Competition in Search Markets with Naive Consumers

Tobias Gamp[†] and Daniel Krähmer[‡]

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Abstract

We study the interplay between quality provision and consumer search in a search market where firms may design products of inferior quality to promote them to naive consumers who fail to fully understand product characteristics. We derive an equilibrium in which both superior and inferior quality is offered and show that as search frictions vanish, the share of firms offering superior goods in the market goes to zero. The presence of inferior products harms sophisticated consumers, as it forces them to search longer to find a superior product. We argue that policy interventions that reduce search frictions such as the standardization of price and package formats may harm welfare. In contrast, reducing the number of naive consumers through transparency policies and education campaigns as well as a minimum quality standard can improve welfare.

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[†]Humboldt University Berlin, Spandauer Straße 1, D-10178 Berlin; email: *tobias.gamp@hu-berlin.de*

[‡]University of Bonn, Institute for Microeconomics, Adenauer Allee 24-42, D-53113 Bonn (Germany); email: *kraehmer@hcm.uni-bonn.de*

1 Introduction

In many markets, consumers need a basic level of (financial, legal, technical, etc.) expertise to accurately evaluate and compare products. For example, investors need a considerable degree of financial literacy to assess the suitability and fee structure of financial products. Or, many consumer electronics display various features (such as the quality of component parts or the compatibility with other devices and apps, but also privacy settings) whose costs and benefits a non-tech savvy consumer may not be able to appreciate at the point of purchase. Other examples are markets for expert services with credence good features (Darby and Karni (1973)) such as complex repair, medical, or legal services. To make an informed choice, consumers should know what service is best suited to solving their particular problem, the skill or expertise of the service provider for the problem at hand, as well as fee arrangements and the relationship between fees and time needed in ultimately providing the service.

In practice, many consumers lack this expertise. This allows sellers to offer inferior products and services and promote them to “naive” non-expert consumers, for example, by hiding (bad) attributes or the true future usage costs, choosing inappropriate services or making exaggerated quality claims.[1] A by now large literature studies firms’ incentives to offer inferior products to target naive consumers (see the literature review below). In this article, we address a novel issue: the interplay between inferior product design and consumer search. This is motivated by the observation that even for “sophisticated” consumers, who are not naive and possess the relevant expertise, evaluating and comparing complex products is time-consuming and involves costly search.[2]

[1]Many financial service products such as credit cards, current accounts and mortgage loans display hidden fees or attributes which some consumers fail to anticipate and account for. See, for example, Ausubel (1991), Agarwal et al. (2013), FCA (2014) and Agarwal et al. (2015) on credit cards, the OFT (2008), Stango and Zinman (2009), Armstrong and Vickers (2012), Stango and Zinman (2014) and the CMA (2016) on current accounts, and Woodward and Hall (2010) and Gurun et al. (2016) on mortgages. Similarly, providers for insurance products such as home warranties “build in wiggle room that can make it easier for them to deny payouts”, Consumer Reports (2019) or OAG (2018). In markets for credence goods overcharging and undertreatment as well as overtreatment are common phenomena. See, for example, the discussion in Dulleck and Kerschbamer (2006) or Beck et al. (2014). Finally, some consumer electronics such as TVs may display hidden bad attributes such as “fake” high dynamic range and refresh rates, lacking compatibility with streaming services or sparse connections, (CNET (2018)). Indeed, manufacturer intentially create “derivative models" – stripped-down versions of pre-existing TVs – with lesser quality, altered warranty, missing features or components, (USA Today (2013)).

[2]Consistent with this observation, consumers in many financial service markets often do not engage in substantial search, but visit only very few competitors, indicating that they find search to be costly. For example, according to the CFPB (2015), “almost half of consumers who took out a home purchase mortgage reported that they seriously considered only a single lender or mortgage broker before applying for a loan” and less than 20 % considered more

Our main results show that stronger competition in the form of smaller search frictions may not weaken but strengthen firms' incentives to offer inferior products. As a consequence, it may be the case that only sophisticated consumers benefit from lower search frictions, whereas naive consumers lose out. When the efficiency loss from consuming inferior instead of superior products is large, reducing search frictions may even reduce total welfare. We also identify a novel channel through which the presence of naive consumers harms sophisticated ones: the latter waste time examining irrelevant inferior products.

We extend the standard model of consumer search markets by Wolinsky (1986) or Anderson and Renault (1999) and allow for the presence of naive consumers. A firm may offer one product of either inferior or superior quality. Similarly to Armstrong and Chen (2009), we interpret inferior quality as a short-cut for hidden (bad) attributes, opaque usage costs, surcharge fees concealed in the fine print, or exaggerated quality claims that a naive consumer fails to fully discount. More precisely, in our model, sophisticated consumers engage in costly search to detect the price, the (true) quality, and the idiosyncratic fit of a product. Naive consumers, in contrast, lack the skills to fully evaluate a product, and only learn a product's price and whether it is inferior or superior but systematically underestimate the difference in quality. Crucially, naive consumers also lack the expertise to evaluate product fit at the time of purchase. This is realistic in the above-mentioned markets for financial products, consumer electronics, and expert services where it is not only difficult for non-expert consumers (such as e.g. the elderly or less educated) to work out the quality of a product but also how it will fit one's needs in the future or, in case of an expert service, its appropriateness for the problem at hand.[3] This will imply that they only search for an attractive combination of price and (subjective) quality.

Our first result shows that, in equilibrium, firms offer both inferior and superior quality, entailing market segmentation. Sophisticated consumers never buy inferior products but naive consumers purchase either product. Intuitively, sophisticated consumers recognize true quality, avoid purchasing inferior products and search until they find a suitable superior product. In contrast, the fact that naive consumers underestimate the difference in quality allows firms who offer inferior quality to compete for naive consumers, and equilibrium prices adjust such that naive consumers consider both inferior and superior products as acceptable. They hence buy from the first firm they visit.[4] In equilibrium, a firm's product choice is driven by a trade-off between selling (high

than two. Another example is the UK retail banking market where according to the OFT (2008) only 15 − 20 % of all consumers make explicit comparisons across products in terms of interest and rates, although 69 % agree that there may be better alternatives available in theory.

[3]As we explain later in more detail, our results are driven by the joint assumption that naive consumers cannot correctly assess both quality and product fit.

[4]The fact that naive consumers search less is observationally somewhat in line with the idea that lower income

cost) superior products at a low mark–up to all consumers and "ripping off" naive consumers with (low cost), high mark–up inferior products.

Our main insight that lower search frictions may have a detrimental effect on the provision of quality in the market is driven by how lower search frictions affect the relative change in profits in the two segments. In fact, we show that in the extreme case when search costs vanish, the share of firms who offer superior quality goes to zero. The underlying reason is that, as search costs vanish, both naive and sophisticated consumers can compare firms essentially for free. While this puts pressure on all firms' mark-up, this effect is more pronounced for firms who offer superior quality. In fact, as search costs vanish, the mark-ups for superior quality are eliminated whereas the mark-ups for inferior quality never erode, ultimately leading all firms to adopt inferior quality.[5] Thus, intense competition in the form of small search costs has the striking consequence that the market will supply almost only inferior quality.

The effect of smaller search costs on the share of firms offering superior quality in the market is clear-cut as search costs vanish, but for non-negligible search costs, there are countervailing effects: On the one hand, as search costs fall, the market becomes more competitive, and the prices of both superior and inferior products fall (if the share of firms offering superior quality does not adjust).[6] Now, because firms offering superior quality operate with larger demand but smaller margins than firms offering inferior quality, a decreasing price level makes offering inferior quality relatively more attractive. On the other hand, as search costs fall, the difference between the mark–ups for inferior and superior quality shrinks which makes offering superior quality at the lower mark–up more attractive. We identify a necessary and sufficient condition for profits from offering superior quality to decrease more than from offering inferior quality in response to a marginal decrease in search costs. In this case, lower search frictions reduce the fraction of firms offering superior quality in equilibrium.[7]

consumers have less slack to engage in search (Mullainathan and Shafir (2013)).

[5]The above intuition is somewhat incomplete because what matters for sophisticated consumers is not nominal, but effective search costs, that is, the costs it takes to find not any, but an superior product. If the number of superior quality firms shrinks, then, all else equal, sophisticated consumer waste time evaluating inferior products which they will not end up buying. As we show, effective search costs are monotone in nominal search costs and, in fact, go to zero if nominal search costs do, although the share of superior quality firms vanishes.

[6]The equilibrium effect of a decrease in search costs on prices is, however, not clear cut. On one hand, as in standard search models, as search costs fall, sophisticated consumers search more so that, all else equal, superior quality firms reduce their prices. On the other hand, the share of firms offering superior quality changes endogenously with search costs which affects the composition of naive and sophisticated customers that make up a firm's demand, possibly resulting in higher or lower prices.

[7]This result is qualitatively consistent with the results in Ellison and Ellison (2009) which documents that in response to the better search technology (in particular for prices), firms in the online market for memory modules

These findings imply that reducing search costs may be undesirable from a welfare point of view, because the efficiency losses from the increased consumption of inefficient inferior products by naive consumers may outweigh the gains due to savings on search costs. Even if this is not the case (because efficiency losses are relatively small), the ensuing welfare gains may be unequally distributed among market participants: Sophisticated consumers gain, because they benefit from lower prices and search costs. In contrast, naive consumers may lose out, because they suffer from the higher prevalence of inferior products.

Our main results thus suggest that policy interventions that aim at reducing search frictions should be viewed with caution, because they may be socially undesirable, or, at least, may not constitute a Pareto improvement from the consumers' point of view. This may include policies that facilitate the comparability of products, such as the harmonization and standardization of price, package, or product information formats[8], or the promotion of online marketplaces. In contrast, we argue that minimum quality standards, which can be interpreted as a cap on hidden or unforeseeable future fees in our model, improve consumer welfare, because naive consumer not only pay lower hidden fees but also incur these fees less frequently, as fewer firms engage in add-on pricing in response.

Our analysis also sheds new light on the effects of information and education campaigns that reduce consumer naivete (such as promoting financial literacy in the context of financial service markets). As we show, this increases welfare (under a plausible sufficient condition), because as a result of a shrinking naive customer base, fewer firms choose inferior quality. In our search framework, this benefits also sophisticated consumers, as they waste less time examining inferior products. In this sense, naive consumers exert a negative externality on sophisticated consumers in our framework.

Related Literature

Our paper is related to a literature in behavioral industrial organization which studies how firms can exploit naive consumers by offering goods with future add-on services (see, e.g., Gabaix and Laibson (2006), Spiegler (2006), Armstrong and Chen (2009), Armstrong and Vickers (2012), Heidhues et al. (2017)). In this literature, stronger competition may move firms towards greater exploitation of naive consumers, because competition impairs firms' mark-ups from sophisticated

began to "bundle low-quality goods with unattractive contractual terms, like providing no warranty and charging a 20 % restocking fee on all return."

[8]For example, in the context of the UK retail banking market, the Office of Fair Trading discusses the harmonization of price and product formats in order to address "the difficulties caused by lack of information (which) are exacerbated by the fact that different banks use different terminology and present interest rates and charges in different ways". On the other hand, the harmonization of price and product formats might also be beneficial if it helps (naive) consumers to identify inferior products.

consumers.[9] While this is similar to our point that lower search frictions may increase firms' exploitation incentives, in our model, in contrast, the (first order) effect of a decline in search costs is to reduce profits in both segments, giving rise to countervailing effects that may also imply less exploitation.

Similar to us, in Armstrong and Chen (2009) naive consumers misperceive quality differences, and inferior and superior quality firms co-exist in equilibrium, but the superior quality segment does not vanish in the limit as competitive forces (the number of firms) grow large. The reason is that Armstrong and Chen (2009) study equilibria in which firms make profits because they may end up as the monopolist in one of the segments. As a result, in equilibrium firms maximize the probability of being the monopolist in one segment by targeting each segment with equal probability as the number of firms is large. In our model, the mark-ups in the two segments are asymmetrically affected as competition becomes strong, and only mark-ups in the superior quality segment erode in the limit.

In terms of policy implications, our observation that facilitating the comparability of products can turn out to harm consumer welfare is related to Piccione and Spiegler (2012) who reach a similar conclusion in a very different model where firms may avoid price comparisons by choosing less common pricing formats.

Our paper also contributes to the literature on consumer search by integrating naive consumers and seller deception into the seminal papers by Wolinsky (1986) and Anderson and Renault (1999). In this framework, lowering search frictions is typically considered socially desirable, even if firms' product choices are endogenous.[10] Related to our normative analysis, Kuksov (2004) shows that lower search costs may give rise to socially excessive product differentiation incentives.[11] In a setting where high quality comes at higher fixed (rather than marginal) costs, Fishman and Levy (2015), like us, find that lower search costs may increase the number of low quality firms in the market because the profits of low quality firms decrease by less than those of high quality firms. In Fishman and Levy (2015), this effect occurs only when the high quality segment is large whereas in our model, this effect is particularly strong when the high quality segment is small. Moreover, the normative implications are different, as in Fishman and Levy (2015), low quality might be efficient.[12]

[9] See, among others, Varian (1980), Armstrong and Chen (2009), de Clippel et al. (2014), Murooka and Schwarz (2018), and Johnen (2020). In a similar vain, the literature on obfuscation shows that firms' incentives to obfuscate become stronger, as the number of firms grows (see, e.g., Carlin (2009) or Chioveanu and Zhou (2013)).

[10] See, e.g., Bakos (1997), Cachon et al. (2008), Bar-Isaac et al. (2012), Larson (2013).

[11] On a related note, it has also been observed that it can be welfare improving if firms may raise search costs through obfuscation (see Gamp (2018) and Taylor (2017)).

[12] More indirectly related are studies on the relationship between the provision of quality and competition in mar-

Our finding that prices may decrease in search costs echoes similar findings in the search literature.[13] In our model, the price non-monotonicity originates in the fact that the size of the segments is endogenous. Somewhat similarly, in Moraga-González et al. (2017), lower search costs may increase prices due to the market entry of consumers whose demand is less elastic.

This paper is organized as follows. The next section introduces the model. Section 3 derives equilibrium conditions. Section 4 to 6 discuss policy implications, and section 7 concludes. All proofs are in the appendix.

2 Model

We consider a search market with a unit mass of firms $k \in [0,1]$ and a unit mass of consumers $i \in [0,1]$.[14] Each consumer seeks to purchase at most one good. Goods are differentiated, and consumer i's utility from purchasing firm k's good is equal to

$$u_{ik} = q_k + \theta_{ik} - p_k, \tag{1}$$

where q_k is the quality chosen, and p_k is the price chosen by firm k. The term θ_{ik} is a consumer-firm specific match-value which represents idiosyncratic product fit. It is common knowledge that θ_{ik} is distributed on the support $[\underline{\theta}, \overline{\theta}]$ with (sufficiently smooth) cumulative density function F and mean (normalized to) zero (hence $\underline{\theta} < 0$), independent and identical across consumers, firms and firms' quality.[15] The probability density function $f = F'$ is log-concave with $f(\overline{\theta}) > 0$ which implies that F has an increasing and unbounded hazard rate[16]

$$h(\theta) \;=\; \frac{f(\theta)}{1 - F(\theta)}. \tag{2}$$

The objective of our analysis is to study market outcomes when there are some naive consumers who lack the expertise to assess products and are vulnerable to sales practices such as hiding (bad) attributes, obfuscating true usage costs, concealing surcharge fees in the fine print,

kets without consumer search. See, e.g., Cooper and Ross (1984) who show that competitive equilibrium may fail to exist when some consumers cannot discern quality, or Hörner (2002) and Kranton (2003) who study conditions under which competition facilitates/aggravates the build-up of a reputation for providing high quality.

[13] See, e.g., Janssen and Moraga-González (2004), Armstrong and Zhou (2011), Janssen and Shelgia (2015), Moraga-González et al. (2017), or Choi et al. (2018).

[14] Assuming a continuum of firms significantly improves the tractability of the model, as in equilibrium no consumer returns to a previously visited firm.

[15] The assumption that F does not depend on a firm's quality q_k simplifies the exposition. For example, our analysis carries over without any changes when the support of the distribution of match-values does not depend on quality.

[16] That f is log-concave is a standard assumption in the search literature and satisfied by most common distributions. See, e.g., Anderson and Renault (1999) and Bagnoli and Bergstrom (2005).

or making exaggerated quality claims. We interpret such practices broadly as amounting to the choice of a good of inferior quality $\underline{q}$ which naive consumers misperceive, as explained below. Alternatively, a firm can offer a good of superior quality $\overline{q} > \underline{q}$. The marginal costs for producing a good of superior respectively inferior quality are denoted with $\overline{c}$ respectively $\underline{c}$. Let $\Delta c = \overline{c} - \underline{c}$. We assume that the superior good is (weakly) socially efficient,

$$\Delta q \geq \Delta c. \tag{3}$$

To purchase a product from a firm, a consumer has to visit it and examine its product which entails a search cost $s > 0$.[17] A consumer may examine several products, one at a time and in a random order and with perfect recall. Prior to his search, a consumer is uninformed about p_k and (q_k, θ_{ik}).

There are two types of consumers. A fraction $\nu^S \in [0,1]$ of consumers is sophisticated who, upon visiting a firm, observe p_k, q_k, and θ_{ik}. In addition, there is a fraction $\nu^N = 1 - \nu^S$ of naive consumers who lack the expertise to both observe product fit and to understand the true quality. Instead, a naive consumer mistakenly believes that all inferior products supply the (gross) utility $\underline{q}^N$ and all superior products supply the (gross) utility $\overline{q}^N \geq \underline{q}^N$. Most of our results will only depend on the difference $\Delta q^N = \overline{q}^N - \underline{q}^N \geq 0$. When $\Delta q^N = 0$, naive consumers see no quality difference between inferior and superior products. Formally, upon visiting a firm k, a naive consumer only observes p_k and q_k^N but not θ_{ik}. Effectively, he expects any product to supply the (normalized) average fit zero.[18]

The sequence of events is as follows. At the outset, firms simultaneously, independently, and once and for all set q_k and p_k. Consumers then search until they purchase a product.[19] Thus, a strategy for a firm is a quality-price combination, and a strategy for a consumer is a search rule that specifies whether to end or continue search contingent on the past search history. Our solution concept is Perfect Bayesian equilibrium with the constraint that naive consumers have incorrect quality perceptions.

In the benchmark without naive consumers, all firms offer the efficient, superior quality in order to maximize the gains from trade, and our model essentially corresponds to Wolinsky (1986)

[17]Our analysis does not change substantially if the consumers groups have different search costs, for example, if sophisticated consumers' search costs are proportional to those of naive consumers.

[18]In effect, a naive consumer can be seen as if to behave like a rational consumer for whom product fit is irrelevant and for whom the true! difference in quality is Δq^N. In contrast to our model, it would then be efficient that such a consumer buys the low quality which would change the economic interpretation of our results significantly.

[19]We thus implicitly assume that the true and perceived surplus from trade is sufficiently large such that consumers have no interest in leaving the market without purchasing a product.

or Anderson and Renault (1999).[20] A similar logic implies that also when some consumers are naive, all firms offer the superior product if the inferior good is inefficient from the perspective of a naive consumer, that is, $\Delta q^N \geq \Delta c$. In what follows, we therefore assume that

$$\Delta q^N < \Delta c. \tag{4}$$

By (3), this implies that

$$\Delta q^N < \Delta q. \tag{5}$$

Applications

Our model of naivete entails that the inabilities to accurately assess product quality (q) and product match (θ) go hand in hand, capturing that assessing either dimension requires the same specific expertise.[21] We now give a few examples:

A natural application are markets for complex financial products such as retirement savings plans, mutual fund investments, mortgages, credit cards or various insurance policies. Consumer groups such as the less educated or the elderly typically lack the necessary financial literacy to understand both a product's fee structure and whether the product actually matches their needs (e.g., risk preferences and risk exposure, liquidity constraints and/or intended usage pattern). Specifically, the difference Δq between superior and inferior quality may then be the result of hidden (add-on) costs such as management, early or late payment fees, or transaction fees. In particular, both products may offer the same innate quality but similarly to Heidhues et al. (2017), a low quality firm charges an unavoidable add-on fee of size Δc which a consumer has to pay ex post. The total loss the consumer incurs is Δq, amounting to social waste $w = \Delta q - \Delta c$ of add-on pricing.

Another application are markets for consumer electronics (TVs, tablets, PCs, etc.) where it is difficult for a non-tech savvy consumer to understand how the various product components affect its ultimate quality. Moreover, these devices typically display various features (such as compatibility with other devices, apps, or standards but also privacy or data transfer settings) whose (non)-usefulness for him a non-tech savvy user only learns to appreciate with future use, but is not aware of at the point of purchase.

A further application are markets for expert services with credence features (Darby and Karni (1973)) such as complex repair, medical, or legal services whose inherent intricacy prevents lay

[20] In particular, forces other than consumer naivety that may induce firms to offer inefficiently low quality, such as fixed investment costs as in Fishman and Levy (2015), are absent from our model.

[21] To permit tractability, we capture this by the extreme assumption that naive consumers do not observe product fit at all. As we explain in more detail below, what is important for our analysis is that products are sufficiently less differentiated from the perspective of naive compared to standard consumers.

persons from fully appreciating both the quality/appropriateness of the service offered as well as the service provider's match with the problem at hand. For example, a mechanic may (have a reputation to) use original or inferior spares and be more or less experienced/specialised in repairing the particular model specification.

In our model, naive consumers are vulnerable to "undertreatment" when they are sold inefficiently low quality, but we emphasize that, with an appropriate change in interpretation, it can also be applied to the problem of "overtreatment" when a consumer is sold inefficiently high quality. In this case, $\overline{q}$ (resp. $\underline{q}$) denotes the efficient (resp. inefficient) quality as before, however, with efficient quality not being superior anymore, that is $\overline{q} < \underline{q}$ and $\overline{c} < \underline{c}$. We then have $\Delta q < 0$ and $\Delta c < 0$, and Assumptions 3 and 4 amount to $0 > \Delta q \geq \Delta c > \Delta q^N$. As it turns out, all our results carry over to this case, because our arguments depend only on Assumptions 3 and 4 but not on $\Delta q > 0$ und $\Delta c > 0$. We impose the latter assumptions only to fix ideas.

3 Segmented market equilibrium

In the first step of our analysis, we establish necessary and sufficient conditions for the existence of an equilibrium which displays a segmentation of the market in an inferior and superior quality segment.

Definition 1 *A triple $(\lambda^*, \overline{p}^*, \underline{p}^*) \in (0,1) \times \mathbb{R}^2$ is a segmented market equilibrium outcome if there is an equilibrium in which*

- ⋄ *a fraction λ^* of firms offers superior quality with $q_k = \overline{q}$ and $p_k = \overline{p}^*$;*
- ⋄ *a fraction $1 - \lambda^*$ of firms offers inferior quality with $q_k = \underline{q}$ and $p_k = \underline{p}^*$;*
- ⋄ *sophisticated consumers never buy inferior quality (regardless of how high θ_{ik} is);*
- ⋄ *naive consumers buy either quality.*

We refer to an equilibrium which supports a segmented market equilibrium outcome as a segmented market equilibrium.

Next, we show that there is a unique segmented market equilibrium if search costs are sufficiently small. Note that a segmented market equilibrium is a symmetric equilibrium in the sense that all firms that offer the same quality charge the same price. As we show in Appendix C, if $\Delta q > \Delta c$, then for sufficiently small search costs, there are no other symmetric equilibria. We begin by deriving the players' optimal strategies in expectation of a segmented market equilibrium.

Sophisticated consumer search

Because consumers search randomly, it is optimal for any consumer to employ a cutoff search rule which is characterized by a reservation utility which is the smallest level of (perceived) utility that a product must supply to induce the consumer to end his search and purchase the product.

As is shown in McCall (1970), the reservation utility is the current utility level that leaves a consumer indifferent between ending search in the current period and visiting a single additional firm. In a segmented market $(\lambda, \overline{p}, \underline{p})$, a sophisticated consumer expects to encounter a superior firm with probability λ and an inferior firm with probability $1-\lambda$. In a candidate segmented market equilibrium, a sophisticated consumer does not buy an inferior product. Then, his reservation utility is given recursively as

$$\hat{U}^S = \lambda \int_{\underline{\theta}}^{\overline{\theta}} \max\{\overline{q} + \theta - \overline{p}, \hat{U}^S\} \, dF(\theta) + (1-\lambda) \cdot \hat{U}^S - s. \tag{6}$$

It will often be convenient to work with the reservation match-value which is defined as the smallest match-value $\hat{\theta} \in \mathbb{R}$ at which a sophisticated consumer stops and buys superior quality offered at $\overline{p}$:

$$\overline{q} + \hat{\theta} - \overline{p} \equiv \hat{U}^S. \tag{7}$$

Define the function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(z) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \max\{\theta - z, 0\} \, dF(\theta). \tag{8}$$

Then (6) and (7) yield:

$$g(\hat{\theta}) = \frac{s}{\lambda}. \tag{9}$$

It follows from Lemma A.1 in the appendix that equation (9) has a unique solution $\hat{\theta} < \overline{\theta}$.

Finally, it is indeed optimal for a sophisticated consumer to refrain from an inferior product for any match-value if $\hat{U}^S \geq \underline{q} + \overline{\theta} - \underline{p}$, or, equivalently:

$$\overline{q} + \hat{\theta} - \overline{p} \geq \underline{q} + \overline{\theta} - \underline{p}. \tag{10}$$

Naive consumer search

From McCall (1970), a consumer's reservation utility is his (perceived) expected utility. In a candidate segmented market equilibrium, a naive consumer buys superior and inferior products, and it is thus optimal for him to buy at the first firm. Because a naive consumer wrongly believes that any firm, including the first one, offers $(\overline{p}, \overline{q}^N)$ with probability λ and $(\underline{p}, \underline{q}^N)$ with probability $1-\lambda$, his reservation utility is therefore given as

$$\hat{U}^N = \lambda \cdot (\overline{q}^N - \overline{p}) + (1-\lambda) \cdot (\underline{q}^N - \underline{p}) - s. \tag{11}$$

It will be useful to express a naive consumer's search rule in terms of a reservation price $\hat{p}(q) \equiv q^N - \hat{U}^N$ which is the maximal price that he is effectively willing to pay for a product with true quality q (and thus perceived quality q^N) given his beliefs in a segmented market equilibrium. With (11):

$$\begin{aligned} \hat{p}(\overline{q}) &= (1-\lambda)\cdot\Delta q^N + \lambda\cdot\overline{p} + (1-\lambda)\cdot\underline{p} + s, \\ \hat{p}(\underline{q}) &= -\lambda\cdot\Delta q^N + \lambda\cdot\overline{p} + (1-\lambda)\cdot\underline{p} + s. \end{aligned} \tag{12}$$

Finally, it is indeed optimal for a naive consumer to buy inferior and superior quality if $\underline{p} \le \hat{p}(\underline{q})$ and $\overline{p} \le \hat{p}(\overline{q})$, or, equivalently:[22]

$$\Delta p \ge \Delta q^N - \frac{s}{\lambda} \quad \text{and} \quad \Delta p \le \Delta q^N + \frac{s}{1-\lambda}. \tag{13}$$

Demand and firm profits

The next lemma states firms' profits and prices in a candidate segmented market equilibrium. Denote by $\pi(q_k, p_k)$ the profit of a firm that sets q_k and p_k, taking as given that all other firms and consumers adopt the strategy as specified in a (candidate) equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$. To ensure that first order conditions are sufficient for profit maximization of superior quality firms, we focus on equilibria in which the reservation match value is interior ($\hat{\theta} \in (\underline{\theta}, \overline{\theta})$) and impose from now on the following regularity condition:

$$\frac{f'(\overline{\theta})}{f(\overline{\theta})} \ge -\frac{\nu^S}{\nu^N}\cdot f(\overline{\theta}). \tag{14}$$

This condition holds with increasing density.[23]

Lemma 1 *If there is a segmented market equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$ such that $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$, then*

$$\overline{p} = \overline{c} + \frac{1+\lambda\frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda}))}, \quad \underline{p} = \overline{p} + \frac{s}{\lambda} - \Delta q^N, \tag{15}$$

and equilibrium profits are given by

$$\pi(\underline{q}, \underline{p}) = \nu^N\cdot(\underline{p} - \underline{c}) \quad \textit{and} \quad \pi(\overline{q}, \overline{p}) = (\nu^N + \frac{\nu^S}{\lambda})\cdot(\overline{p} - \overline{c}). \tag{16}$$

[22] Although a naive consumer will purchase from the first firm he visits in equilibrium, he will continue search off the path when prices are large just as rational consumers, e.g., in Diamond (1971) and Stahl (1989). A naive consumer in our setting is therefore different from a Varian (1980) type of "loyal" consumer who visits only one firm by assumption.

[23] It is standard in the search literature to impose sufficient conditions similar to (14), see, e.g., Anderson and Renault (1999) and Gamp (2017). Wolinsky (1986) requires (only) the hazard rate of F to be increasing which we also impose in our setting. This is consistent with (14) and the fact that our model coincides with Wolinsky (1986) for $\nu^N = 0$, because if ν^N is sufficiently small, (14) is satisfied under the mild assumption $f(\overline{\theta}) > 0$.

To understand (15), recall that in a segmented market equilibrium, naive consumers buy both inferior and superior products whenever they are cheaper than their respective reservation prices. Hence, the demand from naive consumers is (almost everywhere) inelastic. In contrast, a sophisticated consumer only buys a superior good with a sufficiently high match value or a sufficiently low price. Hence, sophisticated consumer demand for superior goods is elastic. Intuitively, since a superior firm serves both consumer types, its price $\overline{p}$ balances the inelastic demand from naive and the elastic demand from sophisticated consumers, and is determined by the first order condition for $\overline{p}$ to be profit maximizing. On the other hand, an inferior firm serves only naive consumers, because sophisticated consumers select not to purchase inferior quality. As a result of this selection effect, it optimally charges the naive consumers' reservation price $\hat{p}(\underline{q})$.

To see what is behind (16), observe that symmetry and the fact that naive consumers buy from the first firm they visit implies that the demand of the ν^N naive consumers is split equally among the unit mass of all firms. In contrast, the demand of the ν^S sophisticated consumers is split equally among only the mass λ of superior quality firms. Therefore, every inferior quality firm has demand ν^N and every superior quality firm has demand $\nu^N + \nu^S/\lambda$. Expression (16) thus simply reflects that a firm's profit is the product of its demand and its mark–up.

For what follows, two observations are important. First, a superior firm's mark–up $\overline{p} - \overline{c}$ is smaller than an inferior firm's mark–up $\underline{p} - \underline{c}$, because $\underline{p} \geq \overline{p} - \Delta q^N$ by (15) and $\Delta q^N < \Delta c$ by (4) and (5). Second, firms make strictly positive profits in equilibrium. In fact, an inferior quality firm's mark-up is bounded from below by $\Delta c - \Delta q^N$, because $\underline{p}^* \geq \overline{p}^* - \Delta q^N$ by (15) and $\overline{p}^* \geq \overline{c}$.

Equilibrium existence and uniqueness

In a segmented market equilibrium, firms make the same profits in each segment. Otherwise, firms in the less profitable segment would want to move to the more profitable one. Intuitively, in choosing their business model, firms face the trade-off between selling inferior products to only naive consumers at a large mark-up or selling superior goods to all consumers at a low mark-up. More formally, treating the share of superior quality firms λ as exogenous for the moment, from (15) and (16), we obtain the difference between an inferior and superior quality firm's profits as

$$\Phi(\lambda) \equiv \overline{\pi} - \underline{\pi} = \frac{\nu^S}{\lambda} \cdot (\overline{p}(\lambda) - \overline{c}) - \nu^N \cdot \left(\frac{s}{\lambda} + \Delta c - \Delta q^N\right). \tag{17}$$

In equilibrium, we must have $\Phi(\lambda^*) = 0$. As we show in the proof of Proposition 1, a (unique) segmented market equilibrium indeed exists if $\Phi(\lambda^*) = 0$ has a (unique) solution $\lambda^* \in (-\frac{s}{\underline{\theta}}, 1)$[24]

[24] This ensures $\lambda^* \in (0,1)$ and $\hat{\theta}^*$ in $(\underline{\theta}, \overline{\theta})$, since by Lemma A.1 in the appendix, $\hat{\theta}^*$ is in $(\underline{\theta}, \overline{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Moreover, notice that because $\lambda^* < 1$, a segmented market equilibrium can thus only exist if search costs are sufficiently small and $-\frac{s}{\underline{\theta}} < 1$, which is a standard condition (for consumer search to take place) in the search literature (see e.g. Wolinsky (1986) and Anderson and Renault (1999)).

which satisfies (10). The next proposition establishes that this is the case if search costs are sufficiently small.

Proposition 1 *For sufficiently small search costs, there is a unique segmented market equilibrium. The equilibrium share of superior quality firms λ^* solves $\Phi(\lambda) = 0$, and given λ^*, equilibrium prices $\overline{p}^*$ and $\underline{p}^*$ are pinned down by (15).*

In what follows, we assume that search costs are sufficiently small so that Proposition 1 applies. Below, we present a numerical example which illustrates that a segmented market equilibrium exists for a large and reasonable range of search costs. Such an assumption is common in the search literature to ensure that consumers visit more than one firm in expectation (see Footnote 24). In our setting, sufficiently small search costs guarantee, in addition, that sophisticated consumers refrain from purchasing inferior products and the co-existence of an inferior and superior quality segment in equilibrium.

To see the latter, suppose first that (almost) all firms offer superior quality. Then, as search costs get close to zero, the mark–up for superior quality firms gets arbitrarily close to zero,[25] whereas the mark–up for inferior quality firms is bounded from below by $\Delta c - \Delta q^N$. Therefore, if (almost) all firms offer superior quality, there is a level of search costs below which superior quality firms earn lower profits than inferior quality firms. On the other hand, if very few firms offer superior quality, then the profit of a superior quality firm is extremely large, since the demand from sophisticated consumers is divided up between very few firms. As a consequence, there is an intermediate share of superior quality firms where the profits in the two segments equalize.

The left panel of Figure 1 shows the maximal search cost $\bar{s}$ for which a segmented market equilibrium exists as a function of the fraction of naive consumers for the case of uniformly distributed match-values. Intuitively, if ν^N is very low, existence becomes less likely, because firms have only weak incentives to offer inferior quality, as most consumers are sophisticated. On the other hand, if ν^N is very high, then most firms offer inferior quality, and thus existence becomes less likely, because sophisticated consumers find it too costly to search for superior quality (only). As a benchmark, when there are only sophisticated consumers and only superior quality firms, the maximal search cost for which an equilibrium exists in which consumers visit more than one firm is 1.[26]

[25]Formally, as $s/\lambda \to 0$, equation (9) and the definition of g imply that $\hat{\theta}$ converges to $\overline{\theta}$. It follows that the mark–up $\overline{p} - c$ given by (15) converges to zero, because the hazard rate is unbounded.

[26]Indeed, for $F = U[-1, 1]$, the expected gain from visiting a single additional superior quality firm if the current firm supplies superior quality and the worst match-value is 1. A sophisticated consumer thus rejects the worst offer only if the search cost is below 1.

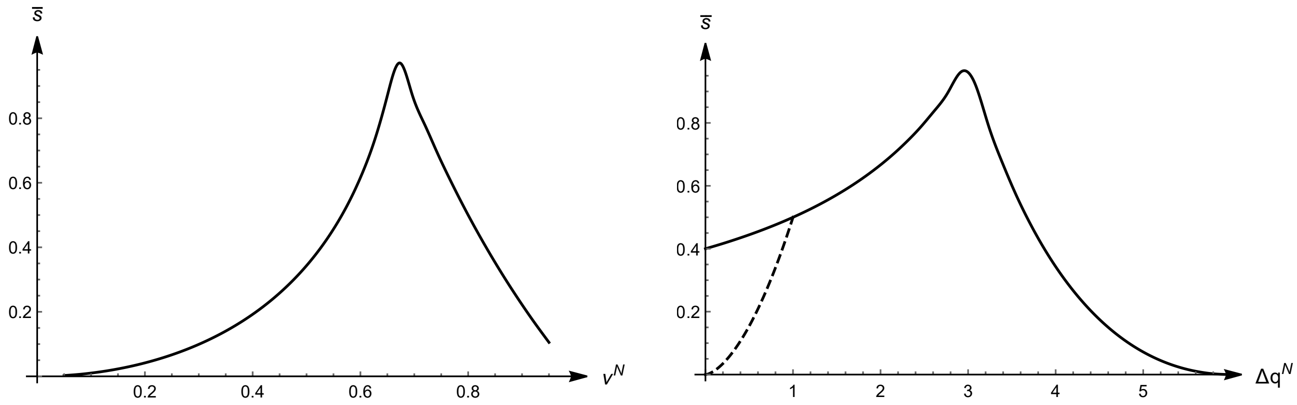

Figure 1: (Solid) The maximal search cost $\bar{s}$ for which a segmented market equilibrium exists as a function of ν^N (left) and Δq^N (right) for uniformly distributed match-values. (Dashed) The maximal search cost for which in equilibrium the superior quality price exceeds the inferior quality one. In both plots $F = U[-1, 1]$, $\Delta q = 8$ and $\Delta c = 6$; in addition, $\Delta q^N = 4$ (left) and $\nu^N = 0.5$ (right).[28]

The right panel of Figure 1 shows the maximal search cost $\bar{s}$ for which a segmented market equilibrium exists as a function of Δq^N. Intuitively, as Δq^N becomes large, existence becomes less likely, because firms have only weak incentives to offer inferior quality, as they cannot sell inferior products at high prices to naive consumers, because these are aware of the large difference in quality between inferior and superior goods. In the example, we set $\Delta c = 6$. Thus, when $\Delta q^N \geq 6$, a segmented market equilibrium fails to exist, because Assumption 4 is violated. In this case, superior quality is efficient from the point of view of both naive and sophisticated consumers so that firms have no incentives to offer inferior one.

Finally, from equation (15), the equilibrium price for superior quality exceeds the price for inferior one if and only if Δq^N exceeds s/λ^*. The dashed line in the right panel of Figure 1 shows the maximal search cost which is consistent with equilibrium existence and satisfies $\Delta q^N \geq s/\lambda^*$. Thus, while the price for superior quality is typically larger than the inferior one, the opposite may be the case when s is large and Δq^N is close to zero.

Remark

Our equilibrium construction is driven by the joint assumption that naive consumers underestimate quality differences and cannot observe product fit (and have therefore less inclination to search). As we argue next, the latter assumption can be somewhat relaxed as long as search incentives remain low. To see this, recall that in equilibrium naive consumers buy both inferior and

[28]For our equilibrium existence, and comparative statics results later on, only the size of s, ν^N, Δq, Δq^N and Δc matters. Without loss of generality, we set $\underline{q} = \underline{q}^N = \underline{c} = 0$ and $\overline{c} = \Delta c$, $\overline{q} = \Delta q$ as well as $\overline{q}^N = \Delta q^N$.

superior quality. Indeed, suppose to the contrary that naive consumers only purchased inferior products. Then, because a naive consumer perceives inferior products as perfect substitutes, he has strict incentives to purchase at a (market-wide) cheapest inferior quality firm, because the best he can hope for by continuing search is to find, again, a cheapest inferior quality firm. Assuming that sophisticated consumers reject inferior products, a cheapest inferior quality firm has hence incentives to raise its price marginally, similarly to the mechanism behind the Diamond (1971) paradox. This logic implies that prices for inferior products adjust so that naive consumers buy superior products as well.

This argument remains valid if naive consumers receive a noisy signal about the firm's true quality or product fit, as long as a naive consumer visiting a (market-wide) cheapest inferior quality firm has a strict incentive to buy even if he has observed the "worst" signal about the firm's product. Intuitively, this is the case if signals are sufficiently noisy, as then the maximum gain from searching for another inferior firm is smaller than the costs to conduct this search. Therefore, our equilibrium construction remains qualitatively robust when naive consumers are able to imperfectly observe product fit.[29]

4 Effects of stronger competition

This section develops the main insight of our paper that stronger competition in the form of lower search costs may have detrimental effects in our setting. A key observation for what follows is that in a segmented market equilibrium, the search costs sophisticated consumers effectively incur depend on the fraction of superior quality firms in the market: If search costs s are one dollar, and among 10 firms there is only a single superior quality firm, a sophisticated consumer has to spend on average 10 dollars to find a superior quality firm, as he wastes 9 dollars visiting inferior quality firms. Thus, the *effective search costs* that a sophisticated consumer faces in equilibrium are the expected search costs

$$\sigma^* \equiv \frac{s}{\lambda^*} \tag{18}$$

[29]Reversely, if naive consumers can observe product fit sufficiently well, then two separate markets for naive and sophisticated consumers emerge: Naive consumer search for a suitable (cheap) inferior product and ignore superior products, as they consider these products as too expensive, whereas sophisticated consumers search for a suitable (expensive) superior product and ignore inferior products. In such a situation, the interaction between the two market segments is rather limited.

to find a superior quality firm. In line with this, observe that superior quality mark–ups depend on search costs only through effective search costs:

$$\overline{p}^* - \overline{c} = \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(g^{-1}(\sigma^*))}. \tag{19}$$

In addition, many of our comparative statics result will be driven by the elasticity of the superior quality mark–up with respect to effective search costs:[30]

$$\epsilon(\sigma) \equiv \frac{\partial(\overline{p}^* - \overline{c})/\partial\sigma}{(\overline{p}^* - \overline{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \text{with } \hat{\theta} = g^{-1}(\sigma). \tag{20}$$

We are now in the position to state the main result of our analysis.

Proposition 2 *(i) As search costs vanish, the share of superior quality firms vanishes:* $\lim_{s \to 0} \lambda^* = 0$. *(ii) The share of superior quality firms increases in s if and only if*

$$\epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c - \Delta q^N}. \tag{C}$$

In a nutshell, the intuition behind part (i) is that vanishing search costs allow consumers to compare prices essentially for free. While this intensifies competition in both segments, the effect is more pronounced in the superior quality segment where, in fact, mark–ups are eliminated. In contrast, an inferior quality firm's mark–up does not vanish and is bounded away from zero.

More precisely, suppose to the contrary that the share of superior quality firms λ^* did not converge to zero, then effective search costs σ^* would tend to zero as s approaches zero, and superior quality firms' mark–ups would erode. Because firms make strictly positive profits in equilibrium, the erosion of mark–ups must be offset by an unbounded increase in the demand of a superior quality firm which, in turn, requires that the share of superior quality firms shrinks to zero (recall that in equilibrium a superior quality firm attracts the demand ν^S/λ^* from sophisticated consumers) – contradicting the hypothesis that it does not.[31]

It is important to note that even though the share of superior firms goes to zero, sophisticated consumers still only buy superior products and split equally among the remaining superior firms. This means that, as search costs go to zero, the equilibrium demand that each of the remaining superior firms derives explodes.

[30] The calculation is provided in Lemma A.4 in the appendix.

[31] We conjecture that our limit result is robust when sophisticated consumers would buy the inferior product in equilibrium with strictly positive but small probability. The reason is that in this case just a small part of an inferior quality firm's demand is elastic and it would still be optimal for it to charge the maximal price that a naive consumer is willing to pay. Hence, its mark-ups would be still bounded away from zero. A full analysis of this case is difficult, however, because a sophisticated consumer's search rule becomes significantly more complicated.

To see shed light on part (ii), note first that condition (C) intuitively means that a 1% increase in effective search costs leads to a sufficiently strong percentage increase in superior quality mark-ups. Because the right hand side of (C) is bounded from above by 1, an intuitive sufficient condition is that a 1% increase in effective search costs leads to a 1% increase in superior quality mark-ups. In Lemma A.6 in the appendix, we show that Condition (C) is met if either $f'(\hat{\theta}^*) \geq 0$ or search costs are sufficiently small.

To see the significance of condition (C), observe that λ^* increases in search costs if, all else equal, profits in the superior quality segment increase by more than those in the inferior quality segment as search costs increase, as this induces more firms to offer superior quality. We now explain that this is the case if and only if condition (C) is met.

The change in profits is driven by two effects. First, as s increases, then all else equal, effective search costs σ increase, and, in turn, superior quality prices increase:

$$\frac{\partial \overline{p}^*}{\partial \sigma} > 0. \tag{21}$$

We refer to $\partial \overline{p}^*/\partial \sigma$ as the "search effect". The search effect is unambiguously positive, because as effective search costs increase, sophisticated consumers become less picky in order to save on search costs, and the critical match-value $\hat{\theta}^*$ goes down (as formally shown in (80) in the appendix). This renders the demand by sophisticated consumers less elastic (as the hazard rate is increasing) which allows superior quality firms to increase prices.[32]

Second, recall that $\underline{p}^* = \overline{p}^* + \sigma^* - \Delta q^N$. Hence, all else equal, an increase in search costs entails, on one hand, an increase in superior and inferior quality prices through the search effect, and, on the other hand, an increase in the price difference $\sigma^* - \Delta q^N$. The first effect benefits superior quality firms relatively more than inferior quality firms, as they derive more demand, whereas the second effect benefits inferior quality firms more.

Intuitively, therefore, superior quality profits increase more than inferior quality profits in response to higher search costs if the search effect is strong in the sense that $\partial \overline{p}^*/\partial \sigma$ is sufficiently large. By definition, the size of the search effect $\partial \overline{p}^*/\partial \sigma$ is proportional to the elasticity of superior quality mark–ups, as

$$\frac{\partial \overline{p}^*}{\partial \sigma} = \epsilon \cdot \frac{\overline{p}^* - \overline{c}}{\sigma^*}. \tag{22}$$

Condition (C) is now the necessary and sufficient condition for superior quality mark–ups, in equilibrium, to be sufficiently elastic so that the search effect is sufficiently strong. Then, in

[32] This feature is analogous to the familiar property that in a search model without naive consumers such as Wolinsky (1986) or Anderson and Renault (1999), prices increase in search costs.

response to an increase in search costs, all prices increase and more firms adopt the business model which generates more demand.

If condition (C) is violated, then the share of superior quality firms decreases in s. In Appendix B, we give a parameterized example to illustrate that this may indeed be the case. The example takes the exponential distribution, which displays a constant hazard rate, and adapts it to our setting with bounded support. Because of the constant hazard rate, $\epsilon(\sigma)$ is zero, and (C) is violated.

Welfare effects

We now study welfare effects of changes in search costs. As is common in behavioral economics models, we take as our welfare measure for naive consumers their true expected utility which is given by

$$U^N \quad = \quad \lambda(\overline{q}-\overline{p}^*)+(1-\lambda)(\underline{q}-\underline{p}^*)-s, \tag{23}$$

since in equilibrium a naive consumer buys from the first firm he visits. While in what follows we mostly focus on changes in welfare, note that a naive consumer's true expected utility may, in fact, be negative if the inferior quality is sufficiently low and $\underline{p}^* > \underline{q}$.[33]

We now argue that an increase in search costs can be socially desirable:

Proposition 3 *Suppose that condition (C) holds. Then total welfare increases in search costs if the efficiency gains from the superior quality, $\Delta q - \Delta c$, are sufficiently large.*

Because prices only affect the division but not the size of total welfare, total welfare is given as the welfare of sophisticated and naive consumers as stated in (7) and (23)[34] net of prices but accounting for the production costs of firms:

$$W \quad = \quad \nu^S \cdot [\overline{q}-\overline{c}+\hat{\theta}^*]+\nu^N \cdot [\lambda^* \cdot (\overline{q}-\overline{c})+(1-\lambda^*)\cdot(\underline{q}-\underline{c})-s]. \tag{24}$$

An increase in search costs affects welfare thus through three channels. First, as we show in Lemma A.5 in the appendix, effective search costs increase in s. Therefore, as argued above, sophisticated consumers end up with a product with a lower match-value ($\hat{\theta}^*$ goes down). Second, the single search that naive consumers conduct becomes costlier. These two effects resemble the welfare effects in standard search models and are unambiguously detrimental to total welfare. Third, however, search costs also affect the quality provision in the market. Under condition

[33] Since a consumer's reservation utility is his (perceived) expected utility, the perceived welfare of a naive consumer is given by (11), which, as a straightforward calculation reveals, is equal to $\underline{q}^N - \underline{p}^*$ in equilibrium. Therefore, the equilibrium comparative statics is the same as for $\underline{p}^*$ which are described in Proposition 4 and 6 below.

[34] A sophisticated consumer's welfare is given by (7), because his reservation utility is his (true) expected utility.

(C), the share of superior quality firms goes up with search costs so that naive consumers, on average, consume the inefficient inferior product less often which in each case increases welfare by $\Delta q - \Delta c$. If the associated efficiency gains are sufficiently large, then an increase in search costs improves total welfare.

The plot in the left panel of Figure 2 shows how an increase in search costs affects total welfare for different levels of Δq for the case of uniformly distributed match-values. From Lemma A.6, the uniform distribution satisfies condition (C), because $f'(\hat{\theta}^*) \geq 0$. Hence, Proposition 3 applies, and, as the plot shows, if Δq (and thus $\Delta q - \Delta c$) is sufficiently large, total welfare increases in s. Note that the changes in Δq correspond to a triplication of the efficiency gains, because $\Delta q - \Delta c$ increases from 1 to 3 as Δq increases from 7 to 9 since $\Delta c = 6$.

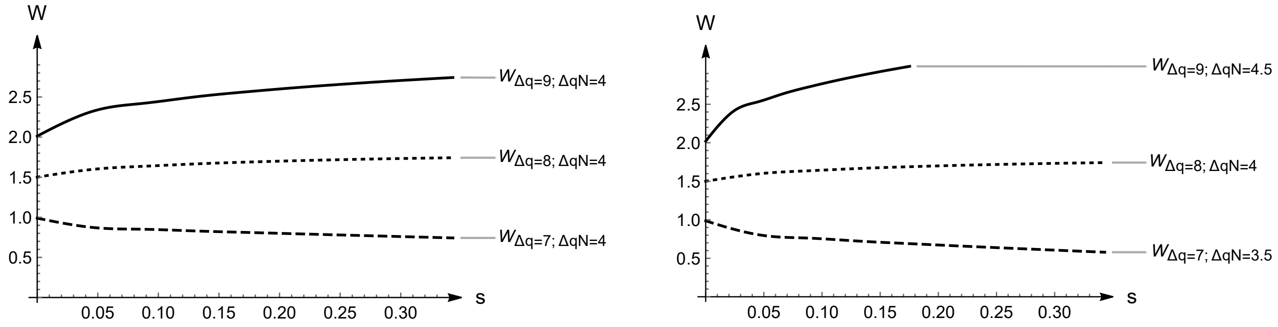

Figure 2: (left) *Total welfare as a function of search costs for different values of* Δq *holding* Δq^N *fixed.* (right) *Total welfare as a function of search costs for different values of* Δq *assuming* $\Delta q^N = 1/2 \cdot \Delta q$. *In both examples, as before,* $F = U[-1, 1]$, $\nu^N = 0.5$ *and* $\Delta c = 6$.

In Proposition 3, we vary the actual quality difference Δq while leaving the perceived quality difference Δq^N fixed. However, it seems plausible that in many situations the perceived quality difference is positively related to the actual quality difference.[35] Formally, Δq^N may be for instance a multiple of Δq. It turns out, however, that in this case it becomes intractable to analytically derive how the welfare response to an increase in s, dW/ds, changes with an increase in Δq. We therefore confine ourselves with illustrating this case in our numerical example. The right panel of Figure 2 shows how an increase in search costs affects total welfare for different levels of Δq, setting $\Delta q^N = 1/2 \cdot \Delta q$. In this case, as Δq increases from 7 to 9, the perceived efficiency gains of inferior quality from the point of view of naive consumers, $\Delta c - \Delta q^N$, drop from 2.5 to 1.5 while the true efficiency gains triple as before. As a result, the range of search costs for which an equilibrium exists shrinks as the plot shows.

The numerically robust feature of our calculations is that dW/ds is approximately flat when $\Delta q - \Delta c = \Delta c - \Delta q^N$, that is, when the true and the perceived efficiency gains are of the same

[35] We thank a referee for pointing this out to us.

size. In the figure, this is the case when $\Delta q = 8$ and $\Delta q^N = 4$. Based on our calculations over a wide range of parameter specifications, we conjecture that dW/ds becomes positive once Δq moves beyond this critical value. Whereas the plots in the right and left panel look qualitatively similar, they suggest that the welfare response to an increase in s is more pronounced if Δq^N is a multiple of Δq.

Firm profits and consumer welfare
We now take a closer look at the division of total welfare and ask how the welfare gains from an increase in search costs are distributed among consumers and firms. To address this question, it is key to understand how equilibrium prices evolve as search costs change, since it is prices which determine the division of welfare. The next proposition states both the effect of higher search costs on equilibrium prices as well as on individual welfare.

Proposition 4 *Suppose condition (C) holds. Then we have:*

(o) Inferior and superior quality prices increase in search costs.

(i) Firm profits increase in search costs.

(ii) Sophisticated consumer welfare decreases in search costs.

(iii) Naive consumer and total consumer welfare increases in search costs if the difference between the true and perceived quality difference, $\Delta q - \Delta q^N$, is sufficiently large.

Parts (i)-(iii) follow straightforwardly from part (o): As to part (i), recall that in equilibrium the demand of an inferior quality firm consists only of naive consumers who visit it first. Hence, its demand is equal to ν^N and thus independent of the share of superior quality firms. Therefore, its profit increases, because its price increases by part (o). As all firms earn the same profit in equilibrium, equilibrium profits increase as search costs increase. Sophisticated consumer welfare (7) unambiguously drops, because a sophisticated consumer suffers from higher superior quality prices and larger effective search costs so that $\hat{\theta}^*$ goes down. A naive consumer benefits from the higher likelihood to encounter a superior quality firm, but suffers from higher market prices, the net effect being positive if Δq is sufficiently large. Formally, naive consumer welfare (11) can be re-written with $\overline{p}^* = \underline{p}^* - s/\lambda^* + \Delta q^N$ as

$$U^N \quad = \quad \underline{q} + \lambda^*(\Delta q - \Delta q^N) - \underline{p}^*. \tag{25}$$

Hence, U^N increases in s if $\Delta q - \Delta q^N$ is sufficiently large, because λ^* increases in s under condition (C). For the same reason, total consumer welfare increase in s when $\Delta q - \Delta q^N$ is sufficiently large, as then the effect on naive consumer welfare outweighs the effect on sophisticated consumer welfare.

While part (o) resembles the price comparative statics in standard search models, we emphasize that condition (C) is critical for it to hold. To understand the intuition, consider first superior quality prices. By (19), the effect of a change in search costs on superior quality prices is given as

$$\frac{d\overline{p}^*}{ds} = \frac{\partial \overline{p}^*}{\partial \sigma}\frac{d\sigma^*}{ds} + \frac{\partial \overline{p}^*}{\partial \lambda}\frac{d\lambda^*}{ds}. \tag{26}$$

Recall that the search effect $\partial \overline{p}^*/\partial \sigma$ is positive by (22) and $d\sigma^*/ds > 0$ by Lemma A.5. This immediately implies that the first term in (26) is positive. The second term in (26) represents a novel effect which is due to the fact that the share of superior quality firms is endogenous in our setting. Recall that a superior quality firm's total demand is composed of the less elastic demand of naive consumers and the more elastic demand of sophisticated consumers. If the number of superior quality firms goes up, sophisticated consumers visit in expectation less firms until they find a satisfying superior product to purchase while naive consumers still buy from the first firm they visit. As a consequence, among those consumers who actually visit a firm, the share of naive consumers increases. Because the demand from naive consumers is less elastic than the demand from sophisticated consumers, prices increase: $\partial \overline{p}^*/\partial \lambda \geq 0$.

In sum, under condition (C), the second term on the right hand side of (26) is positive, and we obtain that superior quality equilibrium prices increase in s. Finally, note that because $\underline{p}^* = \overline{p}^* + \sigma^* - \Delta q^N$ and σ^* increases in s, if superior quality prices increase in s, so do inferior quality prices.

In contrast, when condition (C) is violated, the share of superior quality firms goes down in s, and the second term on the right hand side of (26) is negative. As we show in the example in the appendix, this may imply that superior quality prices actually decrease in search costs.

Proposition 3 suggests that policy interventions that aim at lowering search costs, which benefit sophisticated consumers, may actually harm naive consumers and reduce total welfare, because they might increase the incentives for firms to offer inferior quality. One measure that aims at lowering search costs in practice is the standardization of price and product information so as to make it easier for consumers to evaluate prices and products. The harmonization of price information is a common regulatory tool in the EU for financial services (see the discussion in Piccione and Spiegler (2012)). Search frictions are also reduced by measures that promote e-commerce (such as lowering approval standards to register an online business), certain information disclosure duties imposed on firms which improve consumers' access to product and price information. Information disclosure duties and the standardization of price and product information may, however, also reduce consumer naivete, as they draw the attention of naive consumers on the relevant product characteristics or put them in the position to process the relevant information. The welfare effects resulting from this are described in the next section.

5 Effects of changes in naivete

This section addresses the question how reducing consumer naivete affects (the division of) welfare. As outlined in the introduction, this question has received some attention in the literature, and is relevant to evaluate "transparency policies" such as information or education campaigns (e.g. fostering financial literacy in the context of banking services). In our model, there are two ways to capture a reduction in consumer naivete: either the share of naive consumers, ν^N, falls, or each naive consumer underestimates to a lesser extent the quality difference between inferior and superior products so that $\delta \equiv \Delta q - \Delta q^N > 0$ decreases (as a result of a change in $\overline{q}^N$ or $\underline{q}^N$).

Our next result confirms the intuition that as consumer naivete goes down offering inferior quality becomes less attractive so that the share of inferior quality firms goes down. This is consistent with the existing literature (Gabaix and Laibson (2006), Armstrong and Vickers (2012), Heidhues et al. (2017)). If the inferior product is sufficiently inefficient, this will have the intuitive implication that welfare improves.

Proposition 5 *(i) The share of superior quality firms increases if δ or ν^N decreases.*

(ii) Total welfare increases if δ decreases. Moreover, total welfare increases if ν^N decreases and

(a) a superior product is more efficient than an inferior product irrespective of match values: $\Delta q - \Delta c > \overline{\theta} - \underline{\theta}$, or

(b) the efficiency gains from purchasing a superior product exceed the effective search costs of finding one: $\Delta q - \Delta c > \sigma^*$.

While the argument for why the share of superior quality firms increases as δ decreases is straightforward, it is not obvious that it increases as ν^N decreases. The reason is that apart from a direct effect (more consumers are susceptible to deception), there is an indirect price effect, as the share of naive consumers affects the elasticity of demand of a superior quality firm. As it turns out, however, the direct effect dominates the indirect effect.

As to part (ii), observe first that as δ or ν^N goes down, each naive and each sophisticated consumer generates more surplus: A naive consumer is simply less likely to end up with an inefficient inferior product due to (i). A sophisticated consumer, on the other hand, faces lower effective search costs due to (i) and consequently buys a more suitable product on average ($\hat{\theta}^*$ goes up).

Welfare thus increases as δ goes down. It also increases as ν^N goes down if, in addition, the surplus an additional naive consumer generates is lower than that generated by the sophisticated consumer he replaces. The sufficient conditions stated in parts (a) and (b) of part (ii) ensure that this is the case.

A prominent theme in the literature is how the welfare changes in response to changes in naivete are distributed among sophisticated and naive consumers.[36] A novel negative spillover effect from naive to sophisticated consumers that arises in our framework is that an increase in the number of naive consumers increases the equilibrium effective search costs $\sigma^* = s/\lambda^*$ so that sophisticated consumers have to search longer to find a superior quality firm. This follows from part (i) of Proposition 5. Clearly, consumer welfare also depends on how prices are affected by changes in naivete. While the price effect is, in general, ambiguous, the next proposition provides a sufficient condition so that all prices decrease, and individual welfare increases as Δq^N increases or ν^N decreases. Then, both consumer groups benefit from a policy intervention that reduces consumer naivete.

Proposition 6 *Let $\epsilon(\sigma^*) \geq 1$. Then, as δ increases or ν^N decreases,*

(i) the prices for inferior and superior products drop;

(ii) firm profits decrease;

(iii) sophisticated and naive consumer welfare increase.

Part (ii) and (iii) follow straightforwardly from part (i) and our previous considerations: As Δq^N increases or ν^N decreases, an inferior (and hence a superior) quality firm's profit falls, because it sells inferior products at a lower price to a smaller number of naive consumers. Sophisticated consumer welfare (7) goes up, because a sophisticated consumer benefits from lower superior quality prices by (i) and lower effective search costs as discussed earlier. Finally, naive consumer welfare (25) increases, because a naive consumer is less likely to end up with an inferior product and has to pay less.

The more involved part is (i). By (19), the overall effect of an increase in the share of naive consumers on superior quality prices consists of various effects, and is formally given by

$$\frac{d\overline{p}^*}{d\,\nu^N} \quad = \quad \frac{\partial \overline{p}^*}{\partial\,\nu^N} + \frac{d\overline{p}^*}{d\lambda} \cdot \frac{d\lambda^*}{d\,\nu^N}. \tag{27}$$

[36]In Gabaix and Laibson (2006) and Armstrong and Vickers (2012), sophisticated consumers benefit from being cross-subsidized by naive ones, an effect that is absent in our model. In Heidhues et al. (2017), naive consumers harm sophisticated ones, because their presence pushes up prices and increases deception in the market. In Armstrong and Chen (2009), Kosfeld and Schüwer (2017) and Ispano and Schwardmann (2020) both naive and sophisticated consumer welfare might be non-monotone in the number of naive consumers.

To disentangle this expression, observe first that the direct effect $\partial \overline{p}^*/\partial \nu^N$ is unambiguously positive, because the demand of naive consumers is less elastic than the demand of sophisticated ones. Second, $d\lambda^*/d\nu^N < 0$ by Proposition 5. Therefore, (27) is positive if superior quality prices drop in response to a larger share of superior quality firms, that is, $d\overline{p}^*/d\lambda \leq 0$. Whereas the sign of $d\overline{p}^*/d\lambda$ is, in general, not clear-cut, the condition that $\epsilon(\sigma^*) \geq 1$ turns out to be a sufficient condition for $d\overline{p}^*/d\lambda \leq 0$.[37] Finally, observe that inferior quality prices increase in ν^N if superior quality prices do, because $\underline{p}^* = \overline{p}^* + \sigma^* - \Delta q^N$, and σ^* increases in ν^N, since λ^* decreases in ν^N by Proposition 5. The argument for why inferior and superior quality prices fall as Δq^N increases follows along the same lines with the only difference that there is no direct effect.

6 Add–on price regulation

In this section, we turn to the specific application outlined in the model description where an inferior product is a product with a hidden add-on fee. Many financial service products features such fees, but also expert services come with such fees implicitly, because it is often not disclosed ex ante how many extra services a treatment may require for completion. A widely discussed policy intervention in this context is a cap on, or a ban of, such fees. We now discuss the effects of such an intervention in our setting. Recall that we can interpret Δc as the (maximal) add-on fee that an inferior quality firm charges a consumer ex post. The total loss the consumer incurs is Δq, amounting to social waste $w = \Delta q - \Delta c$ of add-on pricing. Hence, if $w = 0$, the add-on fee is a pure transfer between consumers and firms.

We now consider a regulation that puts a cap on the add-on fee. Formally, this amounts to an increase of the costs to produce inferior quality, because the regulation effectively lowers the margins of inferior quality firms. For simplicity, we assume that the regulation does not change w. An outright ban of the add-on fee, effectively a ban of inferior products, would clearly improve total and consumer welfare. While an add-on fee ban is consistent with some regulatory policies that have been adopted in practice[38], we now consider the effect of marginally capping the (maximal) add-on fee by γ for γ small.

Lemma 2 *For sufficiently small $\gamma > 0$, imposing an add-on fee cap $\overline{\Delta c} = \Delta c - \gamma$ improves total welfare, and improves consumer welfare if $\epsilon(\sigma^*) \geq 1$.*

[37] The underlying reason is that, analogously to the considerations after Proposition 4, an increase in the share of superior quality firms, first, reduces effective search costs, pushing prices down through the search effect, but, second, affects the composition of the demand of superior quality firms, as sophisticated consumers visit in expectation less firms, pushing up prices. The first effect dominates if ϵ is sufficiently large.

[38] For instance, within the EU, consumers are entitled to reimbursement of add-on payments that are not explicitly agreed upon in the initial contract (see Article 22 of the Consumer Rights Directive 2011/83/EU).

Straightforwardly, an add-on fee cap makes charging add-on fees (offering inferior quality) less attractive, as all else equal, it (marginally) reduces the add-on fee a firm can charge naive consumers while not generating any additional demand from sophisticated consumers who still shun it. Hence, the share of superior quality firms increases in response. This improves total welfare, because naive consumers are more likely to purchase the superior (more efficient) quality and effective search costs drop, improving the equilibrium match-value for sophisticated consumers. The same reasoning implies that consumer welfare increases if, in addition, inferior and superior quality prices fall in response to an add-on fee cap and the induced increase in the share of superior quality firms. Similar to Proposition 6, this is the case if $\epsilon(\sigma^*) \geq 1$ holds.[39]

7 Conclusion

In this paper, we integrate consumer naivete in a search market model where firms can offer superior products or inferior products of inefficiently low quality which naive consumer fail to fully recognize. Our main finding is that in the presence of naive consumers, an increase in search frictions may increase firms' incentives to offer superior quality and, as a consequence, may improve total and consumer welfare. The key idea is that the presence of naive consumers prevents competitive forces from working well, because, simply put, producing inferior products which are purchased by naive consumers is a safe profit haven. In fact, it is precisely in seemingly competitive markets with low search frictions where sellers have strong incentives to offer inferior products. Our main results thus suggest that policy interventions that aim at reducing search frictions should be viewed with caution. Instead, we argue that a ban of inferior quality products, minimum quality standards or policy interventions that aim at reducing the share of naive consumers are likely to improve consumer and overall welfare, primarily, because naive consumers are less likely to buy inferior quality, as fewer firms offer it in response.

An interesting avenue for further research is to study firms' incentives to manipulate naive consumers' perceptions in search markets more generally. For example, whereas in our framework, firms design inferior product so that naive consumers misperceive the true utility they supply, firms might also profit if naive consumer misjudge the benefits of continued search. This may provide incentives for a firm to employ sales techniques to manipulate consumers' expectations about search costs, competitors' offers or the probability that its product is still available when the consumer returns.

[39] Our point that a cap on add-on prices can improve consumer welfare is related to Heidhues et al. (2021) who obtain a similar finding in a model where consumers rationally divide their attention between comparing prices across firms ("browsing") and learning the details of few products in depth ("studying").

A Appendix

Lemma A.1 *(i) g is strictly decreasing on $(-\infty, \overline{\theta})$ with $g(-\infty) = \infty$, $g(\underline{\theta}) = -\underline{\theta}$, and $g(\overline{\theta}) = 0$, and $g(z) = 0$ for all $z \geq \overline{\theta}$. Moreover, $g' = -(1-F)$.*
(ii) The inverse of g, denoted by g^{-1}, is well–defined and strictly decreasing on the domain $(0, \infty)$. By convention, we define $g^{-1}(0) = \overline{\theta}$. Note also that $g^{-1}(-\underline{\theta}) = \underline{\theta}$.

Proof of Lemma A.1 To see (i), note that for all $z < \overline{\theta}$, we have $g'(z) = -(1-F(z))$, so that g is strictly decreasing on $(-\infty, \overline{\theta})$. The values of g stated in the lemma follow straightforwardly by plugging in the respective arguments. Part (ii) is an immediate consequence of (i). ■

Proof of Lemma 1 We derive first a firm's profit function. Let $c(\underline{q}) = \underline{c}$ as well as $c(\overline{q}) = \overline{c}$, and

$$\Pi(p,q,c) = \nu^N \cdot (p-c) + \frac{\nu^S}{\lambda} \cdot \frac{1-F\left(\hat{\theta} + (p-\overline{p}) - (q-\overline{q})\right)}{1-F(\hat{\theta})} \cdot (p-c). \tag{28}$$

We show that in a segmented market equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$, the profit of a firm that sets quality q_k and price p_k is then given by

$$\pi(q_k, p_k) = \begin{cases} \Pi(p_k, q_k, c(q_k)) & if \quad p_k \leq \hat{p}(q_k); \\ \Pi(p_k, q_k, c(q_k)) - \nu^N(p_k - c(q_k)) & if \quad p_k > \hat{p}(q_k), \end{cases} \tag{29}$$

where $\hat{p}(q)$ is given by (12).

Indeed, the profit of a firm is given as the product of (a) the mass of consumers who visit the firm in its lifetime, (b) the probability that a visitor actually purchases the good, and (c) the firm's mark–up $(p_k - c)$. We compute (a) and (b) in turn.

As to (a), in a segmented market equilibrium, all naive consumers buy in the first period and leave the market afterwards. Thus, the mass of naive consumers who visit a given firm in its lifetime is equal to ν^N. A sophisticated consumer leaves the market, in a segmented market equilibrium, if he is matched with a superior quality firm, and the match-value exceeds $\hat{\theta}$. This occurs with probability $\lambda \cdot (1 - F(\hat{\theta}))$. Because the mass of sophisticated consumers who visit a given firm in period t is equal to the mass of sophisticated consumers who have not left the market prior to t, this mass is given as $\nu^S \cdot [1 - \lambda(1 - F(\hat{\theta}))]^t$. Hence, the (expected) mass of sophisticated consumers who visit a given firm in its lifetime is equal to

$$\kappa = \nu^S \sum_{t=0}^{\infty} [1 - \lambda(1 - F(\hat{\theta}))]^t = \frac{\nu^S}{\lambda(1 - F(\hat{\theta}))}. \tag{30}$$

As to (b), a visiting sophisticated consumer i buys from firm k if $q_k + \theta_{ik} - p_k \geq \hat{U}^S$. By (7), this is the case if

$$q_k + \theta_{ik} - p_k \geq \overline{q} + \hat{\theta} - \overline{p} \quad \Longleftrightarrow \quad \theta_{ik} \geq \hat{\theta} + (p_k - \overline{p}) - (q_k - \overline{q}). \tag{31}$$

Moreover, all visiting naive consumers buy as long as $p_k \leq \hat{p}(q_k)$. A firm k that sets q_k and $p_k \leq \hat{p}(q_k)$ therefore earns the profit

$$\pi(q_k, p_k) = \nu^N \cdot (p_k - c(q_k)) + \kappa \cdot \left[1 - F\left(\hat{\theta} + (p_k - \overline{p}) - (q_k - \overline{q})\right)\right] \cdot (p_k - c(q_k)). \tag{32}$$

Inserting (30) delivers (29). Finally, if a firm deviates to a price $p_k > \hat{p}(q_k)$, it loses the profits $\nu^N(p_k - c(q_k))$ it would otherwise make from naive consumers.

To show (15), observe that in a segmented market equilibrium, an inferior quality firm derives demand only from naive consumers. Hence, it must charge the maximal price a naive consumer is willing to pay:

$$\underline{p} = \hat{p}(\underline{q}). \tag{33}$$

Inserting this in (12) yields $\underline{p}$ in (15).

As to superior quality prices, by definition (compare (12)), we have $\hat{p}(\overline{q}) = \hat{p}(\underline{q}) + \Delta q^N$. Inserting (33) and (15) yields $\hat{p}(\overline{q}) = \overline{p} - \frac{s}{\lambda}$. Hence, $\overline{p} < \hat{p}(\overline{q})$, and a superior quality firm's profits are given by $\pi(\overline{q}, p) = \Pi(p, \overline{q}, \overline{c})$ for p from an open interval around $\overline{p}$. In particular, $\pi(\overline{q}, p)$ is differentiable at $\overline{p}$ (as $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ by assumption), and the following first order condition is necessary for $\overline{p}$ to be profit maximizing:

$$0 = \frac{\partial \Pi(\overline{p}, \overline{q}, c)}{\partial p} = \nu^N + \frac{\nu^S}{\lambda} - \frac{\nu^S}{\lambda} \cdot \frac{f\left(\hat{\theta}\right)}{1 - F(\hat{\theta})} \cdot (\overline{p} - \overline{c}). \tag{34}$$

Inserting $\hat{\theta} = g^{-1}(\frac{s}{\lambda})$ from (9) and re-arranging terms yields $\overline{p}$ in (15).

The expressions for equilibrium profits in (16) follow immediately by inserting equilibrium prices $\overline{p}$ and $\underline{p}$ and the respective qualities $\overline{q}$ and $\underline{q}$ in (29), taking into account that $\underline{p} \leq \hat{p}(\underline{q})$ and $\overline{p} \leq \hat{p}(\overline{q})$ in a segmented market equilibrium. ■

Lemma A.2 *Let $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ and $\lambda \in (0,1)$ be given, and let*

$$\overline{p} = \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(\hat{\theta})} + c, \quad p_L = (\underline{\theta} - \hat{\theta}) + \overline{p}, \quad p_H = (\overline{\theta} - \hat{\theta}) + \overline{p}. \tag{35}$$

Then the function $\Pi(\cdot, \overline{q}, c)$ as defined in (28) is maximized on the domain (p_L, p_H) by $p^* = \overline{p}$.

Proof of Lemma A.2 Notice first that $p^* = \overline{p}$ is indeed in (p_L, p_H) as $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$. It follows from Lemma 1 that $p^* = \overline{p}$ is a local maximizer. To see that $p^* = \overline{p}$ is a global maximizer, we show that $\Pi(\cdot, \overline{q}, c)$ is quasi-concave on (p_L, p_H). In fact, we show the stronger property that $\Pi(\cdot, \overline{q}, c)$ is log-concave. To see this, let $\tau = \tau(p) = \hat{\theta} + (p - \overline{p})$ and define

$$D(p) \equiv \nu^N + \frac{\nu^S}{\lambda} \cdot \frac{1 - F(\tau)}{1 - F(\hat{\theta})}. \tag{36}$$

Observe that $\Pi(p, \overline{q}, c) = D(p) \cdot (p - c)$, and recall that the product of log-concave functions is log-concave. Because $(p - c)$ is log-concave in p, $\Pi(\cdot, \overline{q}, c)$ is therefore log-concave if D is log-concave. To show this, assume to the contrary that there is a p so that $(\log D)''(p) > 0$. As we show below,

$$(\log D)''(p) > 0 \quad \Rightarrow \quad (\log D)''(p_H) > 0. \tag{37}$$

The right inequality in turn is equivalent to[40]

$$D''(p_H) \cdot D(p_H) - D'(p_H)^2 > 0. \tag{38}$$

With $\alpha \equiv \frac{\nu^S}{\lambda(1 - F(\hat{\theta}))}$, we have

$$D'(p) = -\alpha f(\tau), \quad \text{and} \quad D''(p) = -\alpha f'(\tau). \tag{39}$$

Because $D' < 0$ and $D > 0$, dividing (38) by $D'(p_H) \cdot D(p_H)$ and re-arranging terms yields

$$\frac{f'(\overline{\theta})}{f(\overline{\theta})} < -\frac{\alpha f(\overline{\theta})}{\nu^N + \alpha(1 - F(\overline{\theta}))} = -\frac{\alpha f(\overline{\theta})}{\nu^N} < -\frac{\nu^S}{\nu^N} f(\overline{\theta}), \tag{40}$$

where the final inequality follows from $\alpha > \nu^S$. But this inequality contradicts (14) which establishes that D is log-concave, and hence $\Pi(\cdot, \overline{q}, c)$ is log-concave as desired.

To complete the proof, we have to show (37). By (39) and Footnote 40, we have

$$(\log D)''(p) = \frac{D'(p)}{D(p)} \cdot \left[\frac{f'(\tau)}{f(\tau)} - \frac{D'(p)}{D(p)} \right]. \tag{41}$$

Note that $\frac{D'(p)}{D(p)}$ is negative by (39). Therefore, if the term in the square bracket is decreasing at p whenever $(\log D)''(p)$ is positive, then $(\log D)''(p)$ remains positive once it is positive, and (37) follows. To see that the term in the square brackets is indeed decreasing at p, note that $(\log D)''(p) > 0$ implies that $(\log D)'(p) = \frac{D'(p)}{D(p)}$ is increasing at p, and our assumption that f is log-concave implies that $\frac{f'(\tau)}{f(\tau)}$ is decreasing at p. ∎

Lemma A.3 *There is a segmented market equilibrium outcome $(\lambda^*, \overline{p}^*, \underline{p}^*)$ if and only if there is $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$, $\hat{p}^*(\underline{q}) \in \mathbb{R}$, and $\lambda^* \in (0, 1)$ so that (9), (10), (12), (15) and $\Phi(\lambda^*) = 0$.*

[40] Note that $\frac{d^2}{dp_k^2} \log D = \frac{d}{dp_k}(D'/D) = 1/D^2 \cdot (D''D - D'^2)$.

Proof of Lemma A.3 Necessity ("$\Rightarrow$ ") follows straightforwardly from consumers' and firms' best responses and the equal profit requirement, as developed in the main text. To show sufficiency ("$\Leftarrow$ "), let $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ and $\lambda^* \in (0,1)$ be given and suppose (9), (10), (12), (15) and $\Phi(\lambda^*) = 0$ hold. We first show that consumer behavior is as required in a segmented market equilibrium. By (15), we have $\underline{p}^* = \overline{p}^* + s/\lambda^* - \Delta q^N$ implying (13). Therefore, naive consumers purchase both inferior and superior products, and thus (12) is their optimal search rule. By (10), sophisticated consumers prefer not to purchase inferior products, and thus (9) is their optimal search rule.

Consequently, because consumer behavior conforms with a segmented market equilibrium, a firm's profit function is given by (29). What remains to be shown is that $(\overline{q}, \overline{p}^*)$ and $(\underline{q}, \underline{p}^*)$ with $\overline{p}^*$ and $\underline{p}^*$ as defined in (15) are indeed profit maximizing strategies for firms. Notice that by $\Phi(\lambda^*) = 0$ firms earn equal profits in the two segments so that it suffices to show that $p_k = \overline{p}^*$ is profit maximizing given $\overline{q}$, and $p_k = \underline{p}^*$ is profit maximizing given $\underline{q}$, as in this case, $\Phi(\lambda^*) = 0$ ensures that a deviation to the other segment is not profitable.

We begin with a superior quality firm with $q_k = \overline{q}$. By Lemma 1, we have that

$$\pi(\overline{q}, p_k) \leq \Pi(p_k, \overline{q}, \overline{c}) \quad \text{for all } p_k, \qquad \text{and} \qquad \pi(\overline{q}, \overline{p}^*) = \Pi(\overline{p}^*, \overline{q}, \overline{c}). \tag{42}$$

Hence, Lemma A.2 implies that $\overline{p}^*$ is profit maximizing within the price range $[p_L, p_H]$. Thus, it remains to show that a superior quality firm cannot improve by choosing a price outside of $[p_L, p_H]$.

Consider first a price $p_k < p_L$. At such a price, any sophisticated for any match-value realization strictly prefers purchasing at the current firm k to visiting the next firm. Moreover, any visiting naive consumer buys because $p_k < \overline{p}^* \leq \hat{p}(\overline{q})$ as implied by (13). Thus, the firm's demand would be locally inelastic, and it would be profitable for the firm to slightly increase its price. Hence, $p_k < p_L$ is not optimal.

Consider next a price $p_k > p_H$. At such a price any sophisticated consumer for any match-value realization strictly prefers visiting the next firm to purchasing at the current firm k, and the firm at most sells to naive consumers. So suppose that a naive consumer indeed purchases a superior product at p_k such that the firm makes profits of $\nu^N \cdot (p_k - \overline{c})$. Moreover, notice that if a naive consumer purchases a superior product at p_k, then he also purchases an inferior product at $p_k - \Delta q^N$ since $\Delta q^N < \Delta q$. We infer:

$$\pi(\overline{q}, p_k) = \nu^N \cdot (p_k - \overline{c}) < \nu^N \cdot (p_k - \Delta q^N - \underline{c}) = \pi(\underline{q}, p_k - \Delta q^N) \leq \pi(\underline{q}, \underline{p}^*) = \pi(\overline{q}, \overline{p}^*), \tag{43}$$

where the second inequality follows from $\Delta c > \Delta q^N$, and the third equality follows because at $p_k - \Delta q^N$ an inferior quality firm only derives demand from naive consumers, since a sophisticated consumer, who does not purchase a superior product at p_k, neither purchases an inferior product at $p_k - \Delta q^N$, as $\Delta q > \Delta q^N$. The fourth inequality follows, as $\underline{p}^*$ is optimal given $\underline{q}$ (which we show next), and the final equality follows from $\Phi(\lambda^*) = 0$. This proves that it is optimal for a superior quality firm to charge $\overline{p}^*$.

To see that it is optimal for an inferior quality firm to charge $\underline{p}^*$, consider first a price $p_k > \underline{p}^*$. By (10), an inferior quality firm derives no demand from sophisticated consumers when it charges $\underline{p}^*$ or a larger price $p_k > \underline{p}^*$. Moreover, because $\underline{p}^* = \hat{p}^*(\underline{q})$ by (33), $p_k > \underline{p}^*$ implies $p_k > \hat{p}^*(\underline{q})$, so that the firm neither derives demand from naive consumers. Consequently, a deviation to $p_k > \underline{p}^*$ is not profitable.

Next, consider $p_k < \underline{p}^*$. Observe that because the demand of naive consumers is inelastic up to the price $\hat{p}^*(\underline{q})$ and $\underline{p}^* = \hat{p}^*(\underline{q})$ by (33), setting a price $p_k < \underline{p}^*$ can only be profitable if it generates additional demand from sophisticated consumers, that is, if $p_k < p_H - \Delta q$.[41] Now, at very low prices $p_k < p_L - \Delta q$, also any visiting sophisticated consumer would purchase the firm's product with probability one so that the firm's demand from naive and sophisticated consumers would be perfectly inelastic. Accordingly, it would be profitable to raise prices. Hence, $p_k < p_L - \Delta q$ is not optimal.

Thus, it remains to consider prices $p_k \in [p_L - \Delta q, p_H - \Delta q]$. Because $p_k < \hat{p}^*(\underline{q})$, it holds that

$$\pi(\underline{q}, p_k) = \Pi(p_k, \underline{q}, \underline{c}) \leq \Pi(p_k, \underline{q}, \overline{c} - \Delta q), \tag{44}$$

where the second inequality follows, since $\Pi(p, q, c)$ is decreasing in c and $\Delta q \geq \Delta c$ by (3). From definition (28), we infer that

$$\Pi(p_k, \underline{q}, \overline{c} - \Delta q) = \Pi(p_k + \Delta q, \overline{q}, \overline{c}). \tag{45}$$

Note that $p_k + \Delta q \in [p_L, p_H]$, because $p_k \in [p_L - \Delta q, p_H - \Delta q]$. By Lemma A.2, we thus have

$$\Pi(p_k + \Delta q, \overline{q}, \overline{c}) \leq \Pi(\overline{p}^*, \overline{q}, \overline{c}). \tag{46}$$

With (42) and $\Phi(\lambda^*) = 0$, we conclude that

$$\pi(\underline{q}, p_k) \leq \Pi(\overline{p}^*, \overline{q}, \overline{c}) = \pi(\overline{q}, \overline{p}^*) = \pi(\underline{q}, \underline{p}^*). \tag{47}$$

Hence, a deviation from $\underline{p}^*$ to $p_k \in [p_L - \Delta q, p_H - \Delta q]$ is not profitable for an inferior quality firm. This completes the proof of sufficiency.

Proof of Proposition 1 Observe that to any segmented market equilibrium outcome $(\lambda^*, \overline{p}^*, \underline{p}^*)$ corresponds a unique segmented market equilibrium where the consumers' search strategies are

[41] By definition, p_H is the largest price that a sophisticated is willing to pay for a superior product that displays the best match-value $\overline{\theta}$. Therefore, at $p_k \geq p_H - \Delta q$, an inferior quality firm derives no demand from sophisticated consumers.

pinned down by (9) and (12). Moreover, $\hat{\theta}^*$ and $(\hat{p}^*(\overline{q}), \hat{p}^*(\underline{q}))$ are pinned down by Lemma A.3. The following two claims therefore imply the desired result.

Claim (i): There is a (unique) segmented market equilibrium outcome $(\lambda^*, \overline{p}^*, \underline{p}^*)$ *if and only if* $\Phi(\lambda) = 0$ *has a (unique) solution* $\lambda^* \in (-\frac{s}{\underline{\theta}}, 1)$ *, and (10) is satisfied.*

To see Claim (i), observe that by Lemma A.3, it is sufficient to show that (9), (10), (12), (15) and $\Phi(\lambda^*) = 0$ has a solution $(\hat{\theta}^*, \lambda^*, \hat{p}^*(\underline{q}), \overline{p}^*, \underline{p}^*)$ with $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$, $\hat{p}^*(\underline{q}) \in \mathbb{R}$ and $\lambda^* \in (0,1)$ if and only if $\Phi(\lambda^*) = 0$ with $\lambda^* \in (-\frac{s}{\underline{\theta}}, 1)$ and (10).

Indeed, it follows by Lemma A.1 that $\hat{\theta}^* = g^{-1}(\frac{s}{\lambda^*})$ is a solution to (9) with $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Moreover, (10) and $\Phi(\lambda^*) = 0$ hold by assumption. Finally, the naive consumer's reservation prices and firms prices induced by λ^* are given by (12) and (15).

Claim (ii): If search costs are sufficiently small, then $\Phi(\lambda) = 0$ *has a unique solution* $\lambda^* \in (-\frac{s}{\underline{\theta}}, 1)$*, and the induced prices* $\overline{p}^*$ *and* $\underline{p}^*$ *satisfy (10).*

To prove Claim (ii), we first show that a solution exists. Indeed, an intermediate value argument implies that $\Phi(\lambda) = 0$ has a solution in the range $(-\frac{s}{\underline{\theta}}, 1)$ if

$$\Phi\left(-\frac{s}{\underline{\theta}}\right) > 0 \quad \text{and} \quad \Phi(1) < 0. \tag{48}$$

We show that (48) is satisfied if s is sufficiently small.

To see that the left inequality in (48) is met for small s, insert $\lambda = -\frac{s}{\underline{\theta}}$ into Δ to obtain

$$\Phi\left(-\frac{s}{\underline{\theta}}\right) = \left(-\frac{\nu^S\underline{\theta}}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(-\underline{\theta}))} - \nu^N(-\underline{\theta} + \Delta c - \Delta q^N). \tag{49}$$

Because $\underline{\theta} < 0$, this expression converges to $+\infty$ as $s \to 0$, as desired.

To see that the right inequality in (48) is met for small s, insert $\lambda = 1$ into Δ to obtain

$$\Phi(1) = \frac{1}{h(g^{-1}(s))} - \nu^N(s + \Delta c - \Delta q^N). \tag{50}$$

Recall from (8) that $g(\overline{\theta}) = 0$. Therefore, $\lim_{s\to 0} g^{-1}(s) = \overline{\theta}$, and hence, because $\lim_{\theta\to\overline{\theta}} h(\theta) = \infty$ by assumption, we obtain that $\lim_{s\to 0} \Phi(1) = -\nu^N(\Delta c - \Delta q^N) < 0$, as $\Delta c > \Delta q^N$ by assumption, and this establishes (48).

To show that the solution to $\Phi(\lambda) = 0$ is unique for small s, we show first the auxiliary claim

$$\lim_{s\to 0} \frac{s}{\lambda^*(s)} = 0 \quad \text{for any solution } \lambda^* = \lambda^*(s) \text{ with } \Phi(\lambda^*) = 0. \tag{51}$$

It is clearly sufficient to show that this holds for $\underline{\lambda}^* = \min\{\lambda^* | \Phi(\lambda^*) = 0\}$ (which exists by the continuity of Δ). Indeed, suppose to the contrary that there is a (sub)sequence s_n, $n = 1, 2, \ldots$ with $s_n \to 0$ so that the sequence $s_n/\underline{\lambda}^*(s_n)$ is bounded from below by some $\zeta > 0$, that is, $s_n/\underline{\lambda}^*(s_n) > \zeta$ for all n. This first of all implies that $\lim_{n\to\infty} \underline{\lambda}^*(s_n) = 0$. Moreover, it implies that there is some $\theta_0 < \overline{\theta}$ so that $g^{-1}(\frac{s_n}{\underline{\lambda}^*(s_n)}) < \theta_0$ for all n by Lemma A.1. Together with (17), it thus follows that

$$\lim_{n\to\infty} \Phi(\lambda^*(s_n), s_n) \geq \lim_{n\to\infty} \left\{ \left(\frac{\nu^S}{\lambda^*(s_n)} + \nu^N \right) \cdot \frac{1}{h(\theta_0)} - \nu^N \left(\frac{s_n}{\lambda^*(s_n)} + \Delta c - \Delta q^N \right) \right\} = \infty, \tag{52}$$

a contradiction to the assumption that $\Phi(\lambda^*(s_n), s_n) = 0$ for all n, and this establishes (51).

To complete the proof of uniqueness, we show

$$\frac{\partial \Phi(\lambda^*)}{\partial \lambda} < 0 \quad \text{for any solution } \lambda^* \text{ with } \Phi(\lambda^*) = 0 \text{ and } s \text{ sufficiently small.} \tag{53}$$

This is sufficient for uniqueness by (48). Indeed, to compute $\partial\Delta/\partial\lambda$, observe first that by (15)

$$\begin{aligned}
\frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) &= \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^S} \right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{\partial \hat{\theta}^*}{\partial \lambda} &\quad (54)\\
&= \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^S} \right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{1}{1 - F(\hat{\theta}^*)} \cdot \frac{s}{(\lambda^*)^2}, &\quad (55)
\end{aligned}$$

where in the second line, we have used first that the equilibrium property $g(\hat{\theta}^*) = s/\lambda^*$ implies $\partial\hat{\theta}^*/\partial\lambda^* = -1/g'(\hat{\theta}^*) \cdot (s/(\lambda^*)^2)$, and second that $g' = -(1 - F)$ by Lemma A.1.

Because the hazard rate is increasing, (55) implies $\frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) \leq \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)}$, and hence

$$\begin{aligned}
\frac{\partial \Phi(\lambda^*)}{\partial \lambda} &= \left(-\frac{\nu^S}{(\lambda^*)^2} \right) \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) + \nu^N \cdot \frac{s}{(\lambda^*)^2} &\quad (56)\\
&\leq \left(-\frac{\nu^S}{(\lambda^*)^2} \right) \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} + \nu^N \cdot \frac{s}{(\lambda^*)^2} &\quad (57)\\
&= -\frac{1}{\lambda^*} \Phi(\lambda^*) - \frac{1}{\lambda^*}(\Delta c - \Delta q^N) + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} &\quad (58)\\
&= -\frac{1}{\lambda^*}(\Delta c - \Delta q^N) + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)}, &\quad (59)
\end{aligned}$$

where in the last line, we have used that $\Phi(\lambda^*) = 0$. Now observe that (51) and Lemma A.1 imply that $\hat{\theta}^* = g^{-1}(s/\lambda^*) \to \overline{\theta}$ as $s \to 0$. Since the hazard rate is unbounded, $\lim_{\hat{\theta}^* \to \overline{\theta}} h(\hat{\theta}^*) = \infty$, and expression (59) becomes negative for sufficiently small s, as $\Delta c > \Delta q^N$ by assumption. This establishes (53) and completes the proof of uniqueness.

To complete the proof, we show that for sufficiently small s, the prices $\overline{p}^*$ and $\underline{p}^*$ induced by λ^* satisfy (10). Indeed, (10) is equivalent to

$$\Delta q - \Delta p^* \geq \overline{\theta} - \hat{\theta}^*. \tag{60}$$

From (15), we have $\Delta p^* = \Delta q^N - \frac{s}{\lambda^*}$. Hence, (60) is equivalent to

$$\Delta q - \Delta q^N + \frac{s}{\lambda^*} \geq \overline{\theta} - \hat{\theta}^*. \tag{61}$$

This ineqality is true for small s, because $\Delta q > \Delta q^N$ by assumption and $\lim_{s\to 0} \hat{\theta}^* = \overline{\theta}$ as argued after (59). ■

Lemma A.4 *The elasticity of the superior quality mark–up is given by*

$$\frac{\partial(\overline{p}^*(\sigma) - \overline{c})/\partial\sigma}{(\overline{p}^*(\sigma) - \overline{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \textit{with } \hat{\theta} = g^{-1}(\sigma). \tag{62}$$

Proof of Lemma A.4 Differentiating $\overline{p}^*$ as given in (19) with respect to σ and inserting σ^* yields

$$\frac{\frac{\partial}{\partial\sigma}(\overline{p}^* - \overline{c})}{\overline{p}^* - \overline{c}} = -\frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\partial g^{-1}(\sigma^*)}{\partial\sigma}. \tag{63}$$

Observe that $\partial g^{-1}(\sigma^*)/\partial\sigma = 1/g'(g^{-1}(\sigma^*))$. Together with $g' = -(1 - F)$ by Lemma A.1, we obtain:

$$\frac{\frac{\partial}{\partial\sigma}(\overline{p}^* - \overline{c})}{\overline{p}^* - \overline{c}} \cdot \sigma^* = \frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\sigma^*}{1 - F(g^{-1}(\sigma^*))} \tag{64}$$

The desired result follows then from $\hat{\theta}^* = g^{-1}(\sigma^*)$ by (9). ■

Proof of Proposition 2 As to (i), contrary to the claim suppose that there is a (sub)sequence s_n, $n = 1, 2, \ldots$ with $s_n \to 0$ so that $\lambda^*(s_n)$ is bounded from below by some $\underline{\lambda}$, that is, $\lambda^*(s_n) > \underline{\lambda}$ for all n. Then, $\lim_{n\to\infty} s_n/\lambda^*(s_n) = 0$ holds, and Lemma A.1 implies that $\lim_{n\to\infty} g^{-1}(\frac{s_n}{\lambda^*(s_n)}) = \overline{\theta}$. We can then infer that

$$\lim_{n\to\infty} \Phi(\lambda^*(s_n), s_n) < \left(\frac{\nu^S}{\underline{\lambda}} + \nu^N\right) \cdot \lim_{\theta\to\overline{\theta}} \frac{1}{h(\theta)} - \nu^N \cdot (\Delta c - \Delta q^N) < 0, \tag{65}$$

where the last inequality follows because the hazard rate diverges as θ approaches $\overline{\theta}$ and $\Delta c - \Delta q^N > 0$ by (4). This contradicts the equilibrium condition that $\Phi(\lambda^*(s_n), s_n) = 0$ for all n.

As to (ii), by the equilibrium property $\Phi(\lambda^*, s) = 0$, we have that $d\lambda^*/ds = -(\partial\Phi(\lambda^*)/\partial s)/(\partial\Phi(\lambda^*)/\partial\lambda)$. Because $\partial\Phi(\lambda^*)/\partial\lambda < 0$ by (53), it suffices to show that

$$\frac{\partial\Phi(\lambda^*)}{\partial s} \geq 0 \quad \Leftrightarrow \quad \epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c - \Delta q^N}. \tag{66}$$

Inserting (19) in (17) and differentiating with respect to s yields

$$\frac{\partial\Phi(\lambda^*)}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\overline{p}^*}{\partial\sigma} \frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}, \tag{67}$$

which by the definition of ϵ in (20) is equivalent to

$$\frac{\partial \Phi(\lambda^*)}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot (\overline{p}^* - \overline{c}) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*}. \tag{68}$$

By (17), $\Phi(\lambda^*) = 0$ is equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot (\overline{p}^* - \overline{c}) = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c - \Delta q^N\right), \tag{69}$$

and (68) becomes

$$\frac{\partial \Phi(\lambda^*)}{\partial s} = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c - \Delta q^N\right) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*} = \frac{\nu^N}{s} \cdot \left[\left(\frac{s}{\lambda^*} + \Delta c - \Delta q^N\right) \cdot \epsilon(\sigma^*) - \frac{s}{\lambda^*}\right]. \tag{70}$$

Finally, (70) implies (66), which completes the proof. ■

Lemma A.5 *We have:* $d\sigma^*/ds \geq 0$ *and* $\lim_{s\to 0} \sigma^* = 0$.

Proof of Lemma A.5 We establish $\lim_{s\to 0} \sigma^* = 0$ in (51) in the proof of Proposition 1. To show $d\sigma^*/ds \geq 0$, recall that $\sigma = s/\lambda$, and define the function

$$\tilde{\Phi}(\sigma) = \Phi\left(\frac{s}{\sigma}\right) = \left(\nu^S \cdot \frac{\sigma}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(\sigma))} - \nu^N(\sigma + \Delta c - \Delta q^N). \tag{71}$$

The equilibrium condition $\Phi(\lambda^*) = 0$ becomes $\tilde{\Phi}(\sigma^*) = 0$, and hence

$$\frac{d\sigma^*}{ds} = -\frac{\partial \tilde{\Phi}(\sigma^*)/\partial s}{\partial \tilde{\Phi}(\sigma^*)/\partial \sigma}. \tag{72}$$

Differentiating (71) yields that

$$\frac{\partial \tilde{\Phi}(\sigma^*)}{\partial \sigma} = \frac{\partial \Phi(\lambda^*)}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma} = \frac{\partial \Phi(\lambda^*)}{\partial \lambda} \left(-\frac{s}{\sigma^*}\right), \tag{73}$$

and $\partial \tilde{\Phi}(\sigma^*)/\partial \sigma$ is thus positive, because $\partial \Phi(\lambda^*)/\partial \lambda < 0$ by (53). Moreover, by (71):

$$\frac{\partial \tilde{\Phi}(\sigma^*)}{\partial s} = -\nu^S \frac{\sigma^*}{s^2} \cdot \frac{1}{h(g^{-1}(\sigma^*))}. \tag{74}$$

Because this is negative, it follows that $d\sigma^*/ds$ is positive, as desired. ■

Lemma A.6 *Condition (C) holds if:*
(i) $f'(\hat{\theta}^*) \geq 0$*, or*
(ii) search costs are sufficiently small.

Proof of Lemma A.6 By (66), it suffices to show that $\partial \Phi(\lambda^*)/\partial s \geq 0$ if (i) $f'(\hat{\theta}^*) \geq 0$, or (ii) search costs are sufficiently small.

As to (i), notice first that a straightforward calculation yields that

$$\frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)} = \frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*) \tag{75}$$

and $\epsilon(\sigma^*)$ in (20) becomes

$$\epsilon(\sigma^*) = \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*)\right) \cdot \frac{g(\hat{\theta}^*)}{1 - F(\hat{\theta}^*)} \tag{76}$$

with $\hat{\theta}^* = g^{-1}(\sigma^*)$. Inserting this and $\overline{p}^*$ from (19) in (68) yields

$$\frac{\partial \Phi(\lambda^*)}{\partial s} = \left(\frac{\nu^S}{\lambda^*} + \nu^N\right)\left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1\right)\frac{1}{\lambda^* \cdot (1 - F(\hat{\theta}^*))} - \frac{\nu^N}{\lambda^*}, \tag{77}$$

where we used $g(\hat{\theta}^*)/s = g(g^{-1}(\sigma^*))/s = 1/\lambda^*$. Observe that, as desired, this expression is positive if $f'(\hat{\theta}^*) \geq 0$, because $1 - F(\hat{\theta}^*) < 1$.

As to (ii), note that it suffices to show that (77) is positive as search costs get small. By Lemma A.5, $\sigma^* \to 0$ as $s \to 0$, so that $\hat{\theta}^* \to \overline{\theta}$ as $s \to 0$ by Lemma A.1. We hence find that

$$\lim_{s \to 0}\left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1\right)\frac{1}{(1 - F(\hat{\theta}^*))} = \lim_{\hat{\theta}^* \to \overline{\theta}}\left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1\right)\frac{1}{(1 - F(\hat{\theta}^*))}. \tag{78}$$

Notice that (77) is positive as search costs get small if (78) diverges. The reason why (78) indeed diverges is that, on the one hand, $f'(\hat{\theta}^*)/f(\hat{\theta}^*)$ is bounded from below,[42] while the hazard rate diverges by assumption, and on the other hand, $1 - F(\hat{\theta}^*)$ tends to zero as $\hat{\theta}^* \to \overline{\theta}$. ■

Proof of Proposition 3 By (24),

$$\frac{dW}{ds} = \nu^S \cdot \frac{d\hat{\theta}^*}{ds} + \nu^N \cdot \frac{d\lambda^*}{ds} \cdot (\Delta q - \Delta c) - \nu^N. \tag{79}$$

Hence, if Condition (C) holds, so that $d\lambda^*/ds > 0$ by Proposition 2, then (79) is positive if $\Delta q - \Delta c \geq 0$ is sufficiently large. ■

Proof of Proposition 4 Most arguments are given in the text. What remains to be shown is (a) $\hat{\theta}^*$ decreases in s, (b) $\partial \overline{p}^*/\partial \lambda > 0$ and (c) total consumer welfare increases in s if $\Delta q - \Delta q^N$ is sufficiently large and condition (C) holds. To show (a), by (9), we have $g(\hat{\theta}^*) = \sigma^*$, and hence

$$\frac{d\hat{\theta}^*}{ds} = \frac{d\sigma^*}{ds}\frac{1}{g'(\hat{\theta}^*)}, \tag{80}$$

which is negative, because $g' = -(1 - F)$ by Lemma A.1 and $d\sigma^*/ds \geq 0$ by Lemma A.5. To show (b), note that by (19), we have $\partial \overline{p}^*/\partial \lambda = \nu^N/(\nu^S \cdot h(g^{-1}(\sigma^*))) > 0$. To show (c), total consumer

[42] By log-concavity of f, f'/f is decreasing and $f'(\overline{\theta})/f(\overline{\theta})$ is bounded from below by (14) and $f(\overline{\theta}) > 0$.

welfare is given as $W^C = \nu^S \cdot U^S + \nu^N \cdot U^N$. Inserting (7) and (25), and re-arranging terms using (15) yields

$$W^C \;=\; \nu^S \cdot g^{-1}\left(\frac{s}{\lambda^*}\right) + \overline{q} - \overline{p}^* - \nu^N \cdot \left((1-\lambda^*)\cdot(\Delta q - \Delta q^N) + \frac{s}{\lambda^*}\right). \tag{81}$$

Hence, W^C increases in s if $\Delta q - \Delta q^N$ is sufficiently large, because λ^* increases in s under condition (C). ■

Proof of Proposition 5 As to (i). Because $d\lambda^*/d\nu^N = -(\partial\Phi(\lambda^*)/\partial\nu^N)/(\partial\Phi(\lambda^*)/\partial\lambda)$ and $\partial\Phi(\lambda^*)/\partial\lambda < 0$ by (53), it is sufficient to show that

$$\frac{\partial\Phi(\lambda^*)}{\partial\nu^N} < 0. \tag{82}$$

Recall from (17) that $\Phi = \nu^S/\lambda^* \cdot (\overline{p}^*(\lambda^*) - \overline{c}) - \nu^N \cdot (s/\lambda^* + \Delta c - \Delta q^N)$, and $\nu^S = 1 - \nu^N$. Hence,

$$\begin{aligned} \frac{\partial\Phi(\lambda^*)}{\partial\nu^N} &= -\frac{1}{\lambda^*}\cdot(\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*}\cdot\frac{\partial\overline{p}^*}{\partial\nu^N} - \left(\frac{s}{\lambda^*} + \Delta c - \Delta q^N\right) &(83)\\ &= -\frac{1}{\lambda^*}\cdot\frac{1}{\nu^N}\cdot(\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*}\cdot\frac{\partial\overline{p}^*}{\partial\nu^N}, &(84)\end{aligned}$$

where we have inserted the equilibrium condition $\Phi(\lambda^*) = 0$ in the first line to obtain the second line. Recall from (19) that

$$\overline{p}^* - \overline{c} = \left(1 + \lambda^*\frac{\nu^N}{\nu^S}\right)\cdot\frac{1}{h(\hat{\theta}^*)}, \tag{85}$$

and observe that

$$\frac{\partial\overline{p}^*}{\partial\nu^N} = \frac{\lambda^*}{(1-\nu^N)^2}\cdot\frac{1}{h(\hat{\theta}^*)}. \tag{86}$$

After inserting (85) and (86) in (84) and simplifying terms, we obtain

$$\begin{aligned} \frac{\partial\Phi(\lambda^*)}{\partial\nu^N} &= -\frac{}{\lambda^* h(\hat{\theta}^*)}\left(\frac{1}{\nu^N} + \lambda^*\frac{1}{\nu^S} - \lambda^*\frac{\nu^N}{(1-\nu^N)^2}\right) &(87)\\ &= -\frac{1}{\lambda^* h(\hat{\theta}^*)\nu^N}, &(88)\end{aligned}$$

which establishes (82).

Analogously, $d\lambda^*/d\delta < 0$ follows from $\partial\Phi(\lambda^*)/\partial\delta < 0$ which, in turn, holds because $\partial\Phi(\lambda^*)/\partial\delta = -\nu^N$ by (17).

As to (ii). By (24), a straightforward calculation delivers that

$$\begin{aligned} \frac{dW}{d\nu^N} &= -(\overline{q} - \overline{c} + \hat{\theta}^*) + \nu^S\cdot\frac{d\hat{\theta}^*}{d\nu^N} + \lambda^*\cdot(\Delta q - \Delta c) + \underline{q} - \underline{c} - s + \nu^N\cdot\frac{d\lambda^*}{d\nu^N}\cdot(\Delta q - \Delta c) &(89)\\ &\leq -(1-\lambda^*)\cdot(\Delta q - \Delta c) - \hat{\theta}^* - s, &(90)\end{aligned}$$

where the inequality follows from re–arranging terms, the fact that $d\lambda^*/d\nu^N \leq 0$ by part (i), and because $d\hat{\theta}^*/d\nu^N \leq 0$, which in turn follows from

$$\frac{d\theta^*}{d\nu^N} = \frac{\partial\theta^*}{\partial\lambda}\cdot\frac{d\lambda^*}{d\nu^N} = -\frac{s}{(\lambda^*)^2\cdot g'(\hat{\theta}^*)}\cdot\frac{d\lambda^*}{d\nu^N} \leq 0, \tag{91}$$

because $d\lambda^*/d\nu^N \leq 0$ by part (i) and $g' = -(1-F)$ by Lemma A.1.

Now, from the optimality of the consumer's search rule, $U^S = \overline{q} - \overline{p}^* + \hat{\theta}^*$ is larger than the expected utility that he would obtain if he purchased the first superior product that he encounters, which is $\overline{q} - \overline{p}^* + \mathbb{E}(\theta) - \sigma^*$. Because $\mathbb{E}(\theta) = 0$, this implies that $\sigma^* \geq -\hat{\theta}^*$. Inserting this in (90) yields (recall $s = \sigma^*\cdot\lambda^*$)

$$\frac{dW}{d\nu^N} \leq -(1-\lambda^*)\cdot(\Delta q - \Delta c + \hat{\theta}^*), \tag{92}$$

and $\Delta q - \Delta c \geq -\hat{\theta}^*$ thus implies $dW/d\nu^N < 0$.

Now, observe that this condition is implied by the conditions (a) and (b) in the statement of the proposition. Indeed, (a) $\Delta q - \Delta c \geq \overline{\theta} - \underline{\theta}$ implies $\Delta q - \Delta c \geq -\hat{\theta}^*$, because $\hat{\theta}^* \geq \underline{\theta}$ and $\overline{\theta} > 0$, and (b) $\Delta q - \Delta c \geq \sigma^*$ implies $\Delta q - \Delta c \geq -\hat{\theta}^*$, because $\sigma^* \geq -\hat{\theta}^*$ as argued above.

To show $dW/d\delta < 0$, observe that $dW/d\delta = -dW/d\Delta q^N$, as we assume that the change of δ is caused by a change in $\overline{q}^N$ or $\underline{q}^N$ (and not $\overline{q}$ or $\underline{q}$). From (24), we have $dW/d\Delta q^N = (dW/d\lambda)\cdot(d\lambda^*)/d\Delta q^N)$ and

$$\frac{dW}{d\lambda} = \nu^S\frac{d\theta^*}{d\lambda} + \Delta q - \Delta c,$$

which is strictly positive, because θ^* increases in λ as shown after equation (55) and $\Delta q - \Delta c > 0$ by assumption. The desired result then follows, as $d\lambda^*/d\Delta q^N = -d\lambda^*/d\delta > 0$ by part (i). ■

Proof of Proposition 6 As argued in the text, it suffices to show that

$$\frac{d\overline{p}^*}{d\lambda} \leq 0 \quad \text{if} \quad \epsilon(\sigma^*) \geq 1. \tag{93}$$

Too see this, observe that (19), (20) and a straightforward calculation yield that

$$\frac{d\overline{p}^*}{d\lambda} = \frac{\partial\overline{p}^*}{\partial\lambda} + \frac{\partial\overline{p}^*}{\partial\sigma}\cdot\frac{\partial\sigma^*}{\partial\lambda} = \frac{\nu^N}{\nu^S}\cdot\frac{1}{h(\hat{\theta}^*)} + \epsilon(\sigma^*)\cdot\frac{(\overline{p}^* - \overline{c})}{\sigma^*}\cdot(-\frac{s}{(\lambda^*)^2}). \tag{94}$$

Inserting (19) and simplifying terms yields that

$$\frac{d\overline{p}^*}{d\lambda} = \left(\frac{\nu^N}{\nu^S} - \epsilon(\sigma^*)\cdot(\frac{1}{\lambda^*} + \frac{\nu^N}{\nu^S})\right)\cdot\frac{1}{h(\hat{\theta}^*)}, \tag{95}$$

which establishes (93), as desired. ■

Proof of Lemma 2 As to total welfare, as argued in the text, it suffices to show that the share of superior quality firms increases in response to an add-on fee cap. To see this, observe that by keeping w fixed, a reduction of Δc by γ (as a result of an increase in $\underline{c}$) lowers Δq by γ. But since the difference in inferior and superior quality profits as given in (17) is independent of Δq, the marginal effect of an add-on fee cap on the share of superior quality firms is given by $d\lambda^*/d\underline{c} = -(\partial\Phi(\lambda^*)/\partial\underline{c})/(\partial\Phi(\lambda^*)/\partial\lambda)$. Because $\partial\Phi(\lambda^*)/\partial\lambda < 0$ by (53),

$$\frac{\partial\Phi(\lambda^*)}{\partial\underline{c}} \quad = \quad \nu^N \tag{96}$$

implies $d\lambda^*/d\underline{c} \geq 0$, as desired.

As to consumer welfare, as is argued in the text, if suffices to show that $d\overline{p}^*/d\lambda \leq 0$ if $\epsilon(\sigma^*) \geq 1$. But this follows immediately from (93). ■

B Appendix

We now present an example in which condition (C) is violated. By Proposition 2, this implies that $\frac{d\lambda^*}{ds} < 0$. Moreover, the example has the property that $\frac{dp^*}{ds} < 0$. Let $[\underline{\theta}, \overline{\theta}] = [-1, 1]$, and define

$$f^E(\theta) = \begin{cases} e^{-(\theta+1)} & \theta < 0 \\ e^{-1} & \theta \geq 0. \end{cases} \tag{97}$$

Claim *Let $f = f^E$. Then there is an open set of parameters $s, \overline{q}, \underline{q}, \overline{q}^N, \underline{q}^N, \overline{c}, \underline{c}, \nu^N$ so that a segmented market equilibrium exists with $\hat{\theta}^* < 0$, $\Delta c - \Delta q^N - 1 > 0$ and*

$$\lambda^* = \frac{\frac{\nu^S}{\nu^N} - s}{\Delta c - \Delta q^N - 1} \quad \text{and} \quad \overline{p}^* - \overline{c} = \frac{\Delta c - \Delta q^N - s\frac{\nu^N}{\nu^s}}{\Delta c - \Delta q^N - 1}. \tag{98}$$

In particular, $\frac{d\lambda^*}{ds} < 0$ and $\frac{dp^*}{ds} < 0$.

To show the claim, notice first that $\frac{d\lambda^*}{ds} < 0$ and $\frac{dp^*}{ds} < 0$ are immediate from (98). Next, note that the cumulative distribution function of f^E is

$$F^E(\theta) = \begin{cases} 1 - e^{-(\theta+1)} & \theta < 0 \\ 1 + e^{-1} \cdot (\theta - 1) & \theta \geq 0 \end{cases} \tag{99}$$

and satisfies $h^E(\theta) = 1$ for $\theta \leq 0$. Moreover, the corresponding function g^E as defined by (8) is given by[43]

$$g^E(z) = \begin{cases} \frac{1}{2e} \cdot (2e^{-z} - 1) & z < 0 \\ \frac{1}{2e} \cdot (1 - z)^2 & z \geq 0. \end{cases} \tag{103}$$

[43]To see this, note that (8) implies for $z < 0$:

$$g^E(z) = \int_z^0 (\theta - z)\, e^{-(\theta+1)}\, d\theta + \int_0^1 (\theta - z)\, e^{-1}\, d\theta. \tag{100}$$

We now show that provided $\hat{\theta}^* < 0$, the formulae in (98) satisfies $\overline{p}^* = \overline{p}(\lambda^*)$ as given by (15) and the equilibrium condition $\Phi(\lambda^*) = 0$. In fact, $\hat{\theta}^* < 0$ implies $h^E(\hat{\theta}^*) = 1$, and thus $\Phi(\lambda^*) = 0$ becomes equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) - \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c - \Delta q^N\right) = 0. \tag{104}$$

Rearranging terms then yields the desired expression for λ^*. Inserting $h^E(\theta^*) = 1$ and λ^* from (98) into $\overline{p}^*$ from (15) yields

$$\overline{p}^* - \overline{c} = 1 + \frac{1 - s\frac{\nu^N}{\nu^S}}{\Delta c - \Delta q^N - 1}, \tag{105}$$

which is equivalent to the expression in (98).

From Claim (i) in the proof of Proposition 1[44], it follows then that there is a (unique) segmented market equilibrium with λ^* and $\overline{p}^*$ as given in (98) if there are parameters so that for λ^* as given in (98), we have that (a) $\lambda^* \in (\frac{s}{\underline{\theta}}, 1)$, (b) $\hat{\theta}^* = (g^E)^{-1}(s/\lambda^*)$ indeed satisfies $\hat{\theta}^* < 0$, and (c) condition (10) holds. Moreover, because the equilibrium values are continuous in the parameters, if we perturb the parameters slightly, this does not upset the (strict) inequalities $\lambda^* \in (\frac{s}{\underline{\theta}}, 1)$, $\hat{\theta}^* < 0$, and (10).

To see (a)–(c), let $\nu^S/\nu^N = 1/e$, $s = 1/(2e)$, $\Delta c = 1+2/e$ and $\Delta q^N = 1/e$. By a straightforward but tedious calculation, we obtain

$$\lambda^* = \frac{1}{2}, \quad \hat{\theta}^* = -\ln(3/2), \quad \overline{p}^* = \underline{p}^* = \frac{e}{2} + 1. \tag{106}$$

Clearly, $\lambda^* \in (\frac{s}{\underline{\theta}}, 1)$ and $\hat{\theta}^* < 0$. Moreover, (10) is met if we let $\Delta q > 2$, because $\overline{\theta} - \hat{\theta}^* < 2$. ■

Integration by parts yields

$$\int_z^0 (\theta - z)\, e^{-(\theta+1)}\, d\theta \quad = \quad -(\theta - z)\, e^{-(\theta+1)}\Big|_z^0 + \int_z^0 e^{-(\theta+1)}\, d\theta = (z-1)\, e^{-1} + e^{-(z+1)}. \tag{101}$$

Moreover, $\int_0^1 (\theta - z)\, e^{-1}\, d\theta = e^{-1} \cdot (\frac{1}{2} - z)$. Inserting this in (100) yields the first line in (103.)

For $z \geq 0$, g^E corresponds to the second line in (103), because by (8):

$$g^E(z) = \int_z^1 (\theta - z)\, e^{-1}\, d\theta = e^{-1} \cdot (\frac{1}{2}\theta^2 - z\theta)\Big|_z^1 = e^{-1} \cdot (\frac{1}{2} - z + \frac{1}{2}z^2) = \frac{1}{2e} \cdot (1-z)^2. \tag{102}$$

[44] Strictly speaking, it is not a priori clear that with the density f^E the equilibrium conditions (9), (10), (12), (15) and $\Phi(\lambda^*) \geq 0$ are actually sufficient for equilibrium existence. The reason is that f^E is not globally log-concave (which we use in our sufficiency proof). However, for the specific case that $f = f^E$, the sufficiency proof still goes through with minor changes. We omit the details.

C Online Appendix

Lemma C.1 *Let $\Delta q > \Delta c$. For sufficiently small search costs, there is a unique symmetric equilibrium and this equilibrium is a segmented market equilibrium.*

Proof of Lemma C.1 From Proposition 1, for sufficiently small search costs, there is a unique segmented market equilibrium. It thus suffices to show that for sufficiently small search costs, any symmetric equilibrium is a segmented market equilibrium. To show this, suppose that there is a symmetric equilibrium in which each firm offers $(\overline{q}, \overline{p}^*)$ with probability $\lambda^* \in [0,1]$ and $(\underline{q}, \underline{p}^*)$ with probability $1-\lambda^*$. As before, let $\hat{U}^S$ denote a sophisticated consumer's reservation utility and $\hat{p}(q)$ denote a naive consumer's reservation price. Moreover, as before, let $\hat{\theta} = \hat{U}^S - \overline{q} + \overline{p}^*$ with $\hat{\theta} \in \mathbb{R}$, meaning that a sophisticated consumer buys a superior product in equilibrium if $\theta \geq \hat{\theta}$. Analogously, let $\hat{\theta}_L \equiv \hat{U}^S - \underline{q} + \underline{p}^*$ with $\hat{\theta}_L \in \mathbb{R}$, meaning that a sophisticated consumer buys an inferior product in equilibrium if $\theta \geq \hat{\theta}_L$. Let $\delta \equiv \Delta q - \Delta c > 0$. The proof consists of three steps.

Our first step is to show that for sufficiently small s, there are no equilibria with $\lambda^* = 1$. To show this, notice that an equilibrium with $\lambda^* = 1$ is a generalization of a segmented market equilibrium in which firms offer superior products with probability one in the sense that the equilibrium is also characterized by the equation system (9), (10), (12), (15) and $\Phi(\lambda^*) \geq 0$ from Lemma A.3 with the difference that $\Phi(\lambda^*) = 0$ is replaced by $\Phi(\lambda^*) \geq 0$ because the profits in the superior quality segment must exceed those in the (non-existing) inferior quality segment.[45] From this, it follows that a necessary conditions for the existence of an equilibrium with $\lambda^* = 1$ is that $\Phi(1) \geq 0$ where $\Phi = \overline{\pi} - \underline{\pi}$ from (17) is the difference between inferior and superior quality profits.

Now, for sufficiently small s, from Proposition 1, there is a unique segmented market equilibrium, meaning that there is a unique $\lambda^* \in (0,1)$ such that $\Phi(\lambda^*) = 0$. As we show in (53),

$$\frac{\partial \Phi(\lambda^*)}{\partial \lambda} < 0 \quad \text{for any solution } \lambda^* \text{ with } \Phi(\lambda^*) = 0 \text{ and } s \text{ sufficiently small.} \tag{107}$$

Therefore, $\Phi(\lambda) < 0$ for any $\lambda > \lambda^*$, and, in particular, $\Phi(1) < 0$. We infer that an equilibrium with $\lambda^* = 1$ does not exist for sufficiently small search costs.

Our second step is to show that for sufficiently small s, we have $\hat{\theta}_L \geq \overline{\theta}$, meaning that sophisticated consumers do not buy low quality products. To show this, notice first that for sufficiently small s, from the first step, we have $\lambda^* < 1$, meaning that firms offer inferior products so that $\underline{p}^*$ and $\hat{\theta}_L$ are well-defined. From the definition of $\hat{\theta}_L$, we have

$$\hat{\theta}_L = \hat{U}^S - \overline{q} + \overline{c} + \Delta q - \Delta c + (\underline{p}^* - \underline{c}). \tag{108}$$

[45] The optimal price for a firm that deviates and offers inferior quality is given by (15). The inequality $\Phi(\lambda^*) \geq 0$ then reflects that a deviation to inferior quality and $\underline{p}$ from (15) must not be profitable.

From $\delta = \Delta q - \Delta c$ and the fact that $\underline{p}^* - \underline{c} \geq 0$, because an inferior quality firm's mark-up is positive in equilibrium, it follows that $\hat{\underline{\theta}}_L \geq \overline{\theta}$ for sufficiently small s if

$$\hat{U}^S \geq \overline{q} - \overline{c} + \overline{\theta} - \delta \tag{109}$$

for sufficiently small s.

To show (109), we show below that the equilibrium mass of sophisticated consumers that visit a firm diverges as s vanishes. This implies that the probability that a visiting sophisticated consumer buys at a firm which charges a mark-up bounded away from zero must be either zero or vanish as s vanishes, because otherwise, a firm could obtain profits unbounded from above as search costs vanish.

Now, consider a firm that offers $\overline{q}$ at $\overline{c} + \delta/2$, and thus charges a mark-up bounded away from zero. The probability that a visiting sophisticated consumer buys at this firm is

$$1 - F(\hat{U}^S - \overline{q} + \overline{c} + \delta/2). \tag{110}$$

If (109) is violated, then this probability is bounded away from zero, because $\hat{U}^S < \overline{q} - \overline{c} + \overline{\theta} - \delta$ implies that

$$1 - F(\hat{U}^S - \overline{q} + \overline{c} + \delta/2) \geq 1 - F(\overline{\theta} - \delta/2). \tag{111}$$

We infer that for sufficiently small search costs s, (109) must hold.

Finally, we show that the mass of sophisticated consumers that visit a firm in equilibrium diverges as search costs vanish. Indeed, the probability that a sophisticated consumer buys at a random firm he visits in equilibrium is given by

$$Pr(Buy) = \lambda^*(1 - F(\hat{\theta})) + (1 - \lambda^*)(1 - F(\hat{\theta}_L)). \tag{112}$$

The mass of sophisticated consumers who visit a firm in equilibrium is therefore given by

$$\sum_{t=0}^{\infty} [1 - Pr(Buy)]^t = \frac{1}{Pr(Buy)} = \frac{1}{\lambda^*(1 - F(\hat{\theta})) + (1 - \lambda^*)(1 - F(\hat{\theta}_L))}. \tag{113}$$

This mass diverges as s vanishes if both (a) $\lim_{s \to 0} \lambda^* = 0$ or $\lim_{s \to 0} \hat{\theta} \geq \overline{\theta}$, and (b) $\lim_{s \to 0}(1 - \lambda^*) = 0$ or $\lim_{s \to 0} \hat{\theta}_L \geq \overline{\theta}$ hold.

Next, we show (a), the proof of (b) follows along the same lines. Consider a sophisticated consumer at a (superior quality) firm which supplies the utility $\hat{U}^S = \overline{q} - \overline{p}^* + \hat{\theta}$. From the optimality of $\hat{U}^S$, the expected gain of searching for a single additional superior quality firm must be negative:

$$\int_{\hat{\theta}}^{\overline{\theta}} (\theta - \hat{\theta}) dF - \frac{s}{\lambda^*} \leq 0, \tag{114}$$

where s/λ^* are the expected costs of finding such a firm. If $\lim_{s\to 0}\lambda^* \neq 0$, then $\lim_{s\to 0} s/\lambda^* = 0$, and (114) implies $\lim_{s\to 0}\hat{\theta} \geq \overline{\theta}$. On the other hand, if it is not the case that $\lim_{s\to 0}\hat{\theta} \geq \overline{\theta}$, then (114) implies $\lim_{s\to 0} s/\lambda^* < 0$, and hence $\lim_{s\to 0}\lambda^* = 0$, as desired.

We are now in the position to show that for sufficiently small search costs, any symmetric equilibrium is a segmented market equilibrium. We have established that for sufficiently small search costs, in any equilibrium, firms offer inferior products with positive probability, and sophisticated consumers do not buy them. In particular, firms thus offer superior products with positive probability, hence $\lambda^* > 0$, because sophisticated consumers buy in equilibrium (see footnote 19). Thus, what remains to be shown is that for sufficiently small search costs, naive consumers buy both high and low quality products.

To show this, note that because for sufficiently small s, sophisticated consumers do not buy inferior products, and because firms make strictly positive profits in equilibrium, they must derive demand from naive consumers. This implies that naive consumers buy inferior products.

To show that naive consumers buy superior products, suppose to the contrary that they do not. Then we must have $\overline{p}^* \geq \hat{p}(\overline{q})$. In this case, a naive consumer is indifferent between buying and not buying at $\hat{p}(\underline{q})$ if and only if $\hat{p}(\underline{q}) = \underline{p}^* + s/(1-\lambda^*)$, because he expects to find an (inferior) product offered at $\underline{p}^*$ at expected search costs $s/(1-\lambda^*)$. However, because an inferior quality firm derives demand only from naive consumers, we must have $\underline{p}^* = \hat{p}(\underline{q})$, as naive consumers purchase a product if and only if $p \leq \hat{p}(\underline{q})$, a contradiction to $\hat{p}(\underline{q}) = \underline{p}^* + s/(1-\lambda^*)$. ■

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