UNINSURED UNEMPLOYMENT RISK AND OPTIMAL MONETARY POLICY*

Edouard Challe†

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Abstract. I study optimal monetary policy in an economy wherein households precautionary-save against uninsured, endogenous unemployment risk. In this economy greater unemployment risk strengthens the precautionary motive, causing aggregate demand to fall and feed back to greater unemployment risk. This feedback loop causes the policy prescriptions under perfect insurance to be overturned: the policy rate should be *lowered* after contractionary productivity or cost-push shocks in order to neutralise their inefficient impact on aggregate demand. This policy breaks the feedback loop between unemployment risk and aggregate demand and takes the dynamics of the imperfect-insurance economy close to that of the perfect-insurance benchmark.

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†CREST, CNRS, Ecole Polytechnique, Université Paris-Saclay; Address: Ecole Polytechnique, Département d'Economie, Route de Saclay, 91128 Palaiseau Cedex, France; Email: edouard.challe@gmail.com.

1. Introduction

Several recent contributions have stressed that households' precautionary-saving response to uninsured unemployment risk may generate substantial aggregate volatility, relative to a hypothetical situation of perfect insurance. The reason for this is that greater unemployment risk strengthens the precautionary motive for saving, causing aggregate demand, output and employment to fall, which ultimately feeds back to greater unemployment risk.[1] This feedback loop naturally arises, for example, in New Keynesian models when labour market frictions make unemployment risk endogenously countercyclical and imperfect insurance activates the precautionary motive (Challe et al., 2016; Ravn and Sterk, 2013, 2017). In this paper I ask how *should* the central bank respond to aggregate shocks under those frictions and propagation mechanism, *by how much* does this optimal response differ from that under perfect insurance, and *how effective* is it at stabilising welfare-relevant aggregates. I focus on the two aggregate shocks that have played a prominent role in New Keynesian analyses of optimal monetary policy, namely transitory (but persistent) productivity and "cost-push" shocks (Clarida et al., 1999; Woodford, 2003; Gali, 2008).[2] In both cases I find the optimal policy to critically depend on the degree of insurance and implied strength of the precautionary motive.

Consider first the response to a cost-push shock, i.e., an exogenous increase in production costs that is passed through to final goods prices. With uninsured unemployment risk the optimal response of the policy rate is in general ambiguous. On the one hand, the central bank should act to mitigate the direct inflationary impact of the shock, which typically commands an increase in the policy rate; such is the optimal policy in the Representative-Agent New Keynesian model ("RANK model" henceforth), and I recover this policy in the perfect-insurance limit of the imperfect-insurance economy. On the other hand, the shock deters firms' hiring and sets in motion a *deflationary* feedback loop between unemployment risk and aggregate demand; this calls for a muted, or even reverted, response of the policy rate. Under a parametric restriction that gives the optimal response of the policy rate in closed form, these two effects can be additively decomposed into (i) a perfect-insurance response and (ii) an imperfect-insurance correction, whose size depends on households' average consumption drop upon unemployment. Away from this restriction the contribution of imperfect insurance can be recovered numerically by comparing the optimal response of the policy rate in the imperfect-insurance economy and that in the perfect-insurance benchmark. The calibrated model shows that under imperfect insurance the policy rate should be *lowered*, not raised, after a cost-push shock so as to offset its inefficient impact on aggregate demand. This policy effectively breaks the deflationary spiral and takes the aggregate dynamics of the imperfect-insurance economy close to that of the perfect-insurance benchmark.

Uninsured unemployment risk also crucially affects the optimal response of the policy rate to productivity shocks. A persistent productivity-driven contraction (for example) generates an in-

[1] See, e.g., Beaudry et al. (2016), Challe et al. (2016), Chamley (2014), Den Haan et al. (2017), Heathcote and Perri (2016), Ravn and Sterk (2013, 2017) and Werning (2015).

[2] In New Keynesian models, persistent productivity shocks move the IS curve that determines the dynamics of the output gap, while cost-push shocks move the Phillips curve that determines the dynamics of inflation.

crease in unemployment risk and elicits a precautionary response on the part of the households. The resulting fall in aggregate demand exerts a downward pressure on inflation and the employment gap that the central bank effectively stabilises by lowering the policy rate. This optimal response is the opposite of that in the RANK model, which prescribes a *rise* in the policy rate in order to counter the excess aggregate demand generated by the expected recovery. The calibrated model again shows that the central bank should lower the policy rate after a productivity-driven contraction – even when intertemporal substitution in consumption works against the precautionary motive and pushes for a rise. Again, implementation of the optimal policy successfully undoes much of the propagating effect of imperfect insurance on aggregate dynamics.

The present paper integrates two strands of the literature: one that examines the propagation of aggregate shocks within the extended New Keynesian model with uninsured unemployment risk; and one that derives the optimal monetary policy response to aggregate shocks under the simplifying assumption of perfect insurance. The feedback loop that arises under imperfect insurance, labour market frictions and nominal price stickiness was identified, and quantitatively evaluated, by Challe et al. (2016) and Ravn and Sterk (2013).[3] Den Haan et al. (2017) present a related feedback loop working through nominal wage stickiness. Gornemann et al. (2016) also construct a model with a similar set of frictions, but their focus is on the redistributive role of monetary policy rather than the propagation of aggregate shocks. Werning (2015, Section 3.4) examines the sensitivity of aggregate demand to the nominal interest rate under the same frictions, focusing on the aggregated Euler condition (and bypassing an explicit modelling of firm behaviour).[4]

A common feature of all the above-mentioned papers is to specify the way monetary policy is conducted by means of an exogenous nominal interest rate or a simple nominal interest rate rule. By contrast, in the present paper the central bank sets the policy rate with the aim of tracking a well-defined (constrained-) efficient allocation. This generalises the analysis of optimal monetary policy traditionally undertaken within the RANK benchmark, be it without labour-market frictions (e.g., Clarida et al., 1999; Woodford, 2003; Gali, 2008) or with such frictions (Thomas, 2008; Faia, 2009; Blanchard and Gali, 2010; Ravenna and Walsh, 2011). Braun and Nakajima (2012) studied optimal policy within a New Keynesian model with *exogenous* uninsured idiosyncratic shocks; there is no feedback loop between unemployment risk and aggregate demand under this assumption and, as a consequence, the optimal policy does not differ from that in the RANK model. Bilbiie and Ragot (2017) compute the optimal policy under imperfect insurance and endogenous liquidity but exogenous unemployment risk; again, the deflationary spiral that is the focus of the present paper is absent from theirs. Finally, there is a long tradition, going from Bewley (1980) to Nuño and Thomas (2016), of analysing the optimal inflation rate with uninsured idiosyncratic risk but without aggregate shocks; this differs from the present paper, which is entirely concerned with the

[3] Ravn and Sterk (2017) extensively analyse the implications of this feedback loop for equilibrium uniqueness, the propagation of productivity shocks, and the behaviour of risk premia.

[4] Other papers quantitatively examine the effect of monetary policy under imperfect insurance but exogenous unemployment risk – so that there is no feedback from aggregate demand to unemployment risk. This includes Kaplan et al. (2016), who study the impact of conventional interest-rate changes, and McKay et al. (2016), who examine the effect of forward guidance.

optimal policy response to such shocks.

Section 2 presents the model and its equilibrium. Section 3 characterises the constrained-efficient allocation and associated steady state. Section 5 formulates and solves a linear-quadratic approximation of the optimal policy problem under a particular parametric restriction; this allows to derive closed-form expressions for the optimal policy rate that make the specific role played by imperfect insurance fully transparent. Section 4 calibrates and numerically solves the general model.

2. The model

2.1. Households. Households are of two types: There is a unit measure of "workers", who can be employed or unemployed, and a measure $\nu > 0$ of "firm owners" who manage the firms and collect dividends. All households are infinitely-lived, discount the future at the same factor $\beta \in [0, 1)$, and cannot borrow against future income.

Workers. A worker $i \in [0, 1]$ chooses the consumption sequence $\{c_{i,t+k}\}_{k=0}^{+\infty}$ that maximises $V_t^i = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k u(c_{i,t+k})$, where $c_{i,t} \geq 0$ is consumption, $\mathbb{E}_t$ the expectations operator (over both aggregate and idiosyncratic uncertainty), and $u(\cdot)$ a period utility function such that $u' > 0$ and $u'' < 0$. A worker can be employed or unemployed: employed workers earn the real wage w_t, while unemployed workers earn the home production income $\delta < w_t$. Workers transit randomly between labour market statuses and the associated income risk is uninsurable. The budget and borrowing constraints of worker i at date t are given by, respectively:

$$a_{i,t} + c_{i,t} = e_{i,t} w_t + (1 - e_{i,t}) \delta + R_t a_{i,t-1} \text{ and } a_{i,t} \geq 0 \quad \forall i \in [0, 1], \tag{1}$$

where $a_{i,t}$ is the worker's asset wealth at the end of date t, R_t is the ex post gross return on accumulated assets and $e_{i,t}$ an indicator variable taking value 1 if the worker is employed and zero otherwise. Workers' optimal consumption-saving choices must satisfy the Euler condition $u'(c_{i,t}) \geq \beta \mathbb{E}_t u'(c_{i,t+1}) R_{t+1}$, with an equality if the borrowing constraint is slack and a strict inequality if it is binding.

Firm owners. Every firm owner gets an equal share of the aggregate dividend D_t that results from firms' rents (see below), as well as a home production income $\varpi \geq 0$ and a transfer τ_t. Firm owners are risk neutral and all hold the same asset wealth at the beginning of date 0; since they face no idiosyncratic risk and share the same preferences, they stay symmetric at all times and we denote their common individual consumption and end-of-period asset wealth by c_t^F and a_t^F, respectively. A firm owner thus maximises $V_t^F = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k c_{t+k}^F$ subject to

$$a_t^F + c_t^F = D_t/\nu + R_t a_{t-1}^F + \varpi + \tau_t \text{ and } a_t^F \geq 0. \tag{2}$$

Given their preferences and constraints, the consumption versus saving choice of a firm owner at time t is as follows: either $\mathbb{E}_t \beta R_{t+1} > 1$ and $c_t^F = 0$; or $\mathbb{E}_t \beta R_{t+1} < 1$ and $a_t^F = 0$; or $\mathbb{E}_t \beta R_{t+1} = 1$ and $c_t^F \in \left[0, D_t/\nu + R_t a_{t-1}^F + \varpi + \tau_t\right]$.

2.2. Firms. The production structure has three layers: intermediate goods firms produce out of workers' labour units, which they hire in a frictional labour market with search costs. Those goods are sold to wholesale firms, each of whom turn them into a differentiated good. Finally, wholesale goods are purchased and reassembled by final goods firms, the output of which is used for consumption and search costs.

Final goods sector. There is a representative, competitive final goods firm that produces by combining wholesale inputs according to the function:

$$y_t = \left(\int_0^1 y_{h,t}^{\frac{\theta-1}{\theta}} \, \mathrm{d}h \right)^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $y_{h,t}$ is the quantity of wholesale good h used in production and $\theta > 1$ the cross-partial elasticity of substitution between wholesale inputs. Denoting $p_{h,t}$ as the price of wholesale good h in terms of the final good, the optimal combination of inputs gives the following demands:

$$y_{h,t} = y_t p_{h,t}^{-\theta}, \;\; h \in [0,1], \tag{4}$$

while the zero-profit condition in the final goods sector implies that $\int_0^1 p_{h,t}^{1-\theta} \mathrm{d}j = 1$.

Wholesale sector. Wholesale firm $h \in [0,1]$ turns every intermediate good into a specialised good that is monopolistically supplied to the final goods sector. The profit of wholesale firm h is

$$\Pi_{h,t}^W = y_{h,t}[p_{h,t} - \varphi_t(1-\tau^W)], \tag{5}$$

where φ_t is the price of intermediate goods in terms of the final goods and τ^W a production subsidy to the wholesale sector, financed through a lump sum tax on firm owners.[5]

Wholesale firms face nominal pricing frictions a la Calvo: in every period a fraction $1-\omega \in [0,1]$ of the firms are able to reset their price optimally, while the other firms keep it unchanged. As a result of this irregular price adjustments the distribution of wholesale prices evolves over time, but its dynamics can be summarised by three equations (see Woodford, 2003). The first characterises the optimal real reset price applied by (all) price-resetting firms:

$$p_t^* = \frac{\theta(1-\tau^W)}{\theta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\omega\beta)^k \varphi_{t+k} \Pi_{n=1}^k (1+\pi_{t+n})^{\theta} y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\omega\beta)^k \Pi_{n=1}^k (1+\pi_{t+n})^{\theta-1} y_{t+k}}, \tag{6}$$

where π_t is final goods price inflation. The second equation states how inflation evolves as a function of p_t^*:

$$\pi_t = [\omega^{-1} - (\omega^{-1} - 1)(p_t^*)^{1-\theta}]^{\frac{1}{\theta-1}} - 1. \tag{7}$$

The third equation is the law of motion of the price dispersion index $\Delta_t \equiv \int_0^1 p_{h,t}^{-\theta} \mathrm{d}h$ (≥ 1):

$$\Delta_t = (1-\omega)(p_t^*)^{-\theta} + \omega (1+\pi_t)^{\theta} \Delta_{t-1}, \;\; \Delta_{-1} \text{ given.} \tag{8}$$

[5] This subsidy will serve in Section 3 to correct the steady-state distortion due to monopolistic competition.

I assume that Δ_{t-1} is initially at its steady state value (= 1). From (4)–(5) and the definition of Δ_t, the profits generated by the wholesale sector are:

$$\Pi_t^W = \int_0^1 \Pi_{h,t}^W \mathrm{d}h = y_t[1 - \varphi_t \left(1 - \tau^W\right) \Delta_t]. \tag{9}$$

Intermediate goods sector and labour market flows. Intermediate goods firms hire labour in a frictional labour market with search costs. At the beginning of date t a constant fraction $\rho \in (0,1]$ of existing employment relationships are destroyed, at which point the size of the unemployment pool goes from $1 - n_{t-1}$ to $1 - (1-\rho)\, n_{t-1}$. At the same time intermediate goods firms post v_t vacancies, at a unit cost $c > 0$, a random matching market opens and $m\left(1 - (1-\rho)\, n_{t-1}\right)^{\gamma} v_t^{1-\gamma}$, $\gamma \in (0,1)$, new employment relationships are formed.[6] It follows that the job-finding and vacancy-filling rates are, respectively:

$$f_t = m\left[\frac{v_t}{1-(1-\rho)\, n_{t-1}}\right]^{1-\gamma} \text{ and } \lambda_t = m\left[\frac{v_t}{1-(1-\rho)\, n_{t-1}}\right]^{-\gamma}. \tag{10}$$

The value to firm owners of an employment relationship is given by:

$$J_t = (1-\tau^I)(z_t\varphi_t - w_t + T - \zeta_t) + \beta\,(1-\rho)\,\mathbb{E}_t J_{t+1}. \tag{11}$$

where $\tau^I \in [0,1]$ is a corporate tax rate and T a wage subsidy. ζ_t is a random wage tax evolving as follows:

$$\zeta_t = \mu_\zeta \zeta_{t-1} + \epsilon_{\zeta,t},\ \ \mu_\xi \in [0,1)\,,$$

where $\epsilon_{\zeta,t}$ is a white noise process with mean zero and small bounded support.

The taxes and subsidy τ^I and T will serve the same purpose as the production subsidy τ^W in the wholesale sector: they will be set in such a way that the steady state of the decentralised equilibrium be constrained-efficient. The production subsidy τ^W does not suffice for that because even in steady state the economy has two distortions in addition to monopolistic competition in the wholesale sector: labour market search and imperfect insurance. As will become clear in Section 3, τ^I will correct for the former and T for the latter. The random tax ζ_t perturbs the real marginal cost of intermediate goods firms and is partially pass-through to final-good prices. It will manifest itself as a pure cost-push shock and make the decentralised equilibrium of the stochastic economy generically constrained-inefficient. The net proceeds of all taxes and subsidies to the intermediate goods sector are rebated lump-sum to firm owners.

Under free entry the cost of a vacant job (c) must equate its expected payoff ($\lambda_t J_t$). Then, from (10)–(11) we get the following forward recursion for the job-finding rate:

$$f_t^{\frac{\gamma}{1-\gamma}} = (1-\tau^I)(m^{\frac{1}{1-\gamma}}/c)(z_t\varphi_t - w_t + T - \zeta_t) + \beta\,(1-\rho)\,\mathbb{E}_t f_{t+1}^{\frac{\gamma}{1-\gamma}} \tag{12}$$

[6] This standard timing assumption implies that firms may fill vacancies within the period in which they are opened, while workers may change employment relationship without actually experiencing an unemployment spell.

Since employed workers are separated from their firm with probability ρ at the very beginning of the period, but can immediately find a job with probability f_t, the period-to-period transition rate from employment to unemployment is given by:

$$s_t = \rho (1 - f_t), \tag{13}$$

while aggregate employment evolves as:

$$n_t = f_t (1 - n_{t-1}) + (1 - s_t) n_{t-1}. \tag{14}$$

Finally, the aggregate rent generated by intermediate goods firms at time t is:

$$\Pi_t^I = n_t (1 - \tau^I)(z_t \varphi_t - w_t + T - \zeta_t) - c v_t. \tag{15}$$

Firms' vacancy-posting decisions ultimately depend on the real wage w_t, which under random matching is indeterminate within the bargaining set (see Hall, 2005, for an extensive discussion). The baseline specification throughout the paper is that w_t is equal to its socially efficient level w_t^*, which is derived in Section 3 below. We will also consider an alternative – hence inefficient – wage-setting mechanism in Section 5.

2.3. Central bank. The only assets in the economy are nominal bonds, the interest rate on whom i_t (the "policy rate") being controlled by the central bank. In setting this rate the central bank seeks to maximise a social welfare function ("SWF" henceforth) W_t that aggregates households' intertemporal utilities, giving relative welfare weight $\tilde{\Lambda} > 0$ to firm owners (the SWF is utilitarian in the special case where $\tilde{\Lambda} = 1$).[7] The gross real ex post return that results from the policy rate and the dynamics of inflation is:

$$R_t = (1 + i_{t-1}) / (1 + \pi_t). \tag{16}$$

2.4. Market clearing. Given the measures of workers and firm owners (1 and ν, respectively) and the market and home production of final goods, the market-clearing conditions for bonds and final goods are given by $\int_{[0,1]} a_{i,t} \mathrm{d}i + \nu a_t^F = 0$ and $\int_{[0,1]} c_{i,t} \mathrm{d}i + \nu c_t^F + c v_t = y_t + \delta (1 - n_t) + \nu \varpi$, respectively. The supply of intermediate goods is $z_t n_t$. From (4), the demand for intermediate goods is $\int_{[0,1]} y_{h,t} \mathrm{d}h = \Delta_t y_t$. Hence, clearing of the market for intermediate goods requires:

$$\Delta_t y_t = z_t n_t \tag{17}$$

2.5. Equilibrium: definition and characterisation. An equilibrium is a set of sequences of (i) households' $(\{c_t^F, a_t^F, c_t^i, a_t^F, a_t^i\}_{t=0}^{\infty}, i \in [0,1])$, firms' $(\{y_t, y_{h,t}, , p_t^*\}_{t=0}^{\infty}, h \in [0,1])$ and central bank's $(\{i_t\}_{t=0}^{\infty})$ decisions that are individually optimal given prices; and (ii) aggregate variables $\{v_t, J_t, \lambda_t, f_t, s_t, n_t, \Delta_t, \varphi_t, \pi_t, \Pi_t^W, \Pi_t^I, R_t\}_{t=0}^{\infty}$ that solve equations (7) to (17) together with the free

[7] The SWF is computed in Section 3.1 below.

entry condition $c = \lambda_t J_t$.

Under the assumptions made so far, the model does not generate a distribution of wealth across workers, despite imperfect unemployment insurance (see Ravn and Sterk, 2013, 2017). The reason for this is that employed workers' precautionary-saving behaviour pushes down the real interest rate below households' common rate of time preference. At this interest rate, both unemployed workers and firm owners would like to borrow against future income but face a binding debt limit. It follows that the supply of assets is zero and hence no asset trade actually takes place when workers change employment statuses. This feature of the equilibrium allows the precautionary motive to be operative while maintaining the high level of tractability that is required for the analysis of optimal monetary policy. The existence of this no-trade equilibrium can be established by spelling out the corresponding equilibrium conditions and showing that they hold in steady state. Provided that aggregate shocks have small bounded support, these conditions will also hold in stochastic equilibrium.

The first property of the equilibrium is that employed workers do not face a binding debt limit (because they wish to precautionary-save). Hence their Euler condition holds with equality:

$$\mathbb{E}_t M_{t+1}^e R_{t+1} = 1, \tag{18}$$

where their marginal rate of intertemporal substitution, taking into account both aggregate and idiosyncratic risk, is given by:

$$M_{t+1}^e = \beta \frac{(1 - s_{t+1}) u'(w_{t+1}) + s_{t+1} u'(\delta)}{u'(w_t)}. \tag{19}$$

The latter expression uses the fact that in any equilibrium without asset trading workers consume their current income (δ or w_t). Equations (18)–(19) illustrate the fact that a rise in unemployment risk (s_{t+1}) strengthens employed workers' precautionary motive and pushes down the gross real interest rate (R_{t+1}). Holding the policy rate i_t constant, the fall in R_{t+1} is brought about by deflationary pressures in the current period associated with a rise in expected inflation.

The second feature of the equilibrium under consideration is that unemployed workers face a binding debt limit, i.e., their Euler condition holds with strict inequality:

$$\mathbb{E}_t M_{t+1}^u R_{t+1} < 1, \tag{20}$$

where

$$M_{t+1}^u = \beta \frac{(1 - f_{t+1}) u'(\delta) + f_{t+1} u'(w_{t+1})}{u'(\delta)}.$$

Intuitively, equations (18) and (20) can jointly hold because employed workers face a decreasing expected consumption profile – due to the risk of loosing one's job – while unemployed workers face a rising expected consumption profile – due to the possibility of finding one. Hence current marginal utility is low relative to expected marginal utility for the former, while the opposite is true for the latter.

The third feature of the equilibrium is that firm owners also face a binding debt limit, i.e.,

$$\mathbb{E}_t \beta R_{t+1} < 1. \tag{21}$$

Equations (18) and (21) are mutually consistent because employed workers' precautionary motive take the gross real interest rate down below $1/\beta$, while firm owners face no idiosyncratic income shocks and hence have no reason to self-insure. Thus, instead of accepting a low return on their savings, they turn frustrated borrowers and consume their current income in every period. From (9), (15) and (17), the consumption of a firm owner, after all taxes and subsidies have been rebated lump-sum, is given by:

$$c_t^F = \varpi + n_t(z_t/\Delta_t - w_t) - cv_t. \tag{22}$$

Equation (22) shows that, holding labour market conditions (n_t, v_t, w_t) (hence workers' welfare) fixed, price dispersion Δ_t creates a productive inefficiency that is ultimately borne by firm owners.

From (18)–(19), in the absence of aggregate shocks the gross interest rate R is given by:

$$R = 1 + i = \frac{1}{\beta \left[1 - s + su'(\delta)/u'(w)\right]} < \frac{1}{\beta}. \tag{23}$$

For $f \in (0,1)$ we have $s = \rho(1-f) > 0$ and hence (since $\delta < w$), $M^u, \beta < M^e$. Thus, with $M^e R = 1$ – i.e., employed workers are not borrowing-constrained – we have $M^u, \beta < M^e$ – i.e., both unemployed workers and firm owners are –, and this is also true in the stochastic equilibrium in the vicinity of the steady state.

3. Constrained efficiency

The economy described above is potentially plagued by four distortions: monopolistic competition in the wholesale sector, relative price distortions due to nominal rigidities, crowding-out externalities in the labor market, and imperfect insurance against unemployment risk. In what follows I characterise the constrained-efficient allocation and derives the values of steady-state inflation (π) and the tax instruments (τ^W, T, τ^I) that decentralise this allocation in the absence of aggregate shocks (so that the steady state be undistorted).

3.1. Social welfare function. Since all households consume their current income in every period, the intertemporal utilities of employed workers, unemployed workers and firm owners can be written recursively as follows:

$$V_t^e = u(w_t) + \beta \mathbb{E}_t[(1 - s_{t+1}) V_{t+1}^e + s_{t+1} V_{t+1}^u], \tag{24}$$

$$V_t^u = u(\delta) + \beta \mathbb{E}_t[f_{t+1} V_{t+1}^e + (1 - f_{t+1}) V_{t+1}^u], \tag{25}$$

and

$$V_t^F = \varpi + n_t(z_t/\Delta_t - w_t) - cv_t + \beta \mathbb{E}_t V_{t+1}^F. \tag{26}$$

The SWF is $W_t = n_t V_t^e + (1 - n_t) V_t^u + \Lambda V_t^F$, where $\Lambda = \tilde{\Lambda}\nu$ (see Section 2.3). Using equations (14) and (24) to (26) and rearranging, the SWF can be written as:

$$W_t = U_t + \beta \mathbb{E}_t W_{t+1}, \tag{27}$$

where the flow payoff U_t is given by:

$$U_t = \underbrace{n_t u(w_t) + (1 - n_t) u(\delta)}_{\text{workers}} + \underbrace{\Lambda (\varpi + n_t (z_t/\Delta_t - w_t) - c v_t)}_{\text{firm owners}}. \tag{28}$$

3.2. Constrained-efficient allocation. The constrained-efficient allocation is the sequence $\{\Delta_t, w_t, n_t, v_t\}_{t=0}^{+\infty}$ that maximises W_t in (27)–(28), taking as given the initial conditions (n_{-1}, Δ_{-1}), the law of motion of Δ_t (equation (8)) and the economy-wide relationship between employment and vacancies:

$$n_t = (1 - \rho) n_{t-1} + (1 - (1 - \rho) n_{t-1})^{\gamma} v_t^{1-\gamma}.$$

Solving the latter equation for v_t gives:

$$v_t = \left[\frac{n_t - (1 - \rho) n_{t-1}}{(1 - (1 - \rho) n_{t-1})^{\gamma}} \right]^{\frac{1}{1-\gamma}}, \tag{29}$$

which can be substituted into (28). Equation (29) makes clear that, at any level of employment inherited from the previous period (i.e., $(1 - \rho) n_{t-1}$), raising current employment n_t can only be achieved by raising vacancies and hence the total hiring cost borne by firm owners. On the other hand, inherited employment $(1 - \rho) n_{t-1}$ affects the amount of vacancies needed to reach a particular value of n_t in two ways. First, high past employment reduces the need for new vacancies (the numerator); and second, it reduces the size of the unemployment pool, which makes hiring more difficult and raises the need for new vacancies.

Formally, the constrained-efficient allocation is the solution to

$$W_t (n_{t-1}, \Delta_{t-1}) = \max_{p_t^*, w_t, n_t \geq 0} \{U_t + \beta \mathbb{E}_t W_{t+1} (n_t, \Delta_t)\} \quad \text{s.t. (7), (8), (29).}$$

The value of w_t that maximises W_t is given by:

$$w_t = w^* = u'^{-1} (\Lambda). \tag{30}$$

Intuitively, the real wage must equate the (weighted) marginal utilities of employed workers and firm owners; a constant wage then implements an allocation wherein risk-neutral firm owners efficiently provide income insurance to risk-averse workers.

From (7)–(8), it is clear that $p_t^* = 1 \ \forall t$ is optimal: starting from $\Delta_{-1} = 1$, this sequence ensures that $(\pi_t, \Delta_t) = (0, 1) \ \forall t$, which maximises U_t in (28) in every period.[8] In other words, the

[8] The optimality of $\{p_t^*\}_{t=0}^{\infty} = 1$ can be confirmed by computing the first-order condition with respect to p_t^* and the envelope conditions with respect to Δ_t.

constrained-efficient allocation has zero inflation and symmetric wholesale prices at all times.

Finally, the first-order and envelope conditions with respect to n_t give, respectively:

$$u(w_t) - u(\delta) + \Lambda \left[z_t - w_t - \frac{c}{(1-\gamma)\lambda_t} \right] + \beta \mathbb{E}_t W'_{t+1}(n_t) = 0,$$

and

$$W'_t(n_{t-1}) = \frac{\Lambda c}{\lambda_t} \left(\frac{1-\rho}{1-\gamma} \right) \left[1 - \gamma \frac{n_t - (1-\rho) n_{t-1}}{1-(1-\rho) n_{t-1}} \right].$$

Combining those two expressions, and using (13)–(14) and the fact that $\lambda_t^{-1} = f_t^{\frac{\gamma}{1-\gamma}} / m^{\frac{1}{1-\gamma}}$, gives the following forward recursion for the constrained-efficient job-finding rate:

$$f_t^{*\frac{\gamma}{1-\gamma}} = \frac{(1-\gamma) m^{\frac{1}{1-\gamma}}}{c} \left[z_t - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)} \right] + \beta (1-\rho) \mathbb{E}_t f_{t+1}^{*\frac{\gamma}{1-\gamma}} \left(1 - \gamma f_{t+1}^* \right), \tag{31}$$

from which we recover the constrained-efficient employment level n_t^* using (13)–(14).

It is instructive to compare the constrained-efficient employment dynamics, as determined by (14) and (31), with its dynamics in the decentralised equilibrium, as given by (12) and (14). Since the law of motion (14) is common to both dynamics, this amount to comparing (12) and (31). First, in the actual sticky-price dynamics the flow payoff to intermediate goods firms, and hence the job-finding rate, are affected by variations in intermediate goods prices φ_t, while they are not in the constrained-efficient outcome (where the corresponding price is equal to 1 at all times).

Second, even in the flex-price limit the decentralised equilibrium is generically not constrained-efficient in the absence of appropriate taxes and transfers. On the one hand, imperfect insurance tends to make the decentralised job-finding rate excessively low, since firm owners do not internalise the impact of their hiring intensity on workers' unemployment risk. Formally, this shows up in the fact that $[u(w^*) - u(\delta)]/u'(w^*) > 0$ in (31), which calls for a positive wage subsidy T in (12). On the other hand, search externalities cause intermediate goods firms to crowd out each other in the labour market, which tends to generate excessive hiring. There are two sides to this crowding out: first, a static one operating in the current period, which shows up in the fact that $1-\gamma < 1$ in (31); and second, an intertemporal one coming from the fact that current hiring persists over time (whenever $\rho < 1$) and hence crowds out hiring *in the next period* – which shows up in the term $1 - \gamma f_{t+1}^*$ in (31). Both types of crowding out call for setting $\tau^I > 0$ in (12).

3.3. Constrained-efficient steady state. The restriction that taxes and subsidies operate at constant rates (τ^W, τ^I) or level (T) implies that they cannot, in general, decentralise the constrained-efficient allocation in the presence of aggregate shocks.[9] However, one can at least set those instruments, and also trend inflation, in such a way that (π, τ^W, τ^I, T) decentralise the constrained-efficient allocation *in steady state*. First, as shown above the constrained-efficient allocation has $(p_t^*, \pi_t, \Delta_t) = (1, 0, 1)\ \forall t$, while from (6) we have $\varphi_t = (\theta - 1)/\theta(1-\tau^W)\ \forall t$ in any zero-inflation steady state. Then, comparing (12) and (31), we find that the steady state of the

[9]For example, equation (12) makes clear that a suitable time-varying wage subsidy T_t could undo the impact of inefficient cost-push shocks, which is not the case of a constant subsidy.

decentralised equilibrium is constrained-efficient provided that:

$$\pi = 0, \quad \tau^W = \frac{1}{\theta}, \quad T = \frac{u(w^*) - u(\delta)}{u'(w^*)} \quad \text{and } \tau^I = 1 - \frac{(1-\gamma)\left[1 - \beta(1-\rho) f^*\right]}{1 - \beta(1-\rho)(1-\gamma f^*)}, \tag{32}$$

where f^* solves:

$$f^{*\frac{\gamma}{1-\gamma}}\left[1 - \beta(1-\rho)(1-\gamma f^*)\right] = \frac{(1-\gamma)\, m^{\frac{1}{1-\gamma}}}{c}\left[1 - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)}\right]. \tag{33}$$

Intuitively, $\pi = 0$ eliminates relative price distortions; the production subsidy τ^W corrects for monopolistic competition in the wholesale sector and is greater when wholesale goods are less substitutable (i.e., when wholesale firms have more market power); the hiring subsidy T corrects for the lack of insurance and is greater when the utility cost of falling into unemployment ($u(w^*) - u(\delta)$) is high; and the corporate tax rate τ^I corrects for labour crowding-out and is greater when the elasticity of total matches with respect to vacancies $(1-\gamma)$ is low. In what follows I assume that (32) always holds.

4. Optimal policy with full worker reallocation

4.1. Constrained-efficient, natural, and actual employment levels. In this section, I characterise the optimal policy under a particular restriction: the job-destruction rate ρ is equal to 1 at every point in time, so that all employed workers are reallocated – either towards new firms or towards unemployment – in every period. Under this restriction employment no longer is a state variable, which makes it possible to compute the optimal value of the policy rate, and isolate the specific impact of imperfect insurance, in closed form. For expositional clarity but without loss of generality I also assume in this section that $m = 1$. With $\rho = m = 1$ we have, from equations (10), (14) and (32):

$$f_t = n_t = \lambda_t v_t = v_t^{1-\gamma} \text{ and } \tau^I = \gamma. \tag{34}$$

Equation (31) gives the following expression for the constrained-efficient level of employment:

$$n_t^* = \left[\frac{1-\gamma}{c}\left(z_t - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)}\right)\right]^{\frac{1-\gamma}{\gamma}}, \tag{35}$$

On the other hand, from equations (12) and (32) the *actual* level of employment is given by:

$$n_t = \left[\frac{1-\gamma}{c}\left(\varphi_t z_t - \zeta_t - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)}\right)\right]^{\frac{1-\gamma}{\gamma}}. \tag{36}$$

Finally, the *natural* level of employment – i.e., that which would prevail under flexible prices – is the same as n_t in (36) but with $\varphi_t = 1\ \forall t$.

In the remainder of this section I will use the linearised versions of equations (35) and (36). Using hatted variables to denote first-order level-deviations from the steady state, we have:

$$\hat{n}_t^* = \Phi \hat{z}_t \tag{37}$$

and

$$\hat{n}_t = \hat{n}_t^* + \underbrace{\Phi(\hat{\varphi}_t - \hat{\zeta}_t)}_{\text{“employment gap” } \tilde{n}_t}, \tag{38}$$

where

$$\Phi = \frac{(1-\gamma)^2}{\gamma c} n^{\frac{1-2\gamma}{1-\gamma}} > 0 \quad \text{and } n = \frac{f^*}{f^* + \rho(1-f^*)}.$$

Looking at (38) makes it clear that the central bank cannot replicate the constrained-efficient allocation after a cost-push shock, because it cannot simultaneously close the employment gap $\tilde{n}_t$ and stabilise intermediate goods prices $\hat{\varphi}_t$.

4.2. Linear-quadratic problem. One may now derive the linear-quadratic approximation to the optimal policy problem. Appendix A confirms that $\tilde{n}_t$ in (38) is the welfare-relevant employment gap, in the sense that, to second order, maximising W_t in (27) is equivalent to minimising:

$$L_t = \frac{1}{2}\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2), \tag{39}$$

where

$$\Omega = \frac{\theta n \Phi}{\kappa} > 0 \quad \text{and } \kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega} \geq 0. \tag{40}$$

The constraints faced by the central bank are the bond Euler equation for employed workers (equations (18)–(19)) and the optimality conditions for firms in the wholesale (equations (6)–(8)) and intermediate goods (equation (12)) sectors. Linearising (13) with $\rho = 1$ gives $\hat{s}_t = -\hat{f}_t = -\hat{n}_t$. Linearising the Euler condition for employed workers (equations (18)–(19)) around the zero-inflation steady state gives:

$$\Psi \mathbb{E}_t \hat{n}_{t+1} = \hat{\imath}_t - \mathbb{E}_t \pi_{t+1}, \tag{41}$$

where

$$\Psi = \left[1 - n + \frac{1}{u'(\delta)/u'(w^*) - 1}\right]^{-1} \geq 0.$$

Equation (41) determines the path of the policy rate that implements a given target path of inflation and employment, given workers' precautionary response to the employment risk that they are facing. The strength of this precautionary response is measured by the composite parameter Ψ, which in turn depends on workers' consumption loss upon unemployment. In the perfect-insurance limit ($\delta/w^* \to 1$) we have $\Psi \to 0$, so the precautionary motive vanishes and labour-market risk no longer affects the equilibrium real interest rate. As δ/w^* falls and Ψ increases, the precautionary motive gains strength has a larger impact on the equilibrium real interest rate $\hat{\imath}_t - \mathbb{E}_t \pi_{t+1}$; consequently, it has a larger impact on the policy rate $\hat{\imath}_t$ that the central bank must set in order to reach a given outcome.

Linearising equations (6)–(7)) gives the usual New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \hat{\varphi}_t. \tag{42}$$

One may now use (17) and (38) to express (41) and (42) in terms of the employment gap $\tilde{n}_t$ that enters (39). This gives:

$$\Psi \mathbb{E}_t \tilde{n}_{t+1} = \hat{\imath}_t - \mathbb{E}_t \pi_{t+1} - r_t^*, \tag{43}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa}{\Phi} \tilde{n}_t + \kappa \hat{\zeta}_t, \tag{44}$$

where r_t^* is the efficient interest rate (in terms of deviation from its steady state value $R-1$), i.e., the real interest rate which would equate actual employment $\hat{n}_t$ with its efficient level $\hat{n}_t^*$. From (37) and (41), r_t^* is given by:

$$r_t^* = \Psi \Phi \mu_z \hat{z}_t. \tag{45}$$

Equations (43)–(44), which summarise the optimal behaviour of households and firms, are the two constraints that the central bank faces when attempting to minimise (39). The efficient interest rate in (45) covaries with productivity here because of the precautionary motive: a persistent productivity slump worsens future labour market conditions and urges workers to save more (and all the more so that Ψ is large). To close the employment gap, the central bank should close the interest rate gap, i.e., the difference between the actual and efficient interest rates (the right hand-side of (43)). However, because the inefficiency of the employment level due to cost-push shocks persists even under flexible prices, the efficient interest rate differs from the *natural* rate, which (from (38) and (41)) is given by:

$$r_t^n = r_t^* - \Psi \Phi \mu_\zeta \hat{\zeta}_t. \tag{46}$$

Just like negative productivity shocks, persistent cost-push shocks reduce expected firms' hiring, which raises unemployment risk and workers' precautionary response; this effect of $\hat{\zeta}_t$ on r_t^n adds to the effect of $\hat{z}_t$ on r_t^n working through r_t^*.

4.3. Optimal Ramsey policy. The optimal Ramsey policy is the sequence of policy rates $\{i_{t+k}\}_{k=0}^{+\infty}$ that minimises L_t in (39) subject to (43)–(44). Formally, I first use (39) and (44) to solve for the optimal sequences $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ after one-off, positive innovations $\hat{z}_0$ and $\hat{\zeta}_0$ occurring at $t=0$, and then infer $\{i_t\}_{t=0}^{+\infty}$ from (43). Table 1 shows the optimal paths for inflation and the employment gap.

Table 1. Optimal employment and inflation gaps (see Appendix B for details).

	$\tilde{n}_t$	π_t
$t=0$	$-\Upsilon \theta n \hat{\zeta}_0$	$\Upsilon \hat{\zeta}_0 > 0$
$t=1$	$-\Upsilon \theta n (\alpha + \mu_\zeta) \hat{\zeta}_0$	$\Upsilon (\alpha + \mu_\zeta - 1) \hat{\zeta}_0$
$t \geq 2$	$-\Upsilon \theta n (\sum_{k=0}^{t} \alpha^k \mu_\zeta^{t-k}) \zeta_0$	$\Upsilon [\mu_\zeta^t - (1-\alpha) \sum_{k=0}^{t-1} \alpha^k \mu_\zeta^{t-k}] \zeta_0$

Note: $\Upsilon = \frac{\alpha \kappa}{1 - \alpha \beta \mu_\zeta} > 0$ and $\alpha = \frac{1+\beta+\kappa\theta n/\Phi}{2\beta}[1 - (1 - 4\beta(1+\beta+\frac{\kappa\theta n}{\Phi})^{-2})^{1/2}] \in (0,1)$.

Following a cost-push shock, the central bank promises, and then implements, a durable recession so as to mitigate the impact of the shock on current inflation. The shapes of the optimal paths for inflation and the employment gap mirror those obtained in the baseline RANK model (see Woodford, 2003; Gali, 2008); for example, when $\alpha + \mu_\zeta > 1$ the responses of inflation and the employment gap to the shock are both U-shaped, hence the response of the output gap also is (by equations (8) and (17), to first order the output and employment gap are the same). Productivity shocks do not generate a policy tradeoff, hence neither inflation nor the employment gap respond to $\hat{z}_0$.

From (43) and (45), the path of the policy rate that implements a given (perfect-foresight) sequence $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ is given by:

$$\hat{\imath}_t = \Psi\Phi\mu_z\hat{z}_t + \Psi\tilde{n}_{t+1} + \pi_{t+1}. \tag{47}$$

Using the values of $\tilde{n}_{t+1}$ and π_{t+1} in Table 1 gives the optimal sequence of policy rates:

$$\text{For } t=0: \quad \hat{\imath}_0(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\Upsilon(\alpha + \mu_\zeta - 1)\hat{\zeta}_0}_{\text{perfect-insurance response}} \underbrace{- \Psi\Upsilon\theta n(\alpha + \mu_\zeta)\hat{\zeta}_0 \; + \; \Psi\Phi\mu_z\hat{z}_0,}_{\text{imperfect-insurance correction}}$$

and, for $t \geq 1$:

$$i_t(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\Upsilon[\mu_\zeta^t - (1-\alpha)\textstyle\sum_{k=0}^{t}\alpha^k\mu_\zeta^{t-k}]\hat{\zeta}_0}_{\text{perfect-insurance response}} \underbrace{- \Psi\Upsilon\theta n[\textstyle\sum_{k=0}^{t}\alpha^k\mu_\zeta^{t-k}]\hat{\zeta}_0 \; + \; \Psi\Phi\mu_z^{t+1}\hat{z}_0.}_{\text{imperfect-insurance correction}}$$

The optimal policy responses to productivity and cost-push shocks can be explained as follows. First, the policy rate i_t should perfectly track movements in the efficient interest rate r_t^* that are driven by productivity shocks; for example, a (persistent) productivity-driven contraction ($\hat{z}_0 < 0$) should lead to a persistent cut in the nominal interest rate – and hence an equal fall in the *real* interest rate, since inflation remains at zero all along the optimal path. This response is due to the fact that, under imperfect insurance, a persistent productivity-driven contraction raises unemployment risk and hence strengthens the precautionary motive for saving. In the absence of a policy response employment and inflation would deviate from target *downwards*, while a suitably sized cut in the policy rate can simultaneously close the employment and inflation gaps. Crucially, the size of the cut depends on the extent of imperfect insurance (as encoded in Ψ), because the latter determines the strength of the precautionary motive and hence the size of the fall in aggregate demand that would prevail without the offsetting action of the central bank. This policy response to productivity shocks is in contrast with that obtained under standard calibrations of the RANK model (Clarida et al., 1999; Woodford, 2003). In that model a persistent productivity-driven contraction forecasts high future income growth (on the way to the recovery), against which households wish to borrow; this causes a rise in the efficient interest rate that is adequately tracked by an *increase* in the policy rate.

Second, the strength of the precautionary motive in general affects both the size and sign of the optimal interest-rate response to cost-push shocks. Just as in the RANK model, the optimal policy response is such that both inflation and the employment gap persistently deviate from target (inflation upwards and the employment gap downwards). However, the fall in employment strengthens the precautionary motive and generates deflationary pressures in the current period; this mutes down the optimal response of the policy rate and even reverts it if the precautionary motive is sufficiently strong (i.e., if insurance is sufficiently poor).

Third, the optimal policy brings about a path of $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ that is independent of the degree of insurance – see Table 1, wherein none of the coefficients depend on Ψ – and is thus the same as in the perfect insurance limit (i.e., when $\delta/w^* \to 1$). This means that, under the parameter restriction of this section, implementation of the optimal policy *fully undoes the effect of imperfect insurance on the propagation of aggregate shocks.*

4.4. Optimal discretionary policy. I conclude this section by computing the optimal response of the policy rate under an additional source of inefficiency, namely, the inability of the central bank to commit to future policies. This serves to show that uninsured unemployment risk has the same effect as under commitment of muting (and possibly reverting) the response of the policy rate relative to the perfect-insurance case. Under discretion the central bank chooses period by period the value of $\hat{\imath}_t$ that minimises $\tilde{n}_t^2 + \Omega \pi_t^2$ subject to (43)–(44). I first solve for $(\tilde{n}_t, \pi_t)$ by minimising this loss subject to (44) and then infer i_t from (43). The first step gives:

$$\tilde{n}_t + (\theta n)\, \pi_t = 0. \tag{48}$$

The optimal sequence $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ solves (44) and (48) and is given by:

$$\tilde{n}_t = -\left(\frac{1-\beta\mu_\zeta}{\theta n \kappa} + \frac{1}{\Phi}\right)^{-1} \hat{\zeta}_t \quad \text{and} \quad \pi_t = \left(\frac{1-\beta\mu_\zeta}{\kappa} + \frac{\theta n}{\Phi}\right)^{-1} \hat{\zeta}_t. \tag{49}$$

In as much as the cost-push shock raises inflation, a central bank operating under discretion mitigates the impact of the shock by lowering the current employment gap. Using (47) and (49) gives the response of the policy rate to one-off productivity and cost-push shocks:

$$\hat{\imath}_t(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\left(\frac{\kappa \Phi \mu_\zeta^{t+1}}{\left(1-\beta\mu_\zeta\right)\Phi + \kappa\theta n}\right)\hat{\zeta}_0}_{\text{perfect-insurance response}} \underbrace{- \Psi\left(\frac{\kappa\Phi\theta n \mu_\zeta^{t+1}}{\left(1-\beta\mu_\zeta\right)\Phi + \theta n \kappa}\right)\hat{\zeta}_0 + \Psi\Phi\mu_z^{t+1} z_0.}_{\text{imperfect-insurance correction}}$$

The response to the productivity shock is the same as under commitment since this shock generates no policy tradeoff here. The response to a cost-push shock differs from that under commitment, but imperfect insurance has the same effect as of muting the perfect-insurance response. In the perfect-insurance limit we again recover the standard result that a cost-push shock should be fought by raising the policy rate. As unemployment insurance is reduced (i.e., Ψ rises) this response is dampened or even reverted by households' own precautionary reaction to the shock.

5. Optimal policy with partial worker reallocation

Having analytically identified how the precautionary motive affects optimal policy in the special case of full worker reallocation, I now study numerically the optimal interest-rate response to aggregate shocks in the general case of partial worker reallocation (i.e., $\rho \leq 1$). This implies, first, that in the decentralised equilibrium firms make hiring decisions taking into account the future rents they will earn on currently hired employees (and not only the current rent as in (36)); and second, that the constrained-efficient level of employment incorporates the impact of current aggregate hiring on future aggregate hiring costs (and not only its effect on current aggregate hiring costs as in (35)). Formally, I solve the Ramsey problem of finding the sequence $\{i_t\}_0^\infty$ that maximises W_t subject to equations (6)–(8), (12)–(14), (17), (18)–(19), (30) and (32), after one-off productivity and cost-push shocks occurring at $t = 0$.

5.1. Parameterisation. I interpret the period as a quarter and set $u(c) = \ln c$. The other parameters are calibrated to match a certain number of standard targets – see Table 2 for a summary. Following McKay et al. (2016), the subjective discount factor β is set such that the annualised real interest rate $(1+i)^4 - 1 \simeq 4i$ be equal to 2%. The cross-partial elasticity of substitution θ is set to 6, which generates a markup rate of 20%. The fraction of unchanged wholesale goods prices ω is set to 0.75, so that the mean price duration is a year. Regarding labour market variables, I have four parameters (c, w^*, m and ρ) for four targets (f, s, λ and c/w^*). Quarterly series for f_t and s_t where computed in Challe et al. (2016) by time-aggregating monthly series constructed as in Shimer (2005); their averages are very close to 80% and 5%, respectively. The targets for λ and c/w^* are, respectively, 70% (see, e.g., Den Haan et al., 2000; Walsh, 2005; Monacelli et al. 2015) and 4.5% (Hagedorn and Manovskii, 2008). Finally, γ is set to 2/3, very close to the values estimated by Shimer (2005) and Monacelli et al. (2015).

A key parameter in the model is workers' home production δ, which determines the extent of consumption insurance and hence the strength of the precautionary motive. One possibility would be to parameterise δ to match the unemployment insurance replacement ratio. However, this would most likely underestimate the amount of consumption insurance that households effectively enjoy, by ignoring (i) self-insurance via liquid assets, (ii) other forms of direct insurance, and (iii) the subjective valuation of not working. Following this concern I broadly interpret δ/w^* as the opportunity cost of employment, as estimated by Chodorow-Reich and Karabarbounis (2016); their estimates range from 47% to 96%, and I choose the relatively conservative value of 90%.

In what follows I compare the optimal interest-rate response to aggregate shocks (of normalised size 1 and auto-correlation 0.95) in two economies: the baseline imperfect-insurance model, and a counterfactual perfect-insurance model. The perfect-insurance model has $\delta/w^* \to 1$, while adjusting the other parameters to keep matching all targets other than δ/w^*.[10]

[10] The results are almost unchanged if the parameters other than δ/w^* are not adjusted when computing the perfect-insurance responses.

Table 2. Calibration.

Parameters			*Targets*		
Symb.	Description	Value	Symb.	Description	Value
β	Discount factor	0.989	$4 \times i$	Annual interest rate	2%
θ	Elasticity of subst.	6.000	$1/(\theta - 1)$	Markup rate	20%
ω	Fraction of unchanged price	0.750	$1/(1-\omega)$	Mean price duration	1 year
c	Vacancy cost	0.044	c/w^*	Labour cost of vacancy	4.5%
w^*	Real wage	0.979	f	Job-finding rate	80%
m	matching efficiency	0.765	λ	Vacancy-filling rate	70%
ρ	Job-destruction rate	0.250	s	Job-loss rate	5%
δ	Home production	0.882	δ/w^*	Opportunity cost of empl.	90%

5.2. Baseline scenario. Figures 1 and 2 show the optimal responses of the policy rate, and implied employment and inflation rates, after contractionary cost-push and productivity shocks. One may first check that those responses in the perfect-insurance economy are qualitatively the same as in the RANK model: in both cases the central bank must persistently raise the policy rate in order to dampen the inflationary pressures generated by the shock. Those responses are, however very different under imperfect insurance: in both cases the central bank must persistently lower, not raise, the policy rate. This is, as explained in Section 4 above, because both shocks have negative aggregate demand effects – due to workers' precautionary response to the rise in unemployment risk – that the central bank must actively offset.[11]

5.3. Precautionary motive versus intertemporal substitution in consumption. In the baseline scenario the real wage is equal to its constrained-efficient value, which is constant. This implies that aggregate shocks do not generate any intertemporal substitution in consumption on the part of employed workers. This is in contrast with the workings of the RANK model, wherein the optimal interest-rate response to aggregate shocks is entirely governed by intertemporal substitution in consumption. One would thus like to know how plausible fluctuations in employed workers' income and associated intertemporal substitution in consumption would affect the baseline results. To answer this question I follow Blanchard and Gali (2010) in assuming the following cyclical behaviour for the real wage:

$$w_t = w^* z_t^{\chi}, \ \chi \geq 0. \tag{50}$$

This specification nests the baseline scenario ($\chi = 0$) while making space for (inefficient) wage

[11] Note that, unlike in Section 4, under the optimal policy inflation does not stay at zero at all times after a productivity shock. This is because when $\rho < 1$ the taxes and subsidy (τ^I, T) no longer decentralise the constrained-efficient outcome in the absence of cost-push shocks – they only decentralise it in the absence of *both* aggregate shocks. Consequently, monetary policy cannot exactly replicate the constrained-efficient outcome after a productivity shock.

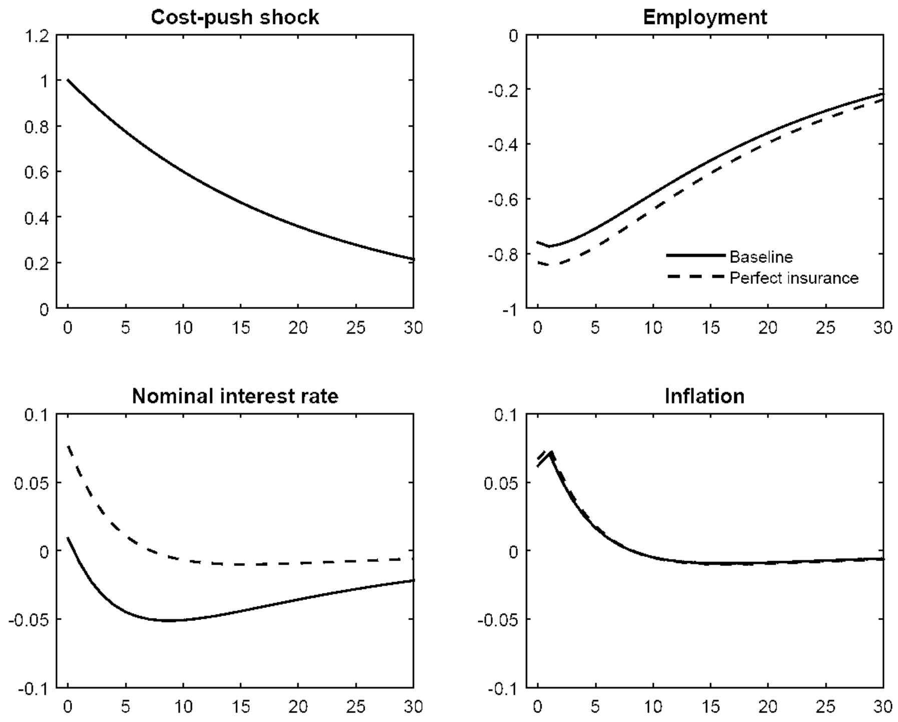

Figure 1: Responses to a cost-push shock.

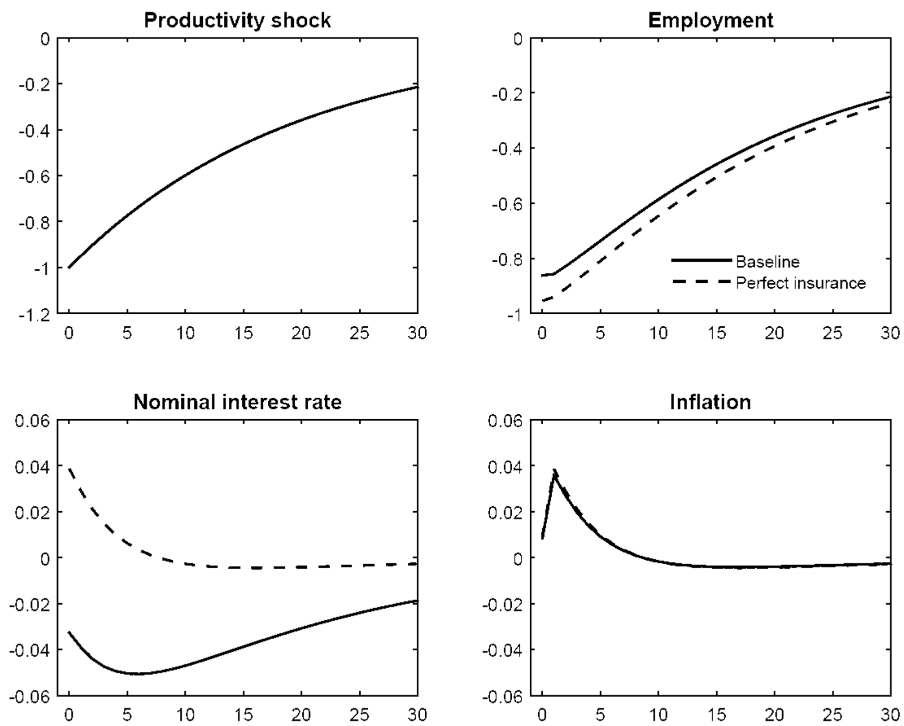

Figure 2: Responses to a productivity shock.

fluctuations.[12]

To examine analytically the implications of wage cyclicality, consider again, momentarily, the case where $\rho = 1$. The Euler condition (41) then becomes:

$$\Psi \mathbb{E}_t \hat{n}_{t+1} - \Xi \hat{w}_t = \hat{\imath}_t - \mathbb{E}_t \pi_{t+1}, \tag{51}$$

where

$$\Xi = -\frac{u''(w^*)}{u'(w^*)}\left[1 - \frac{\mu_z}{1 + (1/n - 1)\left(u'(\delta)/u'(w^*)\right)}\right] > 0 \text{ and } \hat{w}_t = \chi \hat{z}_t.$$

Equation (51) shows that, following a transitory, productivity-driven contraction, two effects compete in determining employed workers' desire to save and implied optimal interest-rate response. On the one hand, unemployment risk increases and the precautionary motive (scaled by the composite parameter Ψ) urges them to save. On the other hand, the real wage transitorily falls and intertemporal substitution in consumption (scaled by the composite parameter Ξ) urges them to borrow. The optimal response of the policy rate must exactly offset the net effect on desired savings so that the goods markets clear without inflationary or deflationary pressures. From (50)–(51) and Table 1, the optimal path of the policy rate after a productivity shock becomes:

$$\hat{\imath}_t(\hat{z}_0) = (\underset{\substack{\uparrow \\ \text{precautionary motive}}}{\Psi\Phi\mu_z} - \underset{\substack{\uparrow \\ \text{intertemporal substitution}}}{\Xi\chi}) \times \mu_z^t \hat{z}_0, \tag{52}$$

which confirms that it depends on the relative strengths of the precautionary versus intertemporal substitution effects.

To quantitatively evaluate the optimal response of the policy rate in the presence of intertemporal substitution in consumption, let us turn back to the general model ($\rho < 1$), calibrated as in the previous section but with the wage equation (50) and $\chi = 1/3$ (See Blanchard and Gali, 2010; Den Haan et al., 2017). The results are shown in Figure 3. As expected, intertemporal substitution in consumption mutes down the policy response to the recession, but the optimal policy still calls for a persistent interest rate cut. How robust is this result? As equation (52) makes clear in the case where $\rho = 1$, holding preferences and technologies the same, the strength of the precautionary motive depends on the opportunity cost of employment δ/w^* (through its effect on Ψ) while the strength of intertemporal substitution in consumption depends on the cyclicality of the wage (i.e., χ). On the one hand, as χ rises, so does the path of the policy rate in the third panel of Figure 3. On the other hand, $\delta/w^* = 0.9$ is conservative, and taking it down also takes down the path of the policy rate. Overall, plausible variations of δ/w^* and χ show that in most cases the policy rate should be cut, or at least not raised, during a recession.

[12] Under our assumptions a benevolent planner who could insulate the effect of wage fluctuations on workers' income through an appropriate time-varying subsidy would do so. Bilateral (e.g., Nash) bargaining between the parties would also generically result in a constrained-inefficient wage, just as a productivity-dependent wage.

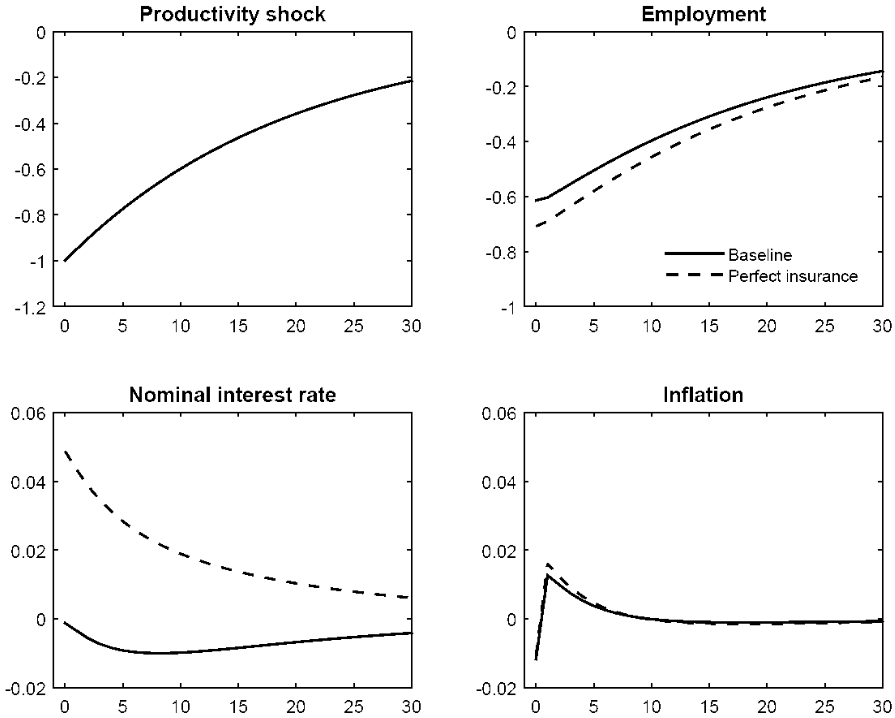

Figure 3: Precautionary motive vs. intertemporal substitution.

6. Conclusion

In this paper, I have computed the optimal interest-rate response to aggregate (productivity and cost-push) shocks in a model economy wherein workers have a precautionary motive against uninsured, endogenous unemployment risk. Using a calibrated version of the model whose perfect-insurance limit replicates the policy prescriptions of the Representative-Agent New Keynesian model, I find those prescriptions to be overturned under imperfect insurance: the policy rate should be lowered after contractionary productivity or cost-push shocks in order to neutralise their inefficient impact on aggregate demand. This policy successfully breaks the feedback loop between unemployment risk and aggregate demand and almost aligns the economy's dynamics with that under perfect insurance.

Of course, this form of policy accommodation requires that the policy rate be unconstrained, i.e., that it never hit a binding zero lower bound – as is formally consistent with our assumption of small aggregate shocks and a model parameterisation implying a positive steady-state interest rate. Extrapolating on this local analysis, the model suggests that under imperfect insurance contractionary "supply" shocks may also put the economy at risk of entering a liquidity trap – inasmuch as the optimal unconstrained policy calls for an interest rate cut – and not only the contractionary "demand" shocks that have usually been considered in the literature.

Appendix to Section 4

A. Derivation of the quadratic loss function With $\rho = m = 1$ and $w_t = w^*$ we have:

$$U_t = u(\delta) + n_t \left[u(w^*) - u(\delta)\right] + \Lambda[\varpi + n_t (z_t/\Delta_t - w^*) - cn_t^{\frac{1}{1-\gamma}}].$$

We will use the facts that $\hat{n}_t^* = \Phi z_t$ and that

$$\frac{\partial U_t}{\partial n_t} = u(w^*) - u(\delta) + \Lambda \left[z_t/\Delta_t - w^* - \frac{c}{1-\gamma} n_t^{\frac{\gamma}{1-\gamma}} \right] = 0.$$

We then get the following quadratic flow utility:

$$\begin{aligned}
U_t &= \underbrace{\{u(w^*) - u(\delta) + \Lambda[1 - w^* - cn^{\frac{\gamma}{1-\gamma}}/(1-\gamma)]\}}_{=0} \hat{n}_t - \frac{\Lambda \gamma c n^{\frac{2\gamma-1}{1-\gamma}}}{2(1-\gamma)^2} \hat{n}_t^2 \\
&\quad + \Lambda z_t \hat{n}_t - \Lambda n (\Delta_t - 1) + \text{t.i.p.} + \mathcal{O}(||\zeta||^3) \\
&\simeq -\frac{\Lambda}{2\Phi} \hat{n}_t^2 + \frac{\Lambda}{\Phi} \hat{n}_t^* \hat{n}_t - \Lambda n (\Delta_t - 1) + \text{t.i.p.} \\
&= -\frac{\Lambda}{2\Phi} \tilde{n}_t^2 - \Lambda n (\Delta_t - 1) + \text{t.i.p., with } \tilde{n}_t = \hat{n}_t - \hat{n}_t^*.
\end{aligned}$$

We now use the facts that (see Woodford, 2003, chapter 6):

$$\Delta_t \simeq 1 + \frac{\theta}{2} \operatorname{Var}(p_t(i)) \text{ and } \sum_{t=0}^{\infty} \beta^t \operatorname{Var}(p_t(i)) = \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \pi_t^2, \text{ with } \kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}.$$

This allows us to write the SWF as follows:

$$\begin{aligned}
W_t &= \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{t+k} \simeq \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[-\frac{\Lambda}{2\Phi} \tilde{n}_{t+k}^2 - \frac{\Lambda n \theta}{2} \operatorname{Var}(p_{t+k}(i)) \right] + \text{t.i.p.} \\
&= -\frac{\Lambda}{2\Phi} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \tilde{n}_{t+k}^2 - \frac{\Lambda n \theta}{2} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \operatorname{Var}(p_{t+k}(i)) + \text{t.i.p.} \\
&= -\frac{\Lambda}{2\Phi} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left(\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2 \right) + \text{t.i.p., with } \Omega = \frac{\theta \Phi n}{\kappa}.
\end{aligned}$$

Maximising W_t is thus equivalent to minimising $L_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2)/2$.

B. Optimal Ramsey policy This adapts Gali (2008, Section 5.1.2) to the present model. The Lagrangian associated with the central bank's problem is:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[\frac{\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2}{2} + \Gamma_{t+k} \left(\pi_{t+k} - \beta \mathbb{E}_t \pi_{t+1+k} - \frac{\kappa}{\Phi} \tilde{n}_{t+k} \right) \right].$$

The first-order conditions with respect to the $\tilde{n}_{t+k}$s and π_{t+k}s are:

$$\begin{aligned} \mathbb{E}_t\tilde{n}_{t+k} - (\kappa/\Phi)\,\mathbb{E}_t\Gamma_{t+k} &= 0 \text{ for all } k \geq 0, \\ \Omega\pi_t + \Gamma_t &= 0, \quad \text{and} \\ -\mathbb{E}_t\Gamma_{t+k} + \Omega\mathbb{E}_t\pi_{t+1+k} + \mathbb{E}_t\Gamma_{t+k+1} &= 0 \text{ for all } k \geq 1. \end{aligned}$$

Using those conditions, dropping the $\mathbb{E}_t$-operator – since we are looking at the response to a one-time shock – and using (40), we find that $\{\tilde{n}_{t+k}, \pi_{t+k}\}_{k=0}^{\infty}$ must satisfy:

$$\text{for } k = 0 : \tilde{n}_t + (\theta n)\,\pi_t = 0; \tag{53}$$

$$\text{for } k \geq 1 : \tilde{n}_{t+k} - \tilde{n}_{t+k-1} + (\theta n)\,\pi_{t+k} = 0. \tag{54}$$

Equations (53) and (54) can be more compactly written as, for all $k \geq 0$:

$$\tilde{n}_{t+k} = -\,(\theta n)\,\hat{p}_{t+k}, \text{ with } \hat{p}_{t+k} \equiv p_{t+k} - p_{t-1}, \tag{55}$$

and where p_{t-1} was the price level before the shock hit. Substituting this expression into (44) and rearranging, we obtain the following difference equation for $\hat{p}_t$:

$$(1 + \beta + \kappa\theta n/\Phi)\,\hat{p}_{t+k} = \hat{p}_{t+k-1} + \beta\hat{p}_{t+k+1} + \kappa\zeta_{t+k}.$$

The stationary solution to this equation is $\hat{p}_{t+k} = \alpha\hat{p}_{t+k-1} + \Upsilon\zeta_{t+k}$, with

$$\Upsilon = \frac{\alpha\kappa}{1 - \alpha\beta\mu_\xi} \quad \text{and} \quad \alpha = \frac{1 + \beta + \frac{\kappa\theta n}{\Phi}}{2\beta}\left[1 - \sqrt{1 - 4\beta\left(1 + \beta + \frac{\kappa\theta n}{\Phi}\right)^{-2}}\,\right] \in (0,1)\,.$$

This solution can be used to recover $\{\tilde{n}_{t+k}, \pi_{t+k}\}_{k=0}^{\infty}$ using (53)–(55). For $k = 0$ we get:

$$\tilde{n}_t = -\,(\theta n)\,\hat{p}_t = -\Upsilon\theta n\zeta_t,$$

where we use of the fact that $\Omega = \theta\Phi n/\kappa$ (see Appendix A). For $k \geq 1$ we have:

$$\tilde{n}_{t+k} = \alpha\tilde{n}_{t+k-1} + \Upsilon\theta n\zeta_{t+k} = -\Upsilon\theta n(\textstyle\sum_{\iota=0}^{k}\alpha^\iota\mu_\zeta^{k-\iota})\zeta_t.$$

Then, we recover the path of inflation using (53)–(54). We obtain:

$$\begin{aligned} \text{for } k &= 0 : \pi_t = -\frac{\tilde{n}_t}{\theta n} = \Upsilon\zeta_t; \\ \text{for } k &= 1 : \pi_{t+1} = \frac{\tilde{n}_t - \tilde{n}_{t+1}}{\theta n} = \Upsilon\left(\alpha + \mu_\zeta - 1\right)\zeta_t \\ \text{for } k &\geq 2 : \pi_{t+k} = \frac{\tilde{n}_{t+k-1} - \tilde{n}_{t+k}}{\theta n} = \Upsilon[\mu_\zeta^k - (1-\alpha)\textstyle\sum_{\iota=0}^{k-1}\alpha^\iota\mu_\zeta^{k-\iota}]\zeta_t \end{aligned}$$

Table 1 summarises the effect of a shock occurring at $t = 0$.

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