

# Procrastination Markets\*

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## Abstract

We develop models of markets with procrastinating consumers where competition operates — or is supposed to operate — both through the initial selection of providers and through the possibility of switching providers. As in other work, consumers fail to switch to better options after signing up with a firm, so at that stage they exert little downward pressure on the prices they pay. Unlike in other work, however, consumers are not keen on starting with the best available offer, so price competition fails at this stage as well. In fact, a competition paradox results: an increase in the number of firms or the intensity of marketing increases the frequency with which a consumer receives switching offers, so it facilitates procrastination and thereby potentially raises prices. By implication, continuous changes in marketing costs can, through a self-reinforcing process, lead to discontinuous changes in market outcomes. Sign-up deals do not serve their classically hypothesized role of returning ex-post profits to consumers, and in some cases even exacerbate the failure of price competition. Consumer procrastination thus emerges as a novel source of competition failure that applies in situations where other theories of competition failure do not.

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# 1 Introduction

A large and growing literature recognizes that individuals procrastinate (i.e., repeatedly put off tasks they know they should perform) in a variety of settings, and the welfare consequences can be disastrous.<sup>1</sup> In markets, an important potential manifestation of procrastination occurs when a consumer fails to switch away from an unfavorable status-quo contract for a key service, such as energy (Hortaçsu et al., 2017, Ito et al., 2017, Office of Gas and Electricity Markets, 2018, 2019a), health insurance (Handel, 2013, Polyakova, 2016), auto insurance (Kiss, 2019), or a credit card (Shui and Ausubel, 2005, Galenianos and Gavazza, 2022). Our goal is to understand the implications of these consumer tendencies for the patterns and degree of competition between firms.

To obtain our insights, we develop and analyze models of markets with procrastinating consumers in which we allow competition to operate both through the initial selection of providers and through the possibility of switching providers. As a starting point, our model predicts that a consumer often fails to switch to better options after signing up with a firm, so at that stage competition for her is soft. This parallels lines of research on switching costs (e.g., Farrell and Klemperer, 2007) and naive consumers (e.g., DellaVigna and Malmendier, 2004), and is widely recognized by regulators (Competition & Markets Authority, 2016b, Canadian Radio-television and Telecommunications Commission, 2017, Financial Conduct Authority, 2017, 2018, Australian Competition & Consumer Commission, 2019). But existing theories — variously nicknamed bargains-and-ripoffs, invest-then-harvest, and the waterbed effect — imply that competition shifts to when consumers choose providers initially, and overall prices may well remain unchanged or even decrease (Taylor, 2003, Cabral, 2016, Ericson, 2020). In contrast, our model says that because a consumer expects to engage in optimal switching behavior in the future and is therefore not keen on starting with the best available offer, price competition at the initial stage is soft or non-existent as well. What is more, a competition paradox results: an increase in the number of firms or the intensity of marketing increases the frequency with which a consumer receives switching offers, so

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<sup>1</sup> We discuss the literature on procrastination in Section 2.2.

it facilitates procrastination and thereby potentially lowers competition and raises prices. By implication, marketing can be self-reinforcing, generating discontinuous jumps in prices in response to continuous changes in marketing costs. And sign-up deals, which in existing theories provide a vehicle for returning ex-post profits to consumers, serve this purpose extremely poorly, while in other senses they exacerbate the failure of competition.

We present our basic model in Section 2. On the demand side, a consumer needs a service from time 0 to time  $T$ , and pays for it per unit of time. At time  $-1$ , she is assigned to an initial offer, but she can switch providers at evenly spaced switching opportunities beginning at time 0. Exchanging her contract carries an immediate effort cost of  $s$ . Being present-biased and naive, the consumer discounts payments and future switching costs by a factor  $\beta < 1$ , but believes that she will not do the same in the future. Turning to the supply side, offers come from firms that understand consumer behavior and have a production cost of zero. The firms simultaneously choose non-negative price pairs consisting of an initial offer capped at  $v$  and a switching offer. From all initial offers, one is randomly chosen to be the consumer's initial offer above. The switching offers of the other firms are randomly allocated to the switching opportunities above, with the wait time between opportunities decreasing in the number of firms and the intensity of their marketing activities. We investigate symmetric pure-strategy Nash equilibria of the game played between the firms.

Our model captures, in a stylized way, the competitive landscape of some important service industries. In one application, the consumer is looking to make a purchase by borrowing a fixed amount that she will be able to repay at time  $T$ . Credit-card issuers make offers consisting of an interest rate on purchases as well as an interest rate on balance transfers. The consumer gets a card to make her purchase, but then receives many solicitations to transfer her balance to a competitor's card. In another application, the consumer is a new resident — or a continuing resident with an expiring contract — who would like to arrange for an essential service such as electricity or gas. She is initially assigned one provider — in the case of a continuing resident, her existing one — but she may have many opportunities to switch providers. Similar logic applies to other subscription services where switching is

possible.

We explore implications of our basic model in Section 3. To begin, we show that the consumer never switches, and she is charged the highest initial price that she sticks with if all switching offers are for a price of zero. This failure to switch means that at least one of two conditions is satisfied. First, under the “no-incentive-to-switch condition,” the consumer thinks that switching is not worth the cost  $s$ . Second, under the “incentive-to-procrastinate condition,” she thinks that switching at the next opportunity dominates switching immediately, so she perpetually procrastinates in switching. For few firms and low-intensity marketing, the no-incentive-to-switch condition determines the equilibrium price, and by standard logic the profit an initial firm can make is constrained by the switching cost augmented by the discount factor ( $s/\beta$ ). For more firms or high-intensity marketing, however, the incentive-to-procrastinate condition determines the equilibrium price, that price is higher, and the competition paradox obtains. The higher is the number of firms or the more intense is marketing, the more frequently the consumer receives switching offers, so the more tempted she is to procrastinate and therefore the higher is the price an initial firm can charge. In this region, a policy that increases competition at the switching stage may or may not be beneficial.

Having established that competition from switching offers cannot discipline markets, we ask whether initial competition can. To do so, we assume that instead of being assigned an initial price at time -1, the consumer selects from multiple initial offers. We show that if the ratio of firms reaching the consumer initially is sufficiently low, then an equilibrium with the same prices as above exists. Intuitively, the consumer would not care for small price cuts because she would reason that she will switch to a still better deal in the future. And instead of offering a deep price cut, a firm prefers to gamble that the consumer chooses it randomly, dampening initial competition.

The competition paradox can generate stark comparative statics when the intensity of marketing is endogenously chosen by firms subject to a marginal cost  $c$ . We show that for  $c$  above a cutoff, the no-incentive-to-switch condition determines the price, and the consumer

pays a total price of  $s/\beta$ ; but for  $c$  below the cutoff, the incentive-to-procrastinate condition is central, the consumer pays a total price of  $Tv$ , and the intensity of marketing each firm chooses is discontinuously higher. Intuitively, at the point where procrastination starts playing a role, the equilibrium price increases, which prompts more intense marketing. Due to the competition paradox, this raises prices further, creating a self-reinforcing mechanism that continues until prices reach the monopoly level.

In Section 4, we consider competition when firms use introductory or sign-up deals rather than permanently low prices to attract consumers. We assume that there are two firms, both firms make offers at time -1, and the consumer has many opportunities to switch to the offer she does not choose initially. With linear prices like in our basic model, these assumptions imply that the equilibrium outcome is competitive. Crucially, however, we modify the model by positing that a firm's offer consists of a temporary free period and a subsequent unit price, with the restriction that the consumer can only take advantage of a firm's sign-up deal once.

We establish that if the total value of the service ( $Tv$ ) is high relative to the switching cost  $s$ , then there are uncompetitive equilibria in which firms charge the monopoly unit price and offer a short introductory deal. This means that sign-up deals do not serve their classically hypothesized role of channeling initial competition and returning ex-post profits to consumers, but they do serve the novel role of stunting competition in the unit price. In equilibrium, the consumer wants to and hence plans to take advantage of both introductory periods, but she procrastinates on switching to the non-chosen offer. If a firm cuts its unit price, the consumer still plans to take advantage of both introductory periods, and now also plans to be on the cheaper unit price for the rest of the time. Because she can achieve this by starting with the more expensive offer, however, she does not care to take the better offer immediately. Similarly, if a firm extends its introductory period, the consumer still plans to take advantage of both deals, and — expecting the same total free period in either case — she does not care which offer she starts with. Furthermore, neither improvement in a firm's offer can induce the procrastinating consumer to switch, so it gives the firm no advantage at all.

In Section 5, we argue that the mechanism limiting competition is robust to alternative psychological assumptions behind procrastination. If the consumer is partially rather than fully naive as modeled by O’Donoghue and Rabin (2001), then prices often remain unchanged. And procrastination due to underestimation of future switching costs (e.g., Tasoff and Letzler, 2014) or overconfidence about memory (e.g., Ericson, 2011) gives rise to the same basic predictions, although these phenomena may drive some subtle specific pricing patterns.

In the uncompetitive equilibria of our markets with undifferentiated products, an indifferent consumer does not respond to better offers. While this may give the results an appearance of fragility, it is exactly our point: procrastinating consumers do not care for better offers, and when consumers do not care for better offers, competition cannot operate reliably. Some factors, such as a consumer heuristic to opt for cheaper deals in these situations, can break indifference in a way that creates competition between perfect substitutes. But such competition is in turn eliminated by minute amounts of differentiation. More generally, we demonstrate in the Appendix that the logic behind competition failure continues to operate in many variants of our model featuring undifferentiated or slightly differentiated products. From this perspective, we interpret our results as saying not that procrastination necessarily prevents competition between undifferentiated products, but that procrastination drastically increases competition frictions.

In Section 6, we discuss related literature. While there are other mechanisms for competition failure, our theory based on procrastination in switching provides a novel mechanism that applies in environments where previous ones — including those based on limitations on sign-up deals, adverse selection, or the inability of consumers to understand or compare prices — do not. Hence, our theory helps explain important examples of high prices for which existing explanations do not give a complete account. For credit cards and utilities, for instance, firms can commit to future prices, and they already offer sign-up deals in which they set low prices for a limited period. It is not obvious why profitable consumers would be relatively unresponsive to these deals (which is required by models of adverse selection)

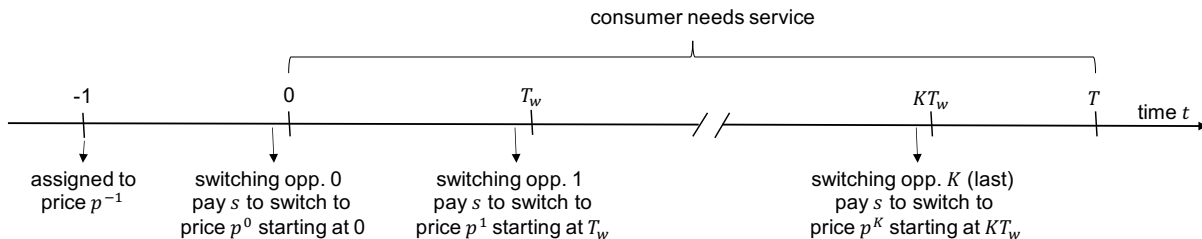


Figure 1: Illustration of Consumer's Problem

or that consumers do not understand them (which is required by models of consumer confusion). Hence, previous research counterfactually predicts that firms would compete more fiercely by extending these deals for longer.

We conclude in Section 7 with some questions for further research. While our model explains broad features of some markets with switching offers, we have not explored how policy should respond to the problems we identify. We suspect that disciplining markets by improving consumer switching behavior is exceedingly difficult, so systems that work without active consumer engagement may be necessary.

## 2 Basic Model of Switching Markets

We first model “switching markets” — i.e., markets for recurring services where switching providers is possible but costly — in which an offer takes the simplest possible form, a linear price.

### 2.1 Setup

*Consumer's Problem (Figure 1).* From time 0 until time  $T$ , the consumer needs a service that she pays for per unit of time. At time  $-1$ , she is automatically assigned to a price  $p^{-1}$ . Shortly before times  $0, T_w, 2T_w, \dots, KT_w < T$ , she receives exploding switching offers for prices  $p^0, p^1, p^2, \dots, p^K$ , respectively, all of which are fixed and observable from the beginning. If she takes up offer  $\kappa \in \{0, \dots, K\}$ , then she pays an instantaneous effort cost

$s > 0$ , and starting subsequently at time  $\kappa T_w$  she pays price  $p^\kappa$  instead of the last price she accepted or was assigned to. We call  $T_w > 0$  the wait time between switching opportunities, and let  $K = \lceil T/T_w \rceil - 1$ ,<sup>2</sup> this ensures that the last contract period,  $T - KT_w$ , is no longer than  $T_w$  either.

*Consumer Behavior.* The consumer is present-biased as modeled by Laibson (1997) and naive as defined by O’Donoghue and Rabin (1999a). To specify her behavior formally, let her switching decision at opportunity  $\kappa$  be  $d_\kappa \in \{0, 1\}$ , where  $d_\kappa = 1$  stands for switching, and let the *total* price she pays between switching opportunities  $\kappa$  and  $\kappa + 1$  (or, for  $\kappa = K$ , the total price she pays after opportunity  $\kappa$ ) be  $p_\kappa$ .<sup>3</sup> At each opportunity  $\kappa$ , the consumer aims to solve

$$\min_{d_\kappa, \dots, d_K} d_\kappa s + \beta \cdot \sum_{\kappa'=\kappa+1}^K d_{\kappa'} s + \beta \cdot \sum_{\kappa'=\kappa}^K p_{\kappa'}, \quad (1)$$

where  $\beta \in (0, 1]$  is her short-run discount factor. Naively, the consumer believes that whatever plan she makes today, she will carry out in the future. Hence, at each decision point  $\kappa = 0, 1, \dots, K$  she solves problem (1) and chooses the  $d_\kappa$  in her solution. We impose the tie-breaking rule that the consumer switches only if she strictly prefers to.

*Game between Firms.* The offers the consumer receives come from  $N \geq 2$  firms that understand consumer behavior and can provide the service at a cost of zero. Firms simultaneously choose price pairs  $(p_I^n, p_S^n)$ , where  $p_I^n \in [0, v]$  is firm  $n$ ’s initial offer and  $p_S^n \in [0, v]$  is firm  $n$ ’s switching offer. Then, one initial offer  $p_I^n$  is randomly (and with equal probability) chosen to be the initial offer  $p^{-1}$  above. For each switching opportunity, one switching offer  $p_S^{n'}$  is randomly (and with equal probability) chosen from the competitors  $n' \neq n$  of the initial firm; these offers remain unchanged if the consumer switches. The consumer’s wait time is  $T_w = T/(m(N - 1))$ , where  $m > 0$  is a firm’s “marketing intensity,” or the frequency with which its offers reach consumers. Here, we take  $m$  to be exogenous, but we endogenize it in Section 3.2. We investigate symmetric pure-strategy Nash equilibria of the game played

<sup>2</sup> For  $x \in \mathbb{R}$ ,  $\lceil x \rceil$  denotes the smallest integer greater than or equal to  $x$ .

<sup>3</sup> Precisely, we consider the cases (i)  $\kappa \leq K - 1$  and (ii)  $\kappa = K$  separately. In case (i), if  $d_{\kappa'} = 0$  for all  $\kappa' \leq \kappa$ , then  $p_\kappa = T_w \cdot p^{-1}$ ; otherwise,  $p_\kappa = T_w \cdot p^j$  for  $j = \arg \max_{\kappa' \leq \kappa} d_{\kappa'} = 1$ . In case (ii), if  $d_{\kappa'} = 0$  for all  $\kappa' \leq K$ , then  $p_K = (T - KT_w) \cdot p^{-1}$ ; otherwise,  $p_K = (T - KT_w) \cdot p^j$  for  $j = \arg \max_{\kappa' \leq K} d_{\kappa'} = 1$ .



between the firms.

## 2.2 Applications and Discussion of Assumptions

Our simple model captures the essential elements of many switching markets. As a first example, consider credit cards. The consumer needs to buy a good on credit that she can pay off at time  $T$ . She has access to one credit card on which she can charge the purchase, but she receives balance-transfer offers from other issuers. Each issuer can specify the interest rate on its card.

Another example is utilities. The consumer moves into a new home and seeks an essential service such as electricity, gas, landline, or broadband. When she moves in, she is randomly assigned an initial supplier. She is, however, in a liberalized market with a number of alternative suppliers, and these suppliers make offers to her to switch. Alternatively, our model applies when the consumer is not moving, but her current contract period is expiring and therefore her provider can change its price. The consumer can switch away before time 0 and receive a competitor’s contract, do nothing and just get the current provider’s new contract, or go with her current provider for a while and switch later.

Consistent with our distinction between the initial price  $p_I^n$  and the switching price  $p_S^n$ , in many applications it is plausible that firms distinguish between new consumers and switching consumers. In the credit-card context, issuers regularly set different terms for purchases and balance transfers. In subscription markets, providers often charge “loyalty penalties” — different prices for continuing (loyal) and new consumers.<sup>4</sup> If instead firms cannot distinguish the two consumer types, then in our basic model intractable mixed-strategy equilibria arise, but the economic mechanisms we identify do not disappear. Substantiating this view, in Appendix A we identify a variant of our model in which the indistinguishability of the two

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<sup>4</sup> For instance, the Competition & Markets Authority (2018) discusses evidence for loyalty penalties in the UK markets for retail domestic energy, home and motor insurance, broadband, mobile tariffs, cash savings, and mortgages, and suggests that loyalty penalties may also arise in other auto-renewal, roll-over or subscription products or services. We can capture loyalty penalties by thinking of  $p_I^n$  and  $p_S^n$  as the prices that firm  $n$  offers to continuing and new consumers, respectively, with a consumer only paying a switching cost if she switches providers.

consumer types does not affect outcomes.

Although below we study competition in initial offers, as a start we assume that there is only one offer the consumer can accept for free. This assumption allows us to identify outcomes when competition derives solely from the possibility of switching. It is also plausible if the consumer is — as in the case of utilities — defaulted into a contract if she does nothing, while she must exert effort to find alternative providers.

Our specification of consumer behavior in terms of naive present bias is motivated both by direct evidence on discounting and beliefs,<sup>5</sup> and by indirect evidence on behavior consistent with the framework. Most importantly, there is overwhelming evidence from both academic and policy circles that consumers often fail to switch to more favorable options in the markets for energy (Competition & Markets Authority, 2016a, Hortacısu et al., 2017, Ito et al., 2017), health insurance (Handel, 2013, Handel and Kolstad, 2015, Polyakova, 2016), credit cards (Shui and Ausubel, 2005, Stango and Zinman, 2015, Galenianos and Gavazza, 2022), cable/satellite TV (Shcherbakov, 2016), mobile-phone subscriptions (Shy, 2002), auto insurance (Kiss, 2019), and mortgages (Keys et al., 2016, Andersen et al., 2020), and attempts to make switching costs very low do not result in high switching rates (Office of Gas and Electricity Markets, 2019a,b). Since a model of rational consumers requires unrealistically high switching costs to generate such low switching rates, it is inconsistent with this evidence.<sup>6</sup> But because models of naive present bias — including those of O’Donoghue and

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<sup>5</sup> In particular, papers by DellaVigna and Paserman (2005), DellaVigna and Malmendier (2006), Paserman (2008), Fang and Silverman (2009), Meier and Sprenger (2010), Carter et al. (2019), and Laibson et al. (2020) document a taste for immediate gratification, while Skiba and Tobacman (2008), Acland and Levy (2015), Fang and Wang (2015), Fedyk (2021), Augenblick and Rabin (2019), Chaloupka et al. (2019), Carrera et al. (2022), Bai et al. (forthcoming), and Kuchler and Pagel (2021) find that individuals are at least partially naive about this taste. On the other hand, Allcott et al. (2022) find that only the least experienced quartile of payday-loan borrowers underestimate their likelihood of future borrowing, while others predict future borrowing correctly on average, suggesting that a model of sophisticated rather than naive present bias better describes these borrowers.

<sup>6</sup> For instance, Handel (2013) estimates that inertia in health-insurance choices costs employees \$2,032 on average; Kiss (2019) estimates that in a standard setting, a switching cost of \$373 is necessary to explain Hungarian drivers’ failure to switch auto-insurance providers; Galenianos and Gavazza (2022) estimate that borrowers’ average cost of examining *one* credit-card offer they receive starts at \$200 and is increasing in the number of examined offers; and Andersen et al. (2020) estimate a psychological mortgage-refinancing cost of \$1,716. Relatedly, Chetty et al. (2014) document that defaults have large effects on the retirement savings of Danish households, and estimate that at least 85% are passive in that they do not adjust their own contributions in response to changes in automatic contributions.

Rabin (1999b,c, 2001, 2008) and, as we explain below, ours — often predict severely costly procrastination, they can account for the failure to switch with much lower switching costs, so they are consistent with the evidence. Strengthening this interpretation, Blumenstock et al. (2018) find that present bias is a more important contributor to the failure to switch than several other hypotheses in the literature. Furthermore, in Section 5.2 we argue that alternative models of procrastination can (within the settings we study) be seen as reinterpretations of our present-bias-based model, so they generate equivalent or similar predictions.

Firms' marketing intensity  $m$  is most straightforwardly interpreted as direct marketing aimed at individual consumers. As a simple example,  $m$  could be the number of mail credit-card solicitations an issuer sends, with the envelope the consumer happens to receive initially and at a switching opportunity randomly determined.<sup>7</sup> More generally,  $m$  can capture any marketing or advertising that attracts the consumer's attention and induces her to consider the firm for her initial purchase or for switching. Such alternatives could include online solicitations and marketing as well as general advertising. In this interpretation,  $T_w$  does not necessarily correspond to the wait time between exploding offers, but could capture the time until the consumer next remembers or finds it convenient to consider switching. Then, there may be a non-trivial wait time even if offers are not exploding.

Our basic model exogenously assumes that total marketing intensity ( $mN$ ) and total marketing intensity for switching ( $m(N-1)$ ) are increasing in the number of firms ( $N$ ). This property will also arise endogenously when firms choose  $m$  strategically (Section 3.2). More subtly, our model assumes that when a firm markets more intensely to initial consumers, it also markets more intensely to switchers. In some cases, this is true by the nature of the market; e.g., a credit-card solicitation specifies interest rates for both purchases and balance transfers. More generally, it is plausible that marketing activities aimed at initial purchases also induce consumers to consider the firm for switching.

The main reason for our restrictive solution concept, symmetric pure-strategy equilib-

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<sup>7</sup> Then, since there are  $m(N-1)$  switching offers that the initial firm's competitors send in total, there are  $m(N-1)$  switching opportunities, so — assuming the opportunities are distributed evenly before time  $T$  — the consumer's wait time is exactly  $T_w = T/(m(N-1))$ .

rium, as well as the unrealistic assumption that the consumer observes all prices at the beginning, is tractability. We are unaware of general analytic methods that would enable us to solve for the behavior of a present-biased consumer facing an arbitrary sequence of prices. Based on this and our own attempts, mixed-strategy equilibria appear intractable, and even the analysis of asymmetric pure-strategy equilibria involves many tedious and non-transparent case distinctions.<sup>8</sup>

### 3 A Paradox and a Failure of Competition

#### 3.1 Basic Effect

To illustrate the logic of equilibrium, we solve for the maximum initial price  $p_I$  that the consumer sticks with if all firms choose a switching price  $p_S^n = 0$ . For the consumer not to switch away at opportunity 0, at least one of two conditions must be satisfied. First, she may prefer never switching to switching at opportunity 0. Second, she may prefer switching at opportunity 1 to switching at opportunity 0, so that she delays switching — she procrastinates — naively thinking that she will switch next time. We analyze each condition in turn.

*No Incentive to Switch.* Switching at opportunity 0 rather than never saves  $\beta T p_I$  in discounted future payments, and incurs immediate cost  $s$ . Hence, the consumer prefers never switching if  $s \geq \beta p_I T$ , or

$$p_I \leq p^{NIS} \equiv \frac{1}{\beta} \cdot \frac{s}{T}. \quad (\text{NIS})$$

Note that if Condition (NIS) is satisfied, then — given that the saving from switching is highest at opportunity 0 — the consumer never benefits from switching, so she never switches.

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<sup>8</sup> We conjecture, however, that the equilibrium outcome we identify does emerge as the unique pure-strategy equilibrium in some natural modifications of our model that simplify consumer or firm behavior. This is the case if at any switching opportunity the consumer can take up any earlier offer, so that her decisions cannot be driven by the risk of losing a good offer at hand. The same is the case if multiple firms make switching offers at each opportunity and there is an (arbitrarily small) share of time-consistent consumers, so that perfect competition for switchers necessarily results.

*Incentive to Procrastinate.* If the consumer switches at opportunity 0, she must pay an immediate effort cost of  $s$ . If she switches at opportunity 1, then she must pay  $p_I$  until then, lowering her discounted utility by  $\beta p_I T_w$ ; and she must pay the switching cost then, lowering her discounted utility by  $\beta s$ . Hence, she prefers switching at opportunity 1 if  $s \geq \beta p_I T_w + \beta s$ , or

$$p_I \leq p^{IP} \equiv \frac{1 - \beta}{\beta} \cdot \frac{1}{T_w} \cdot s = \frac{1 - \beta}{\beta} \cdot m(N - 1) \cdot \frac{s}{T}. \quad (\text{IP})$$

Note that if Condition (IP) holds, then — by the same logic — the consumer procrastinates on switching at all opportunities before the last one. And because Condition (IP) implies that  $s > T_w \beta p_I \geq (T - K T_w) \beta p_I$ , the consumer strictly prefers not to switch at the last opportunity, so she never switches.

We now argue that all firms charging  $p_I^n = p_I^* = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$  and  $p_S^n = p_S^* = 0$  is an equilibrium. The initial firm keeps the consumer forever if  $p_I^n \leq p_I^*$  and loses the consumer immediately if  $v \geq p_I^n > p_I^*$ , so firms have no incentive to deviate on the initial price. And since the initial price is  $p_I^*$ , attracting the consumer at a switching opportunity is impossible for any  $p_S^n \geq 0$ , so firms have no incentive to deviate on the switching price either. In fact, this fully describes the possible equilibrium outcomes:

**Proposition 1** (Equilibrium Outcomes).

- (i) *There is an equilibrium in which  $p_I^n = p_I^* = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$  and  $p_S^n = p_S^* = 0$ .*
- (ii) *In any equilibrium,  $p_I^n = p_I^*$ , and the consumer never switches.*

We illustrate the key implications of Proposition 1 under the assumption that  $v$  is large, which ensures that all the cases below exist. First, suppose that  $\beta = 1$ , i.e., the consumer is a classical time-consistent decisionmaker. Then,  $v > p^{NIS} > p^{IP} = 0$ , so  $p_I^* = p^{NIS}$ , and the consumer pays a total price of  $T p^{NIS} = s$ . In other words, the consumer's switching cost determines the initial firm's market power.

Now suppose that  $\beta < 1$ , i.e., the consumer is time-inconsistent. If  $m(N - 1)$  is small, then  $v > p^{NIS} > p^{IP}$ , so  $p_I^* = p^{NIS}$ , and the consumer pays a total price of  $T p^{NIS} = s/\beta$ . While  $\beta$  enters the formula, a logic similar to that in the time-consistent case is operational:

the switching cost, now augmented by the consumer’s discount factor, determines the initial firm’s market power.

If  $m(N - 1)$  is larger, then  $v > p^{IP} > p^{NIS}$ , so  $p_I^* = p^{IP}$ , and the consumer pays a total price of  $Tp^{IP} = (1 - \beta)m(N - 1)s/\beta$ . This total price is greater than, and moves more than one-to-one with, the switching cost augmented by the discount factor. Furthermore, in what we call the competition paradox, the total price is increasing in the number of firms and the intensity of marketing. With either increase in competition, the consumer receives offers more often, so she can get out of a high-price deal sooner in the future. As a result, she is more prone to procrastination, allowing the initial firm to charge a higher price. Taking this logic to its conclusion, if  $m(N - 1)$  is sufficiently large, then the consumer pays the monopoly price  $v$ .

For existing estimates of  $\beta$  (Augenblick et al., 2015, Laibson et al., 2020, Augenblick and Rabin, 2019, Chaloupka et al., 2019), the total price the consumer pays in the first region above with low  $m(N - 1)$  is between  $s$  and  $2s$ . In the second region with high  $m(N - 1)$ , however, the total price can be an arbitrarily large multiple of  $s$ . Hence, our model is consistent with evidence discussed in Section 2.2 that many consumers fail to switch for price savings that are far larger than any realistic switching cost. The same evidence also says that the markets in question must be in the expensive second region, in which our novel effects obtain.<sup>9</sup>

Due to the competition paradox, policies aimed at increasing competition at the switching stage — typical in the regulation of switching markets<sup>10</sup> — have ambiguous effects on

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<sup>9</sup> While in our model the frequency of switching opportunities is increasing in  $m(N - 1)$ , in some markets the frequency is set by government regulation. This does not affect the main prediction that consumer procrastination can lead to high prices, but in our framework it invalidates the comparative-static prediction that prices increase in the number of firms or the intensity of marketing. In natural variants of our model, however, the same comparative static re-emerges even with a fixed frequency of switching opportunities. If products are differentiated, for instance, an increase in the number of competitors means that the consumer can find a better product for her tastes or needs next time. And if — as we discuss briefly in Section 5.2 — the consumer has imperfect memory, then an increase in the number of marketing messages may mean that she expects more reminders to switch next time. In either case, she sees less of a cost in delaying, again exacerbating her procrastination and thereby increasing prices.

<sup>10</sup> For example, the Office of Gas and Electricity Markets (2019a) investigates in a number of trials how to best activate consumers to consider alternative contracts. Similarly, the Competition & Markets Authority (2018) is concerned about cases in which firms make switching unnecessarily costly by charging high exit fees,

consumer welfare. As an example, consider the introduction of an online tool that helps a consumer find and switch to competing offers. On the one hand, such a tool can lower the switching cost  $s$ . Unless  $p_I^* = v$ , this indeed must lower the price the consumer pays. On the other hand, by providing easy access to switching offers, the tool might lower the wait time  $T_w$  between switching opportunities. This exacerbates the consumer's tendency to procrastinate, so it can increase prices.

We now turn to whether competition at the initial stage lowers prices. We suppose that at time  $-1$ , the consumer chooses between the initial offers of  $N_I \geq 2$  firms selected randomly and with equal probability, and the switching offers of the other  $N_S = N - N_I \geq 1$  firms are allocated to the switching opportunities as above. We define  $p^{IP}$  as in Equation (IP), but with  $N - 1$  replaced by  $N_S$ .

**Proposition 2** (Initial Competition).

(i) If  $\beta = 1$ , then in any equilibrium,  $p_I^n = 0$  and the consumer never switches.

(ii) If  $v > p^{IP} > p^{NIS}$  and

$$\frac{N_I}{N_S} < \frac{(1 - \beta)m}{\beta}, \tag{2}$$

then there is an equilibrium in which all firms offer  $(p_I^n, p_S^n) = (p^{IP}, 0)$ , and an equilibrium in which all firms offer  $(p_I^n, p_S^n) = (0, 0)$ .

As a benchmark, Part (i) confirms that with classical consumers, the competitive outcome obtains. But Part (ii) says that with procrastinators, initial competition may not affect prices at all: if  $\beta < 1$ , the market is in the competition-paradox region, and the number of firms making initial offers is sufficiently small relative to the number of firms making switching offers, then there is an equilibrium with the same outcomes as in Proposition 1. Intuitively, the consumer reasons that she will switch away from any high price, so she does not care about small differences between expensive offers. Then, assuming for instance that she chooses randomly when indifferent, a firm has no incentive to slightly undercut its expensive competitors. Perversely, therefore, in this region competition does not operate

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requiring customers to repeatedly contact the provider, and requiring cancelation through lengthy phonecalls (while sign-up is much easier).

exactly because prices are too high. A lower price is strictly preferred by the consumer only if it obviates the need for switching (i.e., if  $Tp_I \leq s$ ). But such a deep price cut is unprofitable: due to Condition (2), a firm would rather charge  $p_I^*$  and gamble that the consumer chooses it randomly.

While initial competition may be completely ineffective, it may also result in marginal-cost pricing. This multiplicity of equilibria highlights an interesting dichotomy in procrastination markets. If initial prices are low, then the consumer does not plan to switch away from the offer she first takes. She therefore looks for the best initial offer, creating competitive pressure that results in marginal-cost pricing. But if initial prices are high, then the consumer does plan to switch away from the offer she first takes. She therefore does not care about initial prices, and competition in initial prices — which, because she does not switch away, she ultimately pays — is eliminated.<sup>11</sup>

To our knowledge, no previous model generates a failure of competition in the kind of straightforward market setting as ours: where homogeneous consumers who know they will be using a service meet undifferentiated providers who commit to observable linear prices. In models of switching costs with classical consumers, outcomes are competitive if firms commit to a single linear price and consumers choose between multiple offers when signing up. In models with boundedly rational consumers, a failure of competition in linear prices can only occur if consumers are unable to compare prices or believe they will not use the service (see Section 6 for more details).

For presentational purposes, our model demonstrates competition failure for undifferentiated products. As we have emphasized, however, the main message is not that this extreme form of competition failure is robust, but that competition is extremely non-robust

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<sup>11</sup> Imposing that the consumer chooses initial offers she is indifferent between with equal probability selects the equilibrium outcomes that we focus on in the proposition: then, in any equilibrium  $p_I^n \in \{0, p^{IP}\}$ , and the consumer never switches. Imposing only that the consumer's choice when indifferent does not depend on the initial firms' switching prices (which she will never face) implies that in any equilibrium  $p_I^n \in \{0\} \cup [\max\{\frac{s}{T}N_I, p^{NIS}\}, p^{IP}]$ , and the consumer never switches. As before, when initial prices are low enough, the consumer anticipates not switching and hence selects the cheapest initial offer, leading to a competitive outcome. Once prices are high enough, however, the consumer incorrectly foresees switching at  $\kappa = 0$ , and hence is indifferent between high-priced offers. Using this indifference to incentivize firms, we can sustain a range of high-enough initial prices.



to frictions. To substantiate this point, in Appendix A.1 we lay out several variants of our basic model. We show that the logic of Proposition 2 continues to hold either completely unchanged, or with the additional modification that the consumer sees a little bit of differentiation between initial offers. She may, for instance, have a slight convenience benefit from signing up with a particular firm because she has its offer at hand and must look for the other offers, because the firm is more conveniently located, because its registration process seems easier, or because it is her current provider or the default firm assigned to her. Alternatively, she may have a momentary preference for the product, such as when she favors an airline credit card because she is planning a holiday. Even under such differentiation, with time-consistent consumers firms make a profit of at most  $s$ . With naive time-inconsistent consumers, however, firms’ profits can be an arbitrarily large multiple of  $s$ . In particular, such competition failure keeps occurring under plausible conditions in situations where

1. the consumer receives multiple switching offers at each switching opportunity;
2. Condition (2) is not satisfied;
3. the consumer must pay the initial price for a while before she can switch;
4. a share of rational time-consistent consumers (for whom  $\beta = 1$ ) is present; or
5. firms cannot price discriminate between initial consumers and switchers.

Our basic model fits the US credit-card industry in the 1980’s, before many features familiar today were invented. At that time, there were no teaser periods, and annual fees appear to have been largely standardized at a level of \$20, creating an industry with “mass-marketed, straightforward loans.”<sup>12</sup> In particular, a credit-card contract was described largely by a single transparent price — the interest rate — so offers were easy for consumers to compare. Furthermore, many credit-card issuers solicited business nationwide, including for balance transfers (Ausubel, 1991), so most consumers had access to multiple offers. Finally, many

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<sup>12</sup> See Robin Stein (2004), “The Ascendancy of the Credit Card Industry,” Secret History of the Credit Card series, PBS/Frontline. Available at <https://www.pbs.org/wgbh/pages/frontline/shows/credit/more/rise.html>, accessed July 15, 2021.

consumers having borrowed or about to borrow non-trivial sums must have known that they will have outstanding balances for a while, so they knew they would be using the service. In this environment, existing theories predict that a large share of profitable consumers respond to price cuts, so competition ensues. Instead, and consistent with our predictions, interest rates were still very high (Ausubel, 1991).

### 3.2 Endogenous Marketing Intensity

A crucial primitive of our basic model is firms' marketing intensity  $m$ . We now endogenize  $m$  by modifying the model in the following ways. Simultaneously with its pricing decision, firm  $n$  chooses its marketing intensity  $m^n > 0$  at constant marginal cost  $c > 0$ . These marketing decisions determine the consumer's wait time between offers and a firm's chances of reaching the consumer at each decision node. Specifically, firm  $n$ 's initial offer is assigned to the consumer at time  $-1$  with probability  $m^n / (M^{-n} + m^n)$ , where  $M^{-n} = \sum_{n' \neq n} m^{n'}$ . If firm  $n$  is selected initially, then the wait time is  $T_w = T / M^{-n}$ , and at each switching opportunity, firm  $n' \neq n$  makes the switching offer with probability  $m^{n'} / M^{-n}$ .<sup>13</sup> We assume that  $Tv > s/\beta$ ; otherwise, the initial price is always equal to  $v$ . We look for equilibria in which  $p_S^n = 0$  for all  $n$ .

**Proposition 3** (Endogenous Marketing Intensity). *Suppose that  $Tv > s/\beta$ , and let*

$$\bar{c} = \frac{(N-1)^2(1-\beta)s}{N^2\beta}.$$

(i) *If  $c > \bar{c}$ , then the unique equilibrium with  $p_S^n = 0$  has*

$$p_I^n = \frac{s}{\beta T} \quad \text{and} \quad m^n = \frac{(N-1)s}{N^2\beta c}.$$

(ii) *If  $c < \bar{c}$ , then the unique equilibrium with  $p_S^n = 0$  has*

$$p_I^n = v \quad \text{and} \quad m^n = \frac{(N-1)Tv}{N^2c}.$$

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<sup>13</sup> These assumptions imply that the number of switching opportunities is  $K+1 \approx T/T_w = M^{-n}$ , so that independently of other firms' behavior, the expected number of times firm  $n'$  makes a switching offer is approximately  $M^{-n} \cdot (m^{n'} / M^{-n}) = m^{n'}$ .

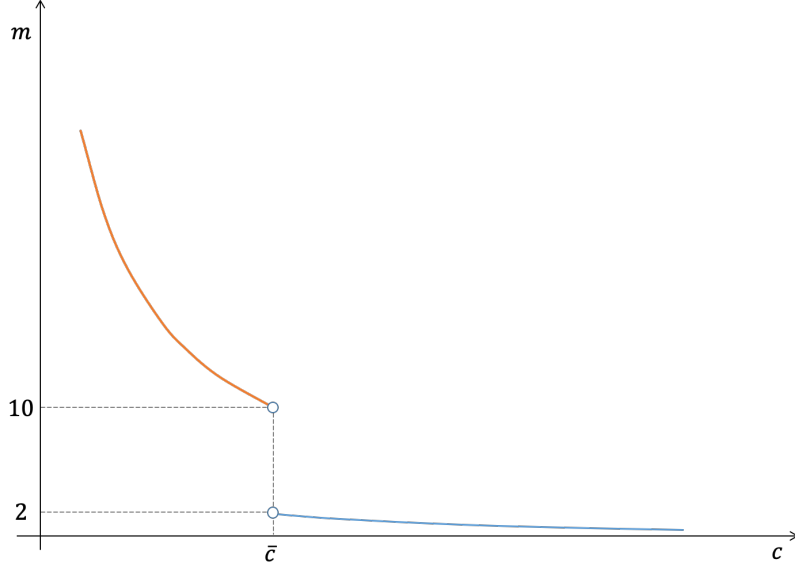


Figure 2: The Number of Offers ( $m$ ) as a Function of the Cost ( $c$ )

Parameters:  $\beta = 0.5$ ,  $N = 2$ ,  $s = 1$ ,  $v = 0.1$ ,  $T = 100$ . A decrease in  $c$  in the high range leads to a slow, steady increase in  $m$ . At the critical  $\bar{c}$ ,  $m$  jumps from 2 to 10. Further decreases in  $c$  lead to continuous, but steep increases in  $m$ .

To understand Proposition 3, we describe industry dynamics as the cost  $c$  decreases, and illustrate this in Figure 2. When  $c$  is high, firms send few offers, so the no-incentive-to-switch condition determines the initial price. Hence, initial prices remain constant at  $p_I^n = s/(\beta T)$ , and a decrease in  $c$  simply leads each firm to send more offers. When  $c$  drops below the threshold  $\bar{c}$ , however, the price and the number of offers each firm sends both jump discontinuously. At this point, there are so many offers that consumer procrastination becomes relevant, and therefore firms can raise their initial prices. This leads each firm to send yet more offers, allowing all of them to increase their initial prices further. The self-reinforcing process operates until initial prices reach  $v$ .<sup>14</sup>

It is worth comparing a firm's profit in this model to that in more standard models. From

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<sup>14</sup> A straightforward implication of Proposition 3 is that in equilibrium,  $m^n N$  and  $m^n (N-1)$  are increasing in  $N$ , confirming the exogenous assumption of our basic model that the consumer's wait time is decreasing in  $N$ . With more firms, each firm has a lower market share. This implies that when a firm increases its marketing effort, it is less likely to cannibalize its own pre-existing marketing, and more likely to steal the consumer from the competition. Even holding prices constant, therefore, the total incentive for marketing is greater. In addition, an increase in  $N$  raises  $\bar{c}$ , which may result in a jump in prices and hence a further increase in marketing intensity.

Proposition 3, the expected profit of a firm is

$$\frac{1}{N} \cdot Tp_I^n - cm^n = \frac{1}{N} \cdot Tp_I^n - \frac{(N-1)Tp_I^n}{N^2} = \frac{Tp_I^n}{N^2}. \quad (3)$$

This implies that even in the second region above, where the price consumers pay per period is constant at  $v$ , firms' profits are declining with the number of firms at the rate  $1/N^2$ . Arguably, therefore, firms do not earn larger net profits than in many other settings. For instance, in the classic model of Salop (1979), profits are also proportional to  $1/N^2$ . But unlike in the Salop model, where entry leads to stiffer price competition that benefits consumers, here competition works through trying to get to consumers first, which does not benefit consumers at all.

The prediction that firms burn the profits from high prices with heavy expenditure on marketing is consistent with the widely recognized fact that credit-card issuers spend a tremendous amount on marketing relative to other firms (e.g., Evans and Schmalensee, 2005).<sup>15</sup> While other models are consistent with high prices and heavy marketing, they typically require multiple disparate assumptions to explain why firms compete hard in the latter but not in the former. In particular, if high prices result from strong brand preferences, then unless it influences brand preferences, marketing should not be very effective in attracting consumers. If high prices result from adverse selection, then marketing is heavy only if the consumers it attracts are not similarly adversely selected. And if high prices are due to consumers' inability to compare prices, then it is not immediately clear why consumers would respond to marketing. In our model, the two parts of the story derive from a single basic assumption and are complements: high prices not only cause, but are also caused by, heavy marketing.

The discontinuity in market outcomes in Proposition 3 is not robust to plausible modifications of our model. For instance, if marketing has increasing marginal cost or  $c$  is heterogeneous, then the discontinuity often disappears. Nevertheless, the result crisply illus-

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<sup>15</sup> For instance, despite a heavy shift toward online marketing, issuers sent 341 million direct mail solicitations *per month* in 2017-2018 (Consumer Financial Protection Bureau, 2019, page 73). And to take a specific issuer, American Express spent \$2.9 billion on marketing and advertising in 2019, which is 10.5% of its total interest and non-interest income (FDIC Consolidated Report, December 31, 2019).

trates the self-reinforcing nature of marketing, which is a general feature of our framework’s logic.

## 4 Sign-Up Deals

Received wisdom says that introductory offers — common in many switching markets — are pro-competitive practices, and one primary channel through which firms return ex-post profits to consumers.<sup>16</sup> Loose logic might suggest that by counteracting the immediate effect of the switching cost, such offers serve an especially beneficial role for procrastinating consumers. We evaluate these conclusions in our framework.

**Setup** We start from our basic model with exogenous  $m$ , focusing for simplicity on a duopoly ( $N = 2$ ). We assume that the consumer can choose either offer at time  $-1$ , she can switch to the non-chosen offer at any switching opportunity, and such a switching opportunity exists ( $K \geq 1$ ). In this version of our model with no additional competition from switching, the unique symmetric pure-strategy equilibrium with linear prices  $(p_I^n, p_S^n)$  involves marginal-cost pricing at the initial stage.<sup>17</sup>

Crucially, however, we modify our setup by assuming that firms offer a sign-up bonus instead of a switching price. Firm  $n$ ’s offer takes the form  $(B^n, p^n)$ , where  $B^n \in [0, T]$  is an introductory time period during which the price is zero, and  $p^n \in [0, v]$  is the subsequent price of the service per unit of time. Since the inducement to switch now comes from the bonus,  $(B^n, p^n)$  serves as firm  $n$ ’s initial as well as switching offer. The consumer can switch to

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<sup>16</sup> Gehrig and Stenbacka (2002) conclude based on their classical switching-cost model that “the strategic use of introductory offers should be promoted.” In the abstract of their review on switching-cost models, Farrell and Klemperer (2007) explain that “[f]irms compete ex ante for [...] ex post power, using penetration pricing, introductory offers, and price wars. Such competition for the market [...] can adequately replace [ex-post] competition, and can even be fiercer than [ex-post] competition.”

<sup>17</sup> Suppose, toward a contradiction, that  $p_I > 0$  in an equilibrium. Arguments akin to those in our previous results imply that the consumer never switches. Now consider the deviation  $p_I^1 = p_S^1 = p_I - \epsilon$ . Then, the consumer does not want to end up with price  $p_I^2 = p_I$ . Independently of whether she plans to end up with  $p_S^2$  or  $p_I^1 = p_S^1$ , she strictly prefers to start with firm 1’s offer, so she takes it with probability 1. Once she takes it, she never switches. For a sufficiently small  $\epsilon > 0$ , therefore, the deviation is profitable. Finally,  $p_I^n = p_S^n = 0$  is clearly an equilibrium.

the non-chosen offer just before the bonus period ends, starting on the new contract exactly when the post-introductory per-unit price would otherwise start to be charged. Thereafter, switching opportunities arrive at evenly spaced intervals of  $T/m$ . The consumer can only collect the sign-up bonus of each firm once.

As in the previous model, our main interest is in situations where the consumer has sufficiently frequent switching opportunities for procrastination to be a dominant consideration, and the total value of the service is high relative to her switching cost. Here, the specific conditions we use are

$$\frac{1 - \beta}{\beta} \cdot \frac{m}{T} \cdot s > v \quad (4)$$

and  $Tv > 8s$ . Condition (4) also guarantees the existence of a pure-strategy equilibrium.

While we argue in Appendix A.2 that cash bonuses lead to similar insights, we focus on bonuses that come in the form of introductory deals because these facilitate a clear contrast with previous models of competition failure. If a firm offers a large cash bonus, it makes losses on consumers who quickly switch away. Hence, previous adverse-selection or arbitrage arguments (Ellison, 2005, Heidhues et al., 2017) also imply that competition might be limited. But competition in introductory deals does not appear to face similar limits. First, a credit-card issuer or electricity provider does not risk large losses by offering a teaser period that is longer than the usual few months. Second, it is not obvious why naive or other profitable consumers would be relatively unresponsive to such offers. In fact, one might think that naive borrowers — often low-income consumers with debt — might be especially attracted by clearly visible or heavily advertised improvements in a credit-card teaser deal. Hence, previous models predict no competition failure.

**Proposition 4** (Sign-Up Deals).

(i) *If  $\beta = 1$ , then in any equilibrium, the consumer pays a total price of zero.*

(ii) *Suppose  $Tv > 8s$  and Condition (4) holds. Then, there are  $\underline{B} > 2s/v$  and  $\bar{B} < T$  such that for any  $B^* \in [\underline{B}, \bar{B}]$ , there exists an equilibrium in which firms offer  $(B^n, p^n) = (B^*, v)$ , the consumer selects randomly at  $t = -1$ , and she never switches. As  $T \rightarrow \infty$ ,  $\underline{B} \rightarrow 2s/v$ .*

Part (i) confirms that with classical consumers, the logic of Bertrand competition is opera-

tional, so consumers obtain the service at marginal cost. But Part (ii) says that with naive present-biased consumers, price competition with identical costs and products may have uncompetitive equilibria. In such an equilibrium, firms charge the monopoly unit price, and offer a sign-up bonus worth at least  $2s$  (a price cut of  $v$  for a period of at least  $2s/v$ ). The consumer collects the sign-up bonus (once), but for large  $T$ , this is negligible relative to her total payment of  $(T - B^*)v$ , so it does little in the way of returning the firm's monopoly ex-post profits to her. Worse, since without sign-up deals marginal-cost pricing always obtains, it is the deals that give rise to the monopoly unit price in the first place.

The intuition is in several parts. First, an introductory period does not prevent procrastination in switching at all. If the consumer switches at the next opportunity rather than the current one, the most she can lose is the payment of her current unit price for the intervening period. The incentive to procrastinate, therefore, is at least as strong as with linear prices above. In combination with Condition (4), this observation implies that consumers do not switch for any feasible pair of initially chosen and initially unchosen offers. As a result, competition between firms is limited to competition for being chosen initially.

Second, in this initial competition the Bertrand logic — whereby an arbitrarily small price cut attracts a discrete share of consumers — fails for the unit price. Although an introductory deal could be worth little relative to the consumer's total payment, it is worth significantly more than the switching cost  $s$ . This implies that the consumer strictly prefers, and hence plans, to take advantage of both deals. If firm 1 lowers its unit price to  $p^1$ , the consumer still plans to take advantage of both deals, now also preferring to pay  $p^1$  for the rest of the time. To achieve this, however, she does not need to start with firm 1's offer; in fact, a natural plan is to start with *firm 2's* offer and switch to firm 1 at the end of the introductory period. Hence, a lower unit price does not give firm 1 any advantage in the consumer's initial choice.

Third, the Bertrand logic fails for the introductory deal as well. If firm 1 raises its introductory period slightly from  $B^*$  to  $B^1$ , the consumer still wants to take advantage of both deals. Furthermore, in whichever order she does so, she pays a price of zero for a

period of  $B^* + B^1$  and a price of  $v$  for the rest of the time. She is therefore indifferent as to which offer to start with. There is then a simple tie-breaking rule that enforces the equilibrium introductory period  $B^*$ : the consumer responds to unilateral decreases in the bonus by choosing the other firm, while she chooses between bonuses of at least  $B^*$  randomly.

While the above tie-breaking rule may appear ad-hoc, it is just one way to illustrate the more general and main point of Proposition 4. Namely, when a procrastinating consumer has no compelling reason to select the better deal, her choice will be determined by other factors, impairing competition. Indeed, some of these other factors can generate logic similar to that with the above tie-breaking rule. For instance, suppose that the consumer is confident that she will remember to switch for a time period of  $B^*$  or shorter, but she thinks she may forget after a longer wait. Then, the consumer strictly prefers to start with an introductory period of  $B^*$  rather than a longer one. In this case, therefore, bonuses are determined by the consumer's (beliefs about her) memory. As another example, suppose that the consumer's switching cost is slightly time-dependent and U-shaped, it is minimized at  $B^*$ , and after  $B^*$  it increases sufficiently slowly for the incentive-to-procrastinate condition to be satisfied. Then, both firms offering  $B^n = B^*$  is the unique symmetric pure-strategy equilibrium with unit prices  $p^n = v$ . In this case, therefore, bonuses are determined by the consumer's convenience considerations.

Again, we are unaware of previous work that predicts such a stark failure of competition in the simplest of circumstances: when consumers are homogeneous and hence no adverse selection operates, firms commit to prices that consumers observe, and there is no constraint on competition in sign-up deals. In models with rational consumers, competition ensues if firms can commit to future prices. In models with rational consumers when firms are unable to commit to future prices, and in models with naive consumers, limits to competition arise only if firms face adverse selection or explicit constraints in initial competition.

In Appendix A.2, we argue that the logic behind competition failure is robust to modifications of our model where

1.  $B^n$  is paid in cash;



2. the price is not capped, but the consumer can cancel and/or go without the service;
3. the long-term discount factor, usually denoted by  $\delta$ , is strictly less than 1;
4. a share of time-consistent consumers (for whom  $\beta = 1$ ) is present;
5. consumers may sometimes have zero switching costs; or
6. firms can offer (but cannot commit to) exploding sign-up bonuses.

As with our previous model, in some of these situations competition failure requires that there is slight initial differentiation between products. Again, therefore, we interpret our results as saying not that procrastination always eliminates competition between perfect substitutes, but that procrastination drastically increases competition frictions.

In our setting with only introductory periods, there is also a competitive equilibrium. To see this, notice that if firm 2 sets  $p^2 = 0$ , then a consumer is only willing to choose firm 1 if firm 1 also offers a price of zero for the entire time. Consistent with equilibrium, therefore, firm 1 is willing to set  $p^1 = 0$  as well. Interestingly, however, there is a profitable deviation, and hence the competitive equilibrium does not survive, if firm 1 can give away a small cash bonus in addition to offering an introductory period. Indeed, suppose that firm 1 offers a cash bonus of more than  $s$ , a short introductory period, and a unit price  $p^n = v$ . Then, the consumer chooses firm 1 planning to collect the bonus and to switch quickly. But she never actually switches, so firm 1 makes a profit. Consistent with this observation, in variants of our model with cash bonuses, only uncompetitive equilibria exist. Since such a small cash bonus appears unlikely to generate strong arbitrage or adverse-selection concerns for firms, the competitive equilibrium is arguably not robust.<sup>18</sup> In this sense, competition failure is stronger in our model with sign-up bonuses than in our basic model above.

Our predictions can contribute to explaining limited competition in some subscription-type markets with little differentiation between products and offers that feature sign-up

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<sup>18</sup> Reinforcing the above point, when firms' marginal costs are positive, the competitive equilibrium fails to exist even if firms can only offer zero-price introductory periods as bonuses. Then, if firm 2 offers marginal-cost pricing, firm 1 can deviate by offering a zero-price introductory period followed by a high price. Like with a cash bonus, the consumer takes firm 1's offer, planning to switch at the end of the introductory period.

deals facing no obvious limit to competition. To continue with our credit-card example, the interest rates consumers paid in the U.S. credit-card market remained very high despite the introduction of teaser periods in the 1990’s (e.g., Ausubel, 1997, DellaVigna and Malmendier, 2004, Stango and Zinman, 2015, Galenianos and Gavazza, 2022). Indeed, Galenianos and Gavazza (2022) note that the market is “reminiscent of monopolistic markets” despite consumers receiving many offers.<sup>19</sup> Similarly, multiple researchers have observed that predictions about how competition would lower retail electricity prices proved too optimistic.<sup>20</sup> Existing models instead predict that firms would compete by extending teaser deals for longer (see Section 6 for more details).

Our model also provides an explanation for the casual observation that retailers of various types have been converting spot markets into subscription markets. For example, there is a long history of selling books (“book clubs”), a more recent trend of selling movies, and a relatively new attempt to sell software, through subscriptions rather than final sales. According to our model, the failure of competition is specific to switching markets — in which consumers must cancel or switch away to stop paying — so firms may prefer such markets even when this is not the natural way to sell their products.

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<sup>19</sup> The balance-weighted credit-card interest rate for balances accruing interest, according to the Federal Reserve’s latest release, is 16.98% (<https://www.federalreserve.gov/releases/g19/current/>, accessed September 14, 2020). Another notable feature of the market is that interest rates are not only high on average, but also highly dispersed across borrowers, even after controlling for borrower and card characteristics (Stango and Zinman, 2015, Galenianos and Gavazza, 2022). Given our focus on pure-strategy equilibria played between symmetric firms — which is sufficient to address our main interest, high prices — our framework does not account for such price dispersion. We conjecture, however, that simple variants result in mixed strategies and therefore dispersed prices. This would be the case in the current model if Condition (4) was not satisfied, and in the previous model if the first opportunity to switch occurred after time 0, and the number of offers a consumer received at time -1 was heterogeneous.

<sup>20</sup> Joskow (2003) notes that “there is a growing perception that . . . [US] retail competition programs have had disappointing results,” which he attributes partly to consumer behavior. In the UK’s deregulated energy market, the Competition & Markets Authority (2016a) found that suppliers earn high profits, estimating an overcharge of £1.4 billion paid by UK customers. See also Agency for the Cooperation of Energy Regulators (2018). These observations lead the UK’s Domestic Gas and Electricity (Tariff Cap) Act of 2018 to reimpose a price cap on standard variable tariffs (Hinson, 2018).

## 5 Alternative Psychological Assumptions

In this section, we consider alternative psychological foundations for procrastination.

### 5.1 Partial Naivete

We first discuss what happens in our basic model when the consumer underestimates, but is not fully naive about, her present bias. To do so, we assume (following O’Donoghue and Rabin, 2001) that the consumer has a point belief  $\hat{\beta}$  about her future short-term discount factor, and  $\beta \leq \hat{\beta} \leq 1$ . In this formulation,  $\hat{\beta} = \beta$  corresponds to sophistication,  $\hat{\beta} = 1$  corresponds to the full naivete we have assumed so far, and  $\beta < \hat{\beta} < 1$  corresponds to partial naivete.

We use the notation from Section 3.1, and start by supposing that  $p_I^* < v$ , so  $p_I^* = \max\{p^{NIS}, p^{IP}\}$ . We argue that for an arbitrarily small amount of naivete — i.e., any  $\hat{\beta} > \beta$  — Conditions (NIS) and (IP) are unaffected, so that the equilibria in Propositions 1 and 2 remain in place. Regarding Condition (NIS), the claim is immediate since the consumer’s beliefs about her future behavior play no role here. Turning to Condition (IP), notice that for any initial price  $p_I \geq p^{IP}$ , a person with a short-term discount factor  $\hat{\beta} > \beta$  strictly prefers switching immediately to switching later. Hence, a consumer with belief  $\hat{\beta}$  must think that if she does not switch now, then she will switch next time. She therefore makes the same comparison as in our basic model, and the same condition results. Intuitively, since  $p^{IP}$  makes the consumer indifferent whether to switch now or next time, an arbitrarily small amount of naivete is sufficient for her to mispredict her behavior.<sup>21</sup>

We now suppose that  $p_I^* = v$ , and derive conditions under which the consumer procrastinates on switching away from this price. Given the above considerations, the equilibria in Propositions 1-3 survive under the same conditions. We define the consumer’s “tolerance for delay”  $d$  as the number of opportunities she is willing to delay switching. If the consumer delays for  $d$  opportunities, then she loses  $\beta d T_w v$  in payments and saves  $(1 - \beta)s$  by pushing

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<sup>21</sup> The conclusion that in a market setting an arbitrarily small amount of O’Donoghue-Rabin-type naivete can have large effects is reminiscent of previous work by DellaVigna and Malmendier (2004) and Heidhues and Köszegi (2010).

the switching cost to the future, so her tolerance for delay is

$$d = \frac{1 - \beta}{\beta} \cdot \frac{1}{T_w} \cdot \frac{s}{v}.$$

Analogously, we define  $\hat{d}$  as the consumer’s perceived future tolerance for delay, which is given by the same formula with  $\beta$  replaced by  $\hat{\beta}$ . The consumer believes that if she delays now and would delay more than  $\hat{d}$  additional opportunities starting next time, then — this being outside her perceived tolerance — she would rather switch next time. Hence, she must believe that if she delays now, she will switch within  $\lfloor \hat{d} \rfloor + 1$  opportunities.<sup>22</sup> If  $\hat{d} < \lfloor d \rfloor$ , then this is within her tolerance for delay  $d$ , so she procrastinates. The condition  $\hat{d} < \lfloor d \rfloor$  can be satisfied even for  $\hat{\beta}$  close to  $\beta$ , especially in a crowded market: for any  $\hat{\beta} > \beta$ , the consumer procrastinates if  $T_w$  is sufficiently small.

## 5.2 Other Models of Procrastination

To model consumer behavior, we have assumed naive present bias, the most widely used microfoundation for procrastination. We argue in this section that other plausible microfoundations lead to similar insights, and might help account for some subtler patterns in firms’ pricing behavior.

*Underestimation of Switching Costs.* Tasoff and Letzler (2014) document that subjects overestimate their probability of redeeming a rebate-like form by 49 percentage points, with additional evidence — e.g., that lowering transaction costs affects redemption but not beliefs — suggesting that the overoptimism is due to an incomplete appreciation of future costs. Similarly, Rodemeier (2020) documents that online shoppers underestimate the hassle cost of claiming a rebate.

To model this, suppose that  $\beta = 1$ , but the consumer has incorrect views about her future switching cost: while the true switching cost at any time is  $s' > s$ , and she understands that her current switching cost is  $s'$ , she believes with certainty that her switching cost at any point in the future will be  $s$ . This leads to the same consumer behavior, and hence the same firm

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<sup>22</sup> For  $x \in \mathbb{R}$ ,  $\lfloor x \rfloor$  denotes the largest integer less than or equal to  $x$ .

behavior, as our model with  $\beta = s/s'$ . Intuitively, underestimating future switching costs has the same effect on behavior as overestimating the future self’s benefit from switching, which in turn is equivalent to overestimating the future self’s discount factor.

*Overconfidence about Memory or Attention.* Consistent with the idea that a consumer is too optimistic about her memory or attention, Ericson (2011) documents that subjects overvalue a payment in six months that they have to remember to claim, and Rogers and Milkman (2016) and Bronchetti et al. (2020) find that people undervalue reminders.

To model this, suppose that the consumer’s switching cost at a moment in time is either  $s_L$  or  $s_H > s_L$ . During any switching opportunity, she faces times with both switching costs, but in order to act, she must also recall the task. Crucially, the consumer never remembers when her switching cost is  $s_L$ ; it might be the case, for instance, that at such times she is engaged in leisure activities and forgets chores. But when the consumer thinks about the future, she naively believes that she will remember the task for both switching-cost realizations. This is equivalent to the model above with  $s' = s_H$  and  $s = s_L$ . Intuitively, overconfidence about memory leads to overconfidence about remembering to switch when the cost is low, which is equivalent to underestimating the future cost of switching.<sup>23</sup>

*Subtle Pricing Patterns.* Alternative sources of procrastination not only have similar basic implications in our settings, but they might also help account for additional observations regarding the details of firms’ pricing strategies. Consider, in particular, some patterns in how firms implement loyalty penalties. In the UK electricity market, customers are subject to price jumps after an initial contract period expires.<sup>24</sup> In the UK home-insurance and auto-insurance markets, companies engage in “price walking:” they increase margins renewal after

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<sup>23</sup> The conditions under which the above models lead to the *exact same* consumer behavior as our present-bias-based model are arguably special. While less extreme versions of our assumptions do not lead to the exact same behavior, the intuitions suggest that they are likely to satisfy the logic of consumer behavior underlying our results: that (i) a consumer overestimates her inclination to switch in the future, and, as a result, (ii) she is prone to not switching now, where (iii) these tendencies become more severe with more options.

<sup>24</sup> At the end of the initial “acquisition” contract, customers who do not actively look for a new contract are often moved to an expensive “default” or standard variable tariff, which is subject to a regulatory price cap (<https://www.ofgem.gov.uk/energy-price-caps/about-energy-price-caps/price-my-energy-bill-capped/default-tariff-price-cap> assessed on November 23, 2020). About 70% of UK energy customers are on such a default contract (Competition & Markets Authority, 2018, page 26).

renewal until a target margin is reached (Financial Conduct Authority, 2019, page 44). In the UK broadband market, in which there is a general downward pricing trend, companies engage in “legacy pricing:” they keep contract details unaltered for out-of-contract customers, while offering much better deals to new customers (Office of Communications, 2019). And German electricity and broadband providers often offer two-year contracts with monthly payments that start out low but revert to the “regular” level after six to twelve months, and then automatically renew at the latter price for passive consumers. We suspect that these specifics are at least in part motivated by firms’ attempts to avoid drawing consumer attention to price hikes. Firms that must (due to consumer-protection regulation) inform consumers of price changes and contract renewal, for example, may want to switch from the introductory price to the regular price well before renewal, and then simply say that terms are unchanged. By the same token, legacy pricing or price walking may be better at flying under a consumer’s radar than large price hikes. At the same time, price jumps may be easier to hide in the electricity market, where bills depend on variable usage and a price jump is not large in absolute terms.

The attempt to take advantage of consumers’ inattention also helps explain a pattern that our present-bias-based framework does not predict: that contracts sometimes lock consumers in for an extended period, such as two years. In our model, this is suboptimal for a firm because the inability to get out of the contract lowers the consumer’s tendency to procrastinate, lowering the price the firm can charge. But a long-term contract may be optimal for a number of reasons. First, if prices are regulated, then even in our model a firm does not benefit from shortening the contract beyond the length that induces procrastination at the regulated price. Second, a longer-term contract may be better at avoiding consumer attention at the point of renewal (when she has not thought about it for a while), especially since, as discussed above, it involves a longer period in which the regular price is charged. To go further, if the consumer is prone to forgetting about switching, then it may be optimal to lock her in again for another long-term contract. Third, long-term contracts may protect firms from non-procrastinating consumers who would, with short-term contracts, take

advantage of sign-up bonuses.

## 6 Related Literature

In this section, we discuss related research not covered elsewhere. While we point out other differences below, our paper is the first to study the market effects of procrastination in switching, the first to predict the procrastination-induced competition paradox, and the first to generate a failure of competition in transparent linear prices or sign-up deals for homogeneous consumers. Hence, our theory provides a novel mechanism for competition failure that applies in situations where other models of competition failure do not. This helps account for some high observed prices for which previous work does not appear to provide a complete explanation.

Because procrastination can be seen in reduced form as making it more difficult for a consumer to switch, our theory is related to models of consumer inertia due to switching costs (e.g., Farrell and Klemperer, 2007) or default effects (Ericson, 2020); and because procrastination leads the consumer to underestimate the price she will pay, our theory is related to models of hidden prices (e.g., Gabaix and Laibson, 2006, Armstrong and Vickers, 2012). These models predict limited competition in price components a consumer cannot avoid or does not appreciate, but they also predict increased price competition when the consumer signs up initially or finally does switch. In our model, in contrast, even initial competition is compromised.<sup>25</sup>

Of course, although in the existing literature it is equally possible that ex-ante competition more than offsets the lack of ex-post competition (Farrell and Klemperer, 2007, Rhodes, 2014, Cabral, 2016), the literature identifies some possible reasons for the offset to be incomplete. A simple insight is that if a sign-up bonus takes the form of a reduction in the initial price or the price of a base good, then a price floor may limit how much ex-post profit can be handed out ex ante. Our model generates high profits without a price floor. Indeed,

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<sup>25</sup> In addition, for competitive models with switching costs to generate high prices, firms must be unable to commit to future prices when consumers sign up initially. In our model, high prices obtain even though they are announced and known to consumers.

while some dimensions of prices — e.g., the annual fee for credit cards — are arguably at a floor, there are other dimensions — e.g., the length of a teaser period — on which there is no binding constraint.

More subtly, adverse selection, whereby a firm offering a better deal disproportionately attracts less profitable consumers, may limit competition in some markets. But it is not clear that these accounts apply to all relevant settings;<sup>26</sup> and indeed, Ausubel (1999) and Agarwal et al. (2010) find evidence of advantageous selection: consumers who accept better deals are better credit risks on average. Our theory generates high prices with homogeneous consumers, and thereby shows that adverse selection is not necessary for high prices to obtain.<sup>27</sup>

The literature on captive consumers posits that a share of consumers do not compare prices and therefore do not respond to price cuts, limiting competition in undifferentiated products. Consumers may be captive due to strong brand preferences (e.g. Shilony, 1977), prohibitively high search costs (e.g. Varian, 1980), or choice complexity (e.g. Piccione and Spiegler, 2012, Spiegler, 2016); or firms may price such that active search is suboptimal given search costs (e.g. Stahl, 1989, Janssen and Moraga-González, 2004). Since our consumers do not respond to price cuts, in simplistic terms they can be thought of as captive, and hence our model can be thought of as providing a microfoundation for the presence of captive consumers. This microfoundation, however, applies in circumstances when positing

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<sup>26</sup> Ellison (2005) develops a model in which consumers are heterogeneous in their elasticities with respect to both base-good and add-on prices, with a positive correlation between the two elasticities. Then, cutting the base-good price attracts mostly consumers who will not buy the profitable add-on, discouraging competition. But paralleling the logic for switching costs, he shows that if consumers are rational and firms can commit to and advertise future prices, then the adverse-selection logic does not hold. Hence, Ellison (2005) informally proposes a modification based on the presence of naive consumers who are less responsive to the sign-up deal than unprofitable sophisticated consumers. Without our model, it is not obvious why naive consumers would be relatively unresponsive. As a somewhat different story, Ausubel (1991) proposes that profitable credit-card consumers might not care about interest rates because they falsely expect not to borrow, whereas consumers who are sensitive to interest rates are likely to be bad credit risks. But a credit-card borrower who is about to buy something expensive or who has already accumulated substantial credit-card debt should — even if she is naive present-biased — realize that she will be carrying debt for a while, and she should value cheap options to carry debt for longer. When there are such consumers, the adverse selection with respect to credit risk must be extreme to generate high prices.

<sup>27</sup> For a completely different limitation on ex-ante competition based on naive consumers, see Johnen (2020).



that consumers simply do not compare prices is implausible. In the credit-card market of the 80's, for example, there was a single key price (the APR) that was easy to see and compare across firms, and there were heavily marketed inducements to switch, so most consumers presumably saw multiple offers. Furthermore, models based on captive consumers (unlike ours) predict low prices when marketing is very cheap, they do not derive a role for sign-up deals in preventing competition, and they cannot explain uniform high prices such as similar profit-making interest rates. Furthermore, although some of these models also predict that entry raises average prices (Stahl, 1989, Janssen and Moraga-González, 2004, Armstrong and Vickers, 2021) the reasoning is entirely different from ours and relies on consumer heterogeneity.<sup>28</sup>

More distantly related, a number of papers have considered the implications of partially or fully naive present bias on contracting in some of the same markets motivating our analysis. In all these papers, firms make high ex-post profits from naive consumers, but competition at the initial stage eliminates their net profits. A long history of work (e.g. DellaVigna and Malmendier, 2004, Murooka and Schwarz, 2018, Johnen, 2019) shows that auto-renewal contracts can be used to exploit a naive present-biased consumer's misperception about her probability of canceling, and Eliaz and Spiegler (2006), Heidhues and Kőszegi (2010), Gottlieb and Zhang (2021), and Citanna and Siconolfi (2023) investigate how firms design contracts with fully or partially naive time-inconsistent agents.

## 7 Conclusion

Given the main message of our paper — that switching markets do not work well — it is natural to ask whether there are interventions that increase consumer welfare. In our model, measures that lower the switching cost  $s$  can strengthen competition. Indeed, unlike in classical models, such measures can even strengthen competition at the initial stage.

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<sup>28</sup> When there are zero-search cost consumers (shoppers) and those with positive search costs, as the number of firms grows a given firm needs to beat more rivals in order to attract shoppers making it relatively more profitable to price high in order to benefit from positive search-cost consumers; as a result, entry can increase average prices offered by firms.

Unfortunately, however, regulators have been unable to lower switching costs to the point where most consumers no longer procrastinate, so in practice this intervention does not work. Similarly, while any policy intervention that mitigates procrastination should raise competition, designing such an intervention is daunting.<sup>29</sup>

Although more research is clearly necessary, the above considerations suggest that relying on consumer engagement to achieve low prices in switching markets is exceedingly difficult. If so, interventions that involve the supply side might be necessary. One simple policy that only minimally changes the economic environment is a type of managed competition: the consumer is initially assigned to the cheapest provider, and if for some reason she wishes, she can later switch providers. Then, firms compete intensely to be assigned (and hence keep) the procrastinating consumer, short-circuiting procrastination and resulting in marginal-cost pricing.<sup>30</sup> A noteworthy example is Ohio’s residential electricity market as described by Joskow (2003). By default, municipalities purchased power on behalf of consumers and switched them to a cheaper supplier when available. Consumers who wanted to make their own choices could opt out, but not many did. Unsurprisingly, many more consumers were on low-price contracts than in other parts of the US.

Our paper focuses on the implications of procrastination on prices when the consumer buys. But the competition paradox might also give rise to important implications when the consumer does *not* buy even though she should. As has been recognized by many researchers, procrastination due to present bias or forgetting might delay a consumer’s investment into retirement (e.g., Carroll et al., 2009, Brown and Previtro, 2020) or preventive care (e.g.,

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<sup>29</sup> For instance, under a model of naive present bias, strict time windows for switching that are spread out over time can mitigate procrastination. But if procrastination is partly due to imperfect memory, such a policy can exacerbate procrastination. If both mechanisms are operational, then one must combine deadlines with reminders (Ericson, 2017). This raises other issues: it might be difficult to time reminders just right, and if the reminders work by drawing the consumer’s attention, then she might pay less attention to other important things.

<sup>30</sup> Formally, we can evaluate this policy in our basic model of Section 3.1. Suppose, toward a contradiction, that a symmetric equilibrium  $p_I^* > 0, p_S^*$  exists. If  $p_I^* > \max\{p^{NIS} + p_S^*, p^{IP} + p_S^*\}$ , then by the same argument as in the proof of Proposition 1, the consumer switches at opportunity 0, so that the initially assigned firm earns zero profits. By deviating and charging  $p_I^* = \max\{p^{NIS} + p_S^*, p^{IP} + p_S^*\}$ , however, a firm can attract and keep the consumer while earning more than  $p_S^*T$ , so this is a profitable deviation. If  $p_I^* \leq \max\{p^{NIS} + p_S^*, p^{IP} + p_S^*\}$ , then the consumer does not switch and hence the standard Bertrand undercutting logic implies that  $p_I^* = 0$  in equilibrium.

Baicker et al., 2015), and the same may be true of investment into the stock market or purchase of life insurance as well. The logic of our model says that relying on a large competitive market cannot solve such underparticipation by procrastinators: having many competitors with good options means that the consumer does not lose much by delaying entry for a little while, so she is prone to procrastinating.

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# Appendix

## A Robustness

In this section, we demonstrate the robustness of our model’s main mechanism — that a procrastinating consumer expects to get good deals in the future, and hence does not care for getting a good deal now — extends to a number of variants. As we have mentioned, the failure of competition sometimes requires slight differentiation between the products. To formalize this, we assume that there is one randomly chosen initial firm for which the consumer gets a one-time sign-up benefit with present value  $V \geq 0$ . In our models in the text, we have implicitly set  $V = 0$ . Below, we continue to focus on this undifferentiated case unless otherwise stated.

### A.1 Basic Model

We elaborate on the variants of our basic model listed in the text.

1. *The consumer receives multiple switching offers at each switching opportunity.* Suppose that at each switching opportunity, multiple firms make offers to the consumer. Independently of the number of such firms, the same uncompetitive equilibrium as in Proposition 2 survives. In the uncompetitive equilibrium in the proposition, firms’ switching offers were already at marginal cost, so the introduction of immediate competitors cannot present an incentive to deviate.

2. *Condition (2) is not satisfied.* Because Proposition 2 is meant to address situations in products are undifferentiated ( $V = 0$ ), it requires Condition (2) for an uncompetitive equilibrium to exist. But if  $V > \beta s$ , then the uncompetitive equilibrium exists for any  $N_I/N_S$ . The consumer knows that by paying a switching cost  $s$  in the future, she can switch to a zero-price option. Hence, if she has an added benefit  $V > \beta s$  from starting with a specific product, she takes that product and sticks with it. This logic also implies that charging  $p_I^n = p^{IP}$  is strictly optimal if  $p_S^n = 0$  for each firm  $n$ , so the uncompetitive

equilibrium exists.

3. *The consumer must pay the initial price for a while before she can switch.* Consider our assumption that the consumer can switch away from the initial firm before paying its price. In some situations, there may be a lag before the first switching opportunity. This *decrease* in competition from switching has the potential to *lower* prices by inducing some competition in initial offers. Still, if the consumer can switch away relatively quickly, then initial competition is mild, so a small amount of differentiation eliminates it. Specifically, suppose the consumer chooses among contract at time 0 and let time  $T_w > 0$  be the first switching opportunity for the consumer. If  $V \geq \beta p^{IP} T_w$ , the consumer prefers to select the firm with the sign-up benefit thinking she would switch at  $T_w$ , while she procrastinates switching in reality. And if Condition (2) holds, as before, firms do not want to lower the price to below  $s/T$ , which is necessary to attract the consumer. Hence, the uncompetitive equilibrium exists. If  $T_w$  is small, it does so even for small  $V$ .

4. *A share of rational time-consistent consumers is present.* Suppose rational time-consistent consumers (for whom  $\beta = 1$ ) are present. For simplicity, we use the variant of our model in which the consumer receives offers from all non-initial firms at every switching opportunity. Then, it is easy to show that an uncompetitive equilibrium with the same prices as in Proposition 2 survives under a modified Condition (2) in which the right-hand side is multiplied by the share of time-inconsistent consumers. Hence, the equilibrium may be robust to the presence of a non-trivial share of time-consistent consumers. This prediction contrasts with a large class of models starting with Varian (1980) in which any positive share of rational consumers induces some competition. Intuitively, a small price cut at time  $-1$  fails to attract time-consistent consumers for the same reason it fails to attract naive consumers, because they (correctly) expect to switch to a better deal in the future.

5. *Firms cannot price discriminate between initial consumers and switchers.* Suppose that in Proposition 1 we have  $p_I^* = v$ ,  $V > \beta s$ , and each firm can make only a single price offer that does not distinguish between initial and switching consumers. Furthermore, *all* firms make an initial offer to the consumer, and all non-chosen firms make switching offers at

every switching opportunity. Then, there is an equilibrium in which all firms charge a price of  $v$ . Unlike above, where consumers fail to take up good deals, now firms do not even offer them. Intuitively, a consumer is not responsive to a price cut initially because she thinks she will switch to that deal later after receiving  $V$  — which she actually does not. As a result, a firm only loses from competing in price.

## A.2 Sign-Up Deals

We now analyze the variants of our model in Section 4 listed in the text. We start with the alternative in which firms offer cash sign-up bonuses, and build the other variants from that model.

1.  *$B^n$  is paid in cash.* Suppose that  $B^n$  is a cash payment instead of a free introductory period. Consistent with the fact that it is very difficult to deliver immediate utility to the consumer at the time of contracting,  $B^n$  (just like  $p^n$ ) accrues in the future. We impose two tie-breaking rules for time  $t = -1$ : (i) if the consumer is indifferent whether to plan on collecting a bonus, she does plan to do so; and given this, (ii) if she is indifferent as to which offer to take, she randomizes with equal probability. All other assumptions remain unchanged. This leaves the existence of the uncompetitive equilibrium unchanged, and makes it more robust:

**Proposition 5** (Cash Sign-Up Deals). *Suppose  $Tv > s$  and Condition (4) holds. There is a unique pure-strategy equilibrium, and in this equilibrium both firms offer  $(B^n, p^n) = (s, v)$ . The consumer takes one of the offers at  $t = -1$  and never switches.*

To build intuition for the market outcome, we illustrate the consumer's behavior in an off-equilibrium situation. Suppose that  $\beta = 1/2$ ,  $s = T = m = 100$ ,  $v = 20$ ,  $T_w = T/m = 1$ , and the two firms offer  $B^1 = 150$ ,  $p^1 = 20$  and  $B^2 = 180$ ,  $p^2 = 1$ . The comparison is clear: firm 2 offers the strictly better deal both in terms of the sign-up bonus and in terms of the unit price. So what does the consumer do? She signs up for *firm 1's* offer, and sticks with it at each switching opportunity. Given that the sign-up bonuses are above  $s$ , the consumer plans to collect both of them. Furthermore, she does not care about the order in which she

collects the bonuses and wants to end up with the lower unit price, so she strictly prefers to take up the *worse* deal first. Then, when she ponders whether to switch immediately or next time, she sees no harm in collecting the other bonus later, so the bonus plays no role in her decision. Because a price difference of  $p^1 - p^2 = 19$  is also insufficient to prevent her from delaying, she procrastinates until she reaches the last switching opportunity. And parameters are such that once there, she no longer thinks switching is worth it.<sup>31</sup>

The logic of Proposition 5 follows the insights from the example. The consumer plans to take advantage of both sign-up bonuses, but then never switches away from the firm she first signs up with. We argue that in this situation, no firm has a profitable deviation. First, firm  $n$  might try to attract the consumer at time  $-1$  by lowering  $p^n$  — but this has the perverse consequence that the consumer now strictly prefers the other offer. Second, firm  $n$  might try to induce the consumer to switch to it by lowering  $p^n$  — but by Condition (4), preventing consumer procrastination is impossible even with  $p^n = 0$ .<sup>32</sup> Third, firm  $n$  might try to attract the consumer at time  $-1$  by increasing  $B^n$  — but this leaves the consumer indifferent as she plans to collect both firms' bonuses, so it is merely a waste of money. Fourth, firm  $n$  might try to induce the consumer to switch to it by raising  $B^n$  — but this has no effect at switching opportunities before the last one, and is too expensive if the consumer takes it at the last opportunity. Finally, if firm  $n$  lowers  $B^n$  to below  $s$ , then the consumer strictly prefers to take advantage of only the rival's deal, so she chooses the rival.

2. *The price is not capped, but the consumer can cancel and/or go without the service.*

We modify our cash-bonus model to consider situations in which the consumer does not have to buy the service, and the price is not capped. We assume that the consumer's value is  $v$  per unit of time, and that in addition to being able to switch at any switching opportunity

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<sup>31</sup> For a formal analysis, see Appendix B.

<sup>32</sup> For a complete picture, we explain what happens if a firm can choose  $p^n < 0$ . In our model in which the consumer is assumed to exit the market at time  $T$ , a profitable deviation arises: firm  $n$  can choose a  $p^n$  that is sufficiently negative to prevent procrastination, and make profits through a negative  $B^n$ . To avoid arbitrage, the firm can even impose a limit on the total consumption to which the negative price applies. Plausibly, however, a consumer faced with a negative price would not cancel her service at time  $T$ , and not knowing the consumer's precise circumstances, the firm cannot require her to cancel it at that time either. In this case, the consumer can collect the same amount of payments from the negative price if she switches now and if she switches next time. Hence, her incentive to switch now is no greater than with a price of zero, so she procrastinates.

$\kappa = 0, \dots, K$ , she can cancel a contract without losing the bonus at any time, including at the same switching opportunity as when signing up. Switching to, taking up, or canceling a contract costs  $s$ . The consumer cannot hold two contracts at the same time. To focus on the case in which the incentive-to-procrastinate condition determines prices, we assume that  $(1 - \beta)m > 1$ . And to guarantee the existence of a pure-strategy equilibrium, we impose a slightly stronger condition than Condition (4):

$$\left( \frac{1 - \beta}{\beta} - \frac{1}{m} \right) \cdot \frac{m}{T} \cdot s > v. \quad (5)$$

**Proposition 6.** *Suppose that Condition (5) holds, and  $(1 - \beta)m > 1$ . There is an equilibrium in which both firms offer*

$$B^n = s, \quad p^n = v + \frac{1 - \beta}{\beta} \cdot \frac{m}{T} \cdot s.$$

*The consumer takes one of the offers, and never switches or cancels.*

In the equilibrium of our Bertrand pricing game identified in Proposition 6, the consumer pays a price *above* her value. To appreciate the result, consider first what a monopolist would do. The monopolist could offer the straightforward contract  $p = v$  and  $B = 0$ , which the consumer would accept. Alternatively, the monopolist could design a tricky contract by (i) choosing  $p > v$  such that the consumer expects to but does not cancel the contract, and (ii) setting  $B = s$  to compensate the consumer for her expected cancellation cost and thereby inducing her to accept. Since canceling next time lowers the discounted cost of canceling from  $s$  to  $\beta s$  but also imposes a loss of  $p - v$  for a time interval of length  $T_w = T/m$ , the monopolist can charge exactly the  $p^n$  identified in the proposition. It is easy to check that for this  $p^n$ , the monopolist prefers the tricky contract.

In light of the above, Proposition 6 says that despite being engaged in Bertrand competition, the firms behave as if they were monopolists. This failure of competition is even more extreme than in the settings of Propositions 4 and 5, where a monopolist would not offer a sign-up bonus, and hence competition increases consumer welfare at least by a bit. Note also that the consumer would never accept a unit price  $p^n > v$  without a sign-up bonus —

in fact, without the bonus there is a unique equilibrium in which  $p^n = 0$  — so the bonuses are again essential for the failure of competition.

The intuition is in several parts. First, the smallest bonus that induces the consumer to participate still equals the cost of cancelling an overly expensive contract. Second, once the consumer has accepted an offer, she is indifferent to taking up the other offer, so her incentive to procrastinate is driven by when to cancel her existing contract. Third, a firm cannot profitably attract the consumer by decreasing  $p^n$ . A small price cut leaves the consumer indifferent, as she thinks she would cancel even the lower-price contract immediately. Similarly to Proposition 2, therefore, in this region competition does not operate exactly because prices are too high. In addition, Condition (5) ensures that a deep price cut is unprofitable for a firm. Fourth, increasing  $B^n$  is unprofitable for the same reason as in Proposition 5: it can only be used to attract the consumer at the last switching opportunity, which is too late to be profitable.

As a potential example, the consumer might sign up for a renewal service such as Netflix or Audible that she values below the price, and then keep her membership forever. Based on previous work on procrastination, such as DellaVigna and Malmendier (2004), it is unsurprising that a procrastinating consumer might not cancel a bad deal. But the possibility that procrastination induces undifferentiated firms engaged in price competition to charge monopoly or even higher prices has not been pointed out in previous research.

3. *The long-term discount factor, usually denoted by  $\delta$ , is strictly less than 1.* The prediction that there is no competition in cash bonuses at all relies on our assumption that  $\delta = 1$ . With  $\delta < 1$ , the consumer prefers more money earlier, so she is attracted by an increase in the bonus. While this induces competition in the sign-up bonuses, the competition can be extremely weak. To illustrate this, consider our model with the possibility of cancellation. We assume that the bonus for a contract taken at time  $-1$  is delivered one period earlier than the bonus for a contract taken at time 0. To describe an extreme case, suppose that starting from a situation in which both firms offer a bonus of  $s$ , firm 1 increases its bonus  $B^1$  to  $Tp^n$  — i.e., it offers to return all of its ex-post profits. If the consumer



starts with firm 1, she receives a bonus of  $Tp^n$  first and a bonus of  $s$  one period later; and if she starts with firm 2, she receives a bonus of  $s$  first and a bonus of  $Tp^n$  one period later. Hence, at time  $-1$  the consumer perceives the benefit of starting with firm 1 to be less than  $\beta(1 - \delta)Tp^n$ . If one period is short, this value can be extremely small. For instance, suppose that  $Tp^n = \$10,000$ , and one period is one month, so that  $\delta$  is the monthly long-run discount factor. Taking Laibson et al.'s (2020) estimate of the annual long-run discount factor, 0.99, as well as their estimate  $\beta = 0.5$ , the value of starting with the larger bonus is  $\beta(1 - \delta) \cdot \$10,000 = 0.5 \cdot (1 - 0.99^{1/12}) \cdot \$10,000 \approx \$4.19$ . Hence, even tiny differentiation between the products prevents the huge bonus from attracting any extra consumers. As a result, a slightly different version of Proposition 6 survives.<sup>33</sup> Intuitively, the consumer's perceived benefit from taking the better contract is only the value of receiving the bonus one month earlier, which is far less than how much the bonus is worth and how much it costs the firm. Competition between the firms is therefore weak.

4. *A share of time-consistent consumers is present.* Take our cash-bonus model, and suppose that time-consistent consumers (for whom  $\beta = 1$ ) are present. These consumers would follow through on an optimal plan to end up with a cheaper unit price, so they create an incentive to compete. Yet if firm  $n$  lowers  $p^n$ , it is not only guaranteed to attract time-consistent consumers, but it is also guaranteed to lose time-inconsistent consumers. Hence, the uncompetitive outcome is robust to the presence of a non-trivial share of time-consistent consumers.

5. *Consumers may sometimes have zero switching costs.* We introduce the possibility that the consumer switches in equilibrium, modifying the model with cash bonuses (and no time-consistent consumers) in the following ways. We assume that the consumer needs the service until time  $2T$ , and at time  $T'$  shortly after switching opportunity  $K$  and before time  $T$ , her switching cost is zero. At time  $T'$ , two firms make offers to provide the service from

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<sup>33</sup> Precisely, suppose that  $\beta s > V > \beta(1 - \delta)Tp^n$ , where  $p^n$  is given in Proposition 6. Then, both firms setting  $B^n = s - V/\beta$  and  $p^n$  is an equilibrium in which the consumer initially plans to cancel the preferred firm's contract at time 0 but procrastinates the cancellation in reality. Each firm sets the bonus so that the consumer who prefers it is willing to sign up. A firm cannot profitably attract the other firm's consumers by increasing its bonus.

time  $T$  onwards. One of these is the firm that was not chosen by the consumer at time  $-1$ , and the other is a new firm. Analogously to competition between the initial firms, if the consumer takes up a firm's offer at time  $T'$ , then subsequently the other firm makes  $K + 1$  evenly spaced switching offers to provide the service from times  $T, T + T_w, \dots, T + KT_w$ . If the consumer does not take up an offer at time  $T'$ , then she remains on her existing contract, and the new firm makes the switching offers. The consumer's switching cost after time  $T'$  is again  $s$ . She can collect the bonus once from each firm before time  $T'$ , and once from each firm starting at time  $T'$ . We impose the same tie-breaking rules for time  $T'$  as for time  $-1$ . At the beginning of the game, firms are randomly assigned to either time  $-1$  or time  $T'$ , and then choose their offers simultaneously.

**Proposition 7** (Switching Behavior). *Suppose  $Tv > s$  and Condition (4) holds. There is an equilibrium in which all three firms offer  $(B^n, p^n) = (s, v)$ . The consumer takes one of the offers at time  $-1$ , switches at time  $T'$ , and never switches again.*

In the equilibrium, competition at time  $T'$  fails for the same reason as at time  $-1$ . Hence, while the consumer does switch, she merely exchanges one bad deal for another. Although we have not found systematic evidence, casual observation suggests that there is some truth in this prediction. Many or most credit cards that successfully attract consumers with balance transfers, for instance, seem to be just average cards with typical high interest rates.

6. *Firms can offer (but cannot commit to) exploding sign-up bonuses.* We argue that offering exploding bonuses is not credible because if a consumer does not take the firm's offer, then it is in the firm's interest to offer it to future potential consumers. We make this point theoretically, but we also note the anecdotal observation familiar to most consumers in market economies: while sellers often claim that their promotions are temporary, the same promotions tend to arrive again in the future.

Suppose that a firm can make different bonus offers to initial and switching consumers. In our cash-bonus model, this gives rise to a profitable deviation: a firm can announce that its bonus is only available to initial consumers, inducing the consumer to start — and stick

with — its offer.<sup>34</sup> But consider a modification of our last model in which the consumer does not observe a switching offer until the relevant switching opportunity. Then, the same equilibrium outcome as in Proposition 7 survives: a firm that does not win the consumer initially wants to attract her at time  $T'$ , so it cannot credibly promise not to pay a bonus to switchers.<sup>35</sup>

## B Analysis of Consumer-Behavior Example in Appendix

### A.2

We provide an analysis of the individual-decisionmaking example right after Proposition 5:  $\beta = 1/2$ ,  $s = T = m = 100$ ,  $v = 20$ ,  $T_w = T/m = 1$ , and two firms offer  $B^1 = 150$ ,  $p^1 = 20$  and  $B^2 = 180$ ,  $p^2 = 1$ .

Because  $B^1, B^2 > s$  and  $p^2 < p^1$ , the consumer cannot do better than collecting both cash bonuses and paying price  $p^2$  for the entire time from 0 to  $T$ . Furthermore, she can achieve this by taking up firm 1's contract and then switching to firm 2 at opportunity 0, and there is no other way to achieve the same payoff. Hence, the consumer strictly prefers to take up firm 1's contract at  $t = -1$ .

At each switching opportunity except for the last one (i.e., at  $\kappa = 0, 1, \dots, 98$ ), the consumer's discounted utility if she switches to firm 2 immediately is

$$-s + \beta [-(T - \kappa T_w)p^2 + B^2] = -100 + \frac{1}{2} [-(100 - \kappa) + 180] = -60 + \frac{1}{2}\kappa,$$

whereas her discounted utility if she switches to firm 2 at the next opportunity is

$$\beta \{-s - T_w p^1 - [T - (\kappa + 1)T_w]p^2 + B^2\} = \frac{1}{2} \{-100 - 20 - [100 - (\kappa + 1)] + 180\} = -\frac{39}{2} + \frac{1}{2}\kappa.$$

Thus, the consumer procrastinates switching at each opportunity  $\kappa = 0, 1, \dots, K - 1$ .

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<sup>34</sup> A related potential deviation is one in which the firm commits to a low marketing intensity  $m$ , in effect telling the consumer that it will not send switching offers. The logic below applies to this deviation as well.

<sup>35</sup> The equilibrium can be supported by off-equilibrium beliefs in which the consumer assumes that a firm's switching offer is the same as its initial offer. Such a belief appears plausible: the consumer reasons that if the firm thinks it can attract her with the off-equilibrium initial contract at time  $-1$ , then the firm must think it can attract her with the same contract at time  $T'$ .

At the last switching opportunity ( $\kappa = K = 99$ ), the consumer's discounted utility if she switches to firm 2 is

$$-s + \beta [-(T - KT_w)p^2 + B^2] = -100 + \frac{1}{2} [-(100 - 99) + 180] = -\frac{21}{2},$$

whereas her discounted utility if she does not switch is

$$\beta [-(T - KT_w)p^1] = \frac{1}{2} (-1 \cdot 20) = -10.$$

Thus, the consumer does not switch at the last switching opportunity either.

## C Proofs

**Proof of Proposition 1.** We begin by establishing that (given the tie-breaking assumption), for any sequence of price offers  $(p^{-1}, \dots, p^K)$ , there is a unique vector of switching decisions  $d_0, \dots, d_K$ . Let  $p_{\geq \kappa} = (p^\kappa, \dots, p^K)$  be the vector of switching prices the consumer faces from switching opportunity  $\kappa$  onwards.

We start by establishing this for a time-consistent agent, i.e., an agent for whom  $\beta = \hat{\beta} = 1$ . We solve the game backwards. Let  $e(p, \kappa; p_{\geq \kappa})$  denote the expenditure from switching opportunity  $\kappa \in \{0, \dots, K\}$  onwards that a time-consistent consumer incurs for the service when the current price she pays (prior to the switching decision) is given by  $p$ . The expenditure at the last switching opportunity is  $e(p, K; p_{\geq K}) = \min\{s + p^K(T - KT_w), p(T - KT_w)\}$ . At switching opportunity  $\kappa < K$ , the consumer switches if and only if at the current price

$$pT_w + e(p, \kappa + 1; p_{\geq \kappa+1}) > s + p^\kappa T_w + e(p^\kappa, \kappa + 1; p_{\geq \kappa+1}). \quad (6)$$

Starting from the penultimate switching opportunity  $K - 1$ , we can thus recursively define the optimal expenditure from  $\kappa$  onwards as

$$e(p, \kappa; p_{\geq \kappa}) = \min\{pT_w + e(p, \kappa + 1; p_{\geq \kappa+1}), s + p^\kappa T_w + e(p^\kappa, \kappa + 1; p_{\geq \kappa+1})\}.$$

This completes the characterisation of a time-consistent consumer's switching behavior.

Since a naive consumer believes that her future selves behave as a time-consistent consumer would, self  $t$  who pays a current price  $p$  switches at opportunity  $\kappa < K$  if and only if

$$\beta p T_w + \beta e(p, \kappa + 1; p_{\geq \kappa+1}) > s + \beta p^\kappa T_w + \beta e(p^\kappa, \kappa + 1; p_{\geq \kappa+1}), \quad (7)$$

and she switches at  $\kappa = K$  if and only if  $\beta p(T - K T_w) > s + \beta p^K(T - K T_w)$ . This completes the characterization of a naive time-inconsistent agent's switching behavior.

(i). We establish that there exists a pure-strategy equilibrium with the properties stated in the proposition. Let every firm offer  $p_I = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$  and  $p_S = 0$ , and let the naive consumer behave as specified above. In this equilibrium, each firm makes positive profits if it is assigned to be the initial firm, and zero profits if it is not assigned to be the initial firm. We have established in the text that on the equilibrium path, for any  $p_I^n \leq \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ , the consumer never switches. We next show that if firm  $n$  deviates and sets a price  $p_S^n > 0$ , the consumer will also not switch. Because a consumer cannot benefit from switching to a higher price if she does not benefit from switching to a price of zero, whenever  $p_I \leq p^{NIS}$ , any  $p_S^n > 0$  cannot attract the consumer. Hence, suppose  $p^{NIS} < p_S^n \leq p^{IP}$ , so the naive consumer procrastinates switching in the candidate equilibrium. Consider a realization of the switching order in which the consumer is offered the deviant firm's switching price at time  $\kappa < K$ . Since the switching price of the deviant firm is  $p_S^n > 0$  and all other firms offer a switching price of zero,  $p^\kappa \geq p^{\kappa+1}$ . Hence, because a time-consistent consumer could always switch in the next period to a price which is weakly lower than  $p^\kappa$  at a cost of  $s$ , for any  $p \geq 0$ , we have

$$e(p, \kappa + 1; p_{\geq \kappa+1}) \leq e(p^\kappa, \kappa + 1; p_{\geq \kappa+1}) + s.$$

The naive consumer refrains from switching at any  $\kappa < K$  if (7) holds for  $p = p^{IP}$  and  $p^\kappa = p_S^n > 0$ , which is equivalent to

$$\beta p^{IP} T_w + \beta e(p^{IP}, \kappa + 1; p_{\geq \kappa+1}) \leq s + \beta p^\kappa T_w + \beta e(p^\kappa, \kappa + 1; p_{\geq \kappa+1}). \quad (8)$$

Using the above bound for  $e(p, \kappa + 1; p_{\geq \kappa+1})$  with  $p = p^{IP}$ , (8) holds because

$$\beta p^{IP} T_w + \beta s \leq s + \beta p^\kappa T_w \quad \iff \quad p^{IP} \leq \frac{1 - \beta}{\beta} \frac{1}{T_w} s + p^\kappa = p^{IP} + p^\kappa.$$

Hence, at  $\kappa < K$ , firm  $n$  cannot attract the naive consumer currently paying  $p^{IP}$  by setting  $p_S^n > 0$ . Next, consider a realization of the switching order in which the consumer is offered the deviant firm's switching price at time  $\kappa = K$ . At the last switching opportunity, the consumer refrains from switching if and only if  $\beta p^{IP}(T - KT_w) \leq s + \beta p^K(T - KT_w)$  or

$$p^{IP} \leq p^K + \frac{s}{\beta} \frac{1}{T - KT_w},$$

which, using that  $(T - KT_w) < T_w$ , follows from the definition of  $p^{IP}$ . Obviously, the deviant firm  $n$  which sets  $p_S^n > 0$  also cannot profitably attract a naive consumer currently paying  $p = 0$  (in case her deviation would induce the consumer to switch prior to  $\kappa$ ), and hence any deviation to  $p_S > 0$  is unprofitable.

Given that rivals set a switching price of zero, we established in the text that the naive consumer does not switch away from the initial offer if and only if  $p_I^n \leq \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ . Hence, setting  $p_I^n = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$  is also optimal, and we established that a symmetric pure-strategy equilibrium with the properties stated in the proposition exists.

(ii). We now show that in any symmetric pure-strategy equilibrium, the consumer never switches and all firms set  $p_I^n = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ .

Let  $(p_I, p_S)$  be the common initial and switching prices in a candidate symmetric pure strategy equilibrium. In this equilibrium, the consumer refrains from switching at switching opportunity  $\kappa = 0$  in case either she has no incentive to switch or she has an incentive to procrastinate. Switching at opportunity 0 rather than never saves the consumer  $\beta T(p_I - p_S)$  in discounted future payments, and has immediate cost  $s$ . Hence, Self 0 prefers never switching if

$$p_I \leq \frac{1}{\beta} \cdot \frac{s}{T} + p_S = p^{NIS} + p_S. \quad (9)$$

Now suppose the condition (9) is violated and consider the consumer's incentive to procrastinate at  $\kappa = 0$ . If it is suboptimal from the perspective of self  $\kappa = 0$  to switch at time  $\kappa = 1$  to the price  $p_S$ , then it is also suboptimal to switch at a later point in time, so the naive consumer plans never to switch; but then (9) would have to be satisfied, contradicting the case we are considering. Hence, the consumer must plan to switch at  $\kappa = 1$  in case she does not switch at  $\kappa = 0$ . If the consumer switches at  $\kappa = 0$ , she must pay an immediate effort

cost of  $s$ . If she switches at  $\kappa = 1$ , then she must pay  $p_I$  until the next switching opportunity  $\kappa = 1$ , lowering her discounted utility by  $\beta(p_I - p_S)T_w$ ; and in this case she must pay the switching cost next time, lowering her discounted utility by  $\beta s$ . Hence, she prefers to stay inactive in case

$$p_I \leq \frac{1 - \beta}{\beta} \cdot \frac{1}{T_w} \cdot s + p_S = p^{IP} + p_S. \quad (10)$$

In case either (9) or (10) holds with a strict inequality, the consumer at time zero strictly prefers not to switch at opportunity  $\kappa = 0$ . Furthermore, we now show that in this case she also strictly prefers not to switch at any opportunity  $\kappa > 0$ . Switching at an opportunity  $\kappa > 0$  saves the consumer strictly less than  $\beta T(p_I - p_S)$ , hence in such a subgame the non-switching condition (9) holds for a larger set of initial prices. The procrastination condition (10) remains the same for all  $\kappa < K$ ; and a consumer who does not switch at  $\kappa = K$  pays the higher price for a shorter time interval and does not have to pay the switching cost at time  $T$ , making not switching even more attractive. It implies that if either (9) or (10) holds with a strict inequality, the consumer strictly prefers not to switch at every future switching opportunity. But then a firm can benefit from slightly raising the initial price  $p_I$ , a contradiction unless  $p_I$  is already at the price ceiling of  $v$ . We conclude that

$$p_I \geq \min\{v, \max\{p^{NIS} + p_S, p^{IP} + p_S\}\}. \quad (11)$$

In case (11) holds with a strict inequality, the consumer strictly prefers to switch at opportunity  $\kappa = 0$  and an initial firm would earn zero profits from consumers assigned to it. The firm, however, could deviate and lower its initial price  $p_I^n$  so the above inequality holds with equality in which case its assigned consumers would not switch, and thus the deviant firm would earn positive profits from consumers assigned to it, a contradiction. We conclude that in a symmetric pure-strategy equilibrium,

$$p_I = \min\{v, \max\{p^{NIS} + p_S, p^{IP} + p_S\}\}, \quad (12)$$

and hence the consumer never switches in a symmetric pure-strategy equilibrium. Furthermore, in case  $v \leq \max\{p^{NIS}, p^{IP}\}$ , it establishes that  $p_I = v$ .

We finally rule out  $p_S > 0$  in case  $v > \max\{p^{NIS}, p^{IP}\}$ . Suppose otherwise, i.e., a symmetric pure-strategy equilibrium with  $p_S > 0$  exists. Then there exists an  $\epsilon \in (0, p_S)$  such that

$$p_I = \min\{v, \max\{p^{NIS} + p_S, p^{IP} + p_S\}\} > \max\{p^{NIS} + \epsilon, p^{IP} + \epsilon\}.$$

Now consider a firm  $n$  that deviates and sets  $p_S^n = \epsilon > 0$ . With positive probability, the consumer is initially assigned to one of firm  $n$ 's rivals and receives firm  $n$ 's switching offer at  $\kappa = 0$ . We next argue that the consumer strictly prefers switching in this case, and since this implies that the deviant firm  $n$  earns positive expected profits from a consumer assigned to its rivals, it establishes the desired contradiction. Because for  $p_S^n = \epsilon$ ,  $p_I > p^{NIS} + p_S^n$  at  $\kappa = 0$ , a consumer with an initial contract  $p_I$  strictly prefers switching to never switching. Note that if all firms' switching prices were  $p'_S = \epsilon$ , then the consumer would also strictly prefer switching at  $\kappa = 0$  to procrastinating and switching at the next opportunity  $\kappa = 1$  because  $p_I > p^{IP} + \epsilon$ . Since the consumer faces the same or higher switching prices at any future period, raising the switching prices of all rival firms simultaneously from  $p'_S = \epsilon$  makes procrastination (weakly) less desirable. Hence, when seeing the deviant firm's switching price at  $\kappa = 0$ , the consumer will switch immediately and pay a strictly positive price, a contradiction. We conclude that  $p_S = 0$ . Thus, (12) implies that  $p_I = \max\{p^{NIS}, p^{IP}\}$  when  $v > \max\{p^{NIS}, p^{IP}\}$ .  $\square$

**Proof of Proposition 2.** The characterization of the consumer behavior from time 0 onwards remains unchanged from that in the proof of Proposition 1, except for the minor adjustments that now the rivals' marketing intensity is  $N_S m$  (rather than  $(N - 1)m$ ), and hence  $T_w = T/(N_S m)$  (rather than  $T_w = T/((N - 1)m)$ ). This also implies that if rivals set  $p_I = p^{IP}$ , setting  $p_S = 0$  is a best response.

(i). Consider a candidate equilibrium in which all firms charge  $(p_I, p_S)$ . When  $\beta = 1$ , the consumer does not procrastinate. We first establish that on the path of play, when all firms charge the same  $p_S$ , the consumer either switches once at  $\kappa = 0$  or not at all: switching at  $\kappa = 0$  to a lower switching price  $p_S < p_I$  yields a benefit of  $(p_I - p_S)T$ , which is strictly



greater than the benefit from switching later. Because switching entails the same switching costs  $s$  independently of when the consumer switches, she thus either switches immediately or not at all. Now suppose for the sake of a contradiction that the consumer switches in equilibrium. Given our tie-breaking rule, this implies that  $Tp_I > Tp_S + s$  and that when charging  $p_I$  firm  $n$  earns zero profits from any consumer initially assigned to it. Firm  $n$ , however, can profitably deviate and set  $p_I^n = \frac{s}{T}$ , in which case the firm earns profits  $s$  from any consumer that is initially assigned to it. We conclude that the consumer does not switch in any equilibrium and hence that  $p_I \leq p_S + \frac{s}{T}$ .

Because the consumer does not switch in equilibrium, she has a strict incentive to choose a firm among those in  $N_I$  with a lowest price  $p_I^N$ . Suppose for the sake of contradiction that  $p_I > 0$ . Then, by the usual Bertrand logic, there exists a profitable deviation: Conditionally on being selected as an initial firm, some firm  $n'$  has an expected market share of no more than  $\frac{1}{N_I}$  and hence earns no more than  $\frac{Tp_I}{N_I}$ . By deviating and setting a price of  $p_I^{n'} = p_I - \epsilon$  this firm  $n'$  earns  $T(p_I - \epsilon)$ , which for sufficiently small  $\epsilon > 0$  is a profitable deviation. We conclude that in any equilibrium  $p_I = 0$ .

(ii). We first prove that a symmetric pure-strategy equilibrium exists in which all firms charge  $(p_I^n, p_S^n) = (p^{IP}, 0)$ . It follows from Part (i) from the proof of Proposition 1 that setting  $p_S = 0$  is part of a best response. We will now verify that setting  $p_I = p^{IP}$  is also optimal. To do so, we first characterize the consumer's choice at time  $-1$  when all firms in  $N_S$  set  $p_S = 0$ . For any offer with price  $p_I^n$  the consumer chooses at time  $-1$ , she anticipates switching to a price  $p_S = 0$  at opportunity  $\kappa = 0$  in case  $Tp_I^n > s$ . Therefore, in case all firms in  $N_I$  set prices above  $p_I^n > \frac{s}{T}$ , the naive consumer — thinking that she will end up switching and paying a price for the service of zero — is indifferent between all initial offers and to construct the equilibrium we suppose from now on that she chooses each firm among  $N_I$  with probability  $\frac{1}{N_I}$  whenever she is indifferent among them. For a price  $p_I^n \leq \frac{s}{T}$  the consumer does not anticipate switching, and in case  $p_I^n < \frac{s}{T}$  she strictly prefers taking the deviating low price offer and sticking with it.

In either case, given that rivals charge  $p_I = p^{IP}$ , for any  $p_I^n \in \left[ \frac{s}{T} - \frac{V}{\beta T}, p^{IP} \right]$  a firm

belonging to the set  $N_I$  earns from the consumer

$$\frac{p_I^n}{N_I} \leq \frac{p^{IP}}{N_I} = \frac{N_S}{N_I} \frac{1-\beta}{\beta} m \frac{s}{T};$$

hence  $p_I^n = p^{IP}$  is optimal for the range of  $p_I^n \in \left[ \frac{s}{T} - \frac{V}{\beta T}, p^{IP} \right]$ . A firm belonging to the set  $N_I$  earns zero from this consumer when  $p_I^n > p^{IP}$ , because then the consumer will switch at time 0, so this cannot be part of a profitable deviation. When  $p_I^n < \frac{s}{T}$ , a firm belonging to the set  $N_I$  earns at most  $Tp_I^n$  from attracting the consumer. Thus, there is no profitable deviation in an initial price in case

$$\underbrace{\frac{N_S}{N_I} \frac{1-\beta}{\beta} m \frac{s}{T}}_{>1} \geq \frac{s}{T},$$

where the left hand side is greater than  $\frac{s}{T}$  by the assumption of the proposition. We conclude that a symmetric pure-strategy equilibrium exists in which all firms charge  $(p_I^n, p_S^n) = (p^{IP}, 0)$ .

Finally, we show that there is also a symmetric pure-strategy equilibrium in which  $(p_I, p_S) = (0, 0)$ . Note first that it is impossible to induce a consumer whose initial contract specifies a price of  $p_I = 0$  to switch, so setting  $p_S = 0$  is a best response. Second, a consumer anticipates not switching and paying a price of zero when selecting an initial offer of  $p_I = 0$ , so a firm that deviates and charges a higher initial price  $p_I^n > 0$  attracts no consumers. Thus, such a deviation is also unprofitable.  $\square$

**Proof of Proposition 3.** We focus on equilibria in which  $p_S^n = 0$  for all  $n$ . In a symmetric pure-strategy equilibrium  $(p_I, p_S, m)$ , there cannot exist a profitable deviation for any firm. This implies that when fixing  $m$  at the equilibrium level, no firm can benefit from setting a different initial or switching price. Recall that  $M^{-n}$  denotes the aggregate marketing intensity by firms other than  $n$ . Fixing  $M^{-n}$  and  $m^n$  at the equilibrium level, it thus follows from Proposition 1 that  $p_I = \min \left\{ v, \max \left\{ \frac{s}{\beta T}, \frac{(1-\beta)M^{-n}s}{\beta T} \right\} \right\}$ .

We now solve for the marketing intensity of firm  $n$  (i.e.,  $m^n$ ) given  $M^{-n}$ . Since firm  $n$ 's probability of being chosen as the initial firm is  $m^n / (M^{-n} + m^n)$ , its expected profit is

$$\frac{m^n}{M^{-n} + m^n} \cdot Tp_I(M^{-n}) - cm^n.$$

The first-order condition with respect to  $m^n$  is

$$\frac{M^{-n}}{(M^{-n} + m^n)^2} \cdot T p_I(M^{-n}) - c = 0, \quad (13)$$

which leads to  $M^{-n} + m^n = \sqrt{M^{-n} T p_I(M^{-n}) / c}$ . In any pure-strategy equilibrium, the total marketing intensity  $M^{-n} + m^n$  is a constant, and since the right-hand side is strictly increasing in the number of rival offers  $M^{-n}$ , the marketing intensity is the same across all firms  $n$ . That is,  $m^n$  must be the same across all firms in any pure-strategy equilibrium.

Setting  $M^{-n} = m^n(N - 1)$ ,  $m^n$  is uniquely determined by (13):

$$m^n = \frac{(N - 1)T}{N^2 c} p_I(m^n(N - 1)),$$

where  $p_I(m^n(N - 1)) = \min \left\{ v, \max \left\{ \frac{s}{\beta T}, \frac{(1-\beta)m^n(N-1)s}{\beta T} \right\} \right\}$ . Given  $Tv > s/\beta$ , note that  $p_I(M^{-n}) = \frac{s}{\beta T}$  if  $M^{-n} < \frac{1}{1-\beta}$ ,  $p_I(M^{-n}) = \frac{(1-\beta)M^{-n}s}{\beta T}$  if  $M^{-n} \in \left[ \frac{1}{1-\beta}, \frac{\beta T v}{(1-\beta)s} \right]$ , and  $p_I(M^{-n}) = v$  if  $M^{-n} > \frac{\beta T v}{(1-\beta)s}$ . Suppose first  $M^{-n} < \frac{1}{1-\beta}$ . Then substituting  $p_I(M^{-n})$  into the above yields  $m^n = \frac{(N-1)s}{N^2 \beta c}$ . In this candidate equilibrium, the condition  $M^{-n} = m^n(N - 1) < \frac{1}{1-\beta}$  holds if and only if  $c > \frac{(N-1)^2(1-\beta)s}{N^2 \beta} = \bar{c}$ . Suppose second  $M^{-n} > \frac{\beta T v}{(1-\beta)s}$ . Then substituting  $p_I(M^{-n})$  into the above yields  $m^n = \frac{(N-1)T v}{N^2 c}$ . In this candidate equilibrium, the condition  $M^{-n} = m^n(N - 1) > \frac{\beta T v}{(1-\beta)s}$  holds if and only if  $c < \frac{(N-1)^2(1-\beta)s}{N^2 \beta} = \bar{c}$ . We thus established that, given  $p_S^n = 0$  for all  $n$ , for both case (i)  $c > \bar{c}$  and case (ii)  $c < \bar{c}$ ,  $m^n$  and  $p_I^n$  are uniquely pinned down as a candidate equilibrium.

Consider case (i) and consider potential deviations by firm  $n$ . Given  $M^{-n} = \frac{(N-1)^2 s}{N^2 \beta c}$ , because  $p_I(M^{-n})$  does not depend on  $m^n$ , it follows from Proposition 1 that choosing  $p_I^n = \frac{s}{\beta T}$  and  $p_S^n = 0$  is a best response for firm  $n$  regardless of  $m^n$ . Given  $p_I^n = \frac{s}{\beta T}$  and  $p_S^n = 0$ , (13) uniquely determines  $m^n = \frac{(N-1)s}{N^2 \beta c}$ . Hence, for firm  $n$ , taking the strategy specified in Proposition 3 (i) is a best response.

Consider case (ii) and consider potential deviations by firm  $n$ . Given  $M^{-n} = \frac{(N-1)^2 T v}{N^2 c}$ , because  $p_I(M^{-n})$  is already at the cap  $v$ , it follows from Proposition 1 that choosing  $p_I^n = v$  and  $p_S^n = 0$  is a best response for firm  $n$  regardless of  $m^n$ . Given  $p_I^n = v$  and  $p_S^n = 0$ , (13) uniquely determines  $m^n = \frac{(N-1)T v}{N^2 c}$ . Hence, for firm  $n$ , taking the strategy specified in Proposition 3 (ii) is a best response.  $\square$

**Proof of Proposition 4.** (i). Suppose for the sake of contradiction that there exists an equilibrium in which the consumer pays a strictly positive total price. We first rule out the existence of such a candidate equilibrium in which the consumer switches contracts. For the consumer to pay a strictly positive price in this case, it must be that  $B^1 + B^2 < T$  and  $\underline{p} = \min\{p^1, p^2\} > 0$ . When facing such contracts, the consumer in any cost-minimizing switching plan pays no more than  $\underline{p}(T - B^1 + B^2)$  and hence one firm, say firm  $i$ , earns at most  $\underline{p}(T - B^1 + B^2)/2$ . When deviating keeping the bonus  $B^i$  fixed and charging  $p^i = \underline{p} - \epsilon$ , the consumer in any optimal plan would purchase at a positive price only from firm  $i$  ensuring that it earns at least  $(\underline{p} - \epsilon)(T - B^1 + B^2)$ , which for sufficiently small  $\epsilon > 0$  is greater than  $\underline{p}(T - B^1 + B^2)/2$ . Thus, a profitable deviation exists and hence there exists no equilibrium in which the consumer (on the path of play) switches and pays a positive total price.

Second, consider a candidate equilibrium in which the consumer does not switch and pays a strictly positive total price of  $p^i(T - B^i)$ . Observe that both firms must have positive demand as otherwise the firm  $j$  with zero demand could deviate to an offer  $(0, p^j)$  with  $p^j < p^i(T - B^i)/T$ ; in that case, any cost-minimizing plan must involve purchasing from firm  $j$  for at least some of the time (as otherwise the consumer pays a total price of  $p^i(T - B^i) > Tp^j$ ), and hence firm  $j$  would earn positive profits, a contradiction.

Given that both firms have positive demand and that the consumer does not switch, it must be both that  $p^1(T - B^1) = p^2(T - B^2)$  (so the consumer is indifferent between the total prices charged) and that  $\min\{T - B^i, B^j\}p^i \leq s$  for  $i = 1, 2$  and  $j \neq i$  (so that the consumer prefers to save the switching cost rather than to collect firm  $j$ 's bonus time in addition). Consider a firm  $i$  that attracts the consumer with probability no greater than  $1/2$ . By deviating to an offer  $(B^i, p^i - \epsilon)$ , firm  $i$  would attract the consumer with probability one and facing a lower price for the service the consumer would strictly prefer not to switch and collect the bonus time from firm  $j$ . Therefore, firm  $i$  would earn  $(p^i - \epsilon)(T - B^i)$ , which for sufficiently small  $\epsilon > 0$  is greater than  $p^i(T - B^i)/2$ , a contradiction. We conclude that there exists no equilibrium in which the consumer pays a strictly positive total price.

(ii). We proceed in three steps. In Step (I), we show that under Condition (4), the

consumer does not switch. In Step (II), we specify the consumer's behavior in the candidate equilibria. In Step (III), we verify it is optimal for the firms to offer the candidate equilibrium contracts.

*Step (I). The consumer does not switch.* We begin by specifying the consumer's switching behavior. Once a consumer chooses the contract  $(B^i, p^i)$ , the consumer procrastinates switching as long as

$$s + \beta \max\{T - B^i - B^j, 0\}p^j \geq \beta s + \beta(p^i - p^j) \min\left\{\frac{T}{m}, T - B^i\right\} + \beta \max\left\{T - \frac{T}{m} - B^i - B^j, 0\right\}p^j.$$

A sufficient condition for the consumer not to switch is that  $s \geq \beta s + \beta(p^i - p^j)\frac{T}{m}$ , or equivalently,

$$s \geq \frac{\beta}{1 - \beta} \frac{T}{m} (p^i - p^j).$$

Since by Condition (4),

$$s > \frac{\beta}{1 - \beta} \frac{T}{m} v \geq \frac{\beta}{1 - \beta} \frac{T}{m} (p^i - p^j), \quad (14)$$

for any pair of offers  $(B^i, p^i), (B^j, p^j)$  a consumer who chooses the contract  $(B^i, p^i)$  ends up procrastinating switching.

*Step (II). Specifying consumer behavior.* When the consumer selects contract  $(B^i, p^i)$  at  $t = -1$ , the consumer either anticipates collecting both bonuses (and hence switching) or staying with a single firm. In case the consumer plans to collect both bonuses and  $p^i \neq p^j$ , in any optimal plan the consumer wants to exclusively pay the price  $\underline{p} = \min\{p^i, p^j\}$ . She can always plan to achieve this by taking the higher-priced firm's bonus first and then switching (according to her plan) immediately at the end of the first bonus period to the lower priced firm. Hence, we suppose that she selects firm  $i$  for which  $p^i > p^j$  in this case at  $t = -1$ .

By the above arguments, she weakly prefers the switching plan if and only if

$$s + \max\{T - B^i - B^j, 0\}\underline{p} \leq (T - B^i)p^i \text{ for } i = 1, 2 \text{ and } i \neq j. \quad (15)$$

We suppose that the consumer plans to switch when it is weakly optimal to do so.

Whenever the consumer plans not to switch, i.e., if (15) is not satisfied, she selects a contract with smaller total payment:

$$(T - B^i)p^i \leq (T - B^j)p^j.$$

If the consumer plans not to switch and is indifferent between the two offers, i.e., if  $(T - B^i)p^i = (T - B^j)p^j$ , then we suppose that she selects each firm with probability 1/2.

If (15) is satisfied — i.e., the consumer plans to switch — and either  $T - B^i - B^j \leq 0$  or  $p^i = p^j$ , then the consumer is indifferent in which order to switch among the firms. When  $B^i = B^* \leq B^j$ , we suppose that the consumer selects each firm with probability 1/2. When  $B^i = B^* > B^j$ , we suppose that the consumer selects firm  $i$ .

*Step (III). There exists no profitable deviation.* Let  $\underline{B} = \frac{T}{2} - \frac{\sqrt{T(T - \frac{8s}{v})}}{2}$  and  $\overline{B} = \frac{T}{2} + \frac{\sqrt{T(T - \frac{8s}{v})}}{2}$ . Having specified the consumer behavior, we now show that both firms offering  $(B^*, p^*) = (B^*, v)$  for a given  $B^* \in [\underline{B}, \overline{B}]$  is indeed an equilibrium.

By Step (I), the firms' demand is fully determined by the contract the consumer selects at  $t = -1$  because the consumer never switches. On the path of play, the consumer chooses either firm with probability 1/2, and hence, equilibrium profits are  $(1/2)(T - B^*)v$ , which are strictly positive since  $T > \overline{B}$ . Any deviation to a contract offer  $(B^i, p^i)$  with  $p^i < v$  for which the consumer plans to take both bonuses is unprofitable, because in that case the consumer selects the contract of firm  $j$  at  $t = -1$  by Step (II) and hence the deviating firm earns zero profits.

We now establish that if there exists a profitable deviation  $(B^i, p^i)$  with  $p^i \in (0, v)$  for which the consumer plans not to switch, then there exists another profitable deviation  $(0, p^{i'})$  for which the consumer plans not to switch. By (15), the consumer plans not to switch from  $(B^i, p^i)$  if and only if

$$(T - B^i)p^i < s + \max\{T - B^i - B^*, 0\}p^i. \quad (16)$$

We now verify that if the deviation  $(B^i, p^i)$  is profitable, so is the deviation  $(0, p^{i'})$  where  $p^{i'} = (T - B^i)p^i/T < p^i$ . If the consumer selects the contract, the firm earns the exact same amount with both deviations; furthermore, the left-hand side of (16) remains unchanged while the right-hand side weakly increases since

$$\max\{T - B^i - B^*, 0\}p^i = \max\{(T - B^i)p^i - B^*p^i, 0\} = \max\{Tp^{i'} - B^*p^i, 0\} \leq \max\{Tp^{i'} - B^*p^{i'}, 0\}.$$

Thus, from now on we restrict attention to deviations  $(0, p^{i'})$  for which the consumer plans

not to switch. Such a deviation is profitable if and only if the deviating price satisfies

$$Tp^{i'} > (1/2)(T - B^*)v.$$

Let  $p^{i''} = (1/2)(T - B^*)(v/T)$  denote the infimum of such prices. Substituting  $p^{i''}$  into (15) leads to the condition in which the consumer plans not to switch only if at the candidate equilibrium bonus  $B^*$

$$s + (T - B^*)^2 \frac{v}{2T} - \frac{1}{2}(T - B^*)v > 0.$$

The left hand side is a convex function in  $B^*$ , which equals zero at if  $B^* = \underline{B}$  or  $B^* = \overline{B}$ . Hence, for any  $B^* \in [\underline{B}, \overline{B}]$  the consumer prefers to switch when offered a contract  $(0, p^{i''})$  rather than selecting and sticking with  $(0, p^{i''})$ . Because the incentives for switching rather than planing to stick with a deviating contract  $(0, p^{i'})$  are increasing in the deviating firm's price, for any  $v > p^{i'} \geq p^{i''}$  the consumer does not select the deviating firm's contract at  $t = -1$ . We conclude that there exists no profitable deviation in which  $p^i < v$  and the consumer does not plan to switch.

We are left to consider any deviation by firm  $i$  in which  $p^i = v$ . By the tie-breaking rule in the last paragraph of Step (II), whenever the consumer plans to switch, a deviating firm strictly decreases its profits. Consider the case in which the consumer plans not to switch. The consumer selects firm  $i$ 's contract  $(B^i, v)$  and plans not to switch only if

$$(T - B^i)v < s + \max\{T - B^i - B^*, 0\}v.$$

In the case  $T - B^i - B^* > 0$ , the condition becomes  $B^*v < s$ . As we next establish, however, this contradicts with the fact that

$$\underline{B}v = \frac{1}{2} \left( Tv - \sqrt{(Tv)^2 - 8sTv} \right) > 2s. \quad (17)$$

To show (17), note that at  $Tv = 8s$  the above inequality holds since  $Tv - \sqrt{(Tv)^2 - 8sTv} = 8s$ . Now, consider  $x - \sqrt{x^2 - 8sx}$  with  $x > 8s$ . The term  $(x - \sqrt{x^2 - 8sx})^2$  must be strictly positive. Note that  $(x - \sqrt{x^2 - 8sx})^2 = 2x(x - 4s - \sqrt{x^2 - 8sx}) > 0$  implies  $x - \sqrt{x^2 - 8sx} > 4s$ ; so (17) holds for all  $Tv > 8s$ .

In the case  $T - B^i - B^* \leq 0$ , the deviating firm's total profits are lower than  $s$ . This is not a profitable deviation, as each firm earns more than  $s$  in any candidate equilibrium:  $(1/2)(T - B^*)v \geq (1/2)(T - \bar{B})v = (1/2)\underline{B}v > s$ , where the last inequality follows from (17).

Finally, we show that as  $T \rightarrow \infty$ ,  $\underline{B} \rightarrow 2s/v$ ; that is

$$\begin{aligned} \lim_{T \rightarrow \infty} \underline{B} &= \frac{1}{2} \lim_{T \rightarrow \infty} T - \sqrt{T \left( T - \frac{8s}{v} \right)} = \frac{1}{2} \lim_{T \rightarrow \infty} \frac{T^2 - T \left( T - \frac{8s}{v} \right)}{T + \sqrt{T \left( T - \frac{8s}{v} \right)}} = \frac{4s}{v} \lim_{T \rightarrow \infty} \frac{1}{\frac{T + \sqrt{T \left( T - \frac{8s}{v} \right)}}{T}} \\ &= \frac{4s}{v} \lim_{T \rightarrow \infty} \frac{1}{1 + \sqrt{\frac{T^2 - T \frac{8s}{v}}{T^2}}} = \frac{4s}{v} \lim_{T \rightarrow \infty} \frac{1}{1 + \sqrt{1 - \frac{8s}{T}}} = \frac{2s}{v}. \end{aligned}$$

□

**Proof of Proposition 5.** Consider the following candidate equilibrium in which both firms offer  $(B^n, p^n) = (B^*, p^*) = (s, v)$ . If the consumer is indifferent whether to plan on collecting a bonus at  $t = -1$ , she does plan to do so. If she is indifferent as to which initial offer to take at  $t = -1$ , she selects each firm with probability  $1/2$ . Between  $t = 0$  and  $t = T$ , whenever the consumer is indifferent, she does not switch.

As a preliminary observation, note that because the consumer cannot collect the bonus twice and she always sees the same switching offer, she both plans to and actually switches at most once independently of whether firms choose equilibrium or non-equilibrium offers. We thus focus on the first time the consumer switches below. Also, since a naive consumer solves an optimization problem, her decision on whether she wants to switch or not at a given switching opportunity is independent of how she thinks she will act in the future whenever she is indifferent; we use the convention below that in such a case she believes her future self will switch immediately when indifferent.

(i). We first show that the above strategies constitute an equilibrium. Since  $p^*T - B^* = Tv - s > 0$ , each firm prefers the equilibrium offer to any offer which results in the firm having no sales. Note that the consumer is indifferent whether to plan to switch between firms in the candidate equilibrium, and by the above equilibrium specification, she keeps believing that her future self will switch at the next switching opportunity. Hence, the consumer thinks that she will receive  $B^*$  from both firms. Given this, a deviation to  $B^i > s$



and  $p^i = v$  does not induce a consumer's switching until the last opportunity. At the last opportunity, the consumer switches to firm  $i$  only if  $-s + \beta B^i \geq 0$  or equivalently  $B^i \geq \frac{s}{\beta}$ , so firm  $i$ 's profits from it are at most  $(T - K\frac{T}{m})p^i - B^i \leq \frac{T}{m}v - \frac{s}{\beta}$ , which is negative by the assumption  $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$ . Hence, a deviation  $B^i > s$  and  $p^i = v$  merely decreases firm  $i$ 's total profits. Also, each firm has no incentive to deviate to  $B^i < s$  and  $p^i = v$ . To see this, suppose that the consumer takes up firm  $i$ 's offer at  $t = -1$ . Because  $p^i = p^*$  and  $B^* = s$ , the consumer is indifferent whether to plan to switch from firm  $i$  to firm  $j$ . Hence, given that the consumer takes up firm  $i$ 's offer at  $t = -1$ , her anticipated total payoff is  $-Tp^i + B^i < -Tv + s$ . When the consumer takes up firm  $j$ 's offer at  $t = -1$  and would not switch thereafter, her anticipated total payoff is  $-Tp^* + B^* = -Tv + s$ . This implies that the consumer would strictly prefer to take up firm  $j$ 's offer at  $t = -1$  and then not switch, and hence,  $B^i < s$  and  $p^i = v$  is not a profitable deviation.

Suppose the rival firm  $j \neq i$  makes the candidate equilibrium offer  $(B^*, p^*) = (s, v)$  and consider deviations by firm  $i$  in which  $p^i < v$ . If the consumer selects firm  $j$ 's offer at  $t = -1$ , the consumer believes that she will switch to offer  $(B^i, p^i)$  at  $t = 0$  if the following intention-to-switch condition holds:

$$-s - Tp^i + B^i \geq -Tv \iff s \leq T(v - p^i) + B^i, \quad (18)$$

in which case, her anticipated total payoff when selecting the rival's offer first is  $-s - Tp^i + B^* + B^i = -Tp^i + B^i$ .

Consider first the case in which (18) does not hold. Then, the consumer will not switch from firm  $j$  to firm  $i$  at any switching opportunity since  $s > T(v - p^i) + B^i$  implies that  $s > \beta(T(v - p^i) + B^i)$ . Because  $p^i < v = p^*$ , the consumer also never switches from firm  $i$  to firm  $j$ . As violating (18) is equivalent to  $-Tp^i + B^i < -Tv + s$ , the consumer strictly prefers to take up firm  $j$ 's offer at  $t = -1$  (and will never switch away from firm  $j$ ). Hence, such a deviation by firm  $i$  is not profitable.

Consider second the case in which (18) holds. The consumer's anticipated total payoff when selecting firm  $i$ 's offer at  $t = -1$  is  $-Tp^i + B^i$ , because the consumer expects not to switch to firm  $j$  given  $p^* = v > p^i$  and  $B^* = s$ . The consumer's anticipated total

payoff when selecting firm  $j$ 's offer at  $t = -1$  and then switch to firm  $i$  at  $t = 0$  is also  $-s - Tp^i + B^* + B^i = -Tp^i + B^i$ . As the consumer is indifferent at  $t = -1$ , by the tie-breaking rule she plans to collect both bonuses and randomly (with equal probability) chooses an offer at  $t = -1$ . Note that (18) implies that  $Tv - s \geq Tp^i - B^i$ , and hence conditionally on a consumer selecting firm  $i$  at  $t = -1$  and not switching, firm  $i$  earns less from the consumer following the deviation. For a profitable deviation to exist, thus, the deviation must induce the consumer to switch from firm  $j$  to firm  $i$  and the firm must earn profits from the consumer's switching decision. Given that the consumer has selected firm  $j$ 's offer  $(B^*, p^*) = (s, v)$  at  $t = -1$ , the consumer does not procrastinate switching to firm  $i$  at switching opportunity  $\kappa < K$  only if

$$-s + \beta \left( -\frac{T}{m}p^i + B^i \right) > \beta \left( -s - \frac{T}{m}v + B^i \right) \iff p_i + \frac{1 - \beta}{\beta} \frac{m}{T} s < v. \quad (19)$$

Condition (4), however, implies that (19) is violated for all  $p^i \in [0, v]$ , and thus the consumer does not switch prior to the last switching opportunity  $K$ . The consumer switches at switching opportunity  $K$  only if

$$-s + \beta \times \underbrace{\left[ -\left( T - K \frac{T}{m} \right) p^i + B^i \right]}_{\equiv -\pi'} \geq -\beta \left( T - K \frac{T}{m} \right) v, \quad (20)$$

where  $\pi'$  are the profits the firm earns from the consumer switching at switching opportunity  $K$ . By rewriting (20), we obtain

$$\pi' \leq \left( T - K \frac{T}{m} \right) v - \frac{s}{\beta} \leq \frac{T}{m} v - \frac{s}{\beta} < 0,$$

where the strict inequality follows from Condition (4). Thus, either the consumer does not switch following the deviation or firm  $i$  makes a loss from the consumer who switches to firm  $i$ . Hence, the deviation is unprofitable.

We conclude that our candidate equilibrium is indeed an equilibrium.

(ii). We show that the above equilibrium is unique in pure strategies, given  $Tv > s$ ,  $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$ , and the tie-breaking rule in which if the consumer is indifferent whether to plan on collecting a bonus at  $t = -1$ , she does plan to do so, and if the consumer is

indifferent as to which initial offer to take at  $t = -1$ , she selects each firm with probability  $1/2$ . We establish uniqueness in eight steps.

*Step (I).* At any switching opportunity except for the last one (i.e., at opportunities  $\kappa = 0, 1, \dots, K - 1$ ), the consumer does not switch. Note that the consumer switches from firm  $j$  to firm  $i$  only if she does not have an incentive to procrastinate, that is,

$$-s + \beta \left( -\frac{T}{m} p^i + B^i \right) \geq \beta \left( -s - \frac{T}{m} p^j + B^i \right) \iff p_i + \frac{1 - \beta}{\beta} \frac{m}{T} s \leq p^j.$$

Because of the assumption  $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$ , however, this condition is not satisfied for any  $p^i, p^j \in [0, v]$ .

*Step (II).* Whenever the consumer switches from firm  $j$  to firm  $i$  at the last switching opportunity  $\kappa = K$ , firm  $i$  earns negative profits. The consumer switches from firm  $j$  to firm  $i$  at the last switching opportunity only if

$$\begin{aligned} -s + \beta \left[ -\left( T - K \frac{T}{m} \right) p^i + B^i \right] &\geq -\beta \left( T - K \frac{T}{m} \right) p^j \\ \iff \left( T - K \frac{T}{m} \right) p^i - B^i &\leq \left( T - K \frac{T}{m} \right) p^j - \frac{s}{\beta}. \end{aligned} \quad (21)$$

Notice that firm  $i$ 's profits from the consumer switching at opportunity  $K$  is  $(T - K \frac{T}{m}) p^i - B^i$ ; and that

$$\left( T - K \frac{T}{m} \right) p^j - \frac{s}{\beta} \leq \frac{T}{m} v - \frac{s}{\beta} < 0,$$

where the strict inequality follows from the assumption that  $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$ . Thus, if a consumer switches at switching opportunity  $K$ , the firm attracting her makes a loss from this consumer.

We thus conclude that independently of whether firms choose equilibrium or non-equilibrium offers, the consumer either does not switch or the firm that attracts a switching consumer makes a loss from it. From now on, we specify properties of pure-strategy equilibrium offers.

*Step (III).* No firms sets  $B^i > s$ . Suppose otherwise. Let firm  $i$  set  $B^i > s$ . Consider first the case in which firm  $j \neq i$  sets  $B^j \geq s$ . In this case, the consumer at  $t = -1$  plans to collect both firms' sign-up bonuses. Suppose firm  $i$  deviates to an offer  $B^{i'}$  and  $p^{i'} = p^i$  for which  $s < B^{i'} < B^i$ . This deviation does not change the consumer's decision at  $t = -1$  because

of the tie-breaking rule, and the deviation also makes the consumer (weakly) less likely to switch from firm  $j$  to firm  $i$  at  $t \geq 0$ . Hence, it is a profitable deviation — a contradiction. Consider second the case in which firm  $j \neq i$  sets  $B^j < s$ . In this case, the consumer at  $t = -1$  plans to collect firm  $i$ 's offer by the tie-breaking rule. Then, using the same deviation as above, firm  $i$  can decrease its sign-up bonus and earn larger profits — a contradiction.

*Step (IV).* *The consumer does not switch at the last switching opportunity following equilibrium offers.* To see this, note that Condition (21) never holds when  $B^i \leq s$  and  $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$ .

*Step (V).* *Both firms earn positive profits in any equilibrium.* Suppose otherwise. Let firm  $i$  earn zero profits. Consider firm  $i$ 's deviation to  $B^{i'} = s + \epsilon$  and  $p^{i'} = v$  for a sufficiently small  $\epsilon > 0$ . At  $t = -1$ , the consumer is either indifferent between which offer to choose or strictly prefers to take up firm  $i$ 's offer (with potentially planning to switch to firm  $j$ ). Also, if the consumer takes up firm  $i$ 's offer at  $t = -1$ , then she never switches as shown in Steps (I) and (IV) above. Hence, by the tie-breaking rule, the consumer takes up firm  $i$ 's offer with positive probability at  $t = -1$ , and given that the consumer selects firm  $i$  at  $t = -1$ , firm  $i$  earns at least  $p^{i'} - B^{i'} = Tv - s - \epsilon$ . For a sufficiently small  $\epsilon > 0$ , firm  $i$ 's profits from this deviation is positive — a contradiction.

Combined with the result that the consumer does not switch at  $t \geq 0$  in any equilibrium, we conclude that the consumer must be indifferent between both offers at  $t = -1$ , and thus chooses each firm's offer with probability  $1/2$  at  $t = -1$  in any pure-strategy equilibrium.

*Step (VI).* *At least one firm  $i$  offers  $B^i = s$ .* Suppose otherwise: both firms offer  $B^i < s$ . Then, the consumer does not plan to switch to another firm at  $t \geq 0$  when she initially selects firm  $n$  which sets  $p^n = \min\{p^1, p^2\}$ . Given that we established that the consumer is indifferent between two firms at  $t = -1$ , by offering a slightly higher  $B^n$ , firm  $n$  can attract the consumer with probability 1 instead of  $1/2$  at  $t = -1$ , and hence can increase its profits — a contradiction.

*Step (VII).* *Whenever firm  $i$  sets  $B^i = s$ , it also sets  $p^i = v$ .* Suppose otherwise:  $B^i = s$  and  $p^i < v$ . By deviating to  $p^{i'} = v$  and  $B^{i'} = B^i = s$ , at  $t = -1$ , the consumer is either

indifferent or strictly prefers to take up firm  $i$ 's offer (with potentially planning to switch to firm  $j \neq i$ ). Hence, firm  $i$  can still attract the consumer with probability at least  $1/2$  at  $t = -1$ , and given that firm  $i$  attracts her at  $t = -1$ , it earns larger profits — a contradiction.

*Step (VIII).* If firm  $i$  sets  $B^i = s$  and  $p^i = v$ , firm  $j \neq i$  also sets  $B^j = s$  and  $p^j = v$ . First, firm  $j$  must set  $p^j = v$  to make positive profits; if firm  $j$  sets  $B^j \leq s$  and  $p^j < v$ , by the tie-breaking rule, the consumer strictly prefers to choose firm  $i$  at  $t = -1$  (with planning to switch to firm  $j$  at  $t = 0$ ), and since she actually never switches, firm  $j$  earns zero profits. Second, if firm  $j$  sets  $B^j < s$  and  $p^j = v$ , then the consumer strictly prefers to choose firm  $i$  at  $t = -1$  (with planning to never switch away from firm  $i$ ), so again firm  $j$  earns zero profits — a contradiction.

We thus conclude that the equilibrium derived in (i) is the unique pure-strategy equilibrium.  $\square$

**Proof of Proposition 6.** Consider the following candidate equilibrium in which both firms offer  $(B^n, p^n) = (B^*, p^*) = (s, v + \frac{1-\beta}{\beta} \frac{m}{T} s)$ . If the consumer is indifferent whether to plan on collecting a bonus at  $t = -1$ , she does plan to do so. If she is indifferent as to which initial offer to take at  $t = -1$ , she selects each firm with probability  $1/2$ . Between  $t = 0$  and  $t = T$ , whenever the consumer is indifferent, she does not take any action.

Since a naive consumer solves an optimization problem, her decision on whether she wants to switch or not at a given switching opportunity is independent of how she thinks she will act in the future whenever she is indifferent; we use the convention below that in such a case she believes her future self will switch and/or cancel a contract at the earliest opportunity whenever indifferent.

To proceed and establish that the above strategies constitute an equilibrium, we proceed in eight steps. As a preliminary observation, Step (I) shows that because the consumer cannot collect the bonus twice and she sees the same switching offer, she both plans to and actually takes up a different contract at most once independently of whether firms choose equilibrium or non-equilibrium offers. Steps (II) to (VI) derive the firms' profits from the candidate equilibrium contract offers. Steps (VII) and (VIII) establish that there is no

profitable deviation.

*Step (I).* Given any pair of contracts, no self plans to hold the same contract twice. Note this implies trivially that the consumer does not hold the same contract twice. We first establish that if a self  $\kappa \in \{0, \dots, K\}$  holding the initial contract  $j$  cancels it without switching then neither self  $\kappa$  nor any future self plans to hold (or holds) a contract twice. Self  $\kappa \in \{0, \dots, K\}$  will only cancel the initial contract  $j$  without switching if  $-s \geq \beta\tau(v - p^j) - \beta\mathbb{I}_{T > \kappa T_w + \tau} s$ , where  $\tau$  stands for the time interval until either  $T$  is reached or the consumer plans to take up a contract again. Note that it holds only if  $p^j > v$ . In this case, no self  $\kappa' \geq \kappa$  plans to take up contract  $j$  again, because she would be strictly better off by not holding any contract until the next time she plans to incur the switching cost. If self  $\kappa$  takes up contract  $i \neq j$ , then we already established that she will not switch back to or otherwise take up contract  $j$  again. For self  $\kappa$  to plan to hold contract  $i$  twice, self  $\kappa$  hence must cancel it and then take it up again at some  $\kappa' > \kappa$ . If  $\tau$  is the time interval between  $\kappa$  and  $\kappa'$ , this requires  $-s \geq \beta\tau(v - p^i) - \beta s$  for the consumer to cancel, which implies that neither self  $\kappa$  nor any future self plans to take up either contract again.

We now consider self  $-1$ 's problem. Let  $\tau^i$  be the overall length of time self  $-1$  plans to hold contract  $i = 1, 2$ , so that  $T - (\tau^1 + \tau^2)$  is the time that the consumer goes with consuming the service. We consider three cases: (a) self  $-1$  plans to take up both contracts; (b) self  $-1$  plans to only take up contract  $i$ ; (c) self  $-1$  plans to take up no contract.

Consider case (a). Self  $-1$ 's payoff induced by a plan in which the consumer takes up both contracts is

$$\beta [\tau^i(v - p^i) + \tau^j(v - p^j) + B^i + B^j - xs],$$

where  $x$  is the number of times self  $-1$  plans to cancel, switch or take up a contract at any switching opportunity  $\kappa \in \{0, \dots, K\}$ .

Observe next that in case  $p^i = \min\{p^1, p^2\} \leq v$ , in any optimal plan satisfying our tie-breaking assumption in which self  $-1$  plans to take up both contracts. Specifically, self  $-1$  takes up a contract  $j$  in  $t = -1$  and plans to switch to a contract  $i$  according to which she pays  $p^i$  in period 0; this ensures that  $x = 1$  and that she receives  $v - p^i$  for the entire period

$T$  (and if  $p^1 = p^2$  she plans to switch at the earliest opportunity due to our tie-breaking rule). Hence, in this case she plans to take both contracts at  $t = -1$ , she does not plan to hold a given contract twice.

We now rule out that some self  $\tilde{\kappa} \in \{0, \dots, K\}$  plans to hold a contract twice. We already establish this for the case in which self  $\kappa$  holding the initial contract cancels it without switching. Suppose, thus, self  $\kappa$  holding the initial contract  $j$  plans to switch to firm  $i$ . Consider first the case in which self  $\kappa$  holding the initial contract  $j$  plans to switch firm  $i$  and then go back to firm  $j$  at  $\kappa'$  without canceling any contract, and then plans to hold contract  $j$  for a time interval of  $\tau \geq 0$  before acting again (or  $T$  is reached). Obviously,  $\tau > 0$  as otherwise the consumer could save on the switching cost to firm  $j$  and either just keep holding contract  $i$  or cancel immediately without switching first. Hence, to plan to switch back at  $\kappa'$ , it must be that  $\tau(v - p^j) > \tau(v - p^i)$  as otherwise she could save on the switching cost, and  $\tau(v - p^j) \geq 0$  as otherwise she would prefer canceling. But this implies that  $p^j < p^i$  and that  $\min\{p^1, p^2\} \leq v$ , which contradicts the fact established above that  $p^j \geq p^i$ . So self  $\kappa$  cannot plan to switch to firm  $i$  and then go back without canceling any contract. Furthermore, following a switch to contract  $i$ , since  $p^i \leq p^j$ , no future self will actually switch back to firm  $j$ . Consider second the case in which self  $\kappa$  holding the initial contract  $j$  plans to switch to firm  $i$  and then cancel contract  $i$  before taking up contract  $j$  again. To prefer to plan to cancel contract  $i$  to taking up contract  $j$ , it must be that  $(v - p^j) < 0$ , which however contradicts the fact that self  $\kappa$  plans to take up the contract at a later date. Furthermore, if some self  $\kappa' > \kappa$  cancels contract  $i$ , we have  $(v - p^j) < 0$  and so no self  $\kappa'' \geq \kappa$  will take up contract  $j$ . We conclude that no  $\tilde{\kappa} \in \{0, \dots, K\}$  plans to hold a contract twice in case (a).

Consider case (b). Self  $-1$ 's payoff induced by a plan in which the consumer takes up a single contract  $i$  is

$$\beta [\tau^i(v - p^i) + B^i - xs],$$

where  $\tau$  is the length of time she plans to hold contract  $i$  and  $x$  is the number of times self  $-1$  plans to cancel or take up contract  $i$  at switching opportunities  $\kappa \in \{0, \dots, K\}$ . Clearly,

if  $p^i \leq v$ , then  $x = 0$ . If  $p^i > v$ , the payoff of taking up contract  $i$  at  $t = -1$  and canceling it at  $t = 0$ , which is  $\beta(B^i - s)$ , dominates that of taking the contract at  $\kappa \geq 0$  and either canceling it immediately (i.e.,  $\beta(B^i - 2s)$ ) or taking up the contract at  $\kappa \geq 0$  and holding it for a non-zero amount of time. In either case, hence, self  $-1$  takes up the contract  $i$  at  $t = -1$ , and we already established that if she cancels contract  $i$  she does not plan to take it up again. We conclude that self  $-1$  does not plan to hold a contract twice.

Because self  $-1$  does not plan to hold a contract twice, planing to cancel the contract at  $\kappa \in \{0, \dots, K\}$  yields payoff

$$\beta [(T - \kappa T_w)(v - p^i) + B^i - s],$$

while holding it to the end yields payoff  $T(v - p^i) + B^i$ . As self  $-1$ 's canceling payoffs are decreasing in  $\kappa$  for  $p^i > v$ , she either plans to cancel immediately or not at all. Suppose the consumer plans not to cancel the contract at all. For the sake of contradiction, suppose furthermore that self  $\kappa$  holding the initial contract  $i$  wants to switch to contract  $j$  or cancel contract  $i$  at  $\kappa$ . Let  $-s + \beta V$  denote self  $\kappa$ 's payoff from its optimal plan, and note that it must be greater than self  $\kappa$ 's payoff of not canceling, i.e.,  $-s + \beta V > \beta(T - \kappa T_w)(v - p^i)$ . But if self  $-1$  would plan to follow the same plan, then it would get a payoff of

$$\beta [\kappa T_w(v - p^i) + B^i - s + V] > \beta [T(v - p^i) + B^i],$$

contradicting that not canceling is optimal for self  $-1$ . We conclude that no self  $\kappa$  holding the initial contract switches to firm  $j$  or cancels the initial contract immediately. And here benefit from switching or canceling at a future date  $\kappa' > \kappa$  are the same as that of self  $-1$ , so self  $\kappa$  must plan to follow the same non-canceling plan as self  $-1$ .

Now suppose that self  $-1$  plans to cancel immediately at switching opportunity  $\kappa = 0$ , which requires  $p^i > v$ . We will argue that no self  $\kappa \in \{0, \dots, K\}$  holding the initial plans to switch to contract  $j$ . Because self  $-1$  does not plan to hold contract  $j$ , it must be that  $B^j < s$  since otherwise she would be at least as well off planing to take up contract  $j$  at  $t = -1$ , and then immediately switching to contract  $i$  at  $\kappa = 0$ ; and when weakly better off taking both contracts self  $-1$  must plan to do so by our tie-breaking assumption. Furthermore,



because self  $-1$  prefers to cancel contract  $i$  rather than to switch to contract  $j$ , it must be that  $\beta(-s + B^j + T(v - p^j)) < 0$ . Now for the sake of contradiction, suppose some self  $\kappa \in \{0, \dots, K\}$  holding the initial contract prefers to switch to contract  $j$  at  $\kappa' \geq \kappa$ . Let  $\tau$  be the amount of time she plans to hold contract  $j$ , and note that since  $B_j < s$ , for planing to switch to be optimal it must be that  $\tau > 0$ . For switching to dominate planing to canceling the contract at  $\kappa'$  a necessary condition is that  $\beta(-s + B^j + \tau(v - p^j)) \geq 0$ , which requires that  $v > p^j$ . But then

$$0 > \beta(-s + B^j + T(v - p^j)) \geq \beta(-s + B^j + \tau(v - p^j)) \geq 0,$$

a contradiction. We conclude that no self  $\kappa$  plans to or does switch to contract  $j$ , and hence either plans to hold contract  $i$  to the end or cancel it. Because if any self plans to cancel contract  $i$  (immediately or with delay) then she does not take it up again, she must take up contract  $j$  twice to hold a contract twice. But if self  $\kappa$  gets a non-negative payoff from planing to taking up contract  $j$  for the first time at some  $\kappa' \geq \kappa$ , self  $-1$  would get a non-negative incremental payoff of taking contract  $j$  up at  $\kappa'$  after canceling the contract at  $\kappa = 0$ , contradicting our tie-breaking rule that self  $-1$  must take up both contract when indifferent. We conclude if self  $-1$  plans to hold only one contract, no self plans to or does take up a contract twice.

Consider case (c). Because self  $-1$  does not plan to take up a contract, no self who does not hold a contract plans to do so either immediately or in the future. If self  $\kappa$  expects a non-negative payoff from a plan that involves her first taking up a contract at  $\kappa' \geq \kappa$ , self  $-1$  can also plan to do so. But then, self  $-1$  must plan to do so by our tie-breaking rule, a contradiction.

We conclude that for any equilibrium or non-equilibrium pair of offers, the consumer never plans to or does hold a contract twice. *We thus from now on focus on the first time the consumer switches or cancels a contract and, take for granted that she never takes up the contract for a second time.*

*Step (II).* Given the candidate equilibrium offers, self  $t = -1$  plans to take up both contract offers. Suppose otherwise. Then she plans to either (a) take no contract or (b) to take one

contract. Consider case (a). Self  $-1$  could instead plan to select firm  $i$ 's contract and then cancel it at  $t = 0$ , yielding a payoff of  $\beta(B^i - s) = 0$ ; hence, since she at least weakly prefers to take a contract, by the tie-breaking assumption she must do so, a contradiction. Next, consider case (b). Let self  $-1$  plan to take up firm  $i$ 's contract at some opportunity. Suppose self  $-1$  plans to take up the contract of firm  $i$  at switching opportunity  $t = 0$  or later; note first that she cannot take the contract at  $t = 0$  because in that case she would be better off taking firm  $i$ 's contract at  $t = -1$  and saving the cost  $s$  of taking up the contract. But then by essentially the same argument as in case (a), she at least weakly prefers to take up the contract of firm  $j \neq i$  at  $t = -1$  and cancel it at  $t = 0$ , and by the tie-breaking assumption she must do so, a contradiction. Hence, self  $-1$  must take up firm  $i$ 's contract at  $t = -1$ . Let the anticipated continuation value from self  $-1$ 's optimal plan starting at  $t = 0$  be  $V$ , so the self  $-1$ 's anticipated payoff is  $\beta V$ . If self  $-1$ , however, selects firm  $j$ 's contract first and then switches to firm  $i$ 's contract at  $t = 0$  and thereafter follows the same plan as before her anticipated payoff is at least  $\beta B^j - \beta s + \beta V = \beta V$ ; thus, by the tie-breaking assumption, she must plan to take both contracts.

*Step (III).* Given the candidate equilibrium contract offers, if the consumer does not have a contract at some switching opportunity  $\kappa$ , then she will neither take up a contract at  $\kappa$  nor plan to take up an equilibrium contract at any future switching opportunity. Suppose, toward a contradiction, that self  $\kappa$  takes up a contract. Let self  $\kappa$  plan to take up the contract immediately and pay the price  $p^i$  for time interval of length  $\hat{T} \in [0, T]$  before either switching to firm  $j$  or canceling the contract. Then, the incremental payoff from holding the contract of firm  $i$  instead of no contract is at most  $-s + \beta B^i + \beta \hat{T}(v - p^i) < 0$ , so self  $\kappa$  gets a higher anticipated payoff when not planning to take up firm  $i$ 's contract. We next show that, if self  $\kappa' < \kappa$  does not have a contract, she does not plan to take up a contract in future. To see why, note that (a) if self  $\kappa'$  plans to take up the contract at  $\kappa$  and immediately cancel it, then her incremental anticipated payoff from doing so is  $\beta(-s + B^i - s) = -\beta s < 0$ ; (b) if self  $\kappa'$  plans to take up the contract at  $\kappa$  and plans to pay  $p^i$  for time interval of length  $\hat{T} \in (0, T]$ , then her incremental anticipated payoff from doing so is no greater than

$\beta(-s + B^i + \hat{T}(v - p^i)) < 0$ ; (c) if she plans to take up the contract and immediately switch to firm  $j$ , then by analogous arguments from (a) and (b), her anticipated incremental payoff from switching to firm  $j$ 's contract itself is negative, and since the incremental anticipated payoff of taking up and immediately switching away from firm  $i$  is zero, her incremental anticipated payoff from taking up both contracts is negative. Hence, if the consumer does not hold a contract at switching opportunity  $\kappa$ , she does not plan to acquire it.

*Step (IV).* The consumer takes up each equilibrium contract offer at  $t = -1$  with probability  $1/2$ . Since the consumer at  $t = -1$  plans to take up both contract offers in the candidate equilibrium, and if she does not hold a contract at  $\kappa = 0$  then earlier selves do not plan to acquire one at  $t = 0$  or thereafter, she must select one contract offer at  $t = -1$ , and by the tie-breaking rule she chooses either firm with probability  $1/2$  in the candidate equilibrium.

*Step (V).* Given the candidate equilibrium contract offers, self  $\kappa$  neither cancels nor switches away from the contract she chose initially at  $t = -1$ . Suppose otherwise. Then the consumer either (a) cancels firm  $i$ 's contract without switching; (b) switches to firm  $j$ 's contract and does not cancel it immediately; or (c) switches to firm  $j$ 's contract and cancels it immediately. In case (a), for any  $\kappa \leq K$ , we already established that once the consumer does not hold a contract, she does not plan to acquire one in the future, so her continuation value after canceling is zero. For any  $\kappa < K$ , if self  $\kappa$  plans to delay canceling to  $\kappa + 1$ , then her change in anticipated payoff is  $-\beta T_w(p^i - v) - \beta s = -s$ , so she weakly prefers delaying to canceling immediately, and does so by the tie-breaking assumption. If  $\kappa = K$ , self  $K$ 's anticipated payoff when not canceling is  $-\beta(T - K\frac{T}{m})(p^i - v) \geq -\beta\frac{T}{m}(p^i - v) > -s$  and hence self  $K$  does not cancel, a contradiction. In case (b), for  $\kappa < K$  self  $\kappa$ 's anticipated payoff (net of any predetermined  $\beta B^i$ ) is  $-s + \beta B^j - \beta T_w(p^j - v) + \beta V = -(1 - \beta)s - \beta T_w(p^* - v) + \beta V$ , where  $V$  is the anticipated continuation value from following self  $\kappa$ 's optimal "contract cancellation" plan from  $\kappa + 1$  onwards. By not switching to firm  $j$  now and following the same cancellation plan in the future, self  $\kappa$ 's anticipated payoff increases to  $-\beta T_w(p^i - v) + \beta V = -\beta T_w(p^* - v) + \beta V$ , and hence, she prefers not to switch. Similarly, since  $-\beta(T - K\frac{T}{m})(p^* - v) > -s + \beta B^j - \beta(T - K\frac{T}{m})(p^* - v)$ , the consumer prefers not

to switch at switching opportunity  $K$ , a contradiction. Finally, in case (c) self  $\kappa$  is better off just canceling rather than switching and canceling because  $-s + \beta B^* < 0$ . We conclude that the consumer neither cancels nor switches following the candidate equilibrium contract offers.

*Step (VI).* Each firm earns  $\frac{1}{2}(Tp^* - B^*)$  in the candidate equilibrium. This follows immediately from Step (IV) and (V). Because Condition (5) implies  $0 < Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta} \frac{m}{T}s) - s] = \frac{1}{2}(Tp^* - B^*)$ , each firm prefers the equilibrium offer to any offer which results in the firm having no sales.

We next turn to the implications of firm  $i$  deviating. We begin by bounding the profits a firm earns when attracting the consumer at the last switching opportunity, which we then use to show that there is no profitable deviation.

*Step (VII).* A deviating firm  $i$  that attracts the consumer at the last switching opportunity  $K$  earns less than  $\frac{T}{m}v - s$  from doing so. At the last switching opportunity, if the consumer does not have a contract, then she takes up firm  $i$ 's offer if and only if one of the following two conditions holds: either self  $K$ 's payoff when she immediately cancels  $i$ 's contract is positive, i.e.,  $-2s + \beta B^i > 0$ , or self  $K$ 's payoff when she takes up and does not cancel  $i$ 's contract is positive, i.e.,  $-s + \beta [(T - K\frac{T}{m})(v - p^i) + B^i] > 0$ . In the former case, firm  $i$ 's profits from attracting the consumer at the last switching opportunity are  $-B^i < -\frac{2s}{\beta} < 0$ . In the latter case, firm  $i$ 's profits from attracting the consumer at the last switching opportunity are at most

$$\left(T - K\frac{T}{m}\right)p^i - B^i < \left(T - K\frac{T}{m}\right)v - \frac{s}{\beta} \leq \frac{T}{m}v - \frac{s}{\beta}. \quad (22)$$

Because Condition (5) implies that  $\frac{m}{\beta T}s > v$ , a firm makes a loss from attracting a consumer who does not have a contract at the last switching opportunity.

At the last switching opportunity, if firm  $i$  attracts the consumer from firm  $j$ , then she either (a) cancels firm  $i$ 's contract immediately or (b) does not cancel firm  $i$ 's contract. Consider case (a). As self  $K$  needs to receive  $\beta B^i \geq s$  to prefer switching and canceling to just canceling firm  $j$ 's contract, firm  $i$  makes a loss from attracting the consumer. Next consider Case (b). In this case self  $K$  needs to prefer switching to continuing to use firm  $j$ 's

contract, i.e.,

$$-s + \beta \left[ \left( T - K \frac{T}{m} \right) (v - p^i) + B^i \right] \geq \beta \left( T - K \frac{T}{m} \right) (v - p^j). \quad (23)$$

By (23) and the fact that if firm  $j$  offers the equilibrium contract  $p^j = p^*$ , the deviant firm  $i$ 's profits conditional on the consumer switching at  $K$  are at most  $(T - K \frac{T}{m})p^i - B^i \leq (T - K \frac{T}{m})p^* - \frac{s}{\beta}$ . Note also that  $(T - K \frac{T}{m})(v + \frac{1-\beta}{\beta} \frac{m}{T} s) - \frac{s}{\beta} \leq \frac{T}{m}(v + \frac{1-\beta}{\beta} \frac{m}{T} s) - \frac{s}{\beta} = \frac{T}{m}v - s$ , which completes the argument for Step (VI).

*Step (VIII). There is no profitable deviation.* We now partition the set of deviant contract offers  $(B^i, p^i)$  by firm  $i$  into those for which: (A)  $p^i > p^*$ ; (B)  $p^i \in (v, p^*)$ ; (C)  $p^i = p^*$  and  $B^i \neq s$ ; and (D)  $p^i \leq p^*$  and the rule out a profitable deviation case by case.

(A). Consider a deviation by firm  $i$  to an offer  $(B^i, p^i)$  for which  $p^i > v + \frac{1-\beta}{\beta} \frac{m}{T} s$ . Recall that if the consumer has canceled firm  $i$ 's contract at switching opportunity  $\kappa$ , by Steps (I) and (III), she neither takes up nor plans to take up firm  $j$ 's candidate equilibrium contract at any future switching opportunity  $\kappa' > \kappa$ . Given that the consumer takes up firm  $i$ 's contract, self  $\kappa$  strictly prefers canceling it immediately at  $\kappa$  to canceling it at any  $\kappa' > \kappa$  because  $-s > -\beta \hat{T}(v - p^i) - \beta s$  for any time interval  $\hat{T} \geq \frac{T}{m}$ . Similarly, given that the consumer takes up firm  $i$ 's contract, self  $\kappa$  strictly prefers to cancel it immediately rather than to plan to switch to firm  $j$  at any switching opportunity  $\kappa' > \kappa$ ; to see why, let  $\hat{T} \geq \frac{T}{m}$  be the amount of time until she switches to firm  $j$  and  $\tilde{T} \geq 0$  be the amount of time she pays  $p^j$ . Suppose first that she plans to switch to firm  $j$  at  $\kappa' > \kappa$  and cancels it at  $\kappa'' \in \{\kappa', \dots, K\}$ . Then, self  $\kappa$ 's anticipated payoff (net of  $\beta B^i$ ) is  $\beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta \tilde{T}(v - p^*) - \beta s \leq \beta \hat{T}(v - p^i) - \beta s < -s$ , so she strictly prefers canceling firm  $i$ 's contract immediately at  $\kappa$ . Suppose next that she plans to switch to firm  $j$  at  $\kappa' > \kappa$  and plans not to cancel firm  $j$ 's contract at any future switching opportunity. Then, self  $\kappa$ 's anticipated payoff (net of  $\beta B^i$ ) is  $\beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta(T - \hat{T} - \kappa \frac{T}{m})(v - p^*) < \beta T(v - p^*) = -(1 - \beta)ms < -s$ , which holds by the assumption that  $(1 - \beta)m > 1$ , so she prefers to cancel immediately. We conclude that, at any switching opportunity  $\kappa$ , the consumer either cancels firm  $i$ 's contract immediately or plans to hold it until  $T$ .

We now argue that firm  $i$ 's profits from attracting the consumer at a switching oppor-

tunity  $\kappa$  are bounded from above by  $Tv$ . If the consumer cancels the contract immediately at a switching opportunity  $\kappa$ , attracting her at  $\kappa$  is unprofitable. Thus, consider the case in which self  $\kappa$  plans to hold it until  $T$ . If self  $\kappa$  plans to hold the contract of firm  $i$  until  $T$ , her payoff is  $\beta(T - \kappa \frac{T}{m})(v - p^i) + \beta B^i - s$ . When planning to hold firm  $i$ 's contract, self  $\kappa$  is willing to take up this contract at  $\kappa$  only if  $\beta B^i + \beta(T - \kappa \frac{T}{m})(v - p^i) \geq 0$ . Thus, the firm's profits from attracting the consumer at  $\kappa$  are  $(T - \kappa \frac{T}{m})p^i - B^i \leq (T - \kappa \frac{T}{m})v \leq Tv$ .

We now argue that firm  $i$ 's profits from attracting the consumer at  $t = -1$  are also bounded from above by  $Tv$ . Recall that self  $-1$  either plans to hold firm  $i$ 's contract until  $T$  or plans to cancel firm  $i$ 's contract at any switching opportunity  $\kappa = 0$ . Because taking up firm  $i$ 's contract at  $t = -1$  and planning to cancel it at  $\kappa = 0$  requires  $B^i \geq s$ , firm  $i$  makes a loss from attracting the consumer at  $t = -1$  in this case. Suppose next that the consumer plans to hold firm  $i$ 's contract until  $T$ . Self  $-1$ 's anticipated payoff at  $t = -1$  from doing so must be non-negative; i.e.,  $\beta B^i + \beta T(v - p^i) \geq 0$ . In this case, the upper bound of firm  $i$ 's total profits (when the consumer takes up firm  $i$ 's contract at  $t = -1$  with probability 1 and does not cancel thereafter) are  $Tp^i - B^i \leq Tv$ .

Hence, both in case firm  $i$  attracts the consumer at  $t = -1$  or thereafter, her profits from attracting the consumer are bound by  $Tv$ . Furthermore, a consumer takes up firm  $i$ 's contract at most once, so the deviant firm  $i$ 's profits are bounded from above by  $Tv$ . Because Condition (5) implies  $Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta} \frac{m}{T}s) - s] = \frac{1}{2}(Tp^* - B^*)$ , a deviation by firm  $i$  to an offer  $(B^i, p^i)$  for which  $p^i > p^*$  is unprofitable.

(B). Consider a deviation by firm  $i$  to an offer  $(B^i, p^i)$  for which  $p^i \in (v, v + \frac{1-\beta}{\beta} \frac{m}{T}s)$ . We first show that the consumer chooses firm  $i$ 's offer with at most probability  $1/2$  at  $t = -1$ . Suppose otherwise, i.e., self  $-1$  chooses firm  $i$ 's offer at  $t = -1$  with probability 1. There are four cases: (a) self  $-1$  plans to cancel firm  $i$ 's offer at  $\kappa \geq 0$ , (b) self  $-1$  plans to switch to firm  $j$ 's offer at  $\kappa = 0$ , (c) self  $-1$  plans to switch to firm  $j$ 's offer at  $\kappa \geq 1$ , and (d) self  $-1$  plans to hold firm  $i$ 's contract until  $T$ . Consider case (a). Because a consumer does not plan to take up a contract more than once by Step (I), after canceling firm  $i$ 's offer, self  $-1$  does not plan to take up firm  $j$ 's equilibrium offer by Step (III) above. But then,

self  $-1$  attains the same anticipated payoff when planning to take up firm  $j$ 's contract at  $t = -1$  and switching to firm  $i$  at  $t = 0$  (and thereafter following the same “cancellation plan” as before). By the tie-breaking assumption, self  $-1$  must plan to take both contracts, a contradiction. Consider case (b). If self  $t = -1$  plans to cancel firm  $j$ 's contract at  $t = 0$ , then her anticipated payoff is  $\beta(B^i + B^j - s)$  and she attains the same anticipated payoff when taking up firm  $j$ 's contract first and then switching to and canceling firm  $i$ 's contract at  $t = 0$ , so our tie-breaking rule contradicts that she takes up firm  $i$ 's contract with probability one. If self  $t = -1$  plans to pay firm  $j$ 's price  $p^j = p^*$  up to time  $\hat{T}$ , self  $-1$ 's anticipated payoff is  $\beta(B^i + B^j - s + \hat{T}(v - p^*) + \mathbb{I}_{\hat{T} < T} s)$ ; but by taking up firm  $j$ 's contract first and then planing to pay the lower price  $p^i$  up to time  $\hat{T}$ , self  $-1$ 's anticipated payoff strictly increase to  $\beta(B^i + B^j - s + \hat{T}(v - p^i) + \mathbb{I}_{\hat{T} < T} s)$ , a contradiction. Consider case (c). Let  $\hat{T} \geq \frac{T}{m}$  be the time until she switches to firm  $j$  and  $\tilde{T} \geq 0$  be the amount of time she pays  $p^j$ . Suppose first self  $t = -1$  plans to switch to firm  $j$  at  $\kappa' \geq 1$  and cancels its contract at  $\kappa'' \in \{1, \dots, K\}$ . Then, her anticipated payoff from following the plan is  $\beta B^i + \beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta \tilde{T}(v - p^*) - \beta s \leq \beta B^i + \beta \hat{T}(v - p^i) - \beta s - \beta s + \beta B^* < \beta(B^i + B^* - 2s)$ , so self  $-1$  strictly prefers to plan to switch to firm  $j$ 's contract at  $t = 0$  and then cancel it immediately. Suppose second that she plans to switch to firm  $j$  at  $\kappa' > \kappa$  and not to cancel its contract. Then, self  $-1$ 's anticipated payoff from following the plan is  $\beta B^i + \beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta(T - \hat{T})(v - p^*) < -\beta s + \beta B^* + \beta B^i + \beta T(v - p^i)$ , so self  $-1$  strictly prefers to plan to take up firm  $j$ 's offer at  $t = -1$ , switch to firm  $i$ 's offer at  $t = 0$ , and hold it until the end, a contradiction. Consider case (d). Then, for the consumer to be willing to take up firm  $i$ 's contract at  $t = -1$ ,  $\beta T(v - p^i) + \beta B^i \geq 0$  must hold. In this case, firm  $i$ 's total profits are at most  $Tp^i - B^i \leq Tv$ . Because Condition (5) implies  $Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta} \frac{m}{T} s) - s] = \frac{1}{2}(Tp^* - B^*)$ , any deviation that induces the consumer to take up firm  $i$ 's contract with probability one is not a profitable deviation. We conclude that self  $t = -1$  chooses firm  $i$ 's offer with at most probability  $1/2$  for any possibly profitable deviation.

We next show that firm  $i$ 's deviating offer does not induce the consumer to switch from

firm  $j$  to firm  $i$  except possibly at the last switching opportunity  $K$ . We first argue that self  $\kappa$  prefers to procrastinate switching from firm  $j$  to firm  $i$  at any switching opportunity  $\kappa < K$ . Note first that if the consumer plans to cancel firm  $i$ 's contract immediately after switching to it, then she prefers to procrastinate switching rather than to do so since

$$\beta \left( -2s + \frac{T}{m}(v - p^*) + B^i \right) > -2s + \beta B^i \iff (1 - \beta)s > 0.$$

Thus, consider the case in which self  $\kappa$  plans to hold firm  $i$ 's contract at least until the next switching opportunity. Again, she prefers to procrastinate switching to switching immediately because

$$\beta \left( -s + \frac{T}{m}(v - p^*) + B^i \right) > -s + \beta \left( \frac{T}{m}(v - p^i) + B^i \right) \iff p^i > v. \quad (24)$$

Hence, the consumer does not switch from firm  $j$  to firm  $i$  at any switching opportunity  $\kappa < K$ .

We now show that if firm  $i$ 's deviation induces the consumer to switch from firm  $j$  to firm  $i$  at the last switching opportunity  $K$ , firm  $i$ 's deviation is unprofitable. If the consumer switches at  $K$ , (23) must hold, which requires that  $(T - K\frac{T}{m})p^i - B^i \leq (T - K\frac{T}{m})p^* - \frac{s}{\beta}$  and hence also that  $Tp^i - B^i \leq Tp^* - \frac{s}{\beta}$ . By Step (VII), firm  $i$  earns at most  $\frac{T}{m}v - s$  conditional on the consumer switching to  $i$ 's contract at  $K$ . Because self  $t = -1$  chooses firm  $i$  with probability at most  $1/2$ , firm  $i$ 's total profits from this deviation are at most

$$\frac{1}{2}(Tp^i - B^i) + \frac{1}{2}\left(\frac{T}{m}v - s\right) \leq \frac{1}{2}\left(Tp^* - \frac{s}{\beta}\right) + \frac{1}{2}\left(\frac{T}{m}v - B^*\right) = \frac{1}{2}(Tp^* - B^*) + \frac{1}{2}\left(\frac{T}{m}v - \frac{s}{\beta}\right). \quad (25)$$

Because Condition (5) implies  $\frac{m}{\beta T}s > v$ , firm  $i$ 's deviation is unprofitable. Hence, we established that for any potentially profitable deviation, after taking up firm  $j$ 's contract the consumer does not switch to firm  $i$ . This implies that in any possibly profitable deviation, firm  $i$  must attract the consumer at  $t = -1$  with probability  $1/2$ .

Suppose such a profitable deviation in which firm  $i$  attracts the consumer at  $t = -1$  with probability  $1/2$  exists. Note that if  $B^i \geq s$ , since the consumer takes up firm  $i$ 's contract with probability  $1/2$  and does not switch away from firm  $j$ 's contract, firm  $i$ 's deviation profits are at most  $\frac{1}{2}(Tp^i - B^i) < \frac{1}{2}(Tp^* - B^*)$ , a contradiction. Hence  $B^i < s$ . If self  $-1$  takes up firm



$i$ 's contract at  $t = -1$  she plans not to cancel it at  $\kappa = 0$ , because otherwise the incremental anticipated payoff from taking firm  $i$ 's contract would be  $\beta(B^i - s) < 0$ . Similarly, because  $p^i > v$ , self  $-1$  does not plan to cancel the contract at any switching opportunity  $\kappa$ , because otherwise the incremental anticipated payoff from taking firm  $i$ 's contract would be negative. In addition, because  $B^i < s$ , the consumer who takes up firm  $i$ 's contract at  $t = -1$  does not plan to switch to firm  $j$ 's contract at  $t = 0$ ; otherwise, self  $-1$  strictly prefers the plan to take up firm  $j$ 's contract at  $t = -1$  and save on the switching costs. Thus, self  $-1$  plans to pay  $p^i$  for a positive amount of time. Let  $\hat{T} \geq \frac{T}{m}$  be the time until she plans to switch to firm  $j$  and  $\tilde{T} \geq 0$  be the time she pays  $p^j$ . Suppose first self  $t = -1$  plans to switch to firm  $j$  at  $\kappa' \geq 1$  and cancels it at  $\kappa'' \in \{1, \dots, K\}$ . Then, her anticipated payoff from following the plan is  $\beta B^i + \beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta \tilde{T}(v - p^*) - \beta s \leq \beta \hat{T}(v - p^i) - \beta(s - B^i) < 0$ , so she strictly prefers the plan in which she takes up no contact. Suppose second that she plans to switch to firm  $j$  at  $\kappa' > \kappa$  and plans not to cancel firm  $j$ 's contract thereafter. Then her anticipated payoff from following this plan is  $\beta B^i + \beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta(T - \hat{T})(v - p^*) < \beta B^i + \beta T(v - p^i)$ , so it is lower than the anticipated payoff when self  $-1$  plans to take up firm  $i$ 's offer at  $t = 0$  and refrain from canceling it at every switching opportunity. We thus established that if the consumer takes up firm  $i$ 's contract at  $t = -1$ , she plans to pay  $p^i$  until  $T$ . For the consumer to take up firm  $i$ 's contract at  $t = -1$ , thus,  $\beta T(v - p^i) + \beta B^i \geq 0$  must hold. This implies that firm  $i$ 's total profits are at most  $Tp^i - B^i \leq Tv$ . Because Condition (5) implies  $Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta} \frac{m}{T}s) - s] = \frac{1}{2}(Tp^* - B^*)$ , this contradicts that the deviation is profitable. We conclude that any deviation to an offer for which  $p^i \in (v, v + \frac{1-\beta}{\beta} \frac{m}{T}s)$  is unprofitable.

(C). Consider a deviation by firm  $i$  to an offer  $(B^i, p^i)$  for which  $B^i \neq s$  and  $p^i = v + \frac{1-\beta}{\beta} \frac{m}{T}s$ . Suppose first  $B^i > s$ . Suppose self  $-1$  would only take one firm's contract, say the contract of firm  $n \in \{i, j\}$  and denote the continuation value of this plan starting at  $t = 0$  by  $V$ , so her anticipated payoff is  $\beta V$ . By taking firm  $n' \neq n$ 's contract at  $t = -1$ , switching to firm  $n$  at  $\kappa = 0$  and then following the same continuation plan as before from  $\kappa = 0$  onwards, self  $-1$  anticipated payoff would become  $\beta(B^{n'} - s + V) \geq \beta V$ ; by our tie-breaking rule, hence, self  $-1$  must plan to take both contracts. Since the consumer plans

to collect both bonuses, and because  $p^i = p^j$ , self  $t = -1$  is indifferent between taking up either contract  $i$  or  $j$  first and thus must select each with equal probability. As before, for any  $\kappa < K$ , if self  $\kappa$  who holds contract  $n$  plans to delay canceling to  $\kappa + 1$ , then her change in anticipated payoff is  $-\beta T_w(p^n - v) - \beta s = -s$ , so she weakly prefers delaying to canceling immediately, and does so by our tie-breaking assumption. If the consumer switches at  $K$ , (23) must hold, which requires that  $(T - K\frac{T}{m})p^i - B^i \leq (T - K\frac{T}{m})p^* - \frac{s}{\beta}$  and hence also that  $Tp^i - B^i \leq Tp^* - \frac{s}{\beta}$ . By Step (VII), firm  $i$  earns at most  $\frac{T}{m}v - s$  conditional on the consumer switching to  $i$ 's contract at  $K$ . Because self  $t = -1$  chooses firm  $i$  with probability at most  $1/2$ , by the exact same calculation as in (25), firm  $i$ 's deviation is unprofitable in case it induces the consumer to switch at  $K$ .

Suppose second  $B^i < s$ . Because the level of firm  $i$ 's bonus is less than the switching cost and  $p^i = p^j > v$ , self  $t = -1$ 's prefers to take up firm  $j$ 's contract and cancels it at  $t = 0$  yielding an anticipated payoff of  $\beta(B^j - s) = 0$  to all other plans in which she signs a contract, and because the consumer weakly prefers this plan to never taking up any contract, she selects firm  $j$ 's contract. Furthermore, at  $p^i = p^* > v$  and  $B^i < B^*$ , no self  $\kappa$  takes up or switches to firm  $i$ 's contract at any switching opportunity. Hence, firm  $i$  has no sales, a contradiction. We conclude that any deviation by firm  $i$  to an offer  $(B^i, p^i)$  for which  $B^i \neq s$  and  $p^i = p^*$  is unprofitable.

(D). Consider a deviation by firm  $i$  to an offer  $(B^i, p^i)$  for which  $p^i \leq v$ . We first show that, for firm  $i$ 's deviation to be profitable, the consumer must choose firm  $i$ 's offer with probability  $1/2$  at  $t = -1$ . Suppose otherwise; then by our tie-breaking assumption she chooses firm  $i$ 's offer at  $t = -1$  with either (a) probability 1 or (b) probability 0. Consider first case (a). By the arguments in Step (III), a consumer who canceled contract  $i$  at or prior to any switching opportunity  $\kappa$  does neither take up or plan to take up contract  $j$  at any switching opportunity  $\kappa' \geq \kappa$ . Because  $p^i \leq v$ , self  $t = -1$ 's anticipated continuation payoff at any switching opportunity  $\kappa$  of simply keeping the contract until  $T$  is non-negative, while planning to cancel yields an anticipated continuation payoff of  $-s$ , planning to switch to firm  $j$  and cancel contract  $j$  immediately yields an anticipated continuation payoff of  $-2s + B^j = -s$ ,

and planing to switch without canceling immediately yields a strictly negative anticipated continuation payoff. Hence, self  $t = -1$  anticipated payoff of selecting firm  $i$ 's contract is  $\beta(B^i + T(v - p^i))$ . Self  $-1$ 's anticipated payoff when selecting firm  $j$ 's contract at  $t = -1$  and planing to switch to firm  $i$  at  $t = 0$  is  $\beta(B^j - s + B^i + T(v - p^i)) = \beta(B^i + T(v - p^i))$ , and thus by our tie-breaking rule self  $-1$  cannot select contract  $i$  with probability greater than  $1/2$  at  $t = -1$ , a contradiction. Because  $p^i \leq v$ , an upper bound to an self  $\kappa \geq 0$  of taking up the contract of firm  $i$  is  $\beta(T(v - p^i) + B^i)$ , and hence a necessary condition for self  $\kappa$  to take up contract  $i$  is that

$$T(v - p^i) + B^i \geq 0. \quad (26)$$

Hence, firm  $i$ 's deviation profits are at most  $(T - \kappa \frac{T}{m})p^i - B^i \leq (T - \kappa \frac{T}{m})v \leq Tv$ . Because Condition (5) implies  $Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta} \frac{m}{T}s) - s] = \frac{1}{2}(Tp^* - B^*)$ , deviating and attracting the consumer at switching opportunities only is not a profitable deviation. We conclude that the consumer must choose firm  $i$ 's offer with probability  $1/2$  (and hence chooses firm  $j$ 's offer with probability  $1/2$  by the tie-breaking rule) at  $t = -1$ .

If the consumer selects firm  $j$ 's offer at  $t = -1$ , she plans to switch to offer  $(B^i, p^i)$  at  $t = 0$  and then not cancel it rather than to merely cancel firm  $j$ 's offer only if (26) holds. Consider first the case in which (26) does not hold. Then, any self  $\kappa \geq 0$  will refrain from taking up firm  $i$ 's offer. Also, self  $-1$ 's anticipated payoff from taking up firm  $i$ 's offer at  $t = -1$  is at most  $T(v - p^i) + B^i < 0$ , so she strictly prefers to not taking up firm  $i$ 's offer at  $t = -1$ . Hence, firm  $i$  has no sales, a contradiction.

Consider second the case in which (26) holds. Because  $p^* > v \geq p^i$  and  $B^* = s$ , the consumer plans to not switch from firm  $i$  to firm  $j$ , so the consumer's anticipated total payoff when selecting firm  $i$ 's offer at  $t = -1$  is  $T(v - p^i) + B^i$ . Self  $-1$ 's anticipated total payoff when selecting firm  $j$ 's offer at  $t = -1$  and then switching to firm  $i$  at  $t = 0$  is also  $-s + T(v - p^i) + B^* + B^i = T(v - p^i) + B^i$ . As the consumer is indifferent at  $t = -1$ , by the tie-breaking rule self  $t = -1$  plans to collect both bonuses. We next argue that the consumer's uniquely optimal plan is to take up firm  $j$ 's offer at  $t = -1$  and then switch to firm  $i$  at  $t = 0$  and then not incur the cost of canceling firm  $i$ 's contract. To see why, suppose

first that the consumer takes up firm  $j$ 's offer at  $t = -1$ . As self  $t = -1$  plans to collect both bonuses, self  $t = -1$  plans to switch to firm  $i$  at some opportunity. Also, because  $p^i \leq v$ , self  $t = -1$  does not plan to cancel firm  $i$ 's contract. Hence, self  $t = -1$ 's anticipated payoff is  $\beta B^* + \beta \hat{T}(v - p^*) - \beta s + \beta B^i + \beta(T - \hat{T})(v - p^i)$ , where  $\hat{T} \geq 0$  is the amount of time until she switches to firm  $i$ . Because  $p^* > v \geq p^i$ , the anticipated payoff is maximized at  $\hat{T} = 0$ , which is  $\beta T(v - p^i) + \beta B^i$ . Suppose second that the consumer takes up firm  $i$ 's offer at  $t = -1$ . Let  $\hat{T} \geq 0$  be the amount of time until she switches to firm  $i$  and  $\tilde{T} \geq 0$  be the amount of time she pays  $p^*$ . As self  $t = -1$  plans to collect both bonuses, self  $t = -1$ 's anticipated payoff is  $\beta B^i + \beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta \tilde{T}(v - p^*) - \beta s \leq \beta T(v - p^i) + \beta B^i - \beta s < \beta T(v - p^i) + \beta B^i$  if self  $t = -1$  plans to cancel firm  $j$ 's contract at some opportunity, and it is  $\beta B^i + \beta \hat{T}(v - p^i) - \beta s + \beta B^* + \beta(T - \hat{T})(v - p^*) \leq \beta(T - K\frac{m}{T})(v - p^i) + K\frac{m}{T}(v - p^*) + \beta B^i < \beta T(v - p^i) + \beta B^i$  if self  $t = -1$  plans to hold firm  $j$ 's contract until the end. Thus, in either case, the anticipated payoff is lower than the case in which self  $t = -1$  plans to take firm  $j$ 's offer, switch to firm  $i$ 's offer at  $t = 0$ , and hold it until the end. These results imply that, for the deviation offer to be profitable, firm  $i$  must attract the consumer at some switching opportunity. But then (26) must hold, implying that firm  $i$ 's total profits are at most  $Tp^i - B^i \leq Tv$ . Because  $Tv < \frac{1}{2}(Tp^* - B^*)$  by Condition (5), firm  $i$ 's deviation is unprofitable.

We conclude that there is no profitable deviation for firm  $i$ , so the candidate equilibrium specified above is indeed an equilibrium.  $\square$

**Proof of Proposition 7.** Consider the following candidate equilibrium in which all three firms offer  $(B^n, p^n) = (B^*, p^*) = (s, v)$ . If the consumer is indifferent whether to plan on collecting a bonus at  $t = -1$  or  $t = T'$ , she does plan to do so. If she is indifferent as to which of the two initial offers to take at  $t = -1$  or which of the two offers she sees at  $t = T'$ , she selects each firm with probability  $1/2$ . At the switching opportunity  $t = 0$  and the following switching opportunities up to opportunity  $K$ , whenever the consumer is indifferent, she does not switch. Similarly, at the switching opportunity  $T$  and any opportunity thereafter, whenever the consumer is indifferent, she does not switch.

As a preliminary observation, note that because the consumer cannot collect the bonus

twice prior to  $T'$  and she always sees the same switching offer prior to  $T'$ , she both plans to and actually switches at most once following any history prior to  $T'$ , independently of whether firms choose equilibrium or non-equilibrium offers. By the same reasoning, she both plans to and actually switches at most once following any history where she choose to act at time  $t > T'$ . We thus focus on the first time the consumer switches for all histories in which she acts at time  $t \in [0, T')$ , and then again on the first time she switches for histories in which she acts at time  $t \in (T', 2T]$ . Also, since the consumer solves a (perceived) optimization problem, her decision on whether she wants to switch or not at a given switching opportunity is independent of how she thinks she will act in the future whenever she is indifferent; we assume that in such a case she believes her future self will switch immediately when indifferent.

Below, we will rename firms after they have been randomly assigned as to whether they make offers first at  $t = -1$  or  $T'$ . We use the convention that the consumer sees the offers of firms 1 and 2 at  $t = -1$ , and that of firm 3 (as well as the initial firm the consumer did not select at  $t = -1$ ) at  $T'$ . Note that firms choose their offers simultaneously after the assignment.

We show that the above strategies constitute an equilibrium. Since  $p^*T - B^* = Tv - s > 0$ , each firm prefers the equilibrium offer to any offer which results in the firm having no sales.

We now establish that a deviation to  $B^i > s$  and  $p^i = v$  is unprofitable. Consider firm 3 first. Because  $p^1 = p^2 = v$  and  $B^1 = B^2 = s$ , when setting  $p^3 = v$  and  $B^3 > s$ , the consumer is indifferent as to whether she collects the bonus only from firm 3 or both firms whose offer she sees in period  $T'$ , and hence she expects to collect both bonuses by our equilibrium-selection assumption. She is, thus, indifferent whether to select firm 3 or its rival in period  $T'$ , and hence selects either offer with equal probability by the tie-breaking rule. Because  $-s + \beta B^i < \beta(-s + B^i)$  for all  $B^i \geq 0$ , following time  $T'$  the consumer does not switch until the last opportunity. At the last opportunity, the consumer switches to firm  $i$  only if  $-s + \beta B^i \geq 0$  or equivalently  $B^i \geq \frac{s}{\beta}$ , so firm  $i$ 's profits from it are at most  $(T - K\frac{T}{m})p^i - B^i \leq \frac{T}{m}v - \frac{s}{\beta}$ , which is negative by the assumption  $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$ . Hence, a

deviation  $B^3 > s$  and  $p^3 = v$  merely decreases firm 3's total profits. We next show that a deviation to  $B^i > s$  and  $p^i = v$  is unprofitable for firms  $i = 1, 2$ . Without loss of generality, consider such a deviation by firm 1. Conditionally on the consumer seeing firm 1's offer at  $T'$ , this deviation is unprofitable in the subgame starting at  $T'$  by the exact same argument as the one for firm 3. No matter how the consumer plans to switch prior to time  $T'$ , she (correctly) believes that she will switch in period  $T'$ . Hence, her continuation payoff derived from decisions at or after  $T'$  are independent of whether or not the consumer switches at the first  $K$  switching opportunity. If a consumer does not select the deviant firm's contract at  $t = -1$ , she will not switch prior to opportunity  $K$ , and if she switches at opportunity  $K$  the firm makes negative profits from the contract up to time  $T$ . Furthermore, at  $t = -1$ , the consumer strictly prefers to take the non-deviant firm's offer so that she is being offered the higher deviant firm's bonus  $B^i > s$  at  $T'$  again. Thus, such a deviation generates non-positive profits. We hence conclude that any deviation in which  $B^i > s$  and  $p^i = v$  is unprofitable for all firms.

When deviating to a contract with  $p^i = v$  and  $B^i < s$ , the consumer both at  $T'$  and at  $t = -1$  strictly prefers to take firm  $i$ 's rival's offer and she does not want to switch at any switching opportunity in which her switching costs are  $s$ . We conclude that a deviation in which  $B^i < s$  and  $p^i = v$  is unprofitable.

We next consider deviations by firm  $i$  in which  $p^i < v$ , supposing the rivals offer the candidate equilibrium offer  $(B^j, p^j) = (s, v)$ . Again, we begin by considering a deviation of firm 3 to an offer in which  $p^3 < v$ . If the consumer selects the offer of firm  $j \neq 3$  at  $t = T'$ , the consumer believes that she will switch to offer  $(B^3, p^3)$  at  $t = T$  only if the following intention-to-switch condition holds:

$$-s - Tp^3 + B^3 \geq -Tv \iff s \leq T(v - p^3) + B^3, \quad (27)$$

in which case, her anticipated total payoff when selecting the rival's offer first is  $-s - Tp^3 + B^j + B^3 = -Tp^3 + B^3$ .

Consider first the case in which (27) does not hold. Then, the consumer will not switch from firm  $j$  to firm 3 at any switching opportunity following  $T'$ . Because  $p^3 < v = p^j$ ,

the consumer also never switches from firm 3 to firm  $j$ . As violating (27) is equivalent to  $-Tp^3 + B^3 < -Tv + s$ , the consumer strictly prefers to take up firm  $j$ 's offer at  $t = -1$  (and will never switch away from firm  $j$ ). Hence, such deviation by firm 3 is unprofitable.

Consider second the case in which (27) holds. The consumer's anticipated total payoff when selecting firm 3's offer at  $t = T'$  is  $-Tp^3 + B^3$ , because the consumer expects not to switch to firm  $j$  given  $p^j = v > p^3$  and  $B^j = s$ . The consumer's anticipated total payoff when selecting firm  $j$ 's offer at  $t = T'$  and then switch to firm 3 at  $t = T$  is also  $-s - Tp^3 + B^j + B^3 = -Tp^3 + B^3$ . As the consumer is indifferent at  $t = T'$ , by the tie-breaking rule she plans to collect both bonuses and randomly chooses an offer at  $t = T'$ . For a profitable deviation to exist, it must induce the consumer to switch from firm  $j$  to firm 3. Given that the consumer has selected  $(B^j, p^j) = (s, v)$  at  $t = T'$ , (27) implies that if the consumer does not have the intention to switch at time  $T$ , then she does not have the intention to switch at any time thereafter, so without loss of generality we focus on whether the consumer switches from firm  $j$  to firm 3 at  $t = T$ .

At  $t = T$ , the consumer switches from firm  $j$  to firm 3 only if the consumer does not prefer procrastinate switching, i.e., only if

$$-s + \beta \left( -\frac{T}{m}p^3 + B^3 \right) \geq \beta \left( -s - \frac{T}{m}v + B^3 \right) \iff p^3 + \frac{1 - \beta}{\beta} \frac{m}{T} s \leq v. \quad (28)$$

However, (28) is not satisfied for any  $p^i \in [0, v]$  if Condition (4) holds. Hence the consumer never switches at  $t = T$  from firm  $j$  to firm 3, and a deviation to any contract in which  $p^3 < v$  is unprofitable for firm 3.

We are left to consider deviations by firm  $i \in \{1, 2\}$  to an offer in which  $p^i < v$ . We first consider deviations for which  $-Tp^i + B^i < -Tv + s$ . In this case, if firm  $i$ 's contract is available at  $T'$ , from period  $T$  on the consumer gets an expected continuation benefit of  $-Tp^i + B^i$  when selecting firm  $i$ 's offer because thereafter the consumer does not want to switch to firm 3 which offers a price of  $p^3 = v > p^i$  and a bonus for switching of  $s$ . When selecting firm 3 and then not switching, on the other hand, from period  $T$  on the consumer gets expected continuation benefit of  $-Tv + s$ , which implies that she selects firm 3's offer at  $T'$ . Furthermore, in this case she has no intention to switch at time  $T$  because

$-s + B^i - Tp^i < -Tv$ , and switching at a later opportunity is even less beneficial. Hence, here expected continuation benefit from period  $T$  on is  $-Tv + s$  when selecting firm  $j \neq i$  at  $t = -1$ . Similarly, when selecting firm  $i$ , the consumer's continuation value from period  $T$  on is  $-Tv + s$  because she strictly prefers switching at  $T'$  to not doing so, and with both offers being equal to  $(s, v)$  at  $T'$ , her continuation value from period  $T$  on is  $-Tv + s$ . Therefore, in this case her choice between firm 1 and firm 2 as well as her switching behavior up to opportunity  $K$  is identical to that in the game of Proposition 5, and hence it follows from the proof of Proposition 5 that a deviation to a contract for which  $-Tp^i + B^i < -Tv + s$  is unprofitable.

We next consider deviations for which  $-Tp^i + B^i \geq -Tv + s$  and  $p^i < v$  by firm  $i \in \{1, 2\}$ . Let  $j \neq i$ ,  $j \in \{1, 2\}$  denote the rival firm  $i$  faces at  $t = -1$ . Since  $p^i < v$ , if the consumer selects firm  $i$ 's contract at  $T'$ , she will not switch away from it thereafter. Hence, her expected continuation value from  $T$  on is  $-Tp^i + B^i$  when doing so. If she selects firm 3's contract and switches at time  $T$ , on the other hand, she gets an expected payoff of  $-s + B^3 - Tp^i + B^i = -Tp^i + B^i$  from  $T$  on. Hence, the consumer weakly prefers to select firm 3 in period  $T'$  and by the tie-breaking rule, she does so with probability of at least  $1/2$ . Furthermore, the profits conditional on the consumer selecting  $i$ 's offer at  $T'$  or thereafter are weakly lower, and thus such a deviation can only increase profits from period  $T$  on if it induces the consumer to switch from firm 3 to firm  $i$  at time  $T$  (or thereafter). The consumer is willing to switch only if she does not prefer procrastinating, i.e., only if

$$-s + \beta \left( -\frac{T}{m}p^i + B^i \right) \geq \beta \left( -s - \frac{T}{m}v + B^i \right) \iff p^3 + \frac{1 - \beta}{\beta} \frac{m}{T}s \leq v,$$

which, however, does not hold by Condition (4). We conclude that the deviation by firm  $i$  does not raise profits from period  $T$  onwards.

Now consider the consumer's choice at period  $t = -1$ . If the consumer selects the deviant firm  $i$ 's offer, she will not switch prior to period  $T'$  because  $p^i < v = p^j$ . At  $T'$ , where the consumer has the opportunity to switch to firm  $j$ 's or firm 3's offer for free, she switches if and only if  $s - Tv \geq -Tp^i$ . Hence, her expected payoff of selecting firm  $i$ 's offer is  $-Tp^i + B^i + \max\{s - Tv, -Tp^i\}$ . If she selects firm  $j$ 's offer, switches at  $t = 0$ , and switches



to firm  $i$ 's offer at  $t = T'$  (or firm 3's offer and then switches at  $t = T$  to firm  $i$ ), the consumer gets a payoff of  $B^j - s + B^i - Tp^i + B^i - Tp^i = 2(B^i - Tp^i)$ , which is weakly greater. Therefore, the naive consumer's expected payoff from selecting contract  $j$  is weakly higher, and by the tie-breaking assumption, she selects firm  $j$ 's offer with probability of at least  $1/2$ . Since the deviation does not increase profits when the consumer sees firm  $i$ 's offer at time  $T'$  or thereafter and the deviation cannot increase the probability that she sees firm  $i$ 's offer at  $T'$ , a necessary condition for the deviation to increase profits is that the consumer switches from firm  $j$  to firm  $i$  prior to opportunity  $K$  when selecting firm  $j$ 's contract first. Given that the consumer expects to switch at  $T'$  in this case (to firm 3 or firm  $i$ ) independently of whether she switched beforehand, her incentives to switch at opportunities prior to  $K$  are the same in the game of Proposition 5, and it follows from the proof that the consumer does not switch, and hence, the deviation is unprofitable for firm  $i$ . We conclude that there is no profitable deviation for which  $p^i < v$ .

As we have established that there exists no profitable deviation in which  $p^i = v$  and no profitable deviation in which  $p^i < v$  for any of the three firms, we conclude that the strategies in which all three firms offer  $(s, v)$  indeed constitute an equilibrium.  $\square$