

# The Automatic Stabilizer Property of Social Insurance Programs\*

Alisdair McKay  
Boston University

Ricardo Reis  
Columbia University and  
London School of Economics

April 29, 2016

## Abstract

Should the generosity of unemployment benefits and the progressivity of income taxes depend on the presence of business cycles? This paper proposes a tractable model where there is a role for social insurance against uninsurable shocks to income and unemployment, as well as inefficient business cycles driven by aggregate shocks through matching frictions and nominal rigidities. We derive an augmented Baily-Chetty formula showing that the optimal generosity and progressivity depend on a macroeconomic stabilization term. Using a series of analytical examples, we show that this term typically pushes for an increase in generosity and progressivity as long as slack is more responsive to social programs in recessions. A calibration to the U.S. economy shows that taking concerns for macroeconomic stabilization into account raises the optimal unemployment benefits replacement rate by 13 percentage points but has a negligible impact on the optimal progressivity of the income tax. More generally, the role of social insurance programs as automatic stabilizers affects their optimal design.

**JEL codes:** E62, H21, H30.

**Keywords:** Countercyclical fiscal policy; Redistribution; Distortionary taxes.

---

\*Contact: amckay@bu.edu and r.a.reis@lse.ac.uk. The first draft of this paper circulated as “Optimal Automatic Stabilizers” in February of 2015. We are grateful to seminar participants at the SED 2015, the AEA 2016, Brandeis, Columbia, the Federal Reserve Bank of Atlanta, and the LSE for useful comments.

# 1 Introduction

The usual motivation behind large social welfare programs, like unemployment insurance or progressive income taxation, is to provide social insurance and engage in redistribution. A large literature therefore studies the optimal progressivity of income taxes typically by weighing the disincentive effect on individual labor supply and savings against concerns for redistribution and for insurance against idiosyncratic income shocks.<sup>1</sup> In turn, the optimal generosity of unemployment benefits is often stated in terms of a Baily-Chetty formula, which weighs the moral hazard effect of unemployment insurance on job search and creation against the social insurance benefits that it provides.<sup>2</sup>

For the most part, this literature abstracts from aggregate shocks, so that the optimal generosity and progressivity do not take into account business cycles. Yet, from their inception, an auxiliary justification for these social programs was that they were also supposed to automatically stabilize the business cycle.<sup>3</sup> Classic work that did focus on the automatic stabilizers relied on a Keynesian tradition that ignores the social insurance that these programs provide or their disincentive effects on employment. More modern work focuses on the positive effects of the automatic stabilizers, but falls short of computing optimal policies.<sup>4</sup>

The goal of this paper is to answer two classic questions—How generous should unemployment benefits be? How progressive should income taxes be?—but taking into account their automatic stabilizer nature. We present a model in which there is both a welfare role for social insurance as well as aggregate shocks and inefficient business cycles. We use it to solve for the ex ante socially optimal replacement rate of unemployment benefits and progressivity of personal income taxes in the presence of uninsured income risks, precautionary savings motives, labor market frictions, and nominal rigidities.

Our first main contribution is to define the welfare benefits of automatic stabilizers formally through a business-cycle variant of the Baily-Chetty formula for unemployment insurance and a similar formula for the optimal choice of progressivity of the tax system that are both augmented

---

<sup>1</sup>Mirrlees (1971) and Varian (1980) are classic references, and more recently see Conesa and Krueger (2006), Heathcote et al. (2014), Krueger and Ludwig (2013), and Golosov et al. (2016).

<sup>2</sup>See the classic work by Baily (1978) and Chetty (2006), and more recently Landais et al. (2013) and Kekre (2015).

<sup>3</sup>Musgrave and Miller (1948) and Auerbach and Feenberg (2000) are classic references, while Blanchard et al. (2010) is a recent call for more modern work in this topic.

<sup>4</sup>See McKay and Reis (2016) for a recent model, DiMaggio and Kermani (2016) for recent empirical work, and IMF (2015) for the shortcomings of the older literature.

by a new macroeconomic stabilization term. This term equals the expectation of the product of the welfare gain from eliminating economic slack with the elasticity of slack with respect to the replacement rate or tax progressivity. Even if the economy is efficient on average, economic fluctuations may lead to more generous unemployment insurance or more progressive income taxes, relative to standard analyses that ignore the automatic stabilizer properties of these programs.

The second contribution is to characterize this macroeconomic stabilization term analytically to understand the different economic mechanisms behind it. Fluctuations in a measure of aggregate economic slack can lead to welfare losses through four separate channels. First, it may create a wedge between the marginal disutility of hours worked and the social benefit of work. This inefficiency appears in standard models of inefficient business cycles, and is sometimes described as a result of time-varying markups (Chari et al., 2007; Galí et al., 2007). Second, when labor markets are tight, more workers are employed raising production but the cost of recruiting and hiring workers rises. The equilibrium level of unemployment need not be efficient as hiring and search decisions do not necessarily internalize these tradeoffs. This is the source of inefficiency common to search models (e.g. Hosios, 1990). Third, the state of the business cycles alters the extent of uninsurable risk that households face both in unemployment and income risk. This is the source of welfare costs of business cycles that has been studied by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007). Finally, with nominal rigidities, slack affects inflation and the dispersion of relative prices, as emphasized by the new Keynesian business cycle literature (Woodford, 2010; Gali, 2011). Our measure isolates these four effects cleanly in terms of separate additive terms in the condition determining the optimal extent of the social insurance programs.

In turn, the effects of benefits and progressivity of taxes on economic slack depends on their direct effect on aggregate demand, as well as on the impact of aggregate demand on equilibrium output. We show that considering macroeconomic stabilization raises the optimal replacement rate of unemployment insurance. This is because, in recessions, economic activity is inefficiently low and aggregate slack is more responsive to the replacement rate for two reasons. First, there are more unemployed workers with high marginal propensities to consume receiving the transfers. Second, the effect of social insurance on precautionary savings motives is larger when there is a greater risk of unemployment. A similar argument applies to progressive taxation because income risk is countercyclical. Our analysis also makes clear the effect of other aggregate demand policies: there

is little role for fiscal policy to stabilize the business cycle if prices are very flexible or if monetary policy is very aggressive.

Our third and final contribution is to assess whether the macroeconomic stabilization term is quantitatively significant. We do so by measuring how much more generous is unemployment insurance and how much more progressive are income taxes in a calibrated economy with aggregate shocks relative to one where these shocks are turned off. We find a large effect on unemployment insurance: with business cycles, the optimal unemployment replacement rate rises from 36 to 49 percent. However, the level of tax progressivity has very little stabilizing effect on the business cycle so the presence of aggregate shocks has almost no effect on the optimal degree of progressivity.

There are large literatures on the three topics that we touch on: business cycle models with incomplete markets and nominal rigidities, social insurance and public programs, and automatic stabilizers. Our model of aggregate demand has some of the key features of new Keynesian models with labor markets (Gali, 2011) but that literature focuses on optimal monetary policy, whereas we study the optimal design of fiscal rules. Our model of incomplete markets builds on McKay and Reis (2016), Ravn and Sterk (2013), and Heathcote et al. (2014) to generate a tractable model of incomplete markets and automatic stabilizers. This simplicity allows us to analytically express optimality conditions for generosity and progressivity, and to, even in the more general case, easily solve the model numerically and so be able to search for the optimal policies. Finally, our paper is part of a surge of work on the interplay of nominal rigidities and precautionary savings, but this literature has mostly been positive whereas this paper's focus is on optimal policy.<sup>5</sup>

On the generosity of unemployment insurance, our work is closest to Landais et al. (2013) and Kekre (2015). They also generalize the standard Baily-Chetty formula by considering the general equilibrium effects of unemployment insurance. The main difference is that while they study how benefits should vary over the business cycle, we focus instead on showing that even when the level of benefits is fixed, the business cycle affects the optimal choice of this level.<sup>6</sup> In this sense, our focus is on automatic stabilizers, an ex ante passive policy, while they consider active stabilization policy. Moreover, our model includes aggregate uncertainty and we also study income tax progressivity.

On income taxes, our work is closest to Bhandari et al. (2013). They are one of the very few

---

<sup>5</sup>See Oh and Reis (2012); Guerrieri and Lorenzoni (2011); Auclert (2016); McKay et al. (2016); Kaplan et al. (2016); Werning (2015).

<sup>6</sup>See also Mitman and Rabinovich (2011), Jung and Kuester (2014), and Den Haan et al. (2015).

studies of optimal income taxes with aggregate shocks and, like us, they emphasize the interaction between business cycles and the desire for redistribution.<sup>7</sup> However, they do not consider unemployment benefits and restrict themselves to flat taxes over income. Moreover, they solve for the Ramsey optimal fiscal policy, which adjusts the tax instruments every period in response to shocks, while we choose the ex ante optimal rules for generosity and progressivity. This is consistent with our focus on automatic stabilizers, which are ex ante fiscal systems, rather than countercyclical policies.

Finally, this paper is related to the modern study of automatic stabilizers and especially our earlier work in [McKay and Reis \(2016\)](#). There, we considered how the actual automatic stabilizers implemented in the US alter the dynamics of the business cycle. Here we are concerned with the optimal fiscal system as opposed to the observed one.

The paper is structured as follows. Section 2 presents the model, and section 3 discusses its equilibrium properties. Section 4 derives the macroeconomic stabilization term in the optimality conditions for the two social programs. Section 5 discusses its qualitative properties, the economic mechanisms that it depends on, and its likely sign. Section 6 calibrates the model, and quantifies the effects of the automatic stabilizer effect. Section 7 concludes.

## 2 The Model

The main ingredients in the model are uninsurable income and employment risks, social insurance programs, and nominal rigidities so that aggregate demand matters for equilibrium allocations. The model makes a series of particular assumptions, which we explain in this section, in order to generate a tractability that is laid out in the next section. The model is set in discrete time, indexed by  $t$ .

### 2.1 Agents and commodities

There are two groups of private agents in the economy: households and firms.

Households are indexed by  $i$  in the unit interval, and their type is given by their productivity  $\alpha_{i,t} \in \mathbb{R}_0^+$  and employment status  $n_{i,t} \in (0, 1)$ . Every period, an independently drawn share  $\delta$  dies,

---

<sup>7</sup>[Werning \(2007\)](#) also studies optimal income taxes with aggregate shocks and social insurance.

and is replaced by newborn households with no assets and productivity normalized to  $\alpha_{i,t} = 1$ .<sup>8</sup> Households derive utility from consumption,  $c_{i,t}$ , and publicly provided goods,  $G_t$ , and derive disutility from working for pay,  $h_{i,t}$ , searching for work,  $q_{i,t}$ , and being unemployed according to the utility function:

$$\mathbb{E}_0 \sum_t \beta^t \left[ \log(c_{i,t}) - \frac{h_{i,t}^{1+\gamma}}{1+\gamma} - \frac{q_{i,t}^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi (1 - n_{i,t}) \right]. \quad (1)$$

The parameter  $\beta$  captures the joint discounting effect from time preference and mortality risk, while  $\xi$  is a non-pecuniary cost of being unemployed.<sup>9</sup>

The final consumption good is provided by a competitive final goods sector in the amount  $Y_t$  that sells for price  $p_t$ . It is produced by combining varieties of goods in a Dixit-Stiglitz aggregator with elasticity of substitution  $\mu/(\mu - 1)$ . Each variety  $j \in [0, 1]$  is monopolistically provided by a firm with output  $y_{j,t}$  by hiring labor from the households and paying the wage  $w_t$  per unit of effective labor.

## 2.2 Asset markets and social programs

Households can insure against mortality risk by buying an annuity, but they cannot insure against risks to their individual skill or employment status. The simplest way to capture this market incompleteness is by assuming that households only trade a single risk-free bond, that has a gross real return  $R_t$  and is in zero net supply. Moreover, we assume that households cannot borrow, so if  $a_{i,t}$  measures their asset holdings:<sup>10</sup>

$$a_{i,t} \geq 0. \quad (2)$$

The focus of our paper is on two social programs. The first is a progressive income tax such

---

<sup>8</sup>The mortality risk allows for a stationary cross-sectional distribution of productivity along with permanent shocks in section 6, but otherwise plays no significant role in the analysis and so will be assumed away in sections 4 and 5.

<sup>9</sup>If  $\hat{\beta}$  is pure time discounting, then  $\beta \equiv \hat{\beta}(1 - \delta)$ .

<sup>10</sup>A standard formulation for asset markets that gives rise to these annuity bonds is the following: A financial intermediary sells claims that pay one unit if the household survives and zero units if the household dies, and supports these claims by trading a riskless bond with return  $\tilde{R}$ . If  $\tilde{a}_i$  are the annuity holdings of household  $i$ , the law of large numbers implies the intermediary pays out in total  $(1 - \delta) \int \tilde{a}_i di$ , which is known in advance, and the cost of the bond position to support it is  $(1 - \delta) \int \tilde{a}_i di / \tilde{R}$ . Because the riskless bond is in zero net supply, then the net supply of annuities is zero  $\int \tilde{a}_i di = 0$ , and for the intermediary to make zero profits,  $R_t = \tilde{R}_t / (1 - \delta)$ .

that if  $z_{i,t}$  is pre-tax income, the after-tax income is  $\lambda_t z_{i,t}^{1-\tau}$ .  $\lambda_t \in [0, 1]$  determines the overall level of taxes, which together with the size of government purchases  $G_t$ , will pin down the size of the government. The object of our study is instead the automatic stabilizer role of the government, so our focus is on  $\tau \in [0, 1]$ . This determines the progressivity of the tax system. If  $\tau = 0$ , there is a flat tax at rate  $1 - \lambda_t$ , while if  $\tau = 1$  everyone ends up with the same after-tax income. In between, a higher  $\tau$  implies a more convex tax function, or a more progressive income tax system.

The second social program is unemployment insurance. A household qualifies as long as it is unemployed ( $n_{i,t} = 0$ ) and collects benefits that are paid in proportion to what the unemployed worker would earn if she were employed. Suppose the worker's productivity is such that she would earn pre-tax income  $z_{i,t}$  if she were employed, then her after-tax unemployment benefit is  $b\lambda_t z_{i,t}^{1-\tau}$ .<sup>11</sup> Our focus is on the replacement rate  $b \in [0, 1]$ , with a more generous program understood as having a higher  $b$ .<sup>12</sup>

Our goal is to characterize the optimal fixed levels of  $b$  and  $\tau$ . Importantly, we consider the ex ante design problem, so  $b$  and  $\tau$  do not depend on time or on the state of the business cycle. This corresponds to our focus on their role as automatic stabilizers, that is programs that can automatically stabilize the business cycle without policy intervention. The literature on empirically measuring the automatic stabilizers makes a sharp distinction between their built-in properties and the feedback rules or discretionary choices that may dictate adjustments to the programs in response to current and past information.<sup>13</sup>

## 2.3 Key frictions

There are three key frictions in the economy that create the policy trade-offs that we analyze.

---

<sup>11</sup>It would be more realistic, but less tractable, to assume that benefits are a proportion of the income the agent earned when she lost her job. But, given the persistence in earning, both in the data and in our model, our formulation will not be quantitatively too different from this case.

<sup>12</sup>In our notation, it may appear that unemployment benefits are not subject to the income tax, but this is just the result of a normalization: if they were taxed and the replacement rate was  $\tilde{b}$ , then the model would be unchanged and  $b \equiv \tilde{b}^{1-\tau}$ .

<sup>13</sup>Perotti (2005) among many others.

### 2.3.1 Productivity risk

Labor income for an employed household is  $\alpha_{it}w_t h_{it}$ . The productivity of households evolves as

$$\alpha_{i,t+1} = \alpha_{i,t}\epsilon_{i,t+1} \text{ with } \epsilon_{i,t+1} \sim F(\epsilon; x_t), \quad (3)$$

and where  $\int \epsilon dF(\epsilon, x_t) = 1$  for all  $t$ , which implies that the average idiosyncratic productivity in the population is constant and equal to one.<sup>14</sup>

The distribution of shocks varies over time so that the model generates cyclical changes in the distribution of earnings risks, as documented by [Storesletten et al. \(2004\)](#) or [Guvenen et al. \(2014\)](#). We capture this dependence through the variable  $x_t$ , which captures the aggregate slack in the economy. As usual, a higher  $x_t$  implies that the economy is tighter, the output gap is positive, or that the economy is closer to capacity or booming. In the next section we will map  $x_t$  to concrete measures of the state of the business cycle like the unemployment rate or the job finding rate. For concreteness, a simple case that maps to many empirical estimates is to have  $F(\cdot)$  be log-normal with  $\text{Var}(\log \epsilon) = \sigma^2(x)$  and  $\mathbb{E}(\log \epsilon) = -0.5\sigma(x)^2$ .

### 2.3.2 Employment risk

The second source of risk is employment. We make a strong assumption that unemployment is distributed i.i.d. across households. Given the high (quarterly) job-finding rates in the US, this is not such a poor approximation, and it reduces the state space of the model. At the start of the period, a fraction  $v$  of households loses employment and must search to regain employment. Search effort  $q_{i,t}$  leads to employment with probability  $M_t q_{i,t}$ , where  $M_t$  is the job-finding rate per unit of search effort and the probability of resulting in a match is the same for each unit of search effort. Therefore, if all households make the same search effort, then aggregate hiring will be  $vM_t q_t$  and as a result the unemployment rate will be:

$$u_t = v(1 - q_t M_t). \quad (4)$$

Each firm begins the period with a mass  $1 - v$  of workers and must post vacancies at a cost to hire additional workers. As in [Blanchard and Galí \(2010\)](#), the cost per hire is increasing in aggregate

---

<sup>14</sup>Since newborn households have productivity 1, the assumption is that they have average productivity.



labor market tightness, which is just equal to the ratio of hires to searchers, or the job-finding rate  $M_t$ . The hiring cost per hire by firm is  $\psi_1 M_t^{\psi_2}$ , denominated in units of final goods where  $\psi_1$  and  $\psi_2$  are parameters that govern the level and elasticity of the hiring costs. Since aggregate hires are the difference between the beginning of period non-employment rate  $v$  and the realized unemployment rate  $u_t$ , aggregate hiring costs are:

$$J_t = \psi_1 M_t^{\psi_2} (v - u_t). \quad (5)$$

We assume a law of large numbers within the firm so the average productivity of hires is 1.

In this model of the labor market, there is a surplus in the employment relationship since, on one side, firms would have to pay hiring costs to replace the worker and, on the other side, a worker who rejects a job becomes unemployed and foregoes the opportunity to earn wages this period. This surplus creates a bargaining set for wages, which is further complicated in our model because labor supply varies on both the intensive and extensive margins and firms face nominal rigidities. There are many alternative models of the wage setting process, from Nash bargaining to wage stickiness, as emphasized by [Hall \(2005\)](#). We assume a convenient rule that simplifies our model:

$$w_t = \bar{w} A_t (1 - J_t/Y_t) x_t^\zeta. \quad (6)$$

The real wage per effective unit of labor depends on three variables, aside from a constant  $\bar{w}$ . First, it increases proportionately with aggregate effective productivity  $A_t$ , as it would in a frictional model of the labor market. Second, it falls when aggregate hiring costs are higher, so that some of these costs are passed from firms to workers. The justification is that when hiring costs rise, the economy is poorer and this raises labor supply, which the fall in wages exactly offsets. Since these costs are quantitatively small, in reality and in our calibrations, this assumption has little effect in the predictions of the model but allows us to not have to carry this uninteresting wealth effect on labor supply throughout the analysis.<sup>15</sup> Third, when the labor market is tighter, wages rise, with an elasticity of  $\zeta$ .

---

<sup>15</sup>Moreover, in the special cases of the model studied in section 5,  $J_t/Y_t$  is a function of  $x_t$  so this term gets absorbed by the next term after a redefinition of  $\zeta$ .

### 2.3.3 Nominal rigidities

The firm that produces each variety uses the production function  $y_{j,t} = \eta_t^A h_{j,t} l_{j,t}$ , where  $h_{j,t}$  are hours per worker and  $l_{j,t}$  the workers in the firm, subject to exogenous productivity shocks  $\eta_t^A$ . The firm's marginal cost is

$$\frac{w_t + \psi_1 M_t^{\psi_2} / h_t}{\eta_t^A}.$$

Marginal costs are the sum of the wage paid per effective unit of labor and the hiring costs that had to be paid, divided by productivity. Under flexible prices, the firm would set a constant markup,  $\mu$ , over marginal cost. The aggregate profits of these firms are distributed among employed workers in proportion to their skill, which can be thought of as representing bonus payments in a sharing economy.<sup>16</sup>

However, individual firms cannot choose their actual price to equal this desired price every period because of nominal rigidities. We consider two separate simple models of nominal rigidities. In the numerical study of section 6, we assume Calvo (1983) pricing, so that every period a fraction  $\theta$  of randomly drawn firms are allowed to change their price, with the remaining  $1 - \theta$  having to keep their price unchanged from the last period. This leads to the dynamics for inflation  $\pi_t \equiv p_t / p_{t-1}$ :

$$\pi_t = \left[ (1 - \theta) / \left[ 1 - \theta \left( \frac{p_t^*}{p_t} \right)^{1/(1-\mu)} \right] \right]^{1-\mu} \quad (7)$$

where  $p_t^*$  is the price chosen by firms that adjust their price in period  $t$ .

In the analytical study of sections 4 and 5 we assume instead a simpler and more transparent canonical model of nominal rigidities, where every period an i.i.d. fraction  $\theta$  of firms can set their prices  $p_{j,t} = p_t^*$ , while the remaining set their price to equal what they expected their optimal price would be:  $p_{j,t} = \mathbb{E}_{t-1} p_t^*$ . Mankiw and Reis (2010) show that most of the qualitative insights from New Keynesian economics can be captured by this simple sticky-information formulation.

---

<sup>16</sup>Given the structure of the labor market, all firms will have workers working the same hours, and will vary only how many workers to hire. Therefore,  $h_{j,t}$  is the same for all  $j$ , and so is the desired price.

## 2.4 Other government policy

Aside from the two social programs that are the focus of our study, the government also chooses policies for nominal interest rates, government purchases, and the public debt. Starting with the first, we assume a standard Taylor rule for nominal interest rates  $I_t$ :

$$I_t = \bar{I} \pi_t^{\omega_\pi} x_t^{\omega_x} \eta_t^I. \quad (8)$$

where  $\omega_\pi > 1$  and  $\omega_x \geq 0$ . The exogenous  $\eta_t^I$  represent shocks to monetary policy.<sup>17</sup>

Turning to the second, government purchases follow the Samuelson (1954) rule such that, absent  $\eta_t^G$  shocks, the marginal utility benefit of public goods offsets the marginal utility loss from diverting goods from private consumption:

$$G_t = \chi C_t \eta_t^G. \quad (9)$$

Finally, let  $T_t$  be the fiscal impact of the social programs, or the difference between income tax revenue and expenditures on unemployment benefits, so that  $G_t - T_t$  is the primary deficit.<sup>18</sup> This is financed by borrowing from abroad an amount  $B_t$  and paying a world interest rate  $R^*$  so that the government budget constraint is:

$$G_t - T_t = B_{t+1} - B_t - (R^* - 1)B_t. \quad (10)$$

This peculiar assumption deserves some explanation. It is well known, at least since Aiyagari and McGrattan (1998), that in an incomplete markets economy like ours, changes in the supply of safe assets will affect the ability to accumulate in precautionary savings. Deficits or surpluses may stabilize the business cycle by changing the cost of self-insurance. By assuming that the changes in public debt during the business cycle do not affect the safe assets available to households, who cannot borrow from abroad, we eliminate this effect. In the same way that we abstracted above from the stabilizing properties of changes in government purchases, this lets us likewise abstract from the stabilizing property of public debt, in order to focus on our two social programs. Consistent with this approach, we further assume that when we vary the generosity of unemployment insurance

<sup>17</sup>As usual, the real and nominal interest rates are linked by the Fisher equation  $R_t = I_t / \mathbb{E}_t [\pi_{t+1}]$ .

<sup>18</sup>That is:  $T_t = \int z_{i,t} - \lambda_t z_{i,t}^{1-\tau} di - \bar{b} \int (1 - n_{i,t}) z_{i,t} di$ .

and the progressivity of income taxes, debt does not change:  $dB_t/db = dB_t/d\tau = 0$  for all  $t$ .<sup>19</sup>

### 3 Equilibrium and the role of policy

Our model combines idiosyncratic risk, incomplete markets, and nominal rigidities, and yet it is structured so as to be tractable enough to investigate optimal policy. This section highlights how our assumptions, with their virtues and limitations, lead to this tractability. We also highlight the role for social insurance policy in the economy, as well as the distortions it creates.

#### 3.1 Inequality and heterogeneity

The following result follows from the particular assumptions we made on the decision problems of different agents and plays a crucial role in simplifying the analysis:

**Lemma 1.** *All households choose the same asset holdings, hours worked, and search effort, so  $a_{i,t} = 0$ ,  $h_{i,t} = h_t$ , and  $q_{i,t} = q_t$  for all  $i$ .*

To prove this result, note that the decision problem of a household searching for a job at the start of the period is:

$$V^s(a, \alpha, \mathcal{S}) = \max_q \left\{ MqV(a, \alpha, 1, \mathcal{S}) + (1 - Mq)V(a, \alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}, \quad (11)$$

where we used  $\mathcal{S}$  to denote the collection of aggregate states. The decision problem of the household at the end of the period is:

$$V(a, \alpha, n, \mathcal{S}) = \max_{c, h, a' \geq 0} \left\{ \log c - \frac{h^{1+\gamma}}{1 + \gamma} + \chi \log(G) - \xi(1 - n) + \beta \mathbb{E} [(1 - v)V(a', \alpha', 1, \mathcal{S}') + vV^s(a', \alpha', \mathcal{S}')] \right\}, \quad (12)$$

$$\text{subject to: } a' + c = Ra + \lambda(n + (1 - n)b) [\alpha(wh + d)]^{1-\tau}. \quad (13)$$

Starting with asset holdings, since no agent can borrow and bonds are in zero net supply, then it must be that  $a_{i,t} = 0$  for all  $i$  in equilibrium, a result also used by [Krusell et al. \(2011\)](#) and

---

<sup>19</sup>A perhaps simpler way to achieve our goal would be to assume that the government always runs a balanced budget, so  $B_t = 0$ . This would have no effect in our analytical results in sections 4 and 5, but it makes it harder to seriously take the model to the data in section 6.

Ravn and Sterk (2013). Turning to hours worked, the intra-temporal labor supply condition for an employed household is

$$c_{i,t}h_{i,t}^\gamma = \lambda_t z_{i,t}^{-\tau} w_t \alpha_{i,t}, \quad (14)$$

where the left-hand side is the marginal rate of substitution between consumption and leisure, and the right-hand side is the after-tax return to working an extra hour to raise income  $z_{i,t}$ . More productive agents want to work more. However, they are also richer and want to consume more. The combination of our preferences and the budget constraint imply that these two effects exactly cancel out so that in equilibrium all employed households work the same hours.

$$h_t^\gamma = \frac{(1-\tau)w_t}{w_t h_t + d_t}, \quad (15)$$

where  $d_t$  is aggregate dividends per employed worker.<sup>20</sup>

Finally, the optimality condition for search effort is:

$$q_{i,t}^k = M_t [V(a_{i,t}, \alpha_{i,t}, 1, \mathcal{S}) - V(a_{i,t}, \alpha_{i,t}, 0, \mathcal{S})]. \quad (16)$$

Intuitively, the household equates the marginal disutility of searching on the left-hand side to the expected benefit of finding a job on the right-hand side, which is the product of the job-finding probability  $M_t$  and the increase in value of becoming employed. Appendix A.1 shows that this increase in value is independent of  $\alpha_{i,t}$ . The key assumption that ensures this is that unemployment benefits are indexed to income  $z_{i,t}$  so the after-tax income with and without a job scales with idiosyncratic productivity in the same way. This then implies that  $q_{i,t}$  is the same for all households.

The lemma clearly limits the scope of our study. We cannot speak to the effect of policy on asset holdings, and differences in labor supply are reduced to having a job or not, which while it is perhaps the first-order source of heterogeneity, ignores diversity in part-time jobs and overtime. At the same time, it has the big payoff of implying that  $\mathcal{S}$  contains only aggregate variables, so we do not need to keep track of cross-sectional distributions to characterize an equilibrium as in Krusell and Smith (1998). Thus, our model can be studied analytically and numerical solutions are easy to

---

<sup>20</sup>To derive this, substitute  $z_{i,t} = \alpha_{i,t}(w_t h_{i,t} + d_t)$  and  $c_{i,t} = \lambda_t z_{i,t}^{1-\tau}$  into (14).

compute. Moreover, arguably the social programs that we study are more concerned with income, rather than wealth inequality, and the vast majority of studies of the automatic stabilizers also ignores any direct effects of wealth inequality (as opposed to income inequality) on the business cycle.

In our model, there is a rich distribution of income and consumption,  $(z_{i,t}, c_{i,t})$ , driven by heterogeneity in employment status  $n_{i,t}$  and skill  $a_{i,t}$ . In section 6, we are able to fit the more prominent features of income inequality in the United States by parameterizing the distribution  $F(\epsilon, x)$ . Moreover, in our model, there is a rich distribution of individual prices and output across firms,  $(p_{j,t}, y_{j,t})$ , driven by nominal rigidities. And finally, the exogenous aggregate shocks to productivity, monetary policy, and government purchases,  $(\eta_t^A, \eta_t^I, \eta_t^G)$ , affect all of these distributions, which therefore vary over time and over the business cycle. In spite of the simplifications and their limitations, our model still admits a rich amount of inequality and heterogeneity.

### 3.2 Quasi-aggregation and consumption

Define  $\tilde{c}_t$  as the consumption of the average-skilled employed agent. This is related to aggregate consumption,  $C_t$ , according to (see Appendix A.2):

$$\tilde{c}_t = \frac{C_t}{\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] (1 - u_t + u_t b)}. \quad (17)$$

Funding higher replacement rates requires larger taxes on those employed, so it reduces their consumption. Likewise, the amount of revenue raised by the progressive tax system depends on the distribution of income as summarized by  $\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right]$ . More dispersed incomes generate higher revenues and allow for lower taxes for a given level of income.

The next property that simplifies our model is proven in Appendix A.2.

**Lemma 2.** *Aggregate consumption dynamics can be computed from*

$$\frac{1}{\tilde{c}_t} = \beta R_t \mathbb{E}_t \left\{ \frac{1}{\tilde{c}_{t+1}} Q_{t+1} \right\} \quad (18)$$

$$\text{with: } Q_{t+1} \equiv [(1 - u_{t+1}) + u_{t+1} b^{-1}] \mathbb{E} \left[ \epsilon_{i,t+1}^{-(1-\tau)} \right] \quad (19)$$

and equation (17).

Without uncertainty on productivity or unemployment,  $Q_{t+1} = 1$ , and this would be a standard Euler equation from intertemporal choice stating that expected consumption growth is inversely related to the product of the discount factor and the real interest rate.

The variable  $Q_{t+1}$  captures how heterogeneity affect aggregate consumption dynamics through precautionary savings motives. The more uncertain is income, the larger is  $Q_{t+1}$  and so the larger are savings motives leading to steeper consumption growth. A more generous unemployment insurance system and a more progressive income tax lower the dispersion of after-tax income growth and reduce the effect of this  $Q_{t+1}$  term.

### 3.3 Policy distortions and redistribution over the business cycle

Social policies not only affect aggregate consumption, but also all individual choices in the economy, introducing both distortions and redistribution.

Combining the optimality condition for hours with the wage rule we arrive at (see Appendix A.2)

$$h_t = [\bar{w}(1 - \tau)]^{\frac{1}{1+\gamma}} x_t^{\frac{\zeta}{1+\gamma}}. \quad (20)$$

A more progressive income tax lowers hours worked by increasing the ratio of the marginal tax rate to the average tax rate.

Moving to search effort, one can show that (see Appendix A.2)

$$q_t^\kappa = M_t \left[ \xi - \frac{h_t^{1+\gamma}}{1 + \gamma} - \log(b) \right]. \quad (21)$$

This states that the marginal disutility of searching for a job is equal to the probability of finding a job times the increase in utility of having a job. At the margin, this increase is equal to the difference between the non-pecuniary pain from being unemployed and the disutility of working, minus the loss in utility units of losing the unemployment benefits. More generous benefits therefore lower search effort. Intuitively, they lower the value of finding a job, so less effort is expended looking for one.

The distribution of consumption in the economy is given by a relatively simple expression:

$$c_{i,t} = \left[ \alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b) \right] \tilde{c}_t \quad (22)$$

The expression in brackets shows that more productive and employed households consume more, as expected. Combined with  $\tilde{c}_t$ , this formula also shows how social policies redistribute income and equalize consumption. A higher  $b$  requires larger contributions from all households, lowering  $\tilde{c}_t$ , but only increases the term in brackets for unemployed households. Therefore, it raises the consumption of the unemployed relative to the employed. In turn, a higher  $\tau$  lowers the cross-sectional dispersion of consumption because it reduces the income of the rich more than that of the poor. The state of the business cycle affects the extent of the redistribution by driving both unemployment and the cross-sectional distribution of productivity risk.

Finally, social programs also affect price dispersion and inflation. Recalling that  $A_t \equiv Y_t / \int l_{jt} dj$ , average aggregate labor productivity, then integrating over the individual production functions and using the demand for each variety it follows immediately that  $A_t = \eta_t^A / S_t$  where the new variable is price dispersion:

$$S_t \equiv \int (p_t(j)/p_t)^{\mu/(1-\mu)} dj \geq 1 \quad (23)$$

$$= (1 - \theta) S_{t-1} \pi_t^{-\mu/(1-\mu)} + \theta \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \quad \text{if Calvo,} \quad (24)$$

$$= \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \left[ \theta + (1 - \theta) \left( \frac{\mathbb{E}_{t-1} p_t^*}{p_t^*} \right)^{\mu/(1-\mu)} \right] \quad \text{if sticky information.} \quad (25)$$

Nominal rigidities lead otherwise identical firms to charge different prices, and this relative-price dispersion lowers productivity and output in the economy. The social insurance system will alter the dynamics of aggregate demand leading to different dynamics for nominal marginal costs, inflation, and price dispersion.

### 3.4 Slack and equilibrium

One missing ingredient to close the model is a concrete definition of how to measure economic slack  $x_t$ .



For the analytical results in the next two sections, we will take a convenient assumption:

$$x_t = M_t. \tag{26}$$

That is, we measure the state of the business cycle by the tightness of the labor market, as captured by the job-finding rate. This is not such a strong assumption since, in the model,  $(h_t, q_t)$  are functions of only  $M_t$  and parameters, as we can see in equations (20) and (21). Moreover, in the special cases considered in sections 4 and 5, also the unemployment rate and the output gap, that is the difference between actual output and that which arises with flexible prices, are also functions of  $M_t$  as the single endogenous variable.

We assume equation (26) because it makes the analytical derivations in the next two sections more transparent, allowing us to carry fewer cross-terms that are of little interest. Most of the results would extend easily to other measures of slack, like the unemployment rate or hours worked, but with longer and more involved algebraic expressions. When we take the model to the data in section 6, by solving it numerically, so that analytical transparency is irrelevant, we will consider different assumptions on  $x_t$ .

An aggregate equilibrium in our economy is then a solution for 18 endogenous variables together with the exogenous processes  $\eta_t^A$ ,  $\eta_t^G$ , and  $\eta_t^I$ . Appendix A.3 lays out the entire system of equations that defines this equilibrium.

## 4 Optimal policy and insurance versus incentives

All agents in our economy are identical ex ante, making it natural to take as the target of policy the utilitarian social welfare function. Using equation (22) and integrating the utility function in equation (1) gives the objective function for policy  $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t$ , where period-welfare is:

$$\begin{aligned} W_t = & \mathbb{E}_i \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] \right) + u_t \log b - \log (1 - u_t + u_t b) \\ & + \log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1 + \gamma} - v \frac{q_t^{1+\kappa}}{1 + \kappa} + \chi \log(G_t) - \xi u_t. \end{aligned} \tag{27}$$

The first line shows how inequality affects social welfare. Productivity differences and unemployment introduce costly idiosyncratic risk, which is attenuated by the social insurance policies. The

second line captures the usual effect of aggregates on welfare. While these would be the terms that would survive if there were complete insurance markets, recall that the incompleteness of markets also affects the evolution of aggregates, as we explained in the previous section.

The policy problem is then to pick  $b$  and  $\tau$  to maximize equation (27) subject to the equilibrium conditions, at date 0 once and for all. As already discussed, in this and the next section, we make the following simplifications on the general problem: (i) log-normal productivity shocks, (ii) no mortality, (iii) sticky information, and (iv) no government spending shocks.<sup>21</sup>

#### 4.1 Optimal unemployment insurance

Appendix B derives the following optimality condition for  $b$ :

**Proposition 1.** *The optimal choice of the generosity of unemployment insurance  $b$  satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_t \left( \frac{1}{b} - 1 \right) \frac{\partial \log(b\tilde{c}_t)}{\partial \log b} \Big|_{x,q} + \frac{d \log \tilde{c}_t}{d \log u_t} \frac{\partial \log u_t}{\partial b} \Big|_x + \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = 0. \quad (28)$$

Equation (28) is closely related to the Baily-Chetty formula for optimal unemployment insurance. The first term captures the social insurance value of changing the replacement rate. It is equal to the percentage difference between the marginal utility of unemployed and employed agents times the elasticity of the consumption of the unemployed with respect to the benefit. This term omits the general equilibrium effects of benefits and focuses instead on the tax and transfer effects of changing benefits. If unemployment came with no differences in consumption, this term would be zero, and likewise if giving higher benefits to the unemployed had no effect on their consumption. But as long as employed agents consume more, and raising benefits closes some of the consumption gap, then this term will be positive and call for higher unemployment benefits.

The second term gives the moral hazard cost of unemployment insurance. It is equal to the product of the elasticity of the consumption of the employed with respect to the unemployment rate, which is negative, and the elasticity of the unemployment rate with respect to the benefit that arises out of reduced search effort. Higher replacement rates induce agents to search less, which

---

<sup>21</sup>To be clear, none of these assumptions are essential: relaxing (i) would lead to similar expressions with expectations against  $F(\cdot)$  in place of  $\sigma^2(\cdot)$ , relaxing (ii) would require an adjustment of the distribution of inequality in a way that is irrelevant for policy, substituting (iii) for sticky prices would require integrating the effects of policy on  $S_t$  over time, and relaxing (iv) would lead to an additional term in all the expressions equal to the difference between the marginal utility of public expenditures and the resource cost of financing those expenditures.

raises equilibrium unemployment, and leads to higher taxes to finance benefits.

In the absence of general equilibrium effects, these would be the only two terms, and they capture the standard trade-off between insurance and incentives in the literature, very close to [Chetty \(2006\)](#). Optimal policy is pinned down by averaging these benefits and costs across states and time. With business cycles and general equilibrium effects, there is an extra macroeconomic stabilization term. The larger this term is, the more generous optimal unemployment benefits should be. We explain this shortly, but first, we turn to the income tax.

## 4.2 Optimal progressivity of the income tax

Appendix [B](#) shows the following:

**Proposition 2.** *The optimal progressivity of the tax system  $\tau$  satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{\beta(1-\tau)}{(1-\beta)} \right] \sigma^2(x_t) - \left( \frac{A_t}{C_t} - h_t^\gamma \right) \frac{h_t(1-u_t)}{(1-\tau)(1+\gamma)} + \frac{d \log \tilde{c}_t}{d \log u_t} \frac{\partial \log u_t}{\partial \tau} \Big|_x + \frac{dW_t}{dx_t} \frac{dx_t}{d\tau} \right\} = 0. \quad (29)$$

The first three terms again capture the familiar trade-off between insurance and incentives. The first term gives the welfare benefits of reducing the dispersion in after-tax incomes, which is increasing in the extent of pre-tax inequality as reflected by  $\sigma^2(x_t)$ . The second and third term give the incentive costs of raising progressivity. The second term is the labor wedge, the gap between the marginal product of labor and the marginal disutility of labor. More progressive taxes raise the wedge by discouraging labor supply, as explained earlier. The third term reflects the effect of the tax system on the unemployment rate taking slack as given. The tax system affects the relative rewards to being employed and therefore alters household search effort and the unemployment rate.

Finally, the fourth term captures the concern for macroeconomic stabilization in a very similar way to the term for unemployment benefits. Without this term, the expression above would match closely those derived by [Heathcote et al. \(2014\)](#). In its presence, a larger stabilization term in [\(29\)](#) justifies a larger labor wedge and therefore a more progressive tax.

### 4.3 The macroeconomic stabilization term

The two previous propositions clearly isolate the automatic-stabilizing role of the social insurance programs in a single term. It equals the product of the welfare benefit of changing slack and the response of slack to policy. If business cycles are efficient, the macroeconomic stabilization term is zero. That is, if the economy is always at an efficient level of slack, so that  $dW_t/dx_t = 0$ , then there is no reason to take macroeconomic stabilization into account when designing the stabilizers. Intuitively, the business cycle is of no concern for policymakers in this case.

Even if business cycles are efficient on average though, the automatic stabilizers can still play a role. This is because:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = \sum_{t=0}^{\infty} \beta^t \left\{ \mathbb{E}_0 \left[ \frac{dW_t}{dx_t} \right] \mathbb{E}_0 \left[ \frac{dx_t}{db} \right] + \text{Cov} \left[ \frac{dW_t}{dx_t}, \frac{dx_t}{db} \right] \right\},$$

so that even if  $\mathbb{E}_0 [dW_t/dx_t] = 0$ , a positive covariance term would still imply a positive aggregate stabilization term and an increase in benefits (or more progressive taxes). This is then the hallmark of a social policy that serves as an automatic stabilizer: it stimulates the economy more in recessions, when slack is inefficiently high. The stronger this effect, the larger the program should be. In the next section, we discuss the sign of this covariance and what affects it.

## 5 Inspecting the macroeconomic stabilization term

Understanding the macroeconomic stabilization term requires understanding separately the effect of slack on welfare,  $dW_t/dx_t$ , and the effect of the social policies on slack,  $dx_t/db$  and  $dx_t/d\tau$ .

### 5.1 Slack and welfare

There are five separate channels through which the business cycle may be inefficient in our model, characterized in the following result:

**Proposition 3.** *The effect of macroeconomic slack on welfare can be decomposed into:*

$$\begin{aligned} \frac{dW_t}{dx_t} = & \underbrace{(1 - u_t) \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t}}_{\text{labor-wedge}} - \underbrace{\frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t}}_{\text{price-dispersion}} + \underbrace{\frac{1}{C_t} \frac{dC_t}{du_t} \frac{du_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \Big|_u}_{\text{Hosios}} \\ & - \underbrace{\left( \xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma} \right) \frac{\partial u_t}{\partial x_t} \Big|_q}_{\text{unemployment-risk}} + \underbrace{\frac{1-b}{1-u_t+u_t b} \frac{du_t}{dx_t} - \frac{\beta(1-\tau)^2}{2(1-\beta)} \frac{d\sigma^2(x_t)}{dx_t}}_{\text{income-risk}} \end{aligned} \quad (30)$$

The first term captures the effect of the labor wedge or markups. In the economy,  $A_t/C_t$  is the marginal product of an extra hour worked in utility units, while  $h_t^\gamma$  is the marginal disutility of working. If the first exceeds the second, the economy is underproducing, and increasing hours worked would raise welfare.

The second term captures the effect of slack on price dispersion. Because of nominal rigidities, aggregate shocks will lead to price dispersion. In that case, changes in aggregate slack will affect inflation, via the Phillips curve, and so price dispersion. This is the conventional channel in new Keynesian models through which the output gap affects inflation and its welfare costs.

The third term captures the standard [Hosios \(1990\)](#) trade-off of hiring more workers. On the one hand, the extra hire lowers unemployment and raises consumption. On the other hand, it increases hiring costs. If hiring is efficient, so the Hosios condition holds, then  $(dC_t/du_t)(du_t/dx_t) = \partial J/\partial x_t$  at all dates, but otherwise changes in slack will affect hirings, unemployment and welfare.

The terms in the second line of equation (30) fix aggregate consumption and focus on inequality and its effect on welfare. If the extent of income risk is cyclical, which the literature since [Storesletten et al. \(2004\)](#) has extensively demonstrated, then raising economic activity reduces income risk and so raises welfare. In our model, there is both unemployment and income risk, so this works through two channels.

The fourth and fifth term capture the effect of slack on unemployment risk. For a given aggregate consumption, more unemployment has two effects on welfare. First there are more unemployed who consume a lower amount. The term  $\xi - \log b - h_t^{1+\gamma}/(1+\gamma)$  is the utility loss from becoming unemployed. Second, those who are employed consume a larger share (dividing the pie among fewer employed people). These are the two effects of unemployment risk. The sixth and final term shows that cutting slack also lowers the variance of skill shocks, which lowers income inequality.

## 5.2 Three special cases

To better understand these different channels of stabilization, and link them to the literature before us, we consider three special cases that correspond to familiar models of fluctuations.

### 5.2.1 Frictional unemployment

Consider the special case where prices are flexible ( $\theta = 1$ ), there is no productivity risk ( $\sigma^2 = 0$ ), and labor supply does not vary on the intensive margin because hours worked are constant ( $\gamma = \infty$ ). The only source of inequality is then unemployment, which in our model becomes a result of the search and matching paradigm. Therefore, equation (30) becomes:

$$\frac{dW_t}{dx_t} = \frac{1}{C_t} \frac{dC_t}{du_t} \frac{du_t}{dx_t} - \frac{1}{C_t} \left. \frac{\partial J_t}{\partial x_t} \right|_u - \left( \xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma} \right) \left. \frac{\partial u_t}{\partial x_t} \right|_q + \frac{1-b}{1-u_t+u_t b} \frac{du_t}{dx_t}. \quad (31)$$

as only the Hosios effect and the unemployment risk are now present.

In this special case, our model captures the main effects in [Landais et al. \(2013\)](#). They discuss the macroeconomic effects of unemployment benefits from the perspective of their effect on labor market tightness by changing the worker's bargaining position and wages on the one hand and, on the other hand, their impact on dissuading search effort.

### 5.2.2 Real Business Cycle effects

Next, we consider the case of flexible prices ( $\theta = 1$ ), constant search effort ( $\kappa = \infty$ ), and exogenous job finding ( $M_t$  exogenous).<sup>22</sup> With nominal rigidities and search removed, what is left is the labor wedge and the effect of cyclical income risk on welfare, so equation (30) simplifies to

$$\frac{dW_t}{dx_t} = (1-u_t) \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{\beta}{1-\beta} (1-\tau)^2 \frac{d}{dx_t} \frac{\sigma^2(x_t)}{2}. \quad (32)$$

In this case, our paper fits into the standard analysis of business cycles in [Chari et al. \(2007\)](#) through the first term, and into the study the costs of business cycles due to income inequality emphasized by [Krebs \(2003\)](#) through the second term.

<sup>22</sup>When  $M_t$  is constant we need to define slack differently from  $x_t = M_t$ . In this case, the role of  $x_t$  is to change the wage and change labor supply on the intensive margin. The wage will need to adjust to clear the labor market as in the three-equation New Keynesian model and then the wage rule, equation (6), becomes the definition of  $x_t$ .

### 5.2.3 Aggregate-demand effects

Traditionally, the literature on automatic stabilizers has focussed on aggregate demand effects following a Keynesian tradition. When there is no productivity risk ( $\sigma^2 = 0$ ), job search effort is constant ( $\kappa = \infty$ ) and the labor market's matching frictions are constant ( $M_t$  is constant), equation(30) simplifies to:

$$\frac{dW_t}{dx_t} = (1 - u_t) \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t},$$

so only the markup effects are present, both through the labor wedge and through price dispersion.

Appendix B.1 shows that a second-order approximation of  $W_t$  around the flexible-price, socially-efficient level of aggregate output  $Y_t^*$  and consumption  $\tilde{c}_t$  transforms this expression into

$$\frac{dW_t}{dx_t} = \left( \frac{Y_t^*}{C_t^*} \right) \left[ \left( \frac{1}{C_t^*} + \frac{\gamma}{Y_t^*} \right) \left( \frac{Y_t^* - Y_t}{Y_t^*} \right) \frac{dY_t}{dx_t} + \left( \frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1} \right) \left( \frac{\mathbb{E}_{t-1} p_t - p_t}{\mathbb{E}_{t-1} p_t} \right) \frac{dp_t}{dx_t} \right],$$

In this case, our model fits into the new Keynesian framework with unemployment developed in [Blanchard and Galí \(2010\)](#) or [Galí \(2011\)](#). Raising slack affects the output gap and the price level, through the Phillips curve, and this affects welfare through the two conventional terms in the expression. The first is the effect on the output gap, and the second the effect on surprise inflation. These are the two sources of welfare costs in this economy.

### 5.3 Social programs and slack

We now turn attention to the second component of the macroeconomic stabilization term, either  $dx_t/db$  in the case of unemployment benefits or  $dx_t/d\tau$  in the case of tax progressivity. We make a few extra simplifying assumptions to obtain analytical expressions that are easy to interpret. First, we assume that aggregate shocks only occur at date 0 and there is no aggregate uncertainty after that, and that there is a balanced budget ( $B_t = 0$ ). This is so that the analysis can be contained to a single period. Second, we assume that household search effort is exogenous and constant ( $\kappa = \infty$ ) as in sections 5.2.2 and 5.2.3, since the role of job search is well described by [Landais et al. \(2013\)](#).

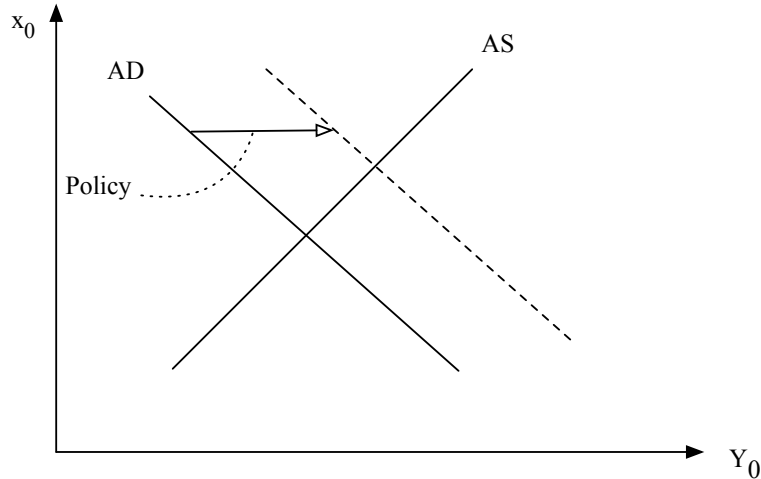


Figure 1: Determination of  $x_0$ .

### 5.3.1 An AD-AS interpretation

With these simplifications, the analysis of the model becomes static, and the household decisions are reduced to consumption and hours worked. Appendix C shows that combining equilibrium in the labor market with the resource constraint and production functions gives an upward sloping relationship between slack  $x$  and output  $Y$ , which we will call the “aggregate supply curve”, for lack of a better term. Likewise combining the aggregate Euler equation, the monetary rule and the Phillips curve gives an “aggregate demand curve.” Both are plotted in figure 1, so that equilibrium is at the intersection of the two curves.

The impact of the social policies on equilibrium slack depends on two features of the diagram. First, how much does a change in policy horizontally shift the AD curve? A bigger shift in AD will lead to a larger effect on slack. Second, what are the slopes of the two curves? If both the AD and the AS are steeper, then a given horizontal shift in the AD brought about by the policies leads to a larger change in slack at the new equilibrium. We discuss shifts and slopes separately.

### 5.3.2 Unemployment benefits and slack

Appendix C proves the following:



**Proposition 4.** *Under the assumptions of section 5.3:*

$$\frac{d \log x_0}{d \log b} = \Lambda^{-1} \left[ \frac{u_0 b}{1 - u_0 + u_0 b} + \frac{u_1 b^{-1}}{1 - u_1(1 - b^{-1})} - \frac{u_1 b}{1 - u_1(1 - b)} \right] \quad (33)$$

where  $\Lambda$  is defined below in lemma 3.

There are two direct effects of raising unemployment benefits on economic slack. The first shows the usual logic of the unemployment insurance system as an automatic stabilizer based on redistribution: aggregate demand responds more strongly to benefits if there are more unemployed workers. This effect arises because the unemployed have a high marginal propensity to consume so the extra benefits lead to an increase in demand.

Second, there is an additional effect coming from expected marginal utility in the future, which depends on the unemployment rate in the next period ( $t = 1$ ). Expected marginal utility is determined by two components. The first is the social insurance that UI provides, lowering uncertainty and precautionary savings and so pushing up aggregate demand today. The second is the higher taxes in order to finance the higher benefits, which reduce consumption. For unemployment rates less than 50% the former effect dominates and the sum of these terms is increasing in the unemployment rate  $u_1$  (see appendix C).

These two effects, captured by the expression in square brackets, shift the AD curve rightwards. As the unemployment rate increases in recessions, both of these effects point towards a counter-cyclical elasticity. The overall effect on tightness is then tempered by the slopes of AD and AS, through the term  $\Lambda$ . We explain this effect below, but since it is common to the effect of more progressive taxes, we turn to that first.

### 5.3.3 Tax progressivity and slack

Appendix C proves:

**Proposition 5.** *Under the assumptions of section 5.3:*

$$\frac{d \log x_0}{d \log \tau} = \Lambda^{-1} \left[ 2\sigma_c^2(x_0)(1 - \tau)\tau - \frac{\partial \log R_0}{\partial \log \tau} \Big|_x + \frac{\partial \log S_0}{\partial \log \tau} \Big|_x + \frac{d \log Y_1}{d \log x_1} \frac{d \log x_1}{d \log \tau} \right] \quad (34)$$

where  $\Lambda$  is defined below in lemma 3.

The first term in between square brackets reflects the effect of social insurance on aggregate demand. When households face uninsurable skill risk, a progressive tax system will raise aggregate demand by reducing the precautionary savings motive. This term will be counter-cyclical if risk increases in a recession as has been documented by [Guvenen et al. \(2014\)](#). The two next terms in the numerator reflect the direct effect of changes in  $\tau$  on the difference between the realized price level and the expected price level, holding slack constant. This has a direct effect on inflation and price dispersion because of price rigidities, and on output through the monetary policy response setting interest rates. Because the policy,  $\tau$ , is known before expectations are formed, these terms are only non-zero as a result of non-linearities in the response of the economy to shocks. The final term reflects the effect of  $\tau$  on marginal cost and slack in the future, which then affects current slack through a wealth effect on aggregate demand.

### 5.3.4 Slopes of AS and AD

Finally, we turn to the term  $\Lambda$ , which reflects the slopes of the AS and AD curves. As the previous two propositions showed, a larger  $\Lambda$  attenuates the effect of the social policies on slack, because it makes both AS and AD flatter. The next lemma, proven in appendix [C](#), describes what determines  $\Lambda$ :

**Lemma 3.** *Under the assumptions of section [5.3](#):*

$$\begin{aligned} \Lambda = & \frac{d \log R_0}{d \log x_0} + (1 - \tau)^2 \sigma_\epsilon^2(x_0) \frac{d \log \sigma_\epsilon^2(x_0)}{d \log x_0} + \frac{1 - b}{1 - u_0 + u_0 b} u_0 \frac{d \log u_0}{d \log x_0} \\ & + \frac{d \log h_0}{d \log x_0} + \frac{d \log(1 - u_0)}{d \log x_0} + \frac{d \log(1 - J_0/Y_0)}{d \log x_0} - \frac{d \log S_0}{d \log x_0} \end{aligned} \quad (35)$$

The first line has the three terms that affect the slope of the AD curve. The first effect involves the real interest rate. A booming economy leads to higher nominal interest rates, both directly via the Taylor rule and indirectly via higher inflation. With nominal rigidities, this raises the real interest rate, which dampens the effectiveness of any policy on equilibrium slack. In other words, a more aggressive monetary policy rule (or more flexible prices) makes AD flatter and so attenuates the effectiveness of social policies.

The second term refers to income risk. If risk is counter-cyclical, this term is negative, so it makes social programs more powerful in affecting slack. The reason is that there is a destabilizing

precautionary savings motive that amplifies demand shocks. In response to a reduction in aggregate demand, labor market tightness falls, leading to an increase in risk and an increase in the precautionary savings motive and so a further reduction in aggregate demand. These reinforcing effects make the AD curve steeper so that social policies become more effective.<sup>23</sup>

The third term in the denominator reflects the impact of economic expansions on the number of employed households, who consume more than unemployed households. Therefore, aggregate demand rises as employment rises in a tighter labor market. This too makes the AD curve steeper and increases the effectiveness of social programs.

The second line in the lemma has the four terms that affect the slope of the AS. Increasing slack raises hours worked or employment, this makes the AS flatter as output increases by relatively more, so it raises  $\Lambda$  and attenuates the effect of social programs. This occurs net of hiring costs, since the fact that they increase with slack works in the opposite direction. Finally, if in a booming economy the efficiency loss from price dispersion decreases, so  $S$  declines, then the AS is likewise flatter and  $\Lambda$  is higher.

#### 5.4 Summary and likely sign

To summarize, there are two main channels through which unemployment benefits or income tax progressivity raise aggregate demand and thereby eliminate slack. These channels are redistribution and social insurance, and both are increasing in the unemployment rate. As unemployment and income risks are counter-cyclical, these forces push for a counter-cyclical elasticities of slack to the social programs, since they dampen the countercyclical fluctuations in the precautionary savings motive. If business cycles are inefficient in the sense that tightness is inefficiently low in a recession, then we expect a positive covariance between  $dW_t/dx_t$  and the elasticities of tightness with respect to policy. This positive covariance implies a positive aggregate stabilization term and more generous unemployment benefits and a more progressive tax system even if the business cycle is efficient on average.

---

<sup>23</sup>Similar reinforcing dynamics arise out of unemployment risk in [Ravn and Sterk \(2013\)](#), [Den Haan et al. \(2015\)](#), and [Heathcote and Perri \(2015\)](#).

## 6 Quantitative analysis

We have shown that business cycles lead to a macroeconomic stabilization term that affects the optimal generosity of unemployment insurance and progressivity of income taxes, and that this term likely makes these programs more generous and progressive, respectively, through multiple channels. We now ask whether these effects are quantitatively significant. To do so, we solve and calibrate the model in sections 2 and 3, without any of the simplifications made in sections 4 to 5 that allowed there for analytical characterizations. We then numerically search for the values of  $b$  and  $\tau$  that maximize the social welfare function, and compare these with the maximal values in a counterfactual economy without aggregate shocks, but otherwise identical.

### 6.1 Calibration and solution of the model

In our baseline case, we define slack as  $x_t = (1 - u_t)/(1 - \bar{u})$  where  $\bar{u}$  is the steady state unemployment rate, instead of the job-finding rate as before. This makes the calibration more transparent since the unemployment rate is a more commonly used indicator of labor market conditions. For similar reasons, we use the Calvo model of price stickiness. We solve the model using global methods as described in Appendix D.2, so that we can accurately compute social welfare. To search for optimal policies, we need to specify the state of the world at the initial date 0, when policies are chosen. We assume the economy starts at the deterministic steady state.

Table 1 shows the calibration of the model, dividing the parameters into different groups. The first group has parameters that we set ex ante to standard choices in the literature. Only the last one deserves some explanation.  $\psi_2$  is the elasticity of hiring costs with respect to labor market tightness, and we set it at 1 as in [Blanchard and Galí \(2010\)](#), in order to be consistent with an elasticity of the matching function with respect to unemployment of 0.5 as suggested by [Petrongolo and Pissarides \(2001\)](#).

Panel B contains parameters individually calibrated to hit some time-series moments. For the preference for public goods, we target the observed average ratio of government purchases to GDP in the US in 1984-2007. For the monetary policy rule, we use OLS estimates of equation (8). The parameter  $v$  determines the probability of not having a job in our model, which we set equal to the sum of quarterly job separation rate, which we construct following [Shimer \(2012\)](#), and the average unemployment rate. Finally, [Güvenen and McKay \(2015\)](#) estimate a version of the income

Symbol	Value	Description	Target
<i>Panel A. Parameters chosen ex ante</i>			
$\delta$	1/200	Mortality rate	50 year expected lifetime
$\frac{\mu}{\mu-1}$	5	Elasticity of substitution	<a href="#">Basu and Fernald (1997)</a>
$\theta$	1/3.5	Prob. of price reset	<a href="#">Klenow and Malin (2010)</a>
$1/\gamma$	1/2	Frisch elasticity	<a href="#">Chetty (2012)</a>
$\psi_2$	1	Elasticity of hiring cost	<a href="#">Blanchard and Galí (2010)</a>
<i>Panel B. Parameters individually calibrated</i>			
$\chi$	0.262	Preference for public goods	$G/Y = 0.207$
$\omega_\pi$	1.66	Mon. pol. response to $\pi$	Estimated Taylor rule
$\omega_u$	0.133	Mon. pol. response to $u$	Estimated Taylor rule
$v$	0.153	Job separation rate	Average value
$F(\epsilon, \cdot)$	mix-normals	Skill-risk process	<a href="#">Guvenen and McKay (2015)</a>
<i>Panel C. Parameters jointly calibrated to steady-state moments</i>			
$\beta$	0.974	Discount factor	3% annual real interest rate
$\bar{w}$	0.809	Average wage	Unemployment rate = 6.1%
$\psi_1$	0.0309	Scale of hiring cost	Recruiting costs of 3% of pay
$1/\kappa$	0.0810	Search effort elasticity	$d \log u / d \log b _x = 0.5$
$\xi$	0.230	Pain from unemployment	Leisure benefit of unemployment
<i>Panel D. Parameters jointly calibrated to volatilities</i>			
$\rho$	0.9	Autocorrelation of shocks	0.9
$\text{StDev}(\eta^A)$	0.724%	TFP innovation	$\text{StDev}(u_t) = 1.59\%$
$\text{StDev}(\eta^I)$	0.244%	Monetary policy innovation	$\text{StDev}(Y   \eta^A) = \text{StDev}(Y   \eta^I)$
$\text{StDev}(\eta^G)$	4.63%	Government purchases innovation	$\text{StDev}(G_t/Y_t) = 1.75\%$
$\zeta$	1.68	Elasticity of wage w.r.t. $x$	$\text{StDev}(h_t) / \text{StDev}(1 - u_t) = 0.568$
<i>Panel E: Automatic stabilizers</i>			
$b$	0.783	UI replacement rate	<a href="#">Rothstein and Valletta (2014)</a>
$\tau$	0.151	Progressivity of tax system	<a href="#">Heathcote et al. (2014)</a>

Table 1: Calibrated parameter values and targets

innovation process that we have specified in equation (3) using a mixture of normals as a flexible parameterization of the distribution  $F(\epsilon'; \cdot)$ . Two of the mixtures shift with a measure of slack to match the observed pro-cyclical skewness of earnings growth rates documented by [Guvenen et al. \(2014\)](#), and we take the unemployment rate to be the measure of slack in our implementation. Appendix D.1 provides additional details.

Panel C instead has parameters chosen jointly to target a set of moments. We target the average unemployment rate between 1960 to 2014, the standard deviation of hours per worker relative to the standard deviation of the employment-population ratio, and recruiting cost of 3 percent of quarterly pay, consistent with [Barron et al. \(1997\)](#). The parameter  $\kappa$  controls the marginal disutility of effort searching for a job, and we set it to target a micro-elasticity of unemployment with respect to benefits of 0.5 as reported by [Landais et al. \(2013\)](#). Last in the panel is  $\xi$ , the non-pecuniary costs of unemployment. In the model, the utility loss from unemployment is  $\log(1/b) - h^{1+\gamma}/(1 + \gamma) + \xi$ , reflecting the loss in consumption, the gain in leisure, and other non-pecuniary costs of unemployment. We set  $\xi = h^{1+\gamma}/(1 + \gamma)$  in the steady state of our baseline calibration so that the benefit of increased leisure in unemployment is dissipated by the non-pecuniary costs.

Panel D calibrates the three aggregate shocks in our model that perturb productivity, monetary policy, and the rule for public expenditures. In each case we assume that the exogenous process is an AR(1) in logs with common autocorrelation. We set the variances to match three targets: (i) the standard deviation of the unemployment rate, (ii) the standard deviation of  $G_t/Y_t$ , and (iii) equal contributions of productivity and monetary shocks to the variance of aggregate output. Finally, we set  $\zeta = 1.68$  to match the standard deviation of hours per worker relative to the standard deviation of the employment-population ratio.

Finally, panel E has the baseline values for the automatic stabilizers. For  $\tau$  we adopt the estimate of 0.151 from [Heathcote et al. \(2014\)](#). For  $b$  we choose  $b^{1/(1-\tau)} = 0.75$ , consistent with a 20-25% decline in household income during unemployment reported by [Rothstein and Valletta \(2014\)](#). Note that this implies interpreting a household as having two workers only one of whom becomes unemployed so  $b^{1/(1-\tau)} = 0.75$  corresponds to a 50 percent replacement rate of one worker's income.

As a check on the model's performance, we computed the standard deviation of hours, output, and inflation in the model, which are 0.74%, 1.67%, and 0.65%. The equivalent moments in the

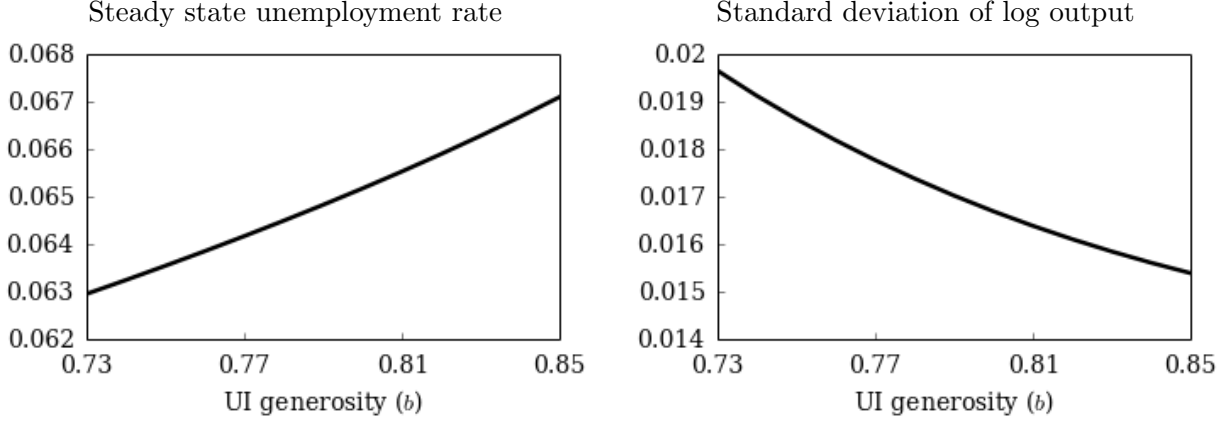


Figure 2: Effects of changing  $b$  for  $\tau = 0.26$

US data 1960-2014 are 0.84%, 1.32%, and 0.55%.

## 6.2 Unemployment insurance

We solve jointly for the optimal  $b$  and  $\tau$ . Our main result is that aggregate shocks increase the optimal  $b$  to 0.805 from 0.752 in the absence of aggregate shocks. Converting the values for  $b$  into pre-tax UI replacement rates based on a two-earner household then we have an optimal replacement rate of 49 percent with aggregate shocks as compared to 36 percent without.<sup>24</sup>

Figure 2 provides a first hint for why this effect is so quantitatively large. It shows the effects of raising  $b$  on the steady state unemployment rate and on the volatility of log output. Raising the generosity of unemployment benefits hurts the incentives for working, so unemployment rises somewhat. However, it has a strong macroeconomic stabilizing effect.

Figure 3 provides a different way to look at this result. Equation (27), repeated here for convenience:

$$\begin{aligned} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \mathbb{E}_i \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] \right) \right] \\ & + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ u_t \log b - \log (1 - u_t + u_t b) \right] \\ & + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1 + \gamma} - v \frac{q_t^{1+\kappa}}{1 + \kappa} + \chi \log(G_t) - \xi u_t \right], \end{aligned}$$

<sup>24</sup>The conversion we apply here is to solve for  $x$  in  $(x/2 + 1/2)^{1-\tau} = b$ .

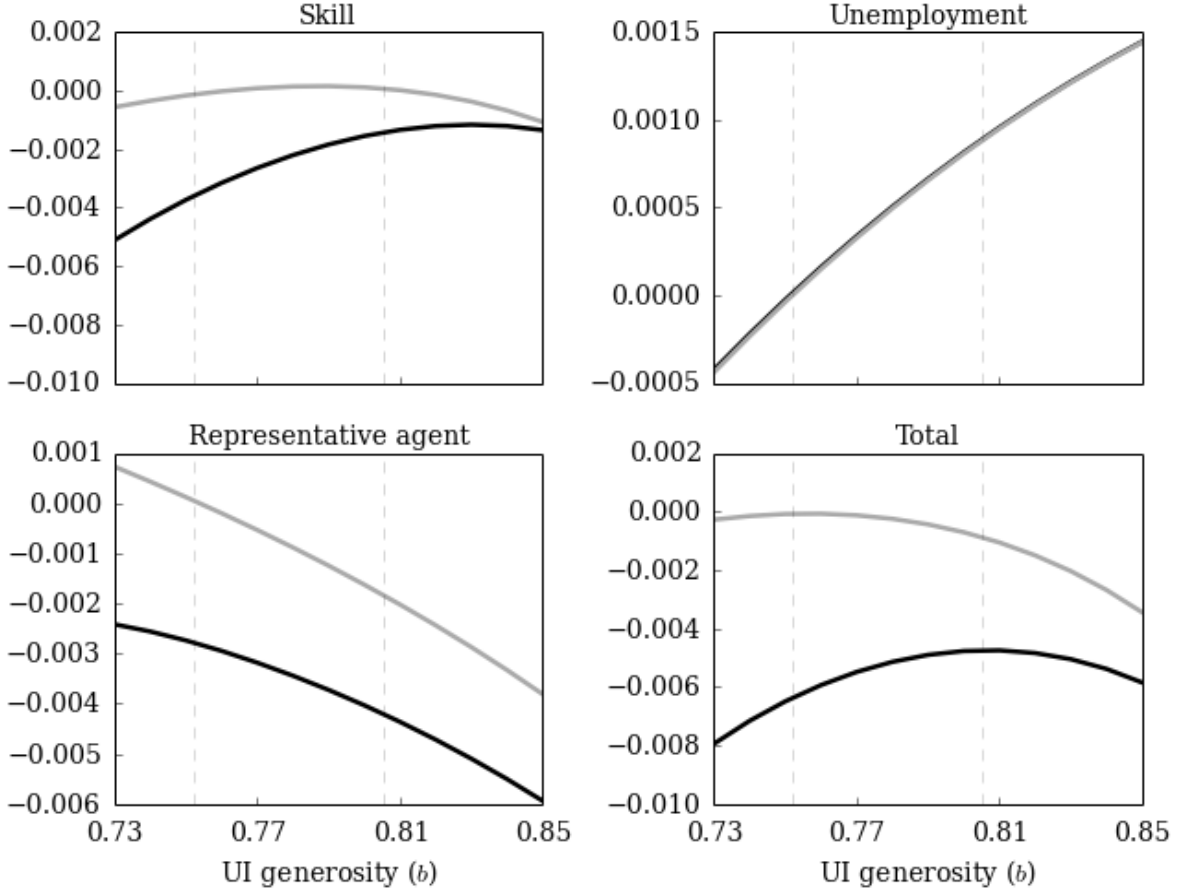


Figure 3: Decomposition of welfare in consumption equivalent units.  $\tau = 0.26$ . With aggregate shocks (black) and without (gray). Vertical lines show the optimal  $b$  with and without aggregate shocks.

shows that welfare is the sum of three components: the welfare loss from income inequality that results from skill risk,  $\alpha_{i,t}$ , the welfare loss from income inequality as a result of unemployment, and the utility that derives from aggregates. Figure 3 plots each of these components in consumption equivalent units and their sum, as we vary  $b$ , both with and without business cycles.<sup>25</sup>

As expected, higher  $b$  provides insurance by lowering idiosyncratic income risk (top-left panel) and unemployment risk (top-right panel). More interesting, the presence of aggregate shocks has

<sup>25</sup>The consumption equivalent units are relative to the social welfare function in the deterministic steady state associated with the policy parameters that are optimal without aggregate shocks. Suppose the additively decomposed social welfare function for a given policy is  $V^A + V^B + V^C$ . Let  $\tilde{V}^A + \tilde{V}^B + \tilde{V}^C$  be the social welfare function of in the steady state with the policy parameters that are optimal without aggregate shocks. If we rescaled the consumption of all households by a factor  $1 + \Delta$ , the latter would become  $\tilde{V}^A + \tilde{V}^B + \tilde{V}^C + \log(1 + \Delta)/(1 - \beta)$ . The top-left panel plots  $\Delta^A = \exp\{(1 - \beta)(V^A - \tilde{V}^A)\} - 1$ . The bottom-right panel plots  $\Delta = \exp\{(1 - \beta)(V^A + V^B + V^C - \tilde{V}^A - \tilde{V}^B - \tilde{V}^C)\} - 1$ . Notice that  $1 + \Delta = (1 + \Delta^A)(1 + \Delta^B)(1 + \Delta^C)$ .



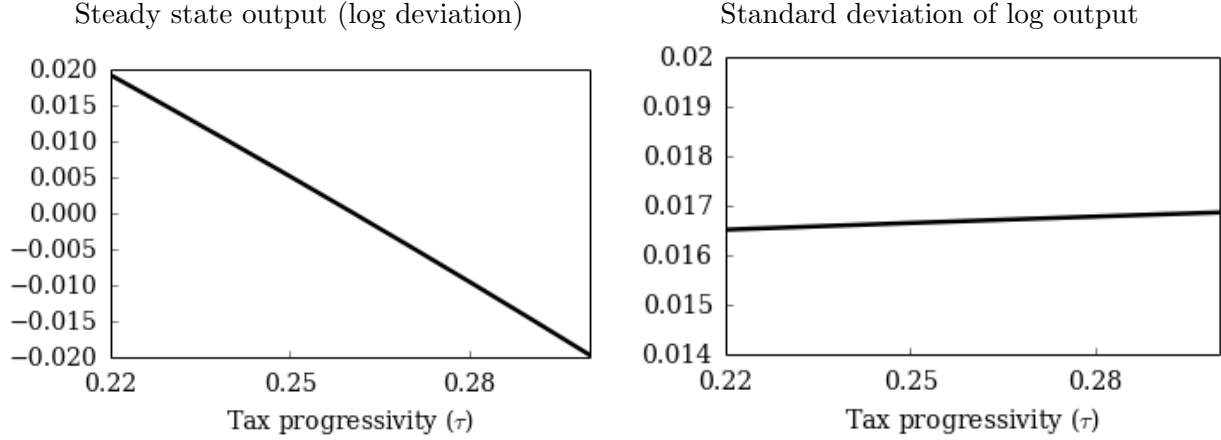


Figure 4: Effects of changing  $b$  for  $\tau = 0.26$ . The left panel shows log output as a deviation from the value at  $\tau = 0.265$ . The right panel uses the same vertical axis as figure 2 for comparison.

a large effect on the first effect, but a negligible one on the second. Idiosyncratic income risk is persistent and it increases in recessions. The reason is that, by stabilizing the business cycle, a higher  $b$  leads to less pre-tax income inequality. The contribution of skill risk to welfare is strongly non-linear in the state of the business cycle and mitigating the most severe recessions leads to a considerable gain in welfare. Instead, the welfare loss from unemployment risk is close to linear in the state of the business cycle so the presence of aggregate risk and the stabilizing effects of benefits have little effect on this component of welfare.

The next two panels confirm that uninsured skill risk drives the results. The welfare coming from aggregates, in the lower-left panel, falls with  $b$  because of the moral hazard effect on the unemployment rate, but again the presence of aggregate shocks has a modest effect on the slope of the relation between welfare and  $b$ . The sum of social welfare, plotted in the lower-right panel of figure 3 mimics the patterns in the top-left column. The optimal unemployment benefit is substantially larger with aggregate shocks, because it stabilizes the business cycle leading to welfare gains by reducing the fluctuations in uninsured risk.

### 6.3 Income tax progressivity

Our second main result is that the optimal tax progressivity is almost unchanged at 0.264 relative to 0.265 without aggregate shocks.

The left panel of figure 4 shows the steady state level of output as a log deviation from the level

under the optimal policy without aggregate shocks. There is a strong negative effect of progressivity on output that works through the disincentive effect on labor supply. At the optimal policy without aggregate shocks, the welfare loss from reducing aggregate output is balanced by the welfare gain from insuring skill risk. When we introduce aggregate shocks, the level of tax progressivity has essentially no effect on the volatility of the business cycle as shown in the right-panel of figure 4 and therefore there is no stabilization benefit of raising progressivity.

## 7 Conclusion

It is common to state that there are stabilizing benefits of unemployment insurance and income tax progressivity, but there are few systematic studies of what factors drive these benefits and how large they are. Moreover, the study of these social programs rarely takes into account this macroeconomic stabilization role, instead treating it a fortuitous side benefits.

This paper tried to remedy this situation. Our theoretical study provided a theoretical characterization of what is an automatic stabilizer. In general terms, an automatic stabilizer is a fixed policy for which there is a positive covariance between the effect of slack on welfare, and the effect of the policy on slack. If a policy tool has this property of stimulating the economy more in recessions, when slack is inefficiently high, then its role in stabilizing the economy calls for expanding the use of the policy beyond what would be appropriate in a stationary environment. We showed what factors drive this covariance, through which economic channels they operate, and what makes them larger or smaller. Overall, we found that the role of social insurance programs as automatic stabilizers affects their optimal design and, in the case of unemployment insurance, it can lead to substantial differences in the generosity of the system.

## References

- Aiyagari, S. R. and McGrattan, E. R. (1998). The optimum quantity of debt. *Journal of Monetary Economics*, 42(3):447–469.
- Auclert, A. (2016). Monetary policy and the redistribution channel. Princeton University manuscript.
- Auerbach, A. J. and Feenberg, D. (2000). The significance of federal taxes as automatic stabilizers. *Journal of Economic Perspectives*, 14(3):37–56.
- Baily, M. N. (1978). Some aspects of optimal unemployment insurance. *Journal of Public Economics*, 10(3):379–402.
- Barron, J. M., Berger, M. C., and Black, D. A. (1997). On-the-job training. *Books from Upjohn Press*.
- Basu, S. and Fernald, J. G. (1997). Returns to scale in us production: Estimates and implications. *Journal of political economy*, 105(2):249–283.
- Bhandari, A., Evans, D., Golosov, M., and Sargent, T. J. (2013). Taxes, Debts, and Redistributions with Aggregate Shocks. NBER working paper 19470.
- Blanchard, O., Dell’Ariccia, G., and Mauro, P. (2010). Rethinking macroeconomic policy. *Journal of Money, Credit and Banking*, 42(s1):199–215.
- Blanchard, O. and Galí, J. (2010). Labor markets and monetary policy: A new keynesian model with unemployment. *American Economic Journal: Macroeconomics*, 2(2):1–30.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383 – 398.
- Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2007). Business Cycle Accounting. *Econometrica*, 75(3):781–836.
- Chetty, R. (2006). A general formula for the optimal level of social insurance. *Journal of Public Economics*, 90(10):1879–1901.

- Chetty, R. (2012). Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply. *Econometrica*, 80(3):969–1018.
- Conesa, J. C. and Krueger, D. (2006). On the optimal progressivity of the income tax code. *Journal of Monetary Economics*, 53(7):1425–1450.
- De Santis, M. (2007). Individual consumption risk and the welfare cost of business cycles. *American Economic Review*, 97(4):1488 – 1506.
- Den Haan, W. J., Rendahl, P., and Riegler, M. (2015). Unemployment (fears) and deflationary spirals. CEPR Discussion Paper No. DP10814.
- DiMaggio, M. and Kermani, A. (2016). The importance of unemployment insurance as an automatic stabilizer. Columbia Business School Research Paper 15-31.
- Gali, J. (2011). *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*. MIT Press, Cambridge, USA.
- Galí, J., Gertler, M., and López-Salido, J. D. (2007). Markups, Gaps, and the Welfare Costs of Business Fluctuations. *The Review of Economics and Statistics*, 89(1):44–59.
- Golosov, M., Troshkin, M., and Tsyvinski, A. (2016). Redistribution and social insurance. *American Economic Review*, 106(2):359–386.
- Guerrieri, V. and Lorenzoni, G. (2011). Credit crises, precautionary savings, and the liquidity trap. NBER Working Paper 17583.
- Guvenen, F., Karahan, F., Ozkan, S., and Song, J. (2015). What do data on millions of u.s. workers reveal about life-cycle earnings risk? University of Minnesota working paper.
- Guvenen, F. and McKay, A. (2015). A parsimonious income process for business cycle analysis. Boston University Working Paper.
- Guvenen, F., Ozkan, S., and Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):pp. 621–660.
- Hall, R. E. (2005). Employment fluctuations with equilibrium wage stickiness. *American economic review*, pages 50–65.

- Heathcote, J. and Perri, F. (2015). Wealth and volatility. Technical report, National Bureau of Economic Research.
- Heathcote, J., Storesletten, K., and Violante, G. L. (2014). Optimal Tax Progressivity: An Analytical Framework. NBER working paper 19899.
- Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *The Review of Economic Studies*, 57(2):279–298.
- IMF (2015). Can fiscal policy stabilize output? In *Fiscal Monitor*, chapter 2, pages 21–49. International Monetary Fund.
- Jung, P. and Kuester, K. (2014). Optimal labor-market policy in recessions. *American Economic Journal: Macroeconomics*, forthcoming.
- Kaplan, G. W., Moll, B., and Violante, G. (2016). Monetary policy according to hank. Princeton University manuscript.
- Kekre, R. (2015). Unemployment insurance in macroeconomic stabilization. Harvard university manuscript.
- Klenow, P. J. and Malin, B. A. (2010). Microeconomic evidence on price-setting. *Handbook of Monetary Economics*, 3:231–284.
- Krebs, T. (2003). Growth and welfare effects of business cycles in economies with idiosyncratic human capital risk. *Review of Economic Dynamics*, 6(4):846–868.
- Krebs, T. (2007). Job displacement risk and the cost of business cycles. *The American economic review*, pages 664–686.
- Krueger, D. and Ludwig, A. (2013). Optimal Progressive Labor Income Taxation and Education Subsidies When Education Decisions and Intergenerational Transfers Are Endogenous. *American Economic Review*, 103(3):496–501.
- Krusell, P., Mukoyama, T., and Smith, Jr., A. A. (2011). Asset prices in a huggett economy. *Journal of Economic Theory*, 146(3):812–844.

- Krusell, P. and Smith, Jr., A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.
- Landais, C., Michaillat, P., and Saez, E. (2013). Optimal Unemployment Insurance over the Business Cycle. Centre for Macroeconomics (CFM) discussion paper 1303.
- Maliar, L. and Maliar, S. (2015). Merging simulation and projection approaches to solve high-dimensional problems with an application to a new keynesian model. *Quantitative Economics*, 6(1):1–47.
- Mankiw, N. G. and Reis, R. (2010). Imperfect information and aggregate supply. In Friedman, B. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3A, chapter 5, pages 183–230. Elsevier: North Holland.
- McKay, A., Nakamura, E., and Steinsson, J. (2016). The Power of Forward Guidance Revisited. *American Economic Review*, forthcoming.
- McKay, A. and Reis, R. (2016). The role of automatic stabilizers in the u.s. business cycle. *Econometrica*, 84(1):141–194.
- Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *Review of Economics Studies*, 38(2):175–208.
- Mitman, K. and Rabinovich, S. (2011). Pro-Cyclical Unemployment Benefits? Optimal Policy in an Equilibrium Business Cycle Model. PIER working paper 11-010.
- Musgrave, R. A. and Miller, M. H. (1948). Built-in flexibility. *American Economic Review*, 38(1):122–128.
- Oh, H. and Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59(Supplement):S50–S64.
- Perotti, R. (2005). Estimating the effects of fiscal policy in OECD countries. Federal Reserve Bank of San Francisco, Conference Proceedings.
- Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic literature*, pages 390–431.

- Ravn, M. and Sterk, V. (2013). Job uncertainty and deep recessions. University College London working paper.
- Rothstein, J. and Valletta, R. G. (2014). Scraping by: Income and program participation after the loss of extended unemployment benefits. IZA Discussion Paper No. 8022.
- Samuelson, P. A. (1954). The review of economics and statistics. *The pure theory of public expenditure*, pages 387–389.
- Shimer, R. (2012). Reassessing the ins and outs of unemployment. *Review of Economic Dynamics*, 15(2):127 – 148.
- Storesletten, K., Telmer, C. I., and Yaron, A. (2001). The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk. *European Economic Review*, 45(7):1311–1339.
- Storesletten, K., Telmer, C. I., and Yaron, A. (2004). Cyclical dynamics in idiosyncratic labor market risk. *Journal of Political Economy*, 112(3):695–717.
- Varian, H. R. (1980). Redistributive taxation as social insurance. *Journal of Public Economics*, 14(1):49–68.
- Werning, I. (2007). Optimal Fiscal Policy with Redistribution. *Quarterly Journal of Economics*, 122(3):925–967.
- Werning, I. (2015). Incomplete Markets and Aggregate Demand. NBER Working Papers 21448, National Bureau of Economic Research, Inc.
- Woodford, M. (2010). Optimal monetary stabilization policy. In Friedman, B. M. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3, chapter 14, pages 723 – 828. Elsevier: North Holland.

# Appendix

## A Supporting material for section 3

### A.1 The value of employment

**Lemma 4.** *Suppose the household's value function in the no-trade equilibrium has the form*

$$V(\alpha, n, \mathcal{S}) = V^\alpha(\alpha, \mathcal{S}) + V^n(a, n, \mathcal{S})$$

for some functions  $V^\alpha$  and  $V^n$ . The choice of search effort is then the same for all searching households regardless of  $\alpha$ .

*Proof of Lemma 4.* In the no-trade equilibrium, the household's search problem is

$$V^s(\alpha, \mathcal{S}) = \max_q \left\{ MqV(\alpha, 1, \mathcal{S}) + (1 - Mq)V(\alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$

Substitute for the value functions to arrive at

$$\begin{aligned} V^s(\alpha, \mathcal{S}) &= \max_q \left\{ Mq [V^\alpha(\alpha, \mathcal{S}) + V^n(a, 1, \mathcal{S})] + (1 - Mq) [V^\alpha(\alpha, \mathcal{S}) + V^n(a, 0, \mathcal{S})] - \frac{q^{1+\kappa}}{1 + \kappa} \right\} \\ V^s(\alpha, \mathcal{S}) &= V^\alpha(\alpha, \mathcal{S}) + \max_q \left\{ MqV^n(1, \mathcal{S}) + (1 - Mq)V^n(0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}, \end{aligned} \quad (36)$$

where we have brought  $V^\alpha(\alpha, \mathcal{S})$  outside the max operator as it appears in an additively separable manner. As there is no  $\alpha$  inside the max operator it is clear that the optimal  $q$  is independent of  $\alpha$ .  $\square$

**Lemma 5.** *In the no-trade equilibrium, the household's value function can be written as*

$$V(\alpha, n, \mathcal{S}) = \frac{1 - \tau}{1 - \beta} \log(\alpha) + n \left[ \log \frac{1}{b} - \frac{h^{1+\gamma}}{1 + \gamma} + \xi \right] + \bar{V}(\mathcal{S}) \quad (37)$$

for some function  $\bar{V}$ .

*Proof of Lemma 5.* Suppose that the value function is of the form given in (37). We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point



of the Bellman equation is in this class by the contraction mapping theorem.  $V^s$  will then be

$$V^s(\alpha, \mathcal{S}) = \frac{1-\tau}{1-\beta} \log(\alpha) + Mq^* \left[ \log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right] - \frac{q^{*1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S})$$

and the choice of  $q^*$  is independent of  $\alpha$  by Lemma 4. Regardless of employment status, the continuation value is

$$\begin{aligned} & (1-v)V(\alpha', 1, \mathcal{S}') + vV^s(\alpha', \mathcal{S}') \\ &= (1-v) \left[ \frac{1-\tau}{1-\beta} \log(\alpha') + \left[ \log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi \right] + \bar{V}(\mathcal{S}') \right] \\ & \quad + v \left[ \frac{1-\tau}{1-\beta} \log(\alpha') + M'q^{*'} \left[ \log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi \right] - \frac{q^{*'}1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}') \right] \\ &= \frac{1-\tau}{1-\beta} \log(\alpha') + (1-v + vM'q^{*'}) \left[ \log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi \right] - (1-v) \frac{q^{*'}1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}') \\ &= \frac{1-\tau}{1-\beta} \log(\alpha') + g(\mathcal{S}'), \end{aligned}$$

where

$$g(\mathcal{S}) \equiv (1-v + vM'q^{*'}) \left[ \log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi \right] - (1-v) \frac{q^{*'}1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}').$$

Turning now to the Bellman equation, in the no-trade equilibrium we have

$$\begin{aligned} V(\alpha, n, \mathcal{S}) &= \log \left[ \lambda(n + (1-n)b) (\alpha(wh + d))^{1-\tau} \right] - n \frac{h^{1+\gamma}}{1+\gamma} - (1-n)\xi + \beta \mathbb{E} \left[ \frac{1-\tau}{1-\beta} \log(\alpha') + g(\mathcal{S}') \right] \\ &= \frac{1-\tau}{1-\beta} \log(\alpha) + n \left[ \log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right] \\ & \quad + \log \left[ \lambda(wh + d)^{1-\tau} \right] + \log b - \xi + \beta \frac{1-\tau}{1-\beta} \mathbb{E} [\log(\epsilon')] + \beta \mathbb{E} [g(\mathcal{S}')]. \end{aligned}$$

Finally,  $\bar{V}(\mathcal{S})$  is given by the second row of the expression above. □

## A.2 Derivations for section 3

*Derivation of (17), (18), (19), and (22).* The Euler equation is

$$\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E} \left[ \frac{1}{c_{i,t+1}} \right].$$

Using  $c_{i,t} = \lambda_t (w_t h_t + d_t)^{1-\tau}$  we can rewrite this as

$$\frac{1}{\lambda_t (w_t h_t + d_t)^{1-\tau}} \geq \beta R_t \mathbb{E}_t \left\{ \frac{(1 - u_{t+1}) + u_{t+1} b^{-1}}{\lambda_{t+1} (w_{t+1} h_{t+1} + d_{t+1})^{1-\tau}} \right\} \mathbb{E}_t \left[ \frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] [n_{i,t} + (1 - n_{i,t})b]. \quad (38)$$

Notice that  $\mathbb{E} \left[ \frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] = \mathbb{E} \left[ \epsilon_{i,t+1}^{\tau-1} \right]$  is common across households. This Euler equation only differs across households is due to the final term involving  $n_{i,t}$ . Assuming the unemployment benefit replacement rate is less than one, this term will be larger for employed than unemployed so in equilibrium all unemployed will be constrained and the Euler equation will hold with equality for all employed.<sup>26</sup>

The consumption of an individual is

$$c_{i,t} = \lambda_t \alpha_{i,t}^{1-\tau} (w_t h_t + d_t)^{1-\tau} (n_{i,t} + (1 - n_{i,t})b). \quad (39)$$

Summing (39) across households we have

$$C_t = \lambda_t (w_t h_t + d_t)^{1-\tau} (1 - u_t + u_t b) \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right]. \quad (40)$$

Solve (40) for  $\lambda_t (w_t h_t + d_t)^{1-\tau}$  and substitute it into (39) to arrive at

$$c_{i,t} = \alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b) \frac{C_t}{(1 - u_t + u_t b) \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right]}. \quad (41)$$

Equations (17) and (22) follow from (41) and the definition of  $\tilde{c}_t$ .

Solve (40) for  $\lambda_t (w_t h_t + d_t)^{1-\tau}$  and substitute it into (38) holding with equality and  $n_{i,t} = 1$  to arrive at

$$\frac{(1 - u_t + u_t b) \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right]}{C_t} = \beta R_t \mathbb{E}_t \left\{ \left[ (1 - u_{t+1}) + u_{t+1} b^{-1} \right] \frac{(1 - u_{t+1} + u_{t+1} b) \mathbb{E}_i \left[ \alpha_{i,t+1}^{1-\tau} \right]}{C_{t+1}} \right\} \mathbb{E} \left[ \epsilon_{i,t+1}^{\tau-1} \right]$$

---

<sup>26</sup>Here we follow [Krusell et al. \(2011\)](#), [Ravn and Sterk \(2013\)](#), and [Werning \(2015\)](#) in assuming that the Euler equation of the employed/high-income household holds with equality. This household is up against its constraint  $a' = 0$  so there could be other equilibria in which the Euler equation does not hold with equality. The equilibrium we focus on is the limit of the unique equilibrium as the borrowing limit approaches zero from below. See [Krusell et al. \(2011\)](#) for further discussion of this point.

and substitute in  $\tilde{c}_t$  using (17)

$$\frac{1}{\tilde{c}_t} = \beta R_t \mathbb{E}_t \left\{ \left[ (1 - u_{t+1}) + u_{t+1} b^{-1} \right] \frac{1}{\tilde{c}_{t+1}} \right\} \mathbb{E} \left[ \epsilon_{i,t+1}^{\tau-1} \right].$$

This can be rearranged to (18) and (19) as  $\mathbb{E} \left[ \epsilon_{i,t+1}^{\tau-1} \right]$  is known at date  $t$ .  $\square$

*Derivation of (20).* Using labor market clearing and the definition of  $A_t$  we can write the aggregate production function as

$$Y_t = A_t h_t (1 - u_t). \quad (42)$$

Output net of hiring costs is paid to employed workers in the form of wage and dividend payments. As the average  $\alpha_{i,t}$  is equal to one we have

$$Y_t - J_t = (w_t h_t + d_t) (1 - u_t). \quad (43)$$

Multiply both sides of (42) by  $(Y_t - J_t)/Y_t$  and substitute for  $Y_t - J_t$  using (43) yields

$$w_t h_t + d_t = A_t h_t \frac{Y_t - J_t}{Y_t}.$$

Substitute this for  $w_t h_t + d_t$  in (15) to arrive at

$$h_t^\gamma = \frac{(1 - \tau) w_t}{A_t h_t \frac{Y_t - J_t}{Y_t}}.$$

Finally use the wage rule, (6), to arrive at (20).  $\square$

*Derivation of (21).* By Lemma 5, the results of Lemma 4 can be applied. Proceed from equation (36) using the functional form for the value function in Lemma 5. This leads to

$$\max_q \left\{ Mq \left[ \log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right] - \frac{q^{1+\kappa}}{1+\kappa} \right\}$$

and the first order condition yields equation (21).  $\square$

### A.3 Equilibrium definition

We first state some addition equilibrium conditions and then state a definition of an equilibrium. Here we focus on the Calvo-pricing version of the model for concreteness.

and the aggregate resource constraint is

$$Y_t - J_t = C_t + G_t - B_{t+1} + R^* B_t. \quad (44)$$

The Fisher equation is

$$R_t = I_t / \mathbb{E}_t [\pi_{t+1}]. \quad (45)$$

The price-setting first order condition leads to

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu (w_s h_s + \psi_1 M_s^{\psi_2}) / (A_s h_s)}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}. \quad (46)$$

Substituting for the value of employment  $V(0, \alpha, 1, \mathcal{S}_t) - V(0, \alpha, 0, \mathcal{S}_t) = -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi$  in (16) yields

$$q_t^\kappa = M_t \left[ \log \left( \frac{1}{b} \right) - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right] \quad (47)$$

The link between  $\tilde{c}_t$  and  $C_t$  depends on the cross-sectional moment  $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$ . This evolves according to

$$\mathbb{E}_i [\alpha_{i,t}^{1-\tau}] = (1-\delta) \mathbb{E}_i [\alpha_{i,t-1}^{1-\tau}] \mathbb{E}_i [\epsilon_{i,t}^{1-\tau}] + \delta. \quad (48)$$

An equilibrium of the economy can be calculated from a system equations in 18 variables and three exogenous processes. The variables are

$$C_t, \tilde{c}_t, u_t, \mathbb{E}_i [\alpha_{i,t}^{1-\tau}], Q_t, R_t, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, \frac{p_t^*}{p_t}, J_t, q_t, M_t, B_t.$$

And the equations are: (4), (5), (6), (7), (8), (9), (17) (18), (19), (20), (24), (42), (44), (45), (46),

(47), (48), and a rule for government borrowing. The exogenous processes are  $\eta_t^A$ ,  $\eta_t^G$ , and  $\eta_t^I$ .

## B Proofs for sections 4 and 5

*Proof of Proposition 1.* As a first step, we will rewrite the term  $\mathbb{E}_i \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] \right)$  in the social welfare function using the no-mortality, log-normal income process. When there is no mortality,  $\delta = 0$ , we can compute the cumulative welfare effect of a change in  $F(\epsilon_{i,t+1}, x_t)$  including the effects on current and future skill dispersion. In particular

$$\begin{aligned} \mathbb{E}_i \log \left( \alpha_{i,t}^{1-\tau} \right) &= \mathbb{E}_i \log \left( \alpha_{i,t-1}^{1-\tau} \epsilon_{i,t}^{1-\tau} \right) \\ &= \mathbb{E}_i \log \left( \alpha_{i,0}^{1-\tau} \epsilon_{i,1}^{1-\tau} \cdots \epsilon_{i,t}^{1-\tau} \right) \\ &= \mathbb{E}_i \log \left( \alpha_{i,0}^{1-\tau} \right) + \mathbb{E}_i \left( \epsilon_{i,1}^{1-\tau} \right) + \cdots + \mathbb{E}_i \left( \epsilon_{i,t}^{1-\tau} \right). \end{aligned}$$

Similarly

$$\begin{aligned} \log \left( \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] \right) &= \log \left( \mathbb{E}_i \left[ \alpha_{i,t-1}^{1-\tau} \right] \mathbb{E}_i \left[ \epsilon_{i,t}^{1-\tau} \right] \right) \\ &= \log \left( \mathbb{E}_i \left[ \alpha_{i,0}^{1-\tau} \right] \mathbb{E}_i \left[ \alpha_{i,1}^{1-\tau} \right] \cdots \mathbb{E}_i \left[ \epsilon_{i,t}^{1-\tau} \right] \right) \\ &= \log \left( \mathbb{E}_i \left[ \alpha_{i,0}^{1-\tau} \right] \right) + \log \left( \mathbb{E}_i \left[ \alpha_{i,1}^{1-\tau} \right] \right) + \cdots + \log \left( \mathbb{E}_i \left[ \epsilon_{i,t}^{1-\tau} \right] \right) \end{aligned}$$

Notice that in this no-mortality case, the date- $t$  loss from skill dispersion can be written

$$\mathbb{E}_i \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] \right) = \mathbb{E}_i \log \left( \alpha_{i,0}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \alpha_{i,0}^{1-\tau} \right] \right) + \sum_{s=1}^t \left[ \mathbb{E}_i \left( \epsilon_{i,s}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \epsilon_{i,s}^{1-\tau} \right] \right) \right].$$

The level of slack at date  $t$ ,  $x_t$ , determines the contribution  $\mathbb{E}_i \log \left( \epsilon_{i,t+1}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \epsilon_{i,t+1}^{1-\tau} \right] \right)$  that will appear in  $W_s$  for all future dates  $s > t$ . Using the log-normal income process, we have

$$\mathbb{E}_i \log \left( \epsilon_{i,t+1}^{1-\tau} \right) - \log \left( \mathbb{E}_i \left[ \epsilon_{i,t+1}^{1-\tau} \right] \right) = -(1-\tau) \frac{\sigma^2(x_t)}{2} + (1-\tau) \frac{\sigma^2(x_t)}{2} - (1-\tau)^2 \frac{\sigma^2(x_t)}{2} = -(1-\tau)^2 \frac{\sigma^2(x_t)}{2}.$$

So the effect of  $x_0$  on  $\sum_{t=0}^{\infty} \beta^t W_t$  is given by

$$-\frac{\beta}{1-\beta} (1-\tau)^2 \frac{\sigma^2(x_t)}{2}.$$

In addition to the social welfare function, (27), the relevant equations of the model are (4), (42), (5), (44), (47), and (20). Differentiating we have

$$\begin{aligned}\frac{dW_t}{du_t} &= \log b + \frac{1-b}{1-u_t+u_tb} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t} + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \\ &= \left( \log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t}\end{aligned}\quad (49)$$

where

$$\begin{aligned}\tilde{c}_t &= \frac{C_t}{\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] (1-u_t+u_tb)} \\ \frac{d\tilde{c}_t}{du_t} &= \frac{C_t(1-b)}{\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] (1-u_t+u_tb)^2} + \frac{1}{\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] (1-u_t+u_tb)} \frac{dC_t}{du_t} \\ &= \frac{C_t(1-b)}{\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] (1-u_t+u_tb)^2} + \frac{1}{\mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] (1-u_t+u_tb)} \left[ -A_t h_t + \psi_1 M_t^{\psi_2} \right] \\ \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} &= \frac{1-b}{1-u_t+u_tb} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t}.\end{aligned}$$

These equations subtly reflect two of our assumptions. First, in (57) and (49) we have omitted any terms related to the welfare effects of changes in  $G_t$  because the Samuelson rule implies the a marginal change in  $G_t$  has no effect on welfare. If we relaxed that assumption we would arrive an additional term that reflects the marginal net benefit of increasing  $G_t$ . Also we have omitted any terms related to changes in foreign borrowing because we assume this is not affected by the stabilizers. If we relaxed this assumption we would arrive at an additional term that reflects the difference between the marginal rate of substitution between  $C_t$  and  $C_{t+1}$  and the world interest rate.

The first-order condition of the social welfare function with respect to  $b$  is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{u_t}{b} - \frac{u_t}{1-u_t+u_tb} + \frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \Big|_x - v q_t^\kappa \frac{\partial q_t}{\partial b} \Big|_x + \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = 0. \quad (50)$$

The first two terms in (50) can be expressed as

$$\begin{aligned}\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} &= u_t \left( \frac{1}{b} - 1 + 1 - \frac{1}{1 - u_t + u_t b} \right) \\ &= u_t \left( \frac{1}{b} - 1 \right) \left( 1 - \frac{u_t b}{1 - u_t + u_t b} \right)\end{aligned}$$

and note that

$$\begin{aligned}\frac{\partial \log(b\tilde{c}_t)}{\partial \log b} \Big|_{x,q} &= \frac{\partial}{\partial \log b} \log \left( \frac{bC_t}{\mathbb{E}_i [\alpha_{i,t}^{1-\tau}] (1 - u_t + u_t b)} \right) \\ &= 1 - \frac{u_t b}{1 - u_t + u_t b}\end{aligned}\tag{51}$$

where the partial derivative on the right hand side of (51) is with respect to  $b$  alone.<sup>27</sup> So we have

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left( \frac{1}{b} - 1 \right) \frac{\partial \log(b\tilde{c}_t)}{\partial \log b} \Big|_{x,q}\tag{52}$$

The third and fourth terms in (50) reflect the budgetary impact of moral hazard:

$$\frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \Big|_x - vq_t^\kappa \frac{\partial q_t}{\partial b} \Big|_x = \left[ \left( \log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \right] \frac{\partial u_t}{\partial b} \Big|_x - vq_t^\kappa \frac{\partial q_t}{\partial b} \Big|_x$$

using (47) this becomes

$$\frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \Big|_x - vq_t^\kappa \frac{\partial q_t}{\partial b} \Big|_x = \left[ \left( \log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \right] \frac{\partial u_t}{\partial b} \Big|_x + \left( -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) \frac{du_t}{dq_t} \frac{\partial q_t}{\partial b} \Big|_x\tag{53}$$

$$= \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \frac{\partial u_t}{\partial b} \Big|_x\tag{54}$$

$$= \frac{d \log \tilde{c}_t}{d \log u_t} \frac{\partial \log u_t}{\partial b} \Big|_x.\tag{55}$$

Substituting (52) and (53) into (50) yields the result. □

---

<sup>27</sup> As  $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$  is an endogenous state that depends on the history of  $x$ , we are taking the partial derivative holding fixed this history.

*Proof of Proposition 2.* We proceed as in the proof of Proposition 1. The first-order condition of the social welfare function with respect to  $\tau$  is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\beta}{1-\beta} (1-\tau) \sigma^2(x_t) + \frac{dW_t}{dh_t} \frac{\partial h_t}{\partial \tau} \Big|_x + \frac{dW_t}{dx_t} \frac{dx_t}{d\tau} \right\} = 0. \quad (56)$$

From (27), (42), and (44) we have

$$\frac{dW_t}{dh_t} = \frac{dW_t}{du_t} \frac{du_t}{dq_t} \frac{dq_t}{dh_t} + \frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma - vq_t^\kappa \frac{dq_t}{dh_t}.$$

From (20) we have

$$\frac{\partial h_t}{\partial \tau} \Big|_x = -\frac{h_t}{(1-\tau)(1+\gamma)}$$

combining these and using (47), (49), and  $du_t/dq_t = -vM_t$  we arrive at

$$\begin{aligned} \frac{dW_t}{dh_t} \frac{\partial h_t}{\partial \tau} \Big|_x &= \frac{dW_t}{du_t} \frac{du_t}{dq_t} \frac{dq_t}{dh_t} \frac{\partial h_t}{\partial \tau} \Big|_x + \frac{A_t(1-u_t)}{C_t} \frac{\partial h_t}{\partial \tau} \Big|_x - (1-u_t)h_t^\gamma \frac{\partial h_t}{\partial \tau} \Big|_x - vq_t^\kappa \frac{dq_t}{dh_t} \frac{\partial h_t}{\partial \tau} \Big|_x \\ &= \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \frac{du_t}{dq_t} \frac{dq_t}{dh_t} \frac{\partial h_t}{\partial \tau} \Big|_x - \left[ \frac{A_t h_t (1-u_t)}{C_t} - (1-u_t)h_t^{1+\gamma} \right] \frac{1}{(1-\tau)(1+\gamma)} \\ &= \frac{d \log \tilde{c}_t}{d \log u_t} \frac{\partial \log u_t}{\partial \tau} \Big|_x - \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{h_t(1-u_t)}{(1-\tau)(1+\gamma)}. \end{aligned}$$

Substituting this into (56) yields the desired result.  $\square$

*Proof of Proposition 3.* Proceeding as in the proof of Proposition 1 we have

$$\begin{aligned} \frac{dW_t}{dx_t} &= \frac{dW_t}{du_t} \frac{du_t}{dx_t} - vq_t^\kappa \frac{dq_t}{dx_t} + \left[ \frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \Big|_u \\ &\quad - \frac{\beta}{1-\beta} (1-\tau)^2 \frac{d}{dx_t} \frac{\sigma^2(x_t)}{2} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t} \end{aligned} \quad (57)$$

where  $dW_t/du_t$  is given by (49). We rearrange (57) to arrive at the desired result. First, note that

$$\frac{du_t}{dx_t} = -vq_t \frac{dM_t}{dx_t} - vM_t \frac{dq_t}{dx_t}.$$



Using this, (47), and (49), (57) then becomes

$$\begin{aligned}
\frac{dW_t}{dx_t} &= \left( - \left( -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \right) \left( -vq_t \frac{dM_t}{dx_t} - vM_t \frac{dq_t}{dx_t} \right) - vM_t \left( -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) \frac{dq_t}{dx_t} \\
&+ \left[ \frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \Big|_u - \frac{\beta}{1-\beta} (1-\tau)^2 \frac{d}{dx_t} \frac{\sigma^2(x_t)}{2} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t} \\
&= vq_t \left( -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) \frac{dM_t}{dx_t} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t} + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \frac{du_t}{dx_t} \\
&+ \left[ \frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \Big|_u - \frac{\beta}{1-\beta} (1-\tau)^2 \frac{d}{dx_t} \frac{\sigma^2(x_t)}{2}.
\end{aligned}$$

This can be rearranged to the desired result by making use of

$$\frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} = \frac{1}{C_t} \frac{dC_t}{du_t} + \frac{1-b}{1-u_t+u_t b}$$

and

$$-vq_t \frac{dM_t}{dx_t} = \frac{\partial u_t}{\partial x_t} \Big|_q.$$

□

### B.1 Derivation for section 5.2.3

We can normalize  $\mathbb{E}_{t-1} p_t = 1$ . A fraction  $\theta$  of firms set the price  $p_t^*$  and the remaining set the price  $\mathbb{E}_{t-1} p_t^* = 1$ . From the definition of the price index we can solve for

$$\left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} = \left( \frac{1 - (1-\theta)p_t^{-1/(1-\mu)}}{\theta} \right)^\mu$$

and substituting into (25) we arrive at

$$S_t = (1-\theta)p_t^{\mu/(\mu-1)} + \theta^{1-\mu} \left( 1 - (1-\theta)p_t^{1/(\mu-1)} \right)^\mu.$$

This makes clear that  $S_t$  is a function of  $p_t$ . Differentiating this function we arrive at

$$S'(1) = (1 - \theta) \frac{\mu}{\mu - 1} \left[ p_t^{1/(\mu-1)} - \theta^{1-\mu} \left( 1 - (1 - \theta)p_t^{1/(\mu-1)} \right)^{\mu-1} p_t^{(2-\mu)/(\mu-1)} \right] = 0$$

$$S''(1) = \frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1}.$$

Next rewrite the welfare function as

$$W_t = \log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1 + \gamma} + \text{t.i.p.}$$

$$C_t = Y_t - J_t - G_t$$

$$Y_t = \frac{\eta_t^A}{S(p_t)} h_t (1 - u_t).$$

Observe that  $u_t$  and  $J_t$  are exogenous in this case as  $M_t$  and  $q_t$  are exogenous. While  $G_t$  is not exogenous, we proceed as if it were because the effect of changes in  $G_t$  have no welfare effects because of the Samuelson rule. Use the production function to rewrite  $h_t$  in terms of  $Y_t$

$$h_t = \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)}.$$

Now rewrite  $W_t$  in terms of  $Y_t$  and  $p_t$

$$W_t = \log(Y_t - J_t - G_t) - \frac{1 - u_t}{1 + \gamma} \left( \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)} \right)^{1+\gamma}.$$

With flexible prices we have  $p = 1$  and  $S = 1$ . The flexible-price, socially-efficient level of output,  $Y^*$ , satisfies

$$\frac{\eta^A}{Y_t^* - J_t - G_t} = \left( \frac{Y_t^*}{\eta_t^A (1 - u_t)} \right)^\gamma.$$

Treating  $W_t$  as a function of  $Y_t$  and  $p_t$ , we take a second-order Taylor approximation around the point  $(Y_t^*, 1)$ . The first-derivatives are

$$W_Y(Y_t^*, 1) = \frac{1}{Y_t - J_t - G_t} - \left( \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)} \right)^\gamma \frac{S(p_t)}{\eta_t^A}$$

$$W_p(Y_t^*, 1) = -(1 - u_t) \left( \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)} \right)^\gamma \frac{Y_t}{\eta_t^A (1 - u_t)} S'(p_t)$$

and both are zero, the former because  $Y^*$  is optimal and the latter because  $S'(1) = 0$ . The second derivatives are

$$\begin{aligned} W_{YY}(Y_t^*, 1) &= -\frac{1}{(C^*)^2} - \gamma(Y_t^*)^{\gamma-1} \left(\frac{S(p_t)}{\eta_t^A}\right)^{1+\gamma} \left(\frac{1}{1-u_t}\right)^\gamma \\ W_{Yp}(Y_t^*, 1) &= -\left(\frac{Y_t^*}{1-u_t}\right)^\gamma \left(\frac{1}{\eta_t^A}\right)^{1+\gamma} (1+\gamma)S(p_t)^\gamma S'(p_t) \\ W_{pp}(Y_t^*, 1) &= -(1-u_t) \left(\frac{Y_t^*}{\eta_t^A(1-u_t)}\right)^{1+\gamma} \left[\gamma S(p_t)^{\gamma-1} S'(p_t) + S(p_t)^\gamma S''(p_t)\right]. \end{aligned}$$

Using  $S(1) = 1$ ,  $S'(1) = 0$ , the expression for  $S''(1)$  above, and the optimality condition for  $Y_t^*$  we arrive at

$$\begin{aligned} W_{YY}(Y_t^*, 1) &= -\frac{1}{(C^*)^2} \left[1 + \gamma \frac{C_t^*}{Y_t^*}\right] \\ W_{Yp}(Y_t^*, 1) &= 0 \\ W_{pp}(Y_t^*, 1) &= -\frac{Y_t^*}{C_t^*} \frac{1-\theta}{\theta} \frac{\mu}{\mu-1}. \end{aligned}$$

By Taylor's theorem we can write

$$W(Y_t, p_t) \approx \frac{1}{2} W_{YY}(Y_t^*, 1) (Y_t - Y_t^*)^2 + \frac{1}{2} W_{pp}(Y_t^*, 1) (p_t - 1)^2$$

Observe that  $Y$  and  $p$  are functions of  $x$ . So when we differentiate with respect to  $x$  we arrive at

$$\frac{dW_t}{dx_t} \approx W_{YY}(Y_t^*, 1) (Y_t - Y_t^*) \frac{dY_t}{dx_t} + W_{pp}(Y_t^*, 1) (p_t - 1) \frac{dp_t}{dx_t}.$$

Substituting for  $W_{YY}$  and  $W_{pp}$  and rearranging yields the result.

## C Proofs for section 5.3

The first step in the analysis is to express the equilibrium in date 0 in terms of as few variables as possible. It follows from (4), (5), (6), and (42) that we can express  $J$ ,  $u$ ,  $Y$ , and  $w$  as functions of  $x$ , and non-policy parameters. It then follows that  $j \equiv J/Y$  is a function of  $x$ .

A firm that sets its price in period 0 will set the optimal markup over current marginal cost

because all prices will be re-optimized in subsequent periods. Therefore the optimal relative price for a firm that updates in period 0 is  $\mu \left( w_0 + \psi_1 M_0^{\psi_2} / h(x_0, \tau) \right) / A_0$  where  $h(x, \tau)$  is given by (20). The price level and inflation rate in period 0 are therefore functions of  $x_0$  and  $\tau$ . It follows from (8) that the same is true of the nominal interest rate  $I_0$  and  $S_0$ . The Fisher equation, (45), and the assumption that the central bank sets inflation to zero in period 1 implies that  $R_0$  is also a function of  $x_0$  and  $\tau$ .

The next step is to derive an ‘‘aggregate demand curve’’ from the aggregate Euler equation. Using equations (17), (18), and (19) along with the log-normal income process we arrive at

$$\frac{(1 - u_t + u_t b)}{C_t} = \beta R_t e^{\sigma_\epsilon^2(x_t)(1-\tau)^2} \mathbb{E}_t \left[ \frac{(1 - u_{t+1}(1 - b)) (1 - u_{t+1}(1 - b^{-1}))}{C_{t+1}} \right].$$

For  $t \geq 0$  the expectation in the Euler equation disappears because outcomes in  $t + 1$  are deterministic. We can re-write the Euler equation as

$$\begin{aligned} C_0 &= \frac{1 - u_0 + u_0 b}{\beta R_0(x_0)} \frac{C_1}{(1 - u_1(1 - b)) (1 - u_1(1 - b^{-1}))} e^{-\sigma_\epsilon^2(x_0)(1-\tau)^2} \\ Y_0 \frac{1 - j_0}{1 + \chi \eta_0^G} &= \\ Y_0 &= \frac{1 + \chi \eta_0^G}{1 + \chi \eta_1^G} \frac{1 - j_1}{1 - j_0} \frac{1 - u_0 + u_0 b}{\beta R(x_0)} \frac{Y_1}{(1 - u_1(1 - b)) (1 - u_1(1 - b^{-1}))} e^{-\sigma_\epsilon^2(M_0)(1-\tau)^2} \end{aligned} \quad (58)$$

Turning to the supply side, the aggregate production function is (42). Under flexible prices, the price-setting first-order condition implies

$$w(x_t) + \frac{\psi_1 x_t^{\psi_2}}{h(x_t, \tau)} = \frac{\eta_t^A}{\mu}.$$

This equation implies  $x_t$  is a function of  $\eta_t^A$ ,  $\tau$ , and non-policy parameters when prices are flexible. We call this function  $x^{\text{Flex}}(\eta_t^A, \tau)$ .

Equations (58) and (42) lead to two expressions for  $Y_0$ . Setting them equal to one another yields an implicit solution for  $x_0$ .  $Y_1$  can be calculated from (42) with  $x_1 = x^{\text{Flex}}(\eta_1^A, \tau)$  because all prices are flexible in period 1. This leads to

$$Y_1 = \eta_1^A (1 - u(x_1)) [\bar{w}(1 - \tau)]^{\frac{1}{1+\gamma}} (x_1)^{\frac{\zeta}{1+\gamma}} \quad (59)$$

Note that  $x_1$  is given by  $x^{\text{Flex}}(\eta_1^A, \tau)$  so (59) makes  $Y_1$  independent of  $x_0$  and  $b$ .

Take the log of equations (58) and (42) and set them equal to one another yields

$$\begin{aligned} & \log\left(\frac{1 - \chi\eta_0^G}{1 - \chi\eta_1^G}\right) + \log(1 - j_1) - \log(1 - j_0) \\ & + \log(1 - u_0 + u_0b) - \log\beta - \log(R(x_0, \tau)) + \log\left(Y_1(x_1^{\text{Flex}}, \tau)\right) - \log(1 - u_1(1 - b)) \\ & - \log(1 - u_1(1 - b^{-1})) - \sigma_\epsilon^2(x_0)(1 - \tau)^2 - \log(\eta_0^A) + \log(S_0) - \log(h_0) - \log((1 - u_0)) = 0. \end{aligned} \quad (60)$$

*Proof of proposition 4.*  $x$  is implicitly defined by (60). Using the implicit function theorem we obtain

$$\frac{dx_0}{db} = - \frac{\frac{u_0}{1-u_0+u_0b} - \frac{u_1}{1-u_1(1-b)} + \frac{u_1b^{-2}}{1-u_1(1-b^{-1})}}{-\frac{1-b}{1-u_0+u_0b} \frac{du_0}{dx_0} - \frac{1}{R_0} \frac{dR_0}{dx_0} - (1-\tau)^2 \frac{d}{dx_0} \sigma_\epsilon^2(x_0) + \frac{1}{S_0} \frac{dS_0}{dx_0} - \frac{1}{h_0} \frac{dh_0}{dx_0} + \frac{1}{1-u_0} \frac{du_0}{dx_0} + \frac{1}{1-j_0} \frac{dj_0}{dx_0}}$$

As an elasticity we have

$$\frac{b}{x_0} \frac{dx_0}{db} = \frac{\frac{u_0b}{1-u_0+u_0b} - \frac{u_1b}{1-u_1(1-b)} + \frac{u_1b^{-1}}{1-u_1(1-b^{-1})}}{\frac{d \log R_0}{d \log x_0} + (1-\tau)^2 \sigma_\epsilon^2(x_0) \frac{d \log \sigma_\epsilon^2(x_0)}{d \log x_0} + \frac{1-b}{1-u_0+u_0b} u_0 \frac{d \log u_0}{d \log x_0} - \frac{d \log S_0}{d \log x_0} + \frac{d \log h_0}{d \log x_0} + \frac{d \log(1-u_0)}{d \log x_0} + \frac{d \log(1-j_0)}{d \log x_0}} \quad (61)$$

□

*Proof that the numerator of (33) is increasing in  $u_0$  and  $u_1$  for  $u < 1/2$ .* Start with  $\frac{u_0b}{1-u_0(1-b)}$ , the derivative with respect to  $u_0$  is

$$\frac{b(1 - u_0(1 - b)) + u_0b(1 - b)}{[1 - u_0(1 - b)]^2} = \frac{b}{[1 - u_0(1 - b)]^2} > 0.$$

Next turn to  $\frac{bu_1(1-u_1)(b^{-2}-1)}{[1+u_1(b^{-1}-1)][1+u_1(b-1)]}$ . We start by dividing by  $b(b^{-2}-1)$ . We will then show the derivative with respect to  $u_1$  is positive

$$\begin{aligned} & \frac{1 - 2u_1}{\{[1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]\}^2} [1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)] \\ & - \frac{u_1(1 - u_1)}{\{[1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]\}^2} [(b^{-1} - 1)(1 + u_1(b - 1)) + (b - 1)(1 + u_1(b^{-1} - 1))] > 0 \end{aligned}$$

multiply both sides by  $\{[1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]\}^2$

$$[1 - 2u_1][1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)] - u_1(1 - u_1)[(b^{-1} - 1)(1 + u_1(b - 1)) + (b - 1)(1 + u_1(b^{-1} - 1))] > 0$$

Rearranging

$$(1 - 2u_1)[1 + (u_1 - u_1^2)(b^{-1} + b - 2)] - u_1(1 - u_1)(b^{-1} + b - 2)(1 - 2u_1) > 0$$

divide both sides  $(1 - 2u_1)$  (recall  $u_1 < 1/2$ ) and rearrange

$$1 + u_1(1 - u_1)(b^{-1} + b - 2) - u_1(1 - u_1)(b^{-1} + b - 2) = 1 > 0.$$

□

*Proof of Claim 5.* Using the implicit function theorem to differentiate (60) we obtain

$$\begin{aligned} \frac{dx_0}{d\tau} &= -\frac{2\sigma_\epsilon^2(x_0)(1 - \tau) + \frac{1}{Y_1} \frac{1}{1+\gamma} \frac{Y_1}{1-\tau} - \frac{1}{h_0} \frac{1}{1+\gamma} \frac{h_0}{1-\tau} + \frac{1}{S_0} \left. \frac{\partial S_0}{\partial \tau} \right|_x - \frac{1}{R_0} \left. \frac{\partial R_0}{\partial \tau} \right|_x + \frac{1}{Y_1} \frac{dY_1}{dx_1} \frac{dx \text{Flex}(A_1, \tau)}{d\tau}}{-\frac{1-b}{1-u_0+u_0b} \frac{du_0}{dx_0} - \frac{1}{R_0} \frac{dR_0}{dx_0} - (1 - \tau)^2 \frac{d}{dx_0} \sigma_\epsilon^2(x_0) + \frac{1}{S_0} \frac{dS_0}{dx_0} - \frac{1}{h_0} \frac{dh_0}{dx_0} + \frac{1}{1-u_0} \frac{du_0}{dx_0} + \frac{1}{1-j_0} \frac{dj_0}{dx_0}} \\ &= -\frac{2\sigma_\epsilon^2(x_0)(1 - \tau) - \frac{1}{R_0} \left. \frac{\partial R_0}{\partial \tau} \right|_x + \frac{1}{S_0} \left. \frac{\partial S_0}{\partial \tau} \right|_x + \frac{1}{Y_1} \frac{dY_1}{dx_1} \frac{dx \text{Flex}(A_1, \tau)}{d\tau}}{-\frac{1-b}{1-u_0+u_0b} \frac{du_0}{dx_0} - \frac{1}{R_0} \frac{dR_0}{dx_0} - (1 - \tau)^2 \frac{d}{dx_0} \sigma_\epsilon^2(x_0) + \frac{1}{S_0} \frac{dS_0}{dx_0} - \frac{1}{h_0} \frac{dh_0}{dx_0} + \frac{1}{1-u_0} \frac{du_0}{dx_0} + \frac{1}{1-j_0} \frac{dj_0}{dx_0}} \end{aligned}$$

As an elasticity we have

$$\frac{\tau}{x_0} \frac{dx_0}{d\tau} = \frac{2\sigma_\epsilon^2(x_0)(1 - \tau)\tau - \left. \frac{\partial \log R_0}{\partial \log \tau} \right|_x + \left. \frac{\partial \log S_0}{\partial \log \tau} \right|_x + \frac{d \log Y_1}{d \log x_1} \frac{d \log x_1}{d \log \tau}}{\frac{d \log R_0}{d \log x_0} + (1 - \tau)^2 \sigma_\epsilon^2(x_0) \frac{d \log \sigma_\epsilon^2(x_0)}{d \log x_0} + \frac{1-b}{1-u_0+u_0b} u_0 \frac{d \log u_0}{d \log x_0} - \frac{d \log S_0}{d \log x_0} + \frac{d \log h_0}{d \log x_0} + \frac{d \log(1-u_0)}{d \log x_0} + \frac{d \log(1-j_0)}{d \log x_0}} \quad (62)$$

□

*Proof of Lemma 3.* See (61) and (62). □

## D Description of methods for section 6

### D.1 Estimated income process

The material in this appendix describes an implementation of the procedure of [Guvenen and McKay \(2015\)](#) and that paper gives more discussion and details. The income process is as follows:  $\alpha_{i,t}$  evolves as in (3). Earnings are given by  $\alpha_{i,t}w_t$  when employed and zero when unemployed. Notice that here we normalize  $h_t = 1$  and subsume all movements in  $h_t$  into  $w_t$ . While this gives a different interpretation to  $w_t$  it does not affect the distribution of earnings growth rates apart from a constant term. The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \sim \begin{cases} N(\mu_{1,t}, \sigma_1) & \text{with prob. } P_1, \\ N(\mu_{2,t}, \sigma_2) & \text{with prob. } P_2, \\ N(\mu_{3,t}, \sigma_3) & \text{with prob. } P_3 \\ N(\mu_{4,t}, \sigma_4) & \text{with prob. } P_4 \end{cases}$$

The tails of  $F$  move over time as driven by the latent variable  $x_t$  such that

$$\begin{aligned} \mu_{1,t} &= \bar{\mu}_t, \\ \mu_{2,t} &= \bar{\mu}_t + \mu_2 - x_t, \\ \mu_{3,t} &= \bar{\mu}_t + \mu_3 - x_t, \\ \mu_{4,t} &= \bar{\mu}_t. \end{aligned}$$

where  $\bar{\mu}_t$  is a normalization such that  $\mathbb{E}_i[\exp\{\epsilon_{i,t+1}\}] = 1$  in all periods.

The model period is one quarter. The parameters are selected to match the median earnings growth, the dispersion in the right tail (P90 - P50), and the dispersion in the left-tail (P50-P10) for one, three, and five year earnings growth rates computed each year using data from 1978 to 2011. In addition we target the kurtosis of one-year and five year earnings growth rates and the increase in cross-sectional variance over the life-cycle. The moments are computed from the Social Security Administration earnings data as reported by [Guvenen et al. \(2014\)](#) and [Guvenen et al. \(2015\)](#). We use these moments to form an objective function as described in [Guvenen and McKay \(2015\)](#).

The estimation procedure simulates quarterly data using the observed job-finding and -separation rates and then aggregates to annual income and computes these moments. To simulate the income process, we require time series for  $x_t$  and  $w_t$ . We assume that these series are linearly related to observable labor market indicators (for details see [Guvenen and McKay, 2015](#)). Call the weights in these linear relationships  $\beta$ . We then search over the parameters  $P$ ,  $\mu$ ,  $\sigma$ , and  $\beta$  subject to the restrictions  $P_2 = P_3$  and  $\sigma_2 = \sigma_3$ .

[Guvenen et al. \(2014\)](#) emphasize the pro-cyclicality in the skewness of earnings growth rates. The estimated income process does an excellent job capturing this as shown in figure XX. The estimated  $\beta$  implies a time-series for  $x_t$  which shifts the tails of the earnings distribution and gives rise the pro-cyclical skewness shown in figure XX. We regress this time-series on the unemployment rate and find a coefficient of 16.7.<sup>28</sup> The fourth component of the mixture distribution occurs with very low probability, and in our baseline specification we set it to zero. This choice is not innocuous, however, because the standard deviation  $\sigma_4$  is estimated to be very large and this contributes to the high kurtosis of the earnings growth distribution. In particular, omitting this component leads to a substantially smaller  $\tau$  as a result of having less risk in the economy. We prefer to omit this from our baseline calibration because the interpretation of these high-kurtosis terms is unclear and we are not entirely satisfied with modeling them as permanent shocks to skill.

The resulting income process that we use in our computations is as follows: The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \sim \begin{cases} N(\mu_{1,t}, 0.0403) & \text{with prob. } 0.9855, \\ N(\mu_{2,t}, 0.0966) & \text{with prob. } 0.00727, \\ N(\mu_{3,t}, 0.0966) & \text{with prob. } 0.00727 \end{cases}$$

with

$$\begin{aligned} \mu_{1,t} &= \bar{\mu}_t, \\ \mu_{2,t} &= \bar{\mu}_t + 0.266 - 16.73(u_t - u^*), \\ \mu_{3,t} &= \bar{\mu}_t - 0.184 - 16.73(u_t - u^*), \end{aligned}$$

---

<sup>28</sup> We regress this estimated time series  $x_t$  on the unemployment rate, which we smooth with an HP filter with smoothing parameter 100,000. If we call this regression function  $f$ , we then proceed with  $F(\epsilon'; f(u))$ .



where  $u^*$  is the steady state is unemployment rate in our baseline calibration.

## D.2 Global solution method

As a first step, we need to rewrite the Calvo-pricing first-order condition recursively:

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s + \psi_1 M_s^{\psi_2} / h_s\right) / A_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}.$$

Define  $p_t^A$  as

$$p_t^A = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s + \psi_1 M_s^{\psi_2} / h_s\right) / A_s$$

and  $p_t^B$  as

$$p_t^B = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s$$

such that

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}.$$

$p_t^A$  and  $p_t^B$  can be rewritten as

$$p_t^A = \mu Y_t \left(w_t + \psi_1 M_t^{\psi_2} / h_t\right) / A_t + (1-\theta) \mathbb{E}_t \left[ \left(\frac{I_t}{\pi_{t+1}}\right)^{-1} \pi_{t+1}^{-\mu/(1-\mu)} p_{t+1}^A \right] \quad (63)$$

$$p_t^B = Y_t + (1-\theta) \mathbb{E}_t \left[ \left(\frac{I_t}{\pi_{t+1}}\right)^{-1} \pi_{t+1}^{-1/(1-\mu)} p_{t+1}^B \right]. \quad (64)$$

The procedure we use builds on the method proposed by [Maliar and Maliar \(2015\)](#) and their application to solving a New Keynesian model. We first describe how we the model for a given grid of aggregate state variables and then describe how we construct the grid.

There are six state variables that evolve according to

$$\begin{aligned}
\mathbb{E}_i \left[ \alpha_{i,t+1}^{1-\tau} \right] &= (1 - \delta) \mathbb{E}_i \left[ \alpha_{i,t}^{1-\tau} \right] \mathbb{E}_i \left[ \epsilon_{i,t+1}^{1-\tau} u_t \right] + \delta \\
\mathbb{E}_i [\log \alpha_{i,t+1}] &= (1 - \delta) [\mathbb{E}_i [\log \alpha_{i,t}] + \mathbb{E}_i [\log \epsilon_{i,t+1} | u_t]] \\
S_{t+1}^A &= S_t \\
\log \eta_{t+1}^A &= \rho^A \log \eta_t^A + \varepsilon_{t+1}^A \\
\log \eta_{t+1}^G &= \rho^G \log \eta_t^G + \varepsilon_{t+1}^G \\
\log \eta_{t+1}^I &= \rho^I \log \eta_t^I + \varepsilon_{t+1}^I,
\end{aligned}$$

where  $S^A$  is the level of price dispersion in the previous period and the  $\varepsilon$  terms are i.i.d. normal innovations.

There are five variables that we approximate with complete second-order polynomials in the state:  $(1/C_t)$ ,  $p_t^A$ ,  $p_t^B$ ,  $J_t$ , and  $V_t$ , where  $V_t$  is the value of the social welfare function. We use (17) and (18) to write the Euler equation in terms of  $C_t$  and this equation pins down  $1/C_t$ .  $p_t^A$  and  $p_t^B$  satisfy (63) and (64).  $V_t$  satisfies

$$V_t = W_t + \beta \mathbb{E}_t [V_{t+1}].$$

$J_t$  satisfies  $J_t = \psi_1 M_t^{\psi_2} (v - u_t)$ . Abusing language slightly, we will refer to these variables that we approximate with polynomials as forward-looking variables.

The remaining variables in the equilibrium definition can be calculated from the remaining equations and all of which only involve variables dated  $t$ . We call these the “static” variables.

To summarize, let  $\mathcal{S}_t$  be the state variables,  $\mathcal{X}_t$  be the forward-looking variables, and  $\mathcal{Y}_t$  be the static variables. The three blocks of equations are

$$\begin{aligned}
\mathcal{S}' &= \mathcal{G}^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon') \\
\mathcal{X} &= \mathbb{E} \mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}') \\
\mathcal{Y} &= \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})
\end{aligned}$$

where  $\mathcal{G}^{\mathcal{S}}$  are the state-transition equations,  $\mathcal{G}^{\mathcal{X}}$  are the forward-looking equations and  $\mathcal{G}^{\mathcal{Y}}$  are the state equations. Let  $\mathcal{X} \approx \mathcal{F}(\mathcal{S}, \Omega)$  be the approximated solution for the forward-looking equations for which we use a complete second-order polynomial with coefficients given by  $\Omega$ . We then opera-

tionalize the equations as follows: given a value for  $\mathcal{S}$ , we calculate  $\mathcal{X} = \mathcal{F}(\mathcal{S}, \Omega)$  and  $\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$ . We then take an expectation over  $\varepsilon'$  using Gaussian quadrature. For each value of  $\varepsilon'$  in the quadrature grid, we compute  $\mathcal{S}' = G^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$ ,  $\mathcal{X}' = \mathcal{F}(\mathcal{S}', \Omega)$  and  $\mathcal{Y}' = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}', \mathcal{X}')$ . We can now evaluate  $\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$  for this value of  $\varepsilon'$  and looping over all the values in the quadrature grid we can compute  $\hat{\mathcal{X}} = \mathbb{E} \mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$ .  $\hat{\mathcal{X}}$  will differ from the value of  $\mathcal{X}$  that was obtained initially from  $\mathcal{F}(\mathcal{S}, \Omega)$ . We repeat these steps for all the values of  $\mathcal{S}$  in our grid for the aggregate state space. We then adjust the coefficients  $\Omega$  part of the way towards those implied by the solutions  $\hat{\mathcal{X}}$ . We then iterate this procedure to convergence of  $\Omega$ .

Evaluating some of the equations of the model involves taking integrals against the distribution of idiosyncratic skill risk  $\epsilon_{i,t+1} \sim F(\epsilon_{i,t+1}, u_t - u^*)$ . We do this using Gaussian quadrature within each of the components of the mixture distribution. We compute expectations over aggregate shocks using Gaussian quadrature as well.

We use a two-step procedure to construct the state on the aggregate state space. We have six aggregate states so we choose the grid to lie in the region of the aggregate state space that is visited by simulations of the solution. We create a box of policy parameters  $[b_L, b_H] \times [\tau_L, \tau_H]$ . For each of the four corners of this box, we use the procedure of [Maliar and Maliar \(2015\)](#) to construct a grid and solve the model. This procedure iterates between solving the model and simulating the solution and constructing a grid in the part of the state space visited by the simulation. This gives us four grids, which we then merge and eliminate nearby points using the techniques of [Maliar and Maliar \(2015\)](#). This leaves us with one grid that we use to solve the model when we evaluate policies. Each of the grids that we construct have 100 points.