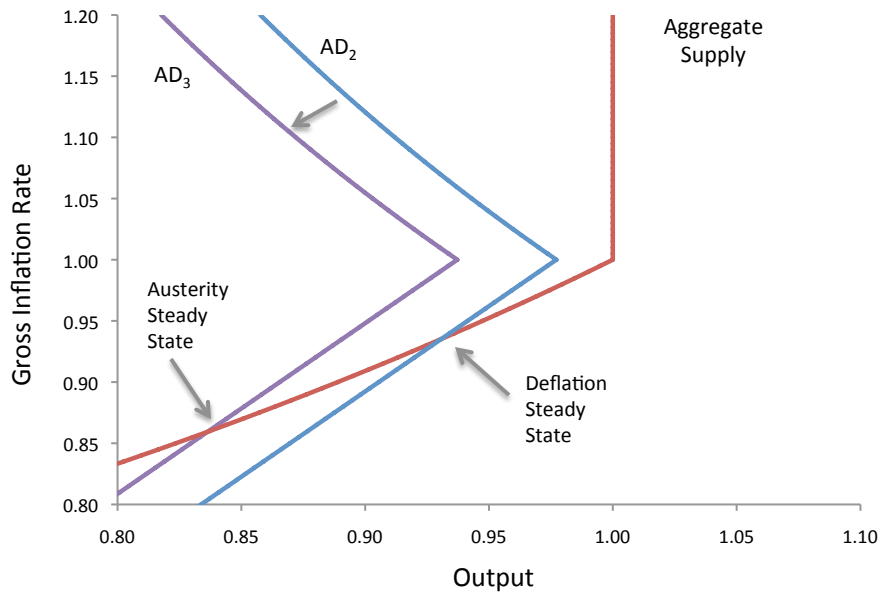
and thereby income. If the model is extended to include investment, this suggests that a paradox of thrift can be illustrated in this setting (see e.g. Eggertsson (2010) for this kind of extension in a similar setting where the natural rate of interest is taken to be exogenous).

Two other more recently discovered paradoxes also appear in the model much in the same way as in the literature on temporary liquidity traps. First, there is the paradox of toil, first illustrated in Eggertsson (2010). This paradox states that if everybody tries to work more, there will be less work in equilibrium. More generally any force that increases the overall productive capacities of the economy - for example, a positive technology shock - can have contractionary effect. This is because a shift out in aggregate supply triggers deflationary pressures, raising the real interest rate and further suppressing aggregate demand. The key behind this result is that aggregate demand is increasing in inflation due to the zero bound (see Figure 5) - a drop in inflation cannot be offset by a cut in the interest rate, so it leads to higher real interest rates.

Figure 5 displays the effect of a labor supply shock that raises labor supply by 4%. This shock shifts the AS curve from AS_1 to AS_2 while leaving the AD curve unchanged and results in an output contraction. A positive productivity shock would have similar effects, shifting down the AS curve and shifting out the FE line. Interestingly, in our environment, an income tax reduction would both shift the AD curve back (by raising the supply of savings at middle-generation households) while shifting out the aggregate supply curve like a positive labor supply shock.

Lastly, there is the paradox of flexibility, as for example in Eggertsson and Krugman (2012). This paradox states that as one increases the degree of nominal flexibility, then output contracts. This is paradoxical since if all prices and wages were flexible, then there would be no contraction

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Figure 5: Paradox of toil
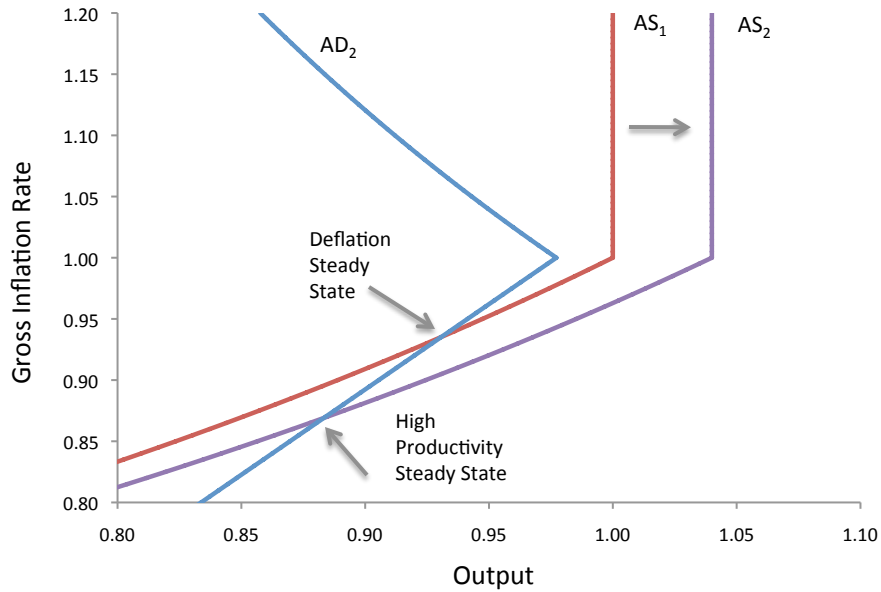
at all. The reason for this is that an increase in price/wage flexibility triggers a drop in expected inflation, thus increasing the real interest rate. This cannot be offset by interest rate cuts due to the zero bound. As wages become more flexible (i.e. a decrease in the parameter γ), the slope of the AS curve steepens as shown in Figure 6. For any given deflation steady state, a decrease in γ will shift the steady state along the AD curve, increasing the rate of deflation, raising real wages, and therefore increasing the shortfall in output.

Figure 6 illustrates the effect of increasing real wage flexibility and the resulting paradox of flexibility. We consider a decrease in γ from 0.5 to 0.42 - an eight percentage point decrease in wage rigidity. This experiment is illustrated in the figure as a change in the slope of the AS curve depicted by the movement from AS_1 to AS_2. Importantly, the full employment line is left unaffected by any change in γ due to the assumption that wages are only downwardly rigid.

Observe that in the limit, as wages are fully flexible, then aggregate demand and supply no longer intersect and there is no equilibrium. This result is in some respect just a restatement of our previous result in Section 3. There is no equilibrium in our model under flexible prices that is consistent with an inflation rate that is below the negative of the natural rate of interest. This is precisely the case here, since the central bank is attempting to target an inflation rate that is too low. No equilibrium exists with that inflation rate under flexible prices, and the same will hold if prices are "flexible enough" (consider the numerical example in Figure 6. As prices become more and more flexible then at some point, the AS and AD curves no longer intersect as our assumption that $\psi > \kappa^{-1}$ ensuring existence in inequality (26) is violated).

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Figure 6: Paradox of flexibility
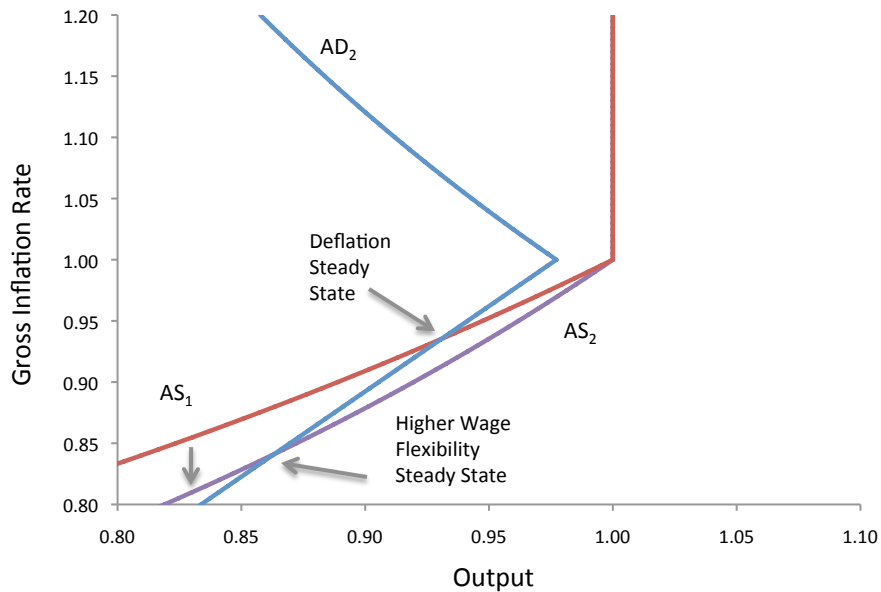
6 Monetary and Fiscal Policy

As shown in the previous sections, a commitment to target low inflation can interact with nominal wage rigidities to generate persistent or possibly permanent shortfalls in output from potential. However, as shown in Figure 2, the intersection of the aggregate demand curve and the full employment line represents another feasible equilibrium if the central bank targets a permanently higher level of inflation. Thus, a permanent increase in the inflation target can restore the economy to full employment by ensuring that the real interest rate equals the natural rate of interest.

Analytically, the ability of a higher inflation target to shift out the AD curve can be seen in equation (25) where an increase in the inflation target decreases the value of Γ^*. Solving equation (25) for Y shows that output moves inversely with Γ^*. Thus, an increase in the inflation target must shift out the aggregate demand curve.

As noted earlier, for a central bank that does not know the natural rate, the choice of a new inflation target should be skewed to the upside. Too low an inflation target in steady state will ensure the continuation of the deflation steady state rather than simply attaining some intermediate level of output. This bound on inflation rates consistent with full employment has been labeled by Krugman variously as the "timidity trap or", in reference to Japan in the late 1990s, as the "law of the excluded middle".

In this environment, commitments to keep nominal rates low are inconsistent with attaining the full employment. Households and firms in the deflation equilibrium expect the nominal rate to remain zero indefinitely. Indeed, interest rate commitments of the variety currently pursued by the

Federal Reserve would be irrelevant in shifting the economy out of the deflationary equilibrium.

While the implications for monetary policy at the zero lower bound differ a bit in our OLG model, the implications for fiscal policy are closer to the lessons from the representative agent New Keynesian model. In particular, government purchases can be a useful tool for reducing the excess of savings and raising real rates above zero. If middle-generation households are taxed to finance some level of government expenditure and the government runs a balanced budget, then an increase in government expenditure reduces the supply of loans and raises the real interest rate. The resulting expression for the real rate is given below:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{(Y_t - G_t - D_{t-1})} \tag{27}$$

where G_t is aggregate government purchases. The expression for the real interest rate in equation (27) is a generalization of the expression for the real interest rate obtained in equation (11), with the real interest rate strictly increasing in the level of government purchases.

It it straightforward to also consider what the multiplier of government spending is in this context for small deviations of government spending from steady state. To a first order approximation this multiplier is given by:

$$\frac{dY}{dG} = \frac{1}{1 - \psi\kappa} > 1$$

which necessarily is greater than 1. The reason why it must be greater than one is the positive effect spending has on inflation. This effect is governed by κ which means that the real interest rate goes down with higher level of spending. If wages are completely fixed then $\kappa = 0$ the multiplier is exactly 1. Recall that this variation of the model is one in which the middle income household is receiving all the income. If the young or the old also receive some income, then this multiplier can be higher, as they will spend every extra dollar of income on consumption and the multiplier starts looking more along the lines of Eggertsson and Krugman (2012).

The effect of transfers is relatively straightforward in this model. What is important is the extent to which transfers move money from the unconstrained agents (the saver) to those that are constrained, namely the young and the old (and possibly constrained middle aged households). These transfers will then be expansionary, an example of which may be various redistribution schemes. Overall the model paints a pretty positive picture for both government spending and redistribution from the point of view of generating economic stabilization.

7 Dynamic Stability

So far, our analysis of secular stagnation has focused exclusively on the permanent state of affairs (the steady state) for output, inflation, real interest rates and real wages. Left unanswered is

whether the transition to this steady state is well behaved once the economy is pushed out of the "normal" situation, for example due to a deleveraging shock or changing demographics. We now consider this question by linearizing the model around the steady state already illustrated and showing that it is stable.

We log-linearize our model around $\Pi = 1$ and a binding zero lower bound. Our model can be distilled into a linearized aggregate demand and linearized aggregate supply curve. Aggregate demand is obtained by linearizing and combining $(11),(13),(7)$ and (22), while aggregate supply is obtained by combining relations (19), (20) and (21):

$$y_t = \bar{D}d_{t-1} + \psi E_t\pi_{t+1} + \psi d_t + \psi g_t \tag{28}$$

$$\pi_t = \frac{\kappa}{1-\gamma}y_t - \frac{\kappa\gamma}{1-\gamma}y_{t-1} \tag{29}$$

where lower case denotes that the variables are expressed in terms of deviations from steady state[11]. We can combine these two equations to yield a first-order stochastic difference equation in terms of y_t only given by:

$$y_t = \frac{\psi\kappa}{1-\gamma+\psi\kappa\gamma}E_t y_{t+1} + \frac{1-\gamma}{1-\gamma+\psi\kappa\gamma}\left(\bar{D}d_{t-1} + \psi d_t + \psi g_t\right)$$

A unique bounded solution now exists for this system as long as $\left(\frac{\psi\kappa}{1-\gamma+\psi\kappa\gamma}\right) < 1$, which obtains as long as $\kappa < \psi^{-1}$. We see that this condition is the same as the condition in (26) which ensures the existence of equilibrium. Thus, if equilibrium exists, then the equilbrium is locally unique and stable. This stands in contrast to the deflation steady state analyzed in Schmitt-Grohé and Uribe (2013).

For a zero inflation or positive inflation steady state where monetary policy is no longer constrained by the zero lower bound, it is possible to show that the Taylor principle $\phi_\pi > 1$ is a sufficient but not a necessary condition for the existence of a unique rational expectations equilibrium. Thus, a simple Taylor rule away from the zero lower bound remains a sensible monetary policy rule.

8   Conclusion

In this paper, we formalize the secular stagnation hypothesis in an overlapping generations model with nominal wage rigidity. We show that, in this setting, any combination of a permanent collateral (deleveraging) shock, slowdown in population growth, or an increase in inequality can lead to a permanent output shortfall by lowering the natural rate of interest below zero on a sustained basis. Absent a higher inflation target, the zero lower bound on nominal rates will bind, real wages will exceed their market clearing rate, and, output will fall below the full employment level.

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[11]Note that g_t is an exogenous shock to population growth, not a shock to government spending.

We also demonstrate that the Keynesian paradoxes of thrift, toil and flexibility continue to hold in our OLG setting. Like the representative agent model, fiscal policy remains an effective tool for shifting back the AD curve towards full employment. However, commitments by a central bank to keep nominal rates low are ineffective if nominal rates are expected to remain low indefinitely.

Our main point is not a prediction that the world as we see it today will remain mired in a recession forever. Instead, the purpose is to clarify the conditions under which this can happen, or more to the point, provide a formalization of the popular secular stagnation hypothesis. Perhaps a main takeaway from the analysis is not just that a permenent recession is possible, but instead that a liquidity trap can be of arbitrary duration and last as long as the particular dynamics that give rise to it (such as a deleveraging shock and/or rise in inequality and/or population growth slowdown). This would suggest that a passive attitude to a recession of this kind is inapproriate.

We anticipate that the mechanisms illustrated in this environment would persist in a richer, quantitative lifecycle model that could be calibrated to match moments of saving and income for the US. In future work, we plan to incorporate capital and investment in this framework and consider the role of fiscal policy financed by changes in the public debt. This framework with negative real interest rates may have further implications for asset prices as well, and also, we hope, provide sharper predictions for the future rather than simply illuminating and suggesting a depressing possibility.

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