Financial Choice and Financial Information[*]

Péter Kondor
London School of Economics

Botond Kőszegi
Central European University

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Abstract

We analyze the implications of increases in the selection of, and information about, derivative financial products in a model in which investors neglect informational differences between themselves and issuers. We assume that investors receive information that is noisy and inferior to issuers' information, and that issuers can select the set of underlying assets when designing a security. In contrast to the received wisdom that diversification is helpful, we show that when custom-designed diversification across a large number of underlying assets is possible, then expected utility approaches negative infinity. Even beyond this limiting case, any expansion in choice induced by either an increase in the maximum number of assets underlying a security, or an increase in the number of assets from which the underlying can be selected, Pareto-lowers welfare. Furthermore, under reasonable conditions an improvement in investor information Pareto-lowers welfare by giving investors the false impression that they can spot good deals. An increase in competition between issuers does not increase welfare, and even increases investors' incentive to acquire welfare-reducing information. Restricting the set of underlying assets the issuer can use—a kind of standardization—raises welfare, and once this policy is adopted, increasing investor information becomes beneficial.

1 Introduction

As a result of intense financial innovation, there is an abundance of derivative financial products available to retail investors and small institutional investors nowadays. Retail investors are able to buy a phenomenal variety of structured retail products and exotic exchange-traded funds (ETF's), and institutional investors have access to custom-designed CDO's and other complex investments.[1] At the same time, the amount of financial information available to small investors has increased drastically, with consumers now being able to base their trading strategies on a deluge of information about investments. The concern arises that if investors are not fully sophisticated, then these developments may not always have been to investors' or society's benefit. Indeed, several researchers have argued that the new derivatives developed for retail investors cannot be fully reconciled with the risk-sharing view of financial innovation, and that deception of consumers or welfare-decreasing speculation must also be taking place (Bergstresser, 2008; Henderson and Pearson, 2011; Simsek, 2013; Celerier and Vallée, forthcoming). Yet there is no systematic analysis of the effect of choice and especially investor information on financial markets when investors are not fully sophisticated.

In this paper, we analyze the implications of increases in the selection of, and information about, derivative financial products in a model in which investors neglect informational differences between themselves and issuers. Our approach is motivated by evidence and arguments that—analogously to the winner's curse in auctions—individuals in general and small investors in particular underestimate the information content of others' actions (Eyster and Rabin, 2005; Eyster et al., 2014; Jin et al., 2015; Enke, 2016). We assume that investors receive information that is noisy and inferior to issuers' information, and that issuers can select the set of underlying assets for a security. In contrast to the received wisdom that diversification is helpful, we show that when arbitrary diversification across a large number of assets is possible, then

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[1] Structured retail products offer directional bets on underlying stocks, exchange rates or indices to retail investors. Celerier and Vallée (forthcoming) estimate that between 2002 and 2010, structured retail products were issued in the amount of €1.5 trillion in Europe only, and they represent a significant fraction of financial wealth (e.g., 8.5% in Belgium). During this period, the average ex-post return before fees was 2%, lower than the corresponding risk-free rate, and an increasing fraction (23% in 2009) exposed investors to complete loss. In Europe, investors purchase these securities from their retail banks. US investors can purchase exotic ETF's directly on the financial market to take leveraged positions or other directional bets on a wide range of underlying assets. Exotic ETF's often provide investors with similar payoff structures as structured securities. Custom-tailored CDOs are fixed-income derivatives with a payment connected to the fraction of default in a specified portfolio of bonds.

expected utility approaches negative infinity. Even beyond this limiting case, any expansion in choice induced by either an increase in the maximum number of assets underlying a security, or an increase in the number of assets from which the underlying can be selected, Pareto-lowers welfare. Furthermore, under reasonable conditions an improvement in investor information Pareto-lowers welfare by making investors more falsely confident in trading. An increase in competition between issuers does not increase welfare, and even increases investors' incentive to acquire welfare-reducing information. Restricting the set of underlying assets the issuer can use—a kind of standardization—raises welfare, and once this policy is adopted, increasing investor information becomes beneficial.

Section 2 presents our model, which has two periods. Investors with log utility in period 2 can only save for that period through risk-neutral competitive issuers. There are I underlying assets whose binary payoffs are resolved in period 2, and issuers and investors start off with the same symmetric prior regarding these payoffs. In period 1, issuers observe private information regarding the payoff of all the assets, and investors observe a noisy version of issuers' information about the payoff of a subset of the assets. An issuer can pick K assets, and offer a derivative security—defined as a map from the payoff of the underlying assets to the payoff of the security—to investors. The issuer's informational advantage could derive, for instance, from understanding the information content of prices in the professional market for derivatives it also uses to hedge the security it offers. As a benchmark, we show that with rational investors the issuer always offers the first-best, constant-payoff security, which insures the investor and eliminates adverse selection. But following Eyster and Rabin (2005), we posit that investors are fully cursed: in considering their investment options, they neglect that an issuer's offer depends on its private information.

We identify basic properties of equilibrium securities in Section 3, first taking as given the underlying assets an issuer chooses. Reminiscent of results in the literature on contracting with heterogeneous priors, the issuer's optimal security offers overly high consumption in states whose probability the investor overestimates and overly low consumption in states whose probability the investor underestimates, thereby inducing suboptimal risk-taking. Conveniently, total investor and social welfare turn out to be a decreasing function of the Kullback-Leibler (KL) divergence of the investor's beliefs from the issuer's beliefs, and an investor's perceived

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welfare turns out to be an increasing function of the KL divergence of the issuer's beliefs from the investor's beliefs. Using this connection, we show that an issuer chooses the underlying K assets to *minimize* investor utility. In particular, the underlying assets are chosen to maximize the distance between the parties' beliefs, and while the investors see this as an opportunity to trade mispriced assets, it in fact leads them to take the most idiosyncratic risk. A crucial aspect of this prediction is that investors ignore how the underlying assets are selected. Although such ignorance is a—not previously noted or studied—logical implication of cursedness, we also mention some direct evidence that individuals neglect related selection effects.

In Section 4, we analyze the welfare implications of financial choice. We first consider what can be thought of as a fully developed financial market—roughly descriptive of custom-designed CDO's and perhaps descriptive of the future of retail investing—where arbitrary diversification across a large number of assets is possible ($K = I$ and I is large). While the possibility of diversification is generally considered beneficial, here it is extremely harmful: as $I \to \infty$, welfare approaches minus infinity. The issuer creates an index of underlying assets—but it is a custom-designed rather than a standard index, tailored to solidify the issuer's informational advantage. Nevertheless, because an investor does not appreciate this design process and the index is composed of many assets, she feels diversified and hence drastically underestimates the probability of low-payoff tail events. As a result, she receives very low welfare.

In addition, we show that the path to the fully developed financial market is monotonic in the sense that any expansion in choice—any increase in K or I—hurts all investors and is hence Pareto-harmful. An expansion in how complex financial derivatives can be made (an increase in K), as well as an expansion in the set of underlying assets that can be used (an increase in I), increases an issuer's scope to minimize investor welfare through its choice of underlying. The latter result has an immediate implication for policy: restricting the set of underlying assets, which we think of as a kind of standardization, raises welfare.

In Section 5, we turn to our main interest in the paper, the effect of providing more information to retail investors. Under the reasonable assumption that there are a large number of underlying assets to choose from but a security can depend only on a few (I is large and $K \ll I$), an improvement in an investor's (inferior) information lowers welfare. Intuitively, under these conditions, the issuer tends to choose underlying assets for which the investor's

information is misleading, pointing in the other direction than the issuer's information. If an investor has better information, she is more confident in her misleading information, leading her to take on more idiosyncratic risk. This generates a kind of discontinuity: if the investor receives exactly the same information as the issuer, then social welfare is maximized and hence discretely higher than without information; but if the investor receives noisy information very close but inferior to the issuer's information, welfare may be discretely lower than without information.

The above insight has several potentially important implications. A positive prediction is that an issuer prefers to write its security on underlying assets in the public eye. A central welfare implication is that the increased availability of financial information may have hurt small investors. In addition, investors who are savvy enough to become more informed (but not less cursed) are worse off than more naive investors. And since giving information to investors that the issuer does not have seems impossible in practice, information-based policies to improve investor and social welfare are likely to backfire by themselves. We also establish, however, that once standardization is adopted, giving investors more information raises welfare. Intuitively, information moves an investor's beliefs closer to the issuers' on average, and without an issuer being able to choose underlying assets on which the investor received misleading information, this increases welfare. Hence, standardization and information provision are complementary policies.

We also consider the effects of competition on our results. A monopolist issuer sells the same securities and hence generates the same total welfare as a competitive issuer, although (due to higher prices) consumers of course receive less of the total surplus. More interestingly, we show that if information is costly and investors make an endogenous decision whether to acquire it, then competition reduces total welfare. Given that competition leaves all of the perceived surplus from the transaction with the investor, it increases the perceived gain from information, and hence increases the incentive for acquiring welfare-reducing information.

Because our basic model abstracts from the classical risk-sharing motive for financial markets in general and derivatives trading in particular, in Section 6 we embed our cursed investors in a standard asset-pricing framework where risk-averse professionals exposed to aggregate risk trade securities with households through competitive intermediaries. Although investors could

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buy securities that expose them to the aggregate risk factor, if they receive noisy information about idiosyncratic assets, they often do not. Since they believe they can spot some very good deals among the many idiosyncratic assets, they trade in idiosyncratic risk, generating the same qualitative results as in our basic model.

In Section 7, we relate our paper to the literatures on financial innovation and on markets with naive consumers. While previous research has found that financial innovation can be harmful when investors have different priors (Simsek, 2013) and that products designed for naive consumers can generate very low welfare (e.g., DellaVigna and Malmendier, 2004; Heidhues and Kőszegi, 2010), no paper has systematically analyzed the effects of choice and especially information in the retail financial market, and how the selection of underlying assets from the vast number of possibilities affects outcomes.

In Section 8, we conclude the paper by pointing to some considerations in security design that are missing from our model. While in our setting the securities investors buy depend only on the parties' information (and preferences), in reality many other considerations, such as the practical ease of hedging and the intuitive plausibility of the trading strategy, seem to play a role.

2 Basic Model

2.1 Setup

Investors and competitive issuers interact over two periods, $t = 1, 2$. In this section, we assume that issuers are risk-neutral and sell securities directly; in Section 6, we identify conditions under which the same outcomes obtain when investors interact with risk-averse professionals through competitive intermediaries. There are I underlying assets such as individual stocks, indices, or exchange rates. Asset i pays $s^i \in \{0, 1\}$ realized in period 2, and the assets are independent. The parties' priors regarding the states are the same, and, denoting probabilities by f, they are $f(s^i = 1) = 1/2$. Our assumption that the assets are independent and symmetric serve to make our points in a clean form, but it will be clear that the mechanisms we identify hold more generally.

In period 1, the issuers receive private signals $y^i \in \{0, 1\}$ about each asset i, where $\Pr(y^i =$

$x|s^i = x) = r > 1/2$. Because we are interested in how issuers use their superior information relative to investors rather than in how issuers with different information trade, we assume that issuers receive the same signals. For a fraction α of the assets, investors also receive a signal $z^i \in \{0,1\}$, where $z^i = y^i$ with probability q, and z^i is pure noise with probability $1-q$. This assumption captures in a simple way the mix of second-hand and unreliable information that is available to small investors. The signals y^i and z^i, including whether z^i is noise and if so what it is, are independent across assets. For most of the paper, we do not impose any assumption on whether the noise in z^i is correlated across investors. For simpler exposition, we assume that issuers observe each investor's z^i and explain in Section 5 that in the empirically most relevant case, relaxing this assumption leaves the results qualitatively unchanged. We denote issuers' and investors' vector of signals by y and z, respectively, and the vector of asset payoffs by s.

While an issuer's informational advantage could derive from multiple sources, one possibility is that—unlike the investor—it observes and takes into account the information content of prices in the professional market for derivatives. This is especially likely to be the case if the issuer also uses the professional market to hedge the security it sells. In fact, a model equivalent to ours arises if issuers have no (direct) informational advantage, but can trade in both the professional market and the retail market, and $f(s^i|y^i)$ is the professional-market price of an Arrow-Debreu security providing a unit of consumption in state s^i. We formally introduce this version of our model in Section 6, and confirm the equivalence.

In period 1, an issuer and an investor can trade a derivative security that pays off in period 2. A security is based on a partition $\mathcal{E}$ of the state space $\{0,1\}^I$, and is given by the vector $(c(E))_{E \in \mathcal{E}}$ of consumption levels conditional on events in the partition. Consistent with the notion that the issuer chooses the assets underlying the security, the issuer can choose any partition that is based on the payoffs of K assets.[2] The parameter K captures the complexity of securities that can be marketed, and hence is a measure of the development of the financial market. The case $K \ll I$ is the best description of the retail market, where the underlying are mostly pre-specified indices or individual stocks, or a combination of a few of these. In particular, this case is consistent with a common practice in the retail financial sector, whereby

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[2] Formally, if the K assets are $i_1, \ldots, i_K$, then the partition consists of all events of the form $\{s|s^{i_1} = j_1, \ldots, s^{i_K} = j_K\}$ with $j_k \in \{0,1\}$.

issuers offer multiple securities with the same functional form but with different stocks as underlyings. A large K—or even $K = I$—is consistent with a situation in which issuers can create very complex securities. This extreme is the best description of the institutional market with custom-designed indices as underlying. It may also be where the retail market is heading in the future.

After observing its private information, issuers choose one underlying $\mathcal{E}$, a security $(c(E))_{E \in \mathcal{E}}$, and a price p at which they offer the security.[3] The issuers can acquire funds at zero interest. Upon observing the offers, investors choose whether to buy a security, and if so, which one. Investors cannot split their wealth between multiple securities.[4] In period 1, an investor's utility is linear with a slope of 1, and in period 2, she has $u(c) = \ln(c)$. Her income in period 2 is 0 in every state, and buying a security is her only opportunity to save.

We assume that investors are fully cursed in the sense of Eyster and Rabin (2005), which in this case means that they neglect to account for disagreements between themselves and issuers in evaluating a security. As a result, when deciding whether to accept an issuer's offer, an investor evaluates the security according to $f(s|z)$, and does not take into account what the security reveals about the issuer's private information. Our major reason for assuming fully rather than partially cursed investors is tractability. But the assumption is also consistent with some recent evidence by Jin et al. (2015) on disclosure games, Turocy and Cason (2015) on interdependent-value auctions, and Enke (2016) on selection, that individuals who are cursed tend to be fully cursed.[5]

We look for the competitive-equilibrium securities in our market, which we define separately for any information z that an investor may have. A competitive-equilibrium security for an investor with information z satisfies two properties: (i) it earns zero expected profits; and (ii) there is no security that the investor strictly prefers and that yields positive expected profits. Note that in our setting, the first-best security has $u'(c(E)) = 1$ for any event E, equating the

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[3] As we will argue, the issuer may choose the functional form of the security before or after observing its private signals is immaterial. It is important, however, that the issuer chooses the underlying assets after observing the signals.

[4] This assumption captures in a simple way that the investor faces limits (either cognitive or practical) in putting together more complicated portfolios than an issuer. In as much as the investor can put together portfolios of multiple securities, that corresponds to an increase in K in our model, which we analyze.

[5] These papers also find that the rest of the population is largely rational. Incorporating rational investors into our model above would not change the results results regarding cursed investors, as it is easy to show that rational investors self-separate and buy the first-best security.

investor's marginal benefit of saving with the social cost of funds. The choice of the underlying partition is immaterial.

2.2   Benchmark: Rational Investor

As a benchmark, we discuss the case of fully rational investors. In this case, we add to the definition of a competitive equilibrium that—akin to a perfect Bayesian equilibrium—investors make Bayesian rational inferences from the securities offered to them. Then, the parties achieve first-best:

Proposition 1. *If investors are rational, then in the unique competitive equilibrium they purchase the first-best security.*

With rational investors, both risk aversion and adverse selection call for a flat security. If the issuer offered a security that is increasing in s^i, for instance, the investor would both dislike it due to the risk it imposes on her, and be worried that the issuer only offers the security because s^i is likely to be low. A direct implication of Proposition 1 is that with rational investors, investor information has no effect on outcomes.

3   Basic Properties of Equilibrium Securities

3.1   An Issuer's Optimal Security Given an Underlying Partition

We think of an issuer's problem by reducing the role of competitors. This both simplifies our analysis, and will make it transparent what the role of competition is. From the perspective of an issuer, we can think of competing security offers in terms of the perceived utility $\underline{u}$ they provide to investors. Clearly, a competitive-equilibrium security must maximize profits when $\underline{u}$ is defined as investors' competitive-equilibrium perceived utility.[6] Hence, we solve for the profit-maximizing security given $\underline{u}$. We first derive some basic properties of an issuer's optimal security taking as given the underlying partition $\mathcal{E}$, and later turn to considering the choice of $\mathcal{E}$. This initial step mirrors many previous analyses of the effects of differences in beliefs on contracting (Harrison and Kreps, 1978; Morris, 1996; Geanakoplos, 2010; Simsek, 2014, for

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[6] Analogously, a monopolist's security maximizes profits when $\underline{u}$ is defined as investors' (perceived) utility from the outside option.

example). Since the investor believes that the probability of event E is $f(E|z)$, the maximum price at which she is willing to buy the security $(c(E))_{E\in\mathcal{E}}$, and hence the price at which the issuer sells the security, is

$$p = \sum_{E\in\mathcal{E}} f(E|z)u(c(E)) - \underline{u}. \tag{1}$$

This implies that in designing the security, the issuer solves

$$\max_{(c(E))_{E\in\mathcal{E}}} p - \sum_{E\in\mathcal{E}} f(E|y)c(E) = \sum_{E\in\mathcal{E}} [f(E|z)u(c(E)) - f(E|y)c(E)] - \underline{u}. \tag{2}$$

Taking the first-order condition with respect to $c(E)$ yields the following lemma:

Lemma 1. *Given the underlying partition, for any $\underline{u}$ the profit-maximizing security—and hence also the competitive-equilibrium security—is given by*

$$c(E) = \frac{f(E|z)}{f(E|y)}. \tag{3}$$

The optimal security induces overconsumption in states that the investor considers more likely than the issuer, and underconsumption in states that the investor considers less likely than the issuer. In other words, the investor takes a bet with the issuer that is a function of their differences in opinion.

3.2 Relative Entropy

With log utility, expected and true welfare conveniently reduce to simple functions. To state this implication of our model, suppose that g and h are probability distributions over $\mathcal{E}$. The *relative entropy* of distribution g with respect to distribution h, or *Kullback-Leibler (KL) divergence* of h from g, is $D_{\mathcal{E}}(g(\cdot)||h(\cdot)) \equiv \sum_{E\in\mathcal{E}} g(E)\ln(g(E)/h(E))$. The KL divergence can be thought of as an unusual distance measure. Like a true metric, it is always non-negative, and it equals zero if and only if $f = g$. But unlike a true metric, it is not symmetric and does not satisfy the triangle inequality. Given the underlying partition $\mathcal{E}$, we denote an investor's true welfare by $W_{\mathcal{E}}(y, z)$, and her perceived welfare by $\widehat{W}_{\mathcal{E}}(y, z)$. In our competitive framework, social welfare equals total investor welfare. Evaluating an investor's expected utility from contract (3) conditional on y and z and subtracting the competitive price (i.e., the issuer's cost

$\sum_{E \in \mathcal{E}} f(E|y)c(E) = \sum_{E \in \mathcal{E}} f(E|y)(f(E|z)/f(E|y)) = \sum_{E \in \mathcal{E}} f(E|z) = 1$) gives the following corollary.

Corollary 1. *The true and perceived welfare are*

$$W_{\mathcal{E}}(y, z) = -D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z)) - 1 \ and$$
$$\widehat{W}_{\mathcal{E}}(y, z) = \ D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y)) - 1, \ respectively.$$

The first part says that the investor's expected welfare is a decreasing function of the KL divergence of the investor's beliefs from the issuers' beliefs. Intuitively, the further are the parties' beliefs, the greater is the bet they take with each other, and hence the lower is welfare. The second part says that perceived welfare is an increasing function of the KL divergence of the issuers' beliefs from the investor's beliefs. Rather than seeing a divergence in beliefs as a bad situation, the investor sees it as an opportunity, since she believes there are mispriced investment opportunities.

3.3   Selection of the Underlying

We now turn to the issuer's choice of the underlying. By the same calculation as in Corollary 1, the issuer's expected profit is $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y)) - 1 - \underline{u}$. This implies that the issuer maximizes the investor's perceived utility from the security. By doing so, the issuer maximizes the price the investor is willing to pay, and hence maximizes profits. Hence, the issuer chooses $\mathcal{E}$ to maximize $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y))$. While it maximizes profits, this choice in fact *minimizes* the investor's true welfare:

Proposition 2. *Among zero-profit securities of the form given by Equation* (3), *the competitive-equilibrium choice of $\mathcal{E}$ minimizes investors' (as well as social) welfare.*

In order to maximize profits, the issuer chooses underlying assets for which its beliefs are furthest from the investor's. These assets allow the issuer to offer the most promising deals to the investor, as the investor believes these assets are the most mispriced. Unfortunately for the investor, exactly because of the large difference in beliefs, this choice of underlying just leads her to bet too much, minimizing her welfare.

One simple point regarding the timing of the issuer's choices is worth emphasizing. We have assumed that the issuer selects both the functional form of the security and the underlying assets after receiving information. But if I is large, the issuer loses little if it pre-specifies the functional form of the security before receiving information, and only selects the K underlying assets after receiving information. In the limit where $I = \infty$, this in fact results in the same expected profit. For instance, the issuer can specify the security assuming that all underlying assets have $y^i = 0, z^i = 1$, and ex post select K assets for which this is the case. Since the issuer knows that it will choose such underlying assets, it knows the precise form the security will take. As a practical example, the issuer can announce before period 1 that it will offer a reverse convertible of the type discussed in Henderson and Pearson (2011), but only in period 1 decide that the underlying stock will be Google.

4   The Welfare Effects of Financial Choice

In this section, we show that an increase in choice has detrimental effects in a number of senses. Our first result concerns what happens when any conceivable derivative product over many underlying can be marketed. This is a situation in which arbitrary diversification is possible, and hence financial markets can be expected to perform well. Instead:

Proposition 3 (Bad Diversification). *Suppose $K = I$. As $I \to \infty$, $W_{\mathcal{E}}(y, z) \to -\infty$.*

While diversification is generally thought to benefit an investor and social welfare, here the opposite is the case. The issuer creates an index of underlying assets—but it is a custom-designed rather than standard index: it depends positively on some assets and negatively on others. Of course, the index is designed to solidify the issuer's informational advantage, being increasing in assets that the issuer has received negative information on, and decreasing assets it has received positive information on. Nevertheless, because the investor does not appreciate this selection process and the index is composed of many assets, she feels diversified and therefore drastically underestimates the probability of low-payoff tail events. As a result, she receives very low welfare. Diversification, when custom-made for cursed investors, is extremely harmful.

An equivalent interpretation of the current version of our model is that the investor—rather

than ignoring the informational content of the offer of an issuer she knows to be informed—falsely believes that an uninformed party is designing the index, when in fact an informed party is. Under this interpretation, the custom-designed security our model predicts appears to capture the flavor of the custom-tailored CDO's that received scrutiny in relation to the ABACUS scandal. As described in more detail in Davidoff et al. (2011), in 2007 Goldman Sachs created a synthetic CDO based on a basket of mortgage-backed bonds. While the investors were under the impression that the underlying bonds had been selected by an independent third party, in reality the hedge-fund manager John Paulson was also involved—and at the same time was speculating on the default of these bonds. Indeed, ABACUS and other similarly created CDOs performed extremely poorly during the financial crisis.[7]

While Proposition 3 says that a fully developed financial market yields extremely low welfare, our next result says that the path to such a situation is monotonic in the sense that any financial innovation is harmful.

Proposition 4 (Financial Innovation is Pareto-Damaging). *Fix the issuers' and the investors' signals.*

1. *An increase in K lowers the welfare of all investors.*
2. *An increase in I lowers the welfare of all investors.*

As we have shown, the issuer selects as underlying assets about which its beliefs are furthest from the investor's. This maximizes the investor's perceived utility among different possible underlying assets, but it *minimizes* her true expected utility. The more assets the issuer can select where it disagrees with the investor (i.e., the larger is K), the more effectively it can minimize the investor's welfare in this fashion (part 1).

This negative welfare implication is a stronger variant of the main welfare result in Simsek (2013). In Simsek's model, assets are linear combinations of underlying normally distributed risks, and investors can form linear portfolios from existing assets. Simsek shows that the addition of a new asset increases speculative variance and thereby lowers social welfare. In his model, however, some investors can benefit from financial innovation. In our case, instead,

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[7] In 2010, the Securities and Exchange Commission (SEC) accused Goldman Sachs of hiding the role and incentives of Paulson in the deal. According to the SEC, Paulson managed to shift the basket of underlying assets toward mortgages he believed would perform especially badly when house prices decline. Goldman Sachs agreed to pay $550 million without admitting or denying wrongdoing.

financial innovation hurts *all* investors; and because in a competitive market issuers make zero profits in any case, this means that financial innovation Pareto-lowers welfare. In Section 6, we show that a version of this result holds when we embed our model into a standard asset-pricing framework.

Beyond the above result, Proposition 4 says that the richer is the set of available underlying assets (i.e., the larger is I), the lower is investors' welfare (part 2). Since the issuer chooses the underlying to minimize investor welfare, an expansion in its options is harmful.

Part 2 has a straightforward implication for policy: restricting the permissible underlying assets that can be used for securities—or, in the extreme, even fully specify the K underlying assets—raises welfare by lowering I. Of course, in the current version of our model the welfare-maximizing policy is to go to the extreme and ban trading in the risky asset altogether. We show in Section 6 that this latter implication does not generalize to the more realistic scenario in which there is also aggregate risk, but the former insight that standardization raises welfare does.

5 The Welfare Effects of Investor Information

5.1 Main Results

We turn to our main interest in the paper: the effects of providing more information to small investors. For most of the section, we restrict attention to the empirically relevant case in which K is relatively small, and I is large. Most structured securities sold to small investors are written on a small number of underlying assets, at least relative to the set of possible underlying assets. Our results are simplest to state when there is a countable number of underlying assets ($I = \infty$), but at the end of the section we state a version of our main point also for large finite I.

To state our main result, we define what it means for a security to be more speculative than another. The security c_1 is more speculative than c_2 if $|\ln(c_1(s_i = 1, s_{-i})/c_1(s_i = 0, s_{-i}))| > |\ln(c_2(s_j = 1, s_{-j})/c_2(s_j = 0, s_{-j}))|$ for all assets i and j underlying the two securities, and all realizations s_{-i} and s_{-j} of the other underlying assets. That is, a security is more speculative than another if it is steeper—either in a positive or in a negative direction—in each of its

underlying assets than the other.

Proposition 5 (Investor Information). *Suppose $I = \infty$. For any K and $\alpha, q \in (0, 1)$, a competitive-equilibrium security is written on K underlying assets about which the investor has information, and the issuers' and investor's information go in opposite directions. Also, an increase in q leads to (i) a decrease in investors' expected utility; and (ii) a more speculative security. Welfare jumps to first best at $\alpha = q = 1$.*

Although the information the investor receives moves her beliefs in the right direction on average, given that her information is noisy and inferior there are bound to be assets for which she receives misleading information. Selecting these assets as the underlying assets allows the issuer to take a bigger bet with the investor, leading to a steeper security and lower investor and total welfare.

Our result that inferior and noisy information lowers welfare implies a discontinuity. If the investor receives the same information as the issuer ($\alpha = 1$ and $q = 1$), welfare is maximized and hence discretely higher than if she receives no information. If the investor receives information that is arbitrarily close in distribution to the issuer's ($\alpha = 1$ and $q \lessgtr 1$), however, welfare is discretely lower than if she receives no information and is in fact (close to) *minimized*. The investor realizes that her signal is uninformative with some probability, but believes that the probability is low, and therefore believes that her information is reliable. As a result, she is especially willing to trade on her information. Of course, the issuer has chosen precisely the rare underlying assets for which the investor's signal is misleading. While this stark example strongly uses our assumption that there are infinitely many assets—and hence the issuer can always find assets for which the investor's information is extremely misleading—the example illustrates that even small amounts of noise in investors' information can drastically lower welfare.

Proposition 5 has potentially far-reaching positive as well as normative implications. On the positive side, the proposition predicts that structured securities and exotic exchange-traded funds are written on underlying assets about which information is readily available to investors. This prediction is broadly consistent with Henderson and Pearson's (2011) observation that the vast majority of structured products are issued for underlying stocks that are commonly known. Furthermore, Bergstresser (2008) and Henderson and Pearson (2011) document that issuance

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is more likely for underlying stocks with high past return and volatility, which researchers (e.g., Barber and Odean, 2008) take as a sign that investors know about these stocks.

An important normative message of Proposition 5 is that the drastically increased availability of financial information may have made many investors worse off. By the same token, public policies aimed solely at improving investor information—which, even if well-conceived, is bound to leave the majority of investors with information that is noisy and inferior to issuers' information—are likely to backfire. Finally, the proposition suggests that more sophisticated—in our case, better-informed—investors are worse off than less sophisticated investors.

A key factor in Proposition 5 is that the investor neglects how the underlying assets are selected. Besides it being an implication of cursedness, some direct evidence also indicates that individuals neglect related selection effects. For instance, Koehler and Mercer (2009) find that most mutual-fund companies selectively advertise their better-performing funds, yet both novice and professional investors *fully* ignore such strategic selection when this is not made transparent to them. Similarly, Brenner et al. (1996) find that individuals do not discount transparently one-sided evidence.

While we have assumed that issuers observe z, in some situations—e.g., when z depends on the investor's subjective interpretation of a news story—they may not. If issuers can offer a menu of securities from which investors select, however, the unobservability of z does not affect the features of the security the investor buys. For any K assets for which investors have information and issuers have received signals equal to 0, an issuer can include in its menu a security that is optimal *assuming* that the investors have observed signals equal to 1. Since the securities are upward-sloping and identical, the investor chooses a security for which she has indeed received favorable signals.

The realistic possibility that issuers offer multiple securities on idiosyncratic risk, and have investors choose according to their information, helps answer the natural question of why investors want to acquire welfare-decreasing information in the first place. To start, since the investor does not understand how the underlying assets are selected, she does not understand that information is welfare-decreasing. Worse, since she believes that she is an expected-utility maximizer, she views information as useful for making decisions. In particular, when many securities are offered, she believes that information will help her select which of the securities

constitute the best deal. As a result, she is willing to expend resources to collect even costly information, exposing herself to the double whammy of paying information-acquisition costs as well as getting a worse security. We explore these information-acquisition incentives in more detail in the next subsection.

For completeness, we establish a variant of Proposition 5 for finite I:

Proposition 6. *For any K, α and $q \in (0,1)$, there is an $\underline{I}$ such that if $I > \underline{I}$, then the investor's expected welfare is lower than if she had no information ($q = 0$).*

When there are only finitely many assets, the issuer is not guaranteed to find underlying assets for which the investor has misleading information. Nevertheless, for a large I it can find sufficiently many such assets with sufficiently large probability that information harms the investor as above.

To conclude this section, we consider the role of information when the issuer cannot select the underlying assets:

Proposition 7 (Investor Information with No Choice of Underlying). *Suppose $K = I$. Then, an investor's expected utility is strictly increasing in q.*

Unlike when it can select from a large number of underlying, the issuer is unable to systematically select underlying events about which the investor's information is misleading. Indeed, investor information moves the parties' beliefs regarding the underlying events closer together on average, mitigating the issuer's incentive to take welfare-decreasing bets against the investor.

Proposition 7 has two economically important implications. First, the assumption that $K = I$ is relevant for understanding a fully developed financial market in which I is large and arbitrary diversification is possible. Although Proposition 3 says that welfare is extremely low in this situation, Proposition 7 adds that at least investor information raises welfare. More importantly, however, the assumption $K = I$ also applies to a situation in which K is low, but the set of underlying assets is restricted by regulation to be similarly low. Then, Proposition 7 qualifies a policy implication we have emphasized earlier in an interesting way: while without standardization an information-based policy is prone to backfire, with standardization it is beneficial. In this sense, standardization-oriented and information-oriented policies are complements.

5.2   The Role of Competition

5.2.1   Basic Welfare Results

We discuss what happens when investors face a monopolistic issuer rather than a competitive issuing industry. We modify the environment of Section 2 by assuming that an investor receives a single take-it-or-leave-it offer from a single monopolistic issuer. For this version of our model, we assume that the investor has an outside option that generates utility $\underline{u}$.

Our analysis in Section 3.1 implies that the issuer's optimal security is independent of $\underline{u}$, so the issuer chooses the same security as in the competitive model. The intuition is simple: in choosing the underlying partition as well as the security, both competitive and monopolistic issuers maximize the investor's perceived expected utility. Since investors are buying the same security as in Section 3, total welfare is unaffected by competition. Competition transfers profits to investors, but investors are still holding a suboptimal amount of risk.

5.2.2   Information Acquisition

One of the main results of our paper is that (noisy and inferior) information lowers investor welfare. As we have noted, an important implication is that investors are willing to expend resources to collect information that makes them worse off even gross of the information-acquisition costs. We compare the incentives for welfare-decreasing information acquisition with and without competition, and find that they are higher under competition. Hence, competition is not only ineffective at increasing welfare, it actually lowers welfare by encouraging harmful information search.

We begin by analyzing the more difficult case, monopoly. When investors may or may not acquire information, the monopolist optimally screens informed and uninformed investors. We extend and slightly modify our basic model to make this screening problem tractable. We suppose that $I = \infty$, and that—having observed its private information—the issuer offers a potentially countable menu of securities. Investors can choose to receive information z^i on an infinite number of underlying assets, but there are also infinitely many assets for which they do not receive information. The issuer knows the assets for which investors have information, but does not know the realized signals z^i when designing its securities. After observing the menu,

investors decide whether to acquire the signals z^i, with investors acquiring either all or none of the available signals. The cost of information acquisition is heterogeneous across investors, with cumulative distribution function $G(\cdot)$ and probability density function $g(\cdot)$. The support of $G(\cdot)$ is $[0, X]$, where X is sufficiently large for the set of investors who acquire information to be interior, and $x + G(x)/g(x)$ is increasing in x. After the relevant subset of investors has acquired information, both informed and uninformed investors either choose not to participate, or choose one security from the issuer's menu.

Note first an important implication of our analysis from the competitive case: since an informed investor's perceived utility from the optimal security is higher than an uninformed investor's, an informed investor is more profitable for the monopolist than an uninformed investor. As a result, the monopolist wants to induce at least some investors to get informed. We identify an optimal way for the monopolist to give incentive $x > 0$ for investors to acquire information. For any K assets for which investors can obtain information and the issuer has received $y^i = 0$, the issuer includes in its menu a security that is optimal *assuming* that the investors have observed signals $z^i = 1$, pricing securities identically to give investors a perceived surplus of x. For uninformed investors, in turn, the monopolist includes in its menu an optimal security written on underlying assets about which no investor has information, pricing the security to leave investors with zero perceived surplus. When facing this menu, the investor knows that if she remains uninformed, she receives a perceived surplus of zero. Furthermore, she realizes that if she becomes informed, she will get a positive signal for some securities, so that her perceived surplus will be x. Intuitively, the investor thinks that by searching for information, she will be able to figure out which of the expensive-looking securities are a good deal. Hence, her incentive to acquire information is x.

In designing its securities, the firm chooses x optimally given the following tradeoff. On the one hand, increasing x leads more investors to acquire information and hence to choose the firm's more profitable product, the one aimed at informed investors. On the other hand, increasing x lowers the firm's margin on its more profitable product.

Based on the above considerations, Proposition 8 identifies the features of the market outcome under monopoly. To state the proposition, we define $d_0^{yz}(q)$ as the relative entropy between $f(s^i|y^i)$ and $f(s^i|z^i)$ for a single primitive asset when the realized signals y^i and z^i

are not equal and z^i is noise with probability $1 - q$.

Proposition 8 (Information Acquisition Under Monopoly)**.** *A profit-maximizing strategy is for the issuer to sell (i) to informed investors a security based on K assets on which investors have opposing information; and (ii) to uninformed investors a security about which no investor has information. The fraction of investors who become informed is $G(x^*)$ defined by*

$$K \cdot (d_0^{yz}(q) - d_0^{yz}(0)) = x^* + \frac{G(x^*)}{g(x^*)}. \tag{4}$$

We now consider the competitive economy. Our analysis in Section 3 applies to informed consumers unchanged, while it applies to uninformed consumers by setting $q = 0$. By Proposition 2, in both cases the investor buys the same security as above. The incentive to acquire information is the difference in the perceived expected utilities these securities offer to the two types of consumers. By Corollary 1, this equals

$$K \cdot (d_0^{yz}(q) - d_0^{yz}(0)),$$

which is $x^* + \frac{G(x^*)}{g(x^*)}$ by Equation (4). Therefore, the fraction of consumers who choose to acquire information is $G\left(x^* + \frac{G(x^*)}{g(x^*)}\right) > G(x^*)$, so that:

Proposition 9 (Competition Increases Information Acquisition)**.** *The fraction of investors who acquire information is greater under competition than under monopoly.*

Since the total welfare generated by a security that an informed investor chooses is lower than the total welfare generated by a security that an uninformed investor chooses, competition lowers welfare. Intuitively, competition generates a greater incentive to acquire welfare-decreasing information by leaving all of the perceived gain from better information in the investor's hands. To profit from consumers' trading on misleading information, a monopolist takes away some of the perceived gain, thereby also lowering the incentive to acquire information in the first place.

Combining our result that an increase in competition leads to an increase in the number of investors acquiring information with our earlier observation that the securities issuers offer to more informed investors are steeper leads to a potentially testable prediction of our model: that

an increase in competition leads parties to trade more speculative securities. Unfortunately, we are not aware of empirical work on this prediction.[8]

It is worth noting an immediate corollary of our analysis:

Corollary 2. *Both under competition and under monopoly, the fraction of informed consumers is increasing in K.*

Corollary 2 says that the more complex are securities, the more consumers are willing to invest in information acquisition. The intuition is simple: with more complex securities, consumers perceive the value of finding out which security is a good buy to be higher. As all information acquisition, the extra information acquisition consumers engage in is harmful. As a result, information acquisition acts as a multiplier on the negative effect of complexity on welfare.

6   Risk-Averse Professionals and Aggregate Risk

Our model in the previous sections was designed to isolate the potential welfare-decreasing effects of financial choice and financial information. In demonstrating these effects, however, we have abstracted away from classical reasons for financial markets: risk-sharing between risk-averse professionals and households when there is aggregate risk in the economy. In this section, we embed our model in a standard asset-pricing framework (e.g., Cochrane, 2009) to allow for such beneficial effects of financial markets. We identify conditions under which our results survive essentially unchanged—with investors being hurt by choice and information, and issuers and professionals not benefiting from it.

We assume a unit mass of representative professionals with utility functions $v_0(\cdot)$ and $v(\cdot)$ in periods 0 and 1, respectively. Professionals receive the deterministic endowment e_0 in period 0 and the stochastic aggregate endowment $e(s^1)$ in period 1, which depends on the first primitive asset, s^1. Hence, s^1 is an aggregate risk factor.

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[8] A comment on the role of our assumption that a monopolist can offer a large menu of securities may be useful. This ensures that when an investor considers whether to get information, she knows that she will find a security written on an underlying asset for which she has received a positive signal. Although we have not formally considered such variants of the model, it seems that if there are fewer securities in the marketplace, then an investor sees less of a chance that she will find a good deal, lowering her incentive to acquire information. To the extent that competition leads to more securities being offered, therefore, this provides an additional reason that competition encourages harmful information acquisition.

The utility function of investors is the same as in our basic model, and we also continue to assume that investors are fully cursed. But we consider two extreme cases that differ in the kind of information investors receive. In case 1, investors receive $z^1 = y^1$ with probability 1, but their other signals are determined as previously, with the restriction that noise in z is uncorrelated across investors. That is, in case 1 investors have the same information as professionals on the aggregate risk, but noisy and inferior information on the other assets. In case 2, investors receive z^1 as previously, but for $i > 1$ receive $z^i = y^i$ with probability 1. That is, in case 2 investors have the same information as professionals regarding all the idiosyncratic assets, but they have different information regarding the aggregate risk factor.

Under case 1, our results are very similar to those with our basic model:

Proposition 10. *Suppose case 1 applies, $K' \geq 1$ and $I = \infty$. Then, (i) increasing K from $K = K'$ to $K = K' + 1$ or (ii) increasing q on $[0, 1)$ Pareto-lowers welfare. However, (iii) restricting the underlying to asset 1 Pareto-increases welfare.*

With risk-averse professionals and aggregate risk, trading in the aggregate risk factor (asset 1) is welfare-improving. Indeed, it may be the case that asset 1 is included among the K underlying assets, and if $K = 1$, it may be the case that the security is based solely on the aggregate risk factor, leading to first-best risk-sharing and benefiting investors and professionals alike. Even so, Part (i) of Proposition 10 says that increasing the complexity of securities is Pareto-damaging, and Part (ii) says that improving investors' inferior information Pareto-lowers welfare. Intuitively, beyond the complexity level at which investors can trade the aggregate risk factor, any increase in the complexity of securities leads investors to take welfare-decreasing bets on idiosyncratic risk. Given our iid primitive assets, investors can do so without effecting the net risk exposure of professionals. In addition, an improvement in investors' information about the idiosyncratic risks lowers welfare for the same reason as before: it makes cursed individuals more confident in their misleading information. As a consequence, Part (iii) says that restricting the underlying to the aggregate risk factor increases utility as it protects investors from the unambiguously welfare decreasing speculative bets on idiosyncratic factors.

In the appendix, we consider whether asset 1 is even included in the underlying of a security an investor purchases. Roughly, this is the case if the potential benefit of risk sharing outweighs

professionals' informational advantage relative to investors who receive misleading information. Otherwise, the investor thinks she can make more favorable idiosyncratic investments, so she chooses not to trade in the aggregate risk factor despite the benefits of risk sharing. In this case, the economy is Pareto inferior even to an economy where investors can buy only riskless investments.[9] It is also worth noting that an improvement in investor information (an increase in q) might move the economy from a situation where investors trade the aggregate risk factor to one where they do not, resulting in a discrete drop in their welfare. As q increases, investors become more confident in being able to spot mispriced idiosyncratic securities, so they switch to trading on idiosyncratic risk.

We now turn to case 2:

Proposition 11. *Suppose case 2 applies. Then, increasing K does not affect social welfare and investors' expected utility strictly increases in q on $[0, 1)$. Restricting the underlying to the aggregate factor does not affect investor's expected utility and social welfare.*

The welfare implications of case 2 are quite different. In this case, investors demand and receive a security where the aggregate factor is the only underlying, independently of K. As we show in the Appendix, in this case investors' utility has two components. First, the security helps investors get exposure to the first-best contract. Second, because they have different information, the security also has a speculative component. This component implies a negative average relative entropy term leading to negative expected utility. As a result, the overall welfare effect of trading securities—relative to a situation where investors can only buy riskless assets—is ambiguous. However, as the information of investors and professionals on the aggregate factor coincides with probability q, increasing q alleviates the damaging effect of the speculative bet. This is analogous to our result in Proposition 7.

Propositions 10 and 11 indicate that once securities are sufficiently complex to allow for trading aggregate risk factors, further complexity cannot be welfare-improving, and is welfare-decreasing so long as professionals have some informational advantage regarding idiosyncratic

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[9] In this case the professionals are not affected by investors' preference to trade idiosyncratic risk. Although each investor is buying a risky contract and professionals are effectively taking the other side of each, professionals are not taking on any risk. They are holding a fully diversified portfolio, which, under our iid assumption is equivalent of a risk-free contract from their point of view. Furthermore, because of investors' log utility, the resulting equilibrium net lending and borrowing between professionals and investors are the same as when only riskless securities can be traded. That is, the risk-free rate and the expected utility of professionals is not affected.

risks. This suggests that beyond a basic level, an expansion in financial choice is welfare-decreasing. Furthermore, any improvement in investor information regarding idiosyncratic risks is welfare-reducing.

Worse, the trading of even basic securities may be welfare-decreasing relative to a situation where investors can buy only riskless investments. Trading securities is welfare-increasing if the benefits of risk sharing are sufficiently large relative to both the average informational advantage professionals have regarding the aggregate risk, and the greatest informational advantage professionals have regarding idiosyncratic risks. Although it is difficult to determine whether this is the case based on primitives, fortunately the two cases have distinct empirical implications. If financial innovation serves primarily to share aggregate risk, then it should expose investors to the few priced aggregate risk factors only. Also, this exposure should be in the same direction in each contract as the sign of the total supply of that risk in the economy. That is, while heterogeneity in the utility functions might validate some variation in the shape of the offered contracts, they should be similar in the provided risk-exposure. If instead financial innovation serves to satisfy the demand of cursed investors for idiosyncratic risks, then we should observe contracts providing a diverse set of risk-exposures, many to non-priced risk-factors.

Celerier and Vallée (forthcoming) and its Appendix describes various examples of the most popular structured retail products between 2002-2010 in European countries. Some of the most popular products are indeed essentially call options on the aggregate stock index, thus consistent with the risk-sharing view. However, the full picture is different. Celerier and Vallée (forthcoming) emphasizes the enormous diversity in the provided risk-exposure of these contracts. Many products use (a small group of) individual stocks as underlyings. These products are inconsistent with the risk-sharing view regardless of their shape. Even those that are written on aggregate market indices often does not fall into the stylized description of the risk-sharing view.[10]

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[10] For example, take the most popular contract in Sweden, Spax Framtid, sold in 325 millions Euros described as follows:

> This is a growth product linked to the performance of DJ Eurostoxx50 index. The performance of the index is observed over every month. At the end of the investment period the negative monthly returns are deducted from the maximum total return of 140%. At maturity the product offers a minimum capital return of 111.25%. (Celerier and Vallée (forthcoming), Appendix, pp. 8.)

7   Related Literature

Our paper is related to the literature on financial innovation as well as that on contracting with boundedly rational agents. To our knowledge, ours is the first paper to analyze the welfare properties of optimally designed securities when investors underestimate the informational content of issuers' actions. We are also not aware of previous work pointing out that inferior information lowers total welfare, and that standardization typically increases welfare.

The literature on rational security design studies the optimal way for a firm to raise capital in the presence of adverse selection (Nachman and Noe, 1994; DeMarzo and Duffie, 1999; Yang, 2013) or moral hazard (Innes, 1990; Hebert, 2015). While these considerations are fundamental in the corporate context, it is plausible to suppose that they are less central in the retail finance context. A retail investor likely finds it difficult to deduce the issuer's private information or incentives from the contract she is offered, or does not even think about these questions. As a result, the issuer is less worried about the investor's inference, radically changing the motives behind security design. By assuming that investors are fully cursed, we focus on analyzing the new considerations that arise.[11]

Of the various views on financial innovation, our work is most related to two strands of the literature arguing that financial innovation facilitates bets in environments where the no-trade theorems do not hold.[12] First, there is a group of papers considering the role of financial innovation with heterogeneous priors. Simsek (2014) establishes that introducing new markets by financial innovation increases the volatility of consumption by introducing new ways of betting. Shen et al. (2014) argue that financial innovation helps reduce the cost of betting by minimizing the associated collateral requirement. Fostel and Geanakoplos (2012) focus on the interaction between financial innovation and endogenous leverage, showing that the sequential introduction of new financial products can result in boom-and-bust cycles.

While the underlying is an aggregate stock index, a potentially useful addition to the investment portfolio of a Swedish investor, the contract actually gives an exposure to the volatility of this index, instead of its level.

[11] A telling sign of the distance between the rational approach and ours is that in Hebert (2015) the optimal security—debt—minimizes relative entropy, whereas in our setting the optimal security maximizes relative entropy.

[12] More classical and less related contributions include Allen and Gale (1994), who argues that new securities help hedging in an incomplete market setting, and Gorton and Pennacchi (1990) and Dang et al. (2012), who show that financial innovation can increase the liquidity of assets by decreasing their sensitivity to private information.

Second, there is a group of papers modeling situations in which individuals are willing to take bets due to various cognitive biases. Gennaioli et al. (2012) study financial innovation when investors are extremely risk-averse but neglect some low-probability risks, leading to the creation of seemingly safe securities that pay less in the neglected state. In contrast, we show that cursedness in general implies the creation of steep, speculative securities. Related to this observation, multiple researchers (e.g. Adrian and Westerfield, 2009; Landier and Thesmar, 2009; de la Rosa, 2011; Gervais et al., 2011) have shown that a principal often gives high-powered incentives to overconfident agents, effectively motivating them with dreams that are unlikely to materialize. It would be interesting to study the questions we raise in this paper in the context of overconfidence. We work with cursedness largely for epistemic reasons: it seems realistic to assume that retail investors buy structured assets not because they think they are better than the professionals they are trading with, but because they do not think through the incentives of the other side.[13,14]

The closest paper to ours is Eyster et al. (2014), which considers the impact of cursed traders in an otherwise standard asymmetric-information rational-expectations-equilibrium model with a fixed set of securities. The paper's main insight is that cursedness can explain the puzzlingly high volume of trade in financial assets. Eyster, Rabin, and Vayanos also consider the effect of investor information. A cursed trader with more precise information has a better estimate of the fundamental value of the asset, but takes riskier positions against rational traders. When this second effect dominates, more precise information makes cursed traders worse off. Due to the issuer's ability to optimize the security, in our setting information *always* lowers the investor's utility, and in addition always lowers total social welfare as well.

8 Conclusion

Our analysis raises several questions regarding the role of cursedness in financial markets. In our model, the shape of an optimal security is determined only by the parties' information and preferences, but in reality other considerations—such as the ease of writing or marketing

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[13] See Eyster and Rabin (2005) for some comparisons between cursedness and overconfidence.

[14] Beyond research on overconfidence, our paper is broadly related to the growing literature on market competition and contracting with naive consumers. See Spiegler (2011) and Kőszegi (2014) for reviews.

contracts or hedging a security in the professional market—also play a role. The exact features of real-life securities, such as their shape or time horizon, are likely also influenced by these additional considerations. Furthermore, while our model assumes exogenously that investors are fully cursed, many or most investors may be only partially cursed, and their degree of cursedness may even depend on market conditions. For instance, if a security is too blatant in taking advantage of cursedness—such as when it is a bet on the flip of a coin the issuer supplies—some investors might be clued in that they should think about the other side's information. In this sense, there appears to be a kind of "plausibility" constraint on these securities—that there should be a plausible reason for the security to make a better return than alternatives. What such plausibility constraint precisely entails is a fruitful topic for future research.

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Proofs—For Online Publication

Proof of Proposition 1. Let us consider the first best security defined by

$$u' \left(c^{FB} \left(E \right) \right) = 1,$$

that is, with log utility, the constant contract $c^{FB} \equiv 1$. Regardless of z and y or $\mathcal{E}$, this contract gives zero profit if it is offered at the price $p = \sum_{E \in \mathcal{E}} f \left(E | y \right) c^{FB} = 1$. Suppose that there is a set Y of y signals, for for which the issuer offers $\tilde{c} \left(E \right)_{E \in \tilde{\mathcal{E}}}$ instead, for some partition $\tilde{\mathcal{E}}$, and $\tilde{c} \left(E \right) \neq 1$ for some event E. If we denote the perceived utility which this contract provides to investors $\underline{u}$, the maximum willingness to pay by rational investors, hence the profit maximizing price of this security is

$$p = \sum_{E \in \mathcal{E}} f \left(E | Y \right) u \left(\tilde{c} \left(E \right) \right) - \underline{u}$$

It implies that the profit of the issuer is

$$\sum_{E \in \mathcal{E}} f \left(E | Y \right) u \left(\tilde{c} \left(E \right) \right) - \underline{u} - \sum_{E \in \mathcal{E}} f \left(E | Y \right) \tilde{c} \left(E \right)$$

and a marginal change of the payoff $\tilde{c} \left(E \right)$ at event any given E would change this profit with

$$u'(\tilde{c} \left(E \right)) - 1.$$

As this is not zero for every E by definition, clearly, $\tilde{c} \left(E \right)$ cannot be a profit maximizing contract. $\qquad \square$

Proof of Lemma 1 In text. $\qquad \square$

For the rest of the proofs we are relying on the following Lemma.

Lemma A.1. *Let us fix an arbitrary K and consider the set of all K long combination of primitive assets. Each element is characterized by the K size vectors s, y and the $K - k$ sized vector z as the corresponding vector of states, and signals where k is the number of assets for which the investor does not observe a signal. For any element, we fix the partition $\mathcal{E}$ implied by all the possible realizations of s. Let $n \leq k$ be the number of assets for which the realization of signal y and z are identical.*

Then

1. *$D_{\mathcal{E}}(f(\cdot | y) || f(\cdot | z))$ and $D_{\mathcal{E}}(f(\cdot | z) || f(\cdot | y))$ depend only on the parameters q, K, k, r and n,*

2. *for any $q, r \in (0, 1)$, $D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))$ and $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y))$ are maximal for the partition(s) $\mathcal{E}$ implied by the K combination(s) with minimal n and, among those, with minimal k,*

3. *for $q = 0$, $D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))$ and $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y))$ are constant in n and k, and increasing in K,*

4. *for any fixed K, $D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))$ and $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y))$ are also strictly increasing in q iff $n < \bar{n}$, .and strictly decreasing if $n > \bar{n}$ for some $0 < \bar{n} < K$, and*

5.
$$\frac{\partial E_{y,z}\left(D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))\right)}{\partial q}, \frac{\partial E_{y,z}\left(D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y))\right)}{\partial q} < 0.$$

Proof of Lemma A.1 Recall that for each element of the s vector, $\Pr\left(s^i = y^i|y^i\right) = r$ and $\Pr\left(s^i = z^i|z^i\right) = qr + (1-q)\frac{1}{2}$ and define

$$d_1^{yz}(q) \equiv r \ln \frac{r}{\left(qr + (1-q)\frac{1}{2}\right)} + (1-r)\ln \frac{1-r}{\left(q(1-r) + (1-q)\frac{1}{2}\right)}$$

$$d_0^{yz}(q) \equiv r \ln \frac{r}{\left(q(1-r) + (1-q)\frac{1}{2}\right)} + (1-r)\ln \frac{(1-r)}{\left(qr + (1-q)\frac{1}{2}\right)}$$

as the relative entropy $D\left(f\left(s^i|y^i\right)||f\left(s^i|z^i\right)\right)$ when $y^i = z^i$ and when $y^i \neq z^i$, respectively. Similarly, let $d_1^{zy}(q)$ and $d_0^{zy}(q)$ be the relative entropy $D\left(f(s^i|z^i)||f\left(s^i|y^i\right)\right)$ for $y^i = z^i$ and when $y^i \neq z^i$, respectively. Then, from the additivity property of relative entropy for independent random variables, for fixed r, we have $D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z)) = d^{yz}(K, n, q, k)$ and $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y)) = d^{zy}(K, n, q, k)$ with the definitions

$$d^{yz}(K, n, q, k) = nd_1^{yz}(q) + (K - k - n)d_0^{yz}(q) + kd_0^{yz}(0) \tag{A.1}$$

$$d^{zy}(K, n, q, k) = nd_1^{zy}(q) + (K - k - n)d_0^{zy}(q) + kd_0^{zy}(0) \tag{A.2}$$

giving result 1. Observing that for $q \in (0, 1)$, $0 < d_1^u(q) < d_0^u(0) < d_0^u(q)$ gives result 2. For $q = 0$, $0 < d_1^u(q) = d_0^u(0) = d_0^u(q)$ implying result 3.

For the comparative static on q, we show only the results on expression $D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))$, the argument for $D_{\mathcal{E}}(f(\cdot|z), f(\cdot|y))$ is analogous. First, note that

$$\frac{\partial d_1^{yz}(q)}{\partial q} = (2r - 1)^2 \frac{q - 1}{1 - q^2(2r - 1)^2} < 0$$

$$\frac{\partial d_0^{yz}(q)}{\partial q} = (2r - 1)^2 \frac{q + 1}{1 - q^2(2r - 1)^2} > 0.$$

which implies, from (A.1), result 4.. For result 5, note also that $\Pr\left(y^i = z^i\right) = \bar{r}\left(q\right)$ where

$$\bar{r}\left(q\right) = \left(r\left(qr + (1-q)\frac{1}{2}\right) + (1-r)\left(q\left(1-r\right) + (1-q)\frac{1}{2}\right)\right).$$

Then,

$$E_{z,y}\left(D\left(\Pr\left(s^i|y^i\right)\,||\,\Pr\left(s^i|z^i\right)\right)\right) = \bar{r}\left(q\right) d_1^{yz}\left(q\right) + (1-\bar{r}\left(q\right)) d_0^{yz}\left(q\right)$$

and

$$\frac{\partial E_{z,y}\left(D\left(\Pr\left(s^i|y^i\right)\,||\,\Pr\left(s^i|z^i\right)\right)\right)}{\partial q} = \frac{\partial\left(\bar{r}\left(q\right)\right)}{\partial q}\left(d_1^{yz}\left(q\right) - d_0^{yz}\left(q\right)\right) + \bar{r}\left(q\right)\frac{\partial d_1^{yz}\left(0\right)}{\partial q} + (1-\bar{r}\left(q\right))\frac{\partial d_0^{yz}\left(q\right)}{\partial q} =$$

$$= (2r-1)\left(\ln\frac{\left(q\left(1-r\right) + (1-q)\frac{1}{2}\right)}{\left(qr + (1-q)\frac{1}{2}\right)} + \frac{4qr\left(1-r\right)\left(2r-1\right)}{1 - q^2\left(2r-1\right)^2}\right) < 0.$$

Given that $d^{yz}\left(K, n, q, k\right)$ is an affine function of $d_1^{yz}\left(q\right)$ and $d_0^{yz}\left(q\right)$, $E_{y,z}\left(D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))\right)$ is an affine function of $E_{z,y}\left(D\left(\Pr\left(s^i|y^i\right)\,||\,\Pr\left(s^i|z^i\right)\right)\right)$, implying the last result. $\qquad\square$

Proof of Proposition 2 Result 2 in Lemma A.1 and Corollary 1 implies that the underlying of the traded contract in a competitive equilibrium is the partition implied by a K-long combination of assets with minimal n and, among those, with minimal k. All such partitions maximize both $D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))$ and $D_{\mathcal{E}}(f(\cdot|z)||f(\cdot|y))$. All such partitions lead to the same welfare and the same perceived utility for investors. $\qquad\square$

Proof of Proposition 3 Given Lemma 1, Lemma A.1 and Corollary 1, for $I = K$, welfare in the competitive equilibrium is

$$E_{z,y}\left(\left[-n d_1^{yz}\left(q\right) - (I - k - n) d_0^{yz}\left(q\right) - k d_0^{yz}\left(0\right)\right]\right)$$

. As $I \to \infty$, the term in the bracket for any fixed n goes to $-\infty$ giving the result. $\qquad\square$

Proof of Proposition 4 Consider Lemma A.1 and Corollary 1 and the effect of larger I or larger K. A larger I implies that there might be a vector with smaller n or k among the K long vectors. A larger K implies that there might be a longer vector with the same n or k. If there is such a vector, it provides a larger perceived utility for the investor, and, at the same time implying a smaller welfare. (If there is no such vector of asset for the given y and z, then the choice of the underlying and welfare does not change. Also, we ignore here the integer problem to simplify the exposition.) $\qquad\square$.

Proof of Proposition 5 Consider Lemma A.1 and Corollary 1. When $I \to \infty$ the probability that there will be a vector of assets with $n = k = 0$ and $\bar{K} = K$ approaches 1.

The corresponding welfare is decreasing in q as by the last statement in Lemma A.1. The speculativeness of the contract when $n = k = 0$ is

$$\frac{\left(\frac{\left(qr+(1-q)\frac{1}{2}\right)}{1-r}\right)^K}{\left(\frac{\left(q(1-r)+(1-q)\frac{1}{2}\right)}{r}\right)^K} = \left(\frac{\left(qr+(1-q)\frac{1}{2}\right)}{\left(q(1-r)+(1-q)\frac{1}{2}\right)}\frac{r}{1-r}\right)^K$$

which is increasing in q. $\qquad\square$

Proof of Proposition 6 Consider Lemma A.1 and Corollary 1. With finite I, and $\alpha = 1$ expected welfare is

$$\sum_{n=0}^{K}\binom{I}{K-n}(1-\bar{r}(q))^{(K-n)}\bar{r}(q)^{I-(K-n)}d^{yz}(K,n,q)$$

$$+\left(1-\sum_{n=0}^{K}\binom{I}{K-n}(1-\bar{r}(q))^{(K-n)}\bar{r}(q)^{I-(K-n)}\right)d^{yz}(K,0,q)$$

where,

$$\bar{r}(q) = \left(r\left(qr+(1-q)\frac{1}{2}\right) + (1-r)\left(q(1-r)+(1-q)\frac{1}{2}\right)\right)$$

The statement comes form the fact that as I grows, the weight of term $d^{yz}(K,0,q)$ gets arbitrarily close to 1, while the weights of $d^{yz}(K,n,q)$ get arbitrarily close to 0 and $d^{yz}(K,0,q) > d^{yz}(K,n,q)$ for every $q > 0$ by Lemma A.1. For $\alpha < 1$, the proof is analogous with slightly more complicated expressions. $\qquad\square$

Proof of Proposition 7 When $K = I$, we have to form expectation over the possible realizations of z, y and the implied realizations of events. For any fixed partition, however,

$$E_{y,z}\left(D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))\right)$$

is an affine function of the the relative entropy of the primitive events, $E_{z,y}\left(D\left(f\left(s^i|y^i\right)||f\left(s^i|z^i\right)\right)\right)$. Given that this term is decreasing in q by Lemma A.1, so does $E_{y,z}\left(D_{\mathcal{E}}(f(\cdot|y)||f(\cdot|z))\right)$. $\qquad\square$

Proof of Proposition 8. We explained in the main text why in equilibrium a fraction of investors are acquiring information, their preferred choice from the offered securities is one with structure $c(E) = \frac{f(E|z)}{f(E|y)}$ written on K assets for which they received information and for which $n = 0$ and they pay a price which implies a surplus $x > 0$. That is, instead of the competitive price of 1, they are paying

$$p^{acq} = d^{zy}(K,0,q,0) - x$$

for the asset, where $d^{zy}(K, 0, q, 0)$ is their perceived utility from the offered asset defined in Lemma A.1. We also argued that the remaining fraction of investors do not acquire information, their preferred choice of offered securities is one with structure $c(E) = \frac{f(E)}{f(E|y)}$ written on K assets for which no investor receives information and they pay a price which implies zero surplus. That is, they are paying

$$p^{n.acq} = d^{zy}(K, 0, 0, 0)$$

for the asset. Here we show how the fraction of these investors and x is determined.

Given the above considerations, the issuer's problem is to choose x to maximize the resulting expected profit

$$\max_{x} \quad G(x)(d^{zy}(K, 0, q, 0) - x - 1) + (1 - G(x))(d^{zy}(K, 0, 0, 0) - 1),$$

where the first term is the expected profit from informed consumers, the second term is the expected profit from uninformed consumers. The first-order condition is

$$g(x)(d^{zy}(K, 0, q, 0) - x - 1) - G(x) - g(x)(d^{zy}(K, 0, 0, 0) - 1) = 0$$

which we can rewrite as (4).

Finally, we show formally that investors' incentive-compatibility conditions hold at this solution. This is obvious for informed investors: their perceived expected utility from choosing a security intended for them is $x > 0$, and their perceived expected utility from choosing a security intended for uninformed investors is 0. For uninformed investors, we have to show that perceived utility change from deviating is smaller than the implied price difference, that is,

$$\sum_{E \subseteq \mathcal{E}} f(E) \ln \frac{f(E|z)}{f(E|y)} - \sum_{E \subseteq \mathcal{E}} f(E) \ln \frac{f(E)}{f(E|y)} = \sum_{E \subseteq \mathcal{E}} f(E) \ln \frac{f(E|z)}{f(E)} \leq p^{acq} - p^{n.acq}.$$

Note that the left hand side is $-D_{\mathcal{E}}(f(E)\|f(E|z)) < 0$ as the securities offered to informed investors are too steep for the uninformed. In contrast, (4) implies $p^{acq} - p^{n.acq} = \frac{G(x)}{g(x)} > 0$, so that the above inequality holds. □

Proof of Proposition 9. In text. □

Proof of Proposition 10 To derive the equilibrium in this extension, we have to consider three subcases determining wether the first primitive asset is included in the underlying. First,

suppose that for a given y

$$\ln \sum_{s^1} f\left(s^1|y\right) \frac{v'\left(e\left(s^1\right) - R^f\right)}{v_0'\left(e_0 + 1\right)} - \sum_{s^1} f\left(s^1|y\right) \ln \frac{v'\left(e\left(s^1\right) - R^f\right)}{v_0'\left(e_0 + 1\right)} < d_0^{zy}\left(q\right) \tag{A.3}$$

holds where R^f, the risk-free rate, solves

$$R^f = \frac{1}{\sum_{s^1} f\left(s^1|y'\right) \frac{v'\left(e\left(s^1\right) - R^f\right)}{v_0'\left(e_0 + 1\right)}}$$

and $d_0^{zy}\left(q\right)$ is defined in Lemma A.1. In the competitive equilibrium investors observing information z receive contracts

$$c^1\left(E\right) = R^f \frac{f\left(E|z\right)}{f\left(E|y\right)}$$

for $E \in \mathcal{E}$ where $\mathcal{E}$ is the partition implied by the K combination of assets with the largest relative entropy given by z and y, just as in the baseline model. These K combinations do not include the first primitive asset. Given that $I = \infty$, each such K combination will have $n = k = 0$ and offered for the competitive price of 1. For example, each investor can receive a security

$$c\left(E\right) = R^f \frac{f\left(E|z = 1_K\right)}{f\left(E|y = 0_K\right)}$$

on a K combination of primitive assets for which $y = 0_K$, a null vector with length K, and $z = 1_K$, a vector of ones with length K. Given our iid assumption on primitive assets, for any event E in a partition implied by such K combination of assets, $f\left(E|y\right)$ measure of active contracts will pay $R^f \frac{f(E|z)}{f(E|y)}$. Hence, by summing over events, the net payment from professionals to investors (through intermediaries) is

$$\sum_{E} f\left(E|y\right) R^f \frac{f\left(E|z\right)}{f\left(E|y\right)} = R^f$$

w.p. 1. This implies that professionals consume $e\left(s^1\right) - R^f$ at date 1 and the pricing kernel is given by

$$\frac{v'\left(e\left(s^1\right) - R^f\right)}{v_0'\left(e_0 + 1\right)}$$

for a given realization of the first primitive asset. Note that the risk-free rate, professionals consumption, and the the pricing kernel is identical to the corresponding object in an economy

where investors can buy the risk-free asset only. However, by the argument in Corollary 1 the utility of investors is

$$\ln R^f - K d_0^{yz}(q) - 1 \tag{A.4}$$

which is smaller than in a saving only economy (by the second term). This is a competitive equilibrium as it leads to zero profit, and the traded contracts provide the largest perceived utility to investors among the potential underlyings not depending on the first primitive asset by the argument in Lemma **1** and Corollary 1. Condition (A.3) ensures that including the first primitive asset instead of any of the others would decrease perceived utility for the investor.

Consider now the subcase when (A.3)

$$\ln \sum_{s^1} f\left(s^1|y\right) \pi\left(s^1\right) - \sum_{s^1} f\left(s^1|y\right) \ln \pi\left(s^1\right) > d_0^{zy}(q) \tag{A.5}$$

holds where R^f, the risk-free rate, is

$$R^f = \frac{1}{\sum\limits_{s^1} f\left(s^1|y\right) \pi\left(s^1\right)}$$

with $\pi\left(s^1\right)$ solving

$$\pi\left(s^1\right) = \frac{v'\left(e\left(s^1\right) - \frac{1}{\pi(s^1)}\right)}{v_0'\left(e_0 + 1\right)} \tag{A.6}$$

Investors observing information z receive a contract

$$c\left(s^1, E\right) = \frac{f\left(E|z\right)}{f\left(E|y\right)\pi\left(s^1\right)}$$

for $E \in \mathcal{E}$ where $\mathcal{E}$ is the partition implied by $K - 1$ combination of assets not including the first asset implying the largest relative entropy given by z and y. (That is, the underlying of $c\left(s^1, E\right)$ is this partition augmented by the potential outcomes of the first primitive asset).The hedging cost of these contracts on the professional market hence the competitive price is also $\sum_E f\left(E|y\right) \pi\left(s^1\right) \frac{f(E|z)}{f(E|y)\pi(s^1)} = 1$. The implied utility for investors is

$$\ln \frac{1}{\pi\left(s^1\right)} - K d_0^{yz}(q) - 1. \tag{A.7}$$

By analogous arguments to the previous case, the implied net payment from professionals to the group of investors is $\frac{1}{\pi(s_1)}$. Therefore, the pricing kernel implied by the aggregate consumption of professionals is $\pi\left(s^1\right)$, consistently with the previous expressions. In this case, the

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consumption level of professionals, the pricing kernel and the risk-free rate is the same as the corresponding objects in an economy with first best risk-sharing. Condition (A.3) ensures that replacing the first primitive asset to any of the others would decrease perceived utility for the investor.[15]

In any of these subcases, given (A.4),(A.7), $\frac{\partial d_0^{yz}(q)}{\partial q} > 0$ from Lemma A.1 and the fact that K and q does not influence the consumption of professionals in equilibrium, social welfare is smaller when $K = K' + 1$ than in $K = K'$, and decreasing in q. Also, restricting the underlying to the aggregate state would imply the allocation of first best risk sharing (with investors' consumption $c\left(s^1\right) = \frac{1}{\pi(s^1)}$ where $\pi\left(s^1\right)$ is the pricing kernel given by (A.6)), clearly increasing the expected utility of all. □

Proof of Proposition 11 For Case 2, conjecture and verify that given the pair of signal realizations $(\bar{y}, \bar{z})$ which professionals and investors observe for the aggregate risk factor s^1, in equilibrium each investor choose contract and consumption

$$c(s_1|\bar{y}, \bar{z}) = \frac{f\left(s^1|z = \bar{z}\right)}{f\left(s^1|y = \bar{y}\right)\pi\left(s^1|\bar{y}, \bar{z}\right)}$$

where $\pi\left(s^1|\bar{y}, \bar{z}\right)$ is professionals' pricing kernel given implicitly by

$$\pi\left(s^1|\bar{y}, \bar{z}\right) = \frac{v'\left(e\left(s^1\right) - \frac{f\left(s^1|z=\bar{z}\right)}{f\left(s^1|y=\bar{y}\right)\pi\left(s^1|\bar{y},\bar{z}\right)}\right)}{v_0'\left(e_0 + 1\right)} \tag{A.8}$$

and R^f is the risk free rate given by

$$R^f = \frac{1}{\sum_{s^1} f\left(s^1|y = \bar{y}\right)\pi\left(s^1|\bar{y}, \bar{z}\right)}.$$

The argument for why this is a competitive equilibrium is analogous of Case 1. The implied expected utility of investors is

$$-E_{z,y}\left(D\left(\Pr\left(s^i|y^i\right)\|\Pr\left(s^i|z^i\right)\right)\right) + E_{z,y}\left(\ln\frac{1}{\pi\left(s^1|\bar{y}, \bar{z}\right)}\right)$$

which is insensitive to K, but increasing in q by Lemma A.1. □

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[15]If neither (A.5) nor (A.3) holds, then in equilibrium both $c\left(s^1, E\right)$ and $c\left(E\right)$ contracts are traded as defined above implying that the net position of professionals is a weighted average of $\frac{1}{\pi(s^1)}$ and $R^f = \frac{1}{\sum_{s^1} f\left(s^1|y'\right)\pi\left(s^1\right)}$, and the pricing kernel $\pi\left(s^1\right)$ is determined by this weighted average. The weight is such that the condition (A.5) implied by the new pricing kernel holds for equality. That is, investors will be indifferent between the optimal securities based on portfolios including and not including the aggregate asset. The proof of this case otherwise is analogous to the other cases.