Human Capital Risk, Contract Enforcement, and the Macroeconomy*

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Abstract

We use data from the Survey of Consumer Finance and Survey of Income Program Participation to show that young households with children are under-insured against the risk that an adult member of the household dies. We develop a macroeconomic model with human capital risk, limited pledgability of human capital (limited contract enforcement), and age-dependent returns to human capital investment. We show analytically that, consistent with the life insurance data, in equilibrium young households are under-insured against human capital risk – they choose to use marginal resources for consumption or investment rather than purchase insurance. A calibrated version of the model can quantitatively account for the life-cycle variation of life-insurance, financial wealth, earnings, and consumption observed in the US data. Our analysis implies that a reform that makes consumer bankruptcy more costly, like the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005, leads to a substantial increase in the volume of both credit and insurance.

Keywords: Human Capital Risk, Limited Enforcement, Insurance

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1. Introduction

For many households, human capital comprises the largest component of their wealth. For example, as shown in figure 1, households in their twenties and early thirties derive a yearly flow of income from their human capital (labor earnings) that by far exceeds their entire stock of net financial assets and housing wealth (net worth). Human capital is also subject to hard-to-diversify risk which is, in the case of disability or mortality risk, potentially catastrophic in magnitude. In this paper, we provide empirical evidence that young households are not well-insured against one important type of human capital risk, namely the risk of the death of a family member, even though life insurance markets exist and are competitive. Specifically, the median young household with children buys life insurance that covers between 10 and 40 percent of the net losses associated with the death of a family member, whereas older households are close to fully insured (see figure 2).[1] We further argue that the observed under-insurance pattern emerges naturally as a result of the life-cycle profile of human capital returns combined with the non-pledgability of human capital, the latter being the outcome of a bankruptcy laws that limits wage garnishment. Finally, we show that our approach has important macroeconomic implications.

Our argument for the under-insurance of young households proceeds intuitively as follows. As is well known, labor earnings increase rapidly from the level achieved upon entering the job market reaching a peak in late middle age (see figure 3). Following a long tradition in labor economics, we interpret the high earnings growth of young households as the result of investment in high-return post-schooling human capital. Thus, young households have access to a risky investment opportunity (human capital) with high expected returns, but they also desire to smooth consumption and have little assets beyond human capital. Consequently, young households have a strong motivation for borrowing, which is, however, tightly constrained by their lack of collateralizable assets and their inability to pledge future earnings as collateral on their debts. On the margin, young households prefer to either consume or invest in human capital rather than purchase insurance. Hence, young households under-invest in risky human capital *and* under-insure against human capital risk even if

[1] Our measure of net losses takes into account social security survivor benefits, progressive income taxes, and implicit insurance from the possibility of re-marriage. See Section 2 for details.

insurance is priced in an actuarially fair manner.

To develop the argument more formally, we present a theory of the life-cycle accumulation of human capital and financial capital, the allocation of financial capital across assets, and the decision to purchase insurance against human capital risk, when human capital is non-pledgable. Households in the model are heterogeneous, differing in their age, labor market status, marital status, number of dependents, and holdings of their human capital and financial assets. We then calibrate the model to match a number of features of the US data, as well as the main features of the US chapter 7 bankruptcy code. Specifically, the model is calibrated to match i) the empirical life-cycle profile of median household earnings, ii) estimates of labor market risk obtained by the empirical literature, iii) the empirical life-cycle profile of mortality risk and rates of demographic transition (such as marriage and childbirth), and iv) the human capital losses associated with the death of a spouse estimated in this paper.

Using the calibrated model economy, we provide a quantitative assessment of the theory. We emphasize three main findings. First, in equilibrium young households are severely under-insured against human capital risk, whereas older households are almost fully insured. In other words, our quantitative analysis suggests that realistic life-cycle variations in human capital returns combined with the basic features of the US bankruptcy code generate substantial under-insurance of young households. Second, the model provides an accurate account of the empirical life-cycle pattern of life insurance holdings and the life-cycle profile of under-insurance we constructed from the data. Third, the model also produces life-cycle profiles of financial wealth and human wealth that are in line with the data. We take the last two results as corroboration of our theory since the model has not been calibrated to match the corresponding targets.

We also argue that our approach has important macro-economic implications. There has been a long-standing debate among academic scholars and policy makers with regard to the relative merits of alternative consumer bankruptcy codes. In the US, this debate has led to legislation making it more costly to declare bankruptcy, such as the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005. In this paper, we add to this debate by exploring a channel that has not been studied by the previous quantitative literature on consumer bankruptcy: making it more costly to declare bankruptcy not only increases the volume of credit, but also the amount of insurance purchased by households. In the human capital model analyzed here, it further increases economic growth since it leads to more investment in the high-return asset. For the calibrated version of the model, we find that these effects are substantial leading to on overall welfare gain of the order of 0.5% of lifetime consumption.

In sum, in this paper we make an empirical contribution by providing evidence of under-insurance in the life insurance market, and we make a contribution to the quantitative macro literature by showing that a calibrated version of the model can account for the observed life-cycle patterns of insurance, earnings, financial wealth, and consumption. In addition to these substantive contributions, this paper also makes a methodological contribution by developing a tractable framework and demonstrating how to surmount the non-convexity in the household decision problem that is inherent in production economies with limited pledgability with risk averse agents. The tractability of the model is indispensable for our quantitative analysis since the computation of the solution to the households decision problem would otherwise be extremely time-consuming. Furthermore, it allows us to derive analytical results about the age-insurance relationship for a simplified version of the model.[2]

The rest of this paper is organized as follows. Following a review of the literature, Section 2 describes a number of aspects of the US data on life insurance and presents our measures of under-utilization of life insurance. Section 3 presents our theoretical model, and derives our underinsurance result analytically for a special case of the model. Section 4 describes our calibration while Section 5 describes our results. Section 6 establishes that the results of the model are robust to a number of changes to the model, while Section 7 concludes.

Literature Our paper is related to four strands of the literature. First, there is the

[2]The quantitative macro literature on limited commitment/enforcement with labor market risk has so far not analyzed models with life-cycle heterogeneity or endogenous human capital accumulation. There is, of course, quantitative work on incomplete-market models with life-cycle heterogeneity and human capital investment, but the computation of optimal household decision rules is much less complex in the incomplete-market case since there a fewer assets and therefore fewer portfolio choices.

literature on life insurance. Like Hendel and Lizzeri (2003), we emphasize commitment problems in the market for life insurance, but in contrast we focus on the life-cycle and macroeconomic implications of borrowing constraints that emerge endogenously as a result of limited pledgability. Our explanation of the life-cycle pattern of life insurance holdings also contrasts with the preference-based explanation of Hong and Rios-Rull (2012), which we further discuss in Section 6 below. We also go beyond Hong and Rios-Rull (2012) by providing a detailed account of the income losses in the case of death of the spouse and show that the insurance needs of young households are substantially larger than the insurance needs of middle-aged households (see Section 2.3 below). Our focus on young families with children differs from Koijen, Nieuwerburgh, and Yogo (2012), who focus on elderly men using data from the Health and Retirement Survey.[3]

Second, our paper relates to the literature on risk sharing in models with limited enforcement/commitment. This literature has so far not analyzed the human capital channel,[4] which is the focus of our analysis. Our theoretical contribution to this literature is to show how to avoid the non-convexity problem in a class of limited enforcement production economy models (see also Wright, 2001). We also show that a calibrated macro model with physical capital and limited contract enforcement can generate substantial lack of consumption insurance once we introduce life-cycle considerations. In contrast, Cordoba (2004) and Krueger and Perri (2006) provide a quantitative analysis based on a model with physical capital and no life-cycle, and find that consumption insurance in equilibrium is almost perfect.[5]

Our paper is also related to the voluminous literature on macroeconomic models with

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[3]Using life insurance data, Bernheim et al. (2003) find, as we do, under-insurance for those who are most exposed to risk, and dub this empirical finding the "under-insurance puzzle". Inkmann and Michaelides (2012) use life insurance data and a portfolio choice model without labor income (risk) to estimate the bequest motive. Fang and Kung (2010) analyze the effect of income and health shocks on the lapsation of insurance policies.

[4]Andofatto and Gervais (2006) and Lochner and Monge (2011) analyze models with human capital and endogenous borrowing constraints, due to enforcement problems, but they abstract from risk and therefore cannot address the issues that are central to the current paper.

[5]Krueger and Perri (2006) match the cross-sectional distribution of consumption fairly well, but the implied volatility of individual consumption is negligible in their model. See also Broer (2012) for a detailed discussion of the quantitative implications of limited commitment models without a life-cycle component.

incomplete markets, and in particular studies of human capital accumulation (Krebs, 2003, Guvenen, Kuruscu, and Ozkan, 2011, and Huggett, Ventura, and Yaron, 2011) and the life-cycle profile of consumption (Heathcote, Storesletten, and Violante, 2009, and Kaplan and Violante, 2010). This literature has shown that the incomplete-market framework is a powerful tool for understanding the observed life-cycle behavior of income, consumption, and human capital. In this paper, we show that a model with one financial friction, limited contract enforcement, explains equally well the life-cycle pattern of income, consumption, and human capital. Further, in Section 6 below we argue that a related incomplete-market model with a life insurance market has difficulty explaining the life insurance data, and show that, at least in the context of our discussion of the reform of the US bankruptcy code, the policy implication of these two classes of models differ significantly.

Finally, Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), and Livshits, MacGee, and Tertilt (2007) have also analyzed the consequences of reforming the consumer bankruptcy code based on models with equilibrium default and no insurance markets. In these papers, an increase in the cost of bankruptcy increases borrowing and reduces default, which leads to a reduction in risk sharing since default is a means towards smoothing consumption across states of nature. In contrast, in our model an increase in the cost of bankruptcy increases borrowing *and* improves risk sharing since households can take better advantage of existing insurance markets, an effect that has also been studied in the theoretical contributions of Attanasio and Rios-Rull (2000) and Krueger and Perri (2010). In this paper, we show that the increase in equilibrium insurance is substantial and that there are interesting life-cycle implications.

2. Empirical Facts

2.1 Life Insurance

Our primary source of data on life insurance holdings is the Survey of Consumer Finance (SCF). Our data are drawn from the 6 surveys of the SCF conducted between 1992 and 2007. The unit of observation is the "family", which corresponds to our concept of a household, and we measure the household's age as the age of the household head. We focus our attention on married households with at least one child since they constitute a group of households that we can identify in our data as a group with a clear motive for purchasing life insurance (see also the discussion and references in Inkmann and Michaelides, 2012). We construct life-cycle profiles by computing median household values for each age group and survey (calendar time) first and then use a centered 5-year age bin, where we remove possible time effects using time dummies as in Huggett et al. (2011).[6] Further details on the data, definition of variables, and sample selection are provided in the Appendix.

Life insurance contracts can be approximately divided into Term Insurance and Whole Life Insurance.[7] Term insurance contracts only offer insurance against the death event, whereas whole life insurance contracts offer a combination of insurance and saving. We use the face value (amount of money paid in the case of death) of all insurance contracts, term insurance and whole life insurance, to construct the amount of insurance owned by a household, subtracting the savings component of whole life policies as reported in the SCF. The SCF presents total holdings for the household, and hence these data reflect the total payout from the death of both spouses (we present data on life insurance holdings by spouse from an alternative data source in Section 4 below).

Figure 4 shows the empirical life-cycle profile of life insurance purchases of married households with children. The first line plots the median across all such households and shows that households in their early 20's have roughly \$15 thousand dollars of life insurance. This rises to about \$150 thousand by the time these households reach their 40's, and decline to \$50 thousand as households reach retirement age. The second line plots the median across only those households that have purchased some life insurance. Amongst these households, the young purchase around \$85 thousand in life insurance, rising quickly to \$200 thousand before declining slowly down to \$75 thousand in their early 60's.

The inverted u-shaped pattern of both series seems to indicate that young households are under-insured. However, it could also mean that young households simply need less insurance. To establish whether households are underinsured against the risk of death of

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[6] We have also used cohort-dummies, with similar results.

[7] Universal life insurance is grouped with whole life insurance.

a spouse, we need to take a stand on the "appropriate" level of insurance. One approach would be to use a model to deduce optimal holdings in the absence of market frictions, and use this as a measure of full insurance. We return to this approach later in the paper after presenting our model. In subsection 2.3 we turn to an alternative approach in which we estimate the size of the loss of human wealth following the death of a spouse, which we proxy by the present value of income losses taking into account the implicit insurance that results from the possibility of re-marriage, social security survivor benefits, and progressive income taxation.

2.2 Life Insurance – Two Issues

Life insurance contracts can be divided into insurance that households purchase directly from insurance companies and insurance that is obtained through employment or membership in organizations (group insurance). If the amount of group insurance offered by the employer exceeds the amount households want to hold, then these households are "involuntarily" over-insured and the insurance holdings observed in the data are not the outcome of the optimal insurance choice by households. Clearly, the phenomenon of involuntary over-insurance can only occur for households who have not purchased any individual life insurance from insurance companies. Although the SCF does not distinguish between group insurance and insurance purchased individually, data on employer provided life insurance are available from the Survey of Income and Program Participation (SIPP). Based on data drawn from the SIPP, we find that for each age between 23 and 60, the median household with kids holds substantially more life insurance than the amount of insurance provided by the employer. Further, for the median household with children the amount of employer-provided life insurance is roughly constant over the life-cycle and the shape of the life-cycle profile of total (group plus individual) life insurance holdings is therefore not much affected by the presence of group life insurance. See the Appendix for more details. Thus, we conclude that the consideration of insurance purchases as voluntary is appropriate to a first approximation. Hong and Rios-Rull (2012) come to a similar conclusion after analyzing data drawn from the International Survey of Consumer Financial Decisions.

Like much of the previous literature on life insurance (Hendel and Lizzeri 2003, Hong and Rios-Rull 2012, and Koijen et al 2012), we model the market for life insurance as a competitive market with actuarially fair pricing. This seems reasonable given the large number of competing providers and the lack of regulatory inference, and has found support in the data.[8] We also follow the bulk of this literature in abstracting from considerations of asymmetric information. We argue that this is reasonable given that moral hazard problems appear small, and that adverse selection is limited by the requirement of a medical exam and the provision of a medical history with the risk that a policy will be voided if health information is not fully disclosed. Further, the available empirical evidence suggests that adverse selection is not of first-order importance in the market for life insurance (see, for example, Cawley and Philipson 1999, and Koijen et al 2012).[9]

2.3 Human Capital Risk and Underinsurance

Human capital is subject to a significant amount of idiosyncratic risk. In this paper, we divide these risks into labor market risk and demographic risk, with a particular focus on the mortality risk of spouses. We view labor market risk as that risk which affects observed labor earnings, which includes job displacement risk and some forms of disability risk. We follow a substantial literature (e.g. Huggett et al. 2011, Krebs 2003) and set the parameters describing human capital risk so that the implied labor market earnings process is consistent with estimates of permanent labor market risk obtained by the empirical literature (Carroll and Samwick 1997, Meghir and Pistaferri 2004, Storesletten et al. 2004), and defer a discussion of the details until we calibrate the model in Section 4.

Demographic risk captures the effects of marriage, divorce, childbirth, and death of a spouse on household earnings. Rates of marriage and childbirth are calibrated using data from the SIPP. As a result of the small numbers of young widows and widowers in the SIPP, we cannot reliably estimate a life-cycle profile of re-marriage rates for widows. We therefore follow the macro literature on life insurance (Hong and Rios-Rull, 2012) and use

[8]For example, Winter (1981) finds little evidence of discrimination in the pricing of life insurance policies.

[9]By contrast, there is considerable evidence of adverse selection in the market for annuities, where a medical exam is not required (Brugiavini 1990 and Friedman and Warshawski 1990). See also Society of Actuaries (2012) for details on the range of data collected by life insurers.

re-marriage rates of divorcees as a proxy for the re-marriage rates of widows/widowers, but introduce an adjustment to take into account what is known about re-marriage rates of widows and widowers from the economics and sociology literatures.[10] See the Appendix for details. Mortality risk is chosen to match the year-to-year average survival rates for the period 1991-2000 from the US life-tables for the respective group.

Computation of the size of the loss associated with the death of a spouse is also inhibited by the relative paucity of data on young widows and widowers. We approximate the amount of household human capital lost in the case of death by the (expected) present value of the after-tax earnings differential between married households and the corresponding single household after including Social Security survivor benefits. We use the income tax and survivor benefit schedules from the year 2000, the mid point of our sample. In the Appendix we provide a detailed account of our approach.

Figure 5 plots the ratio of human capital losses in the event of death of a spouse to household labor earnings for the sample of married households with kids. To allow comparability with the life insurance data described above (which is aggregated for one household) the loss associated with the death of the household head is added to the loss from the death of their spouse. The first line depicts the loss of labor pre-tax earnings without allowing for the possibility that a widow or widower can remarry and shows that young households with children lose roughly 30 years of earnings following the death of a spouse. The second line includes the effect of taxes and social security survivor benefits on lost earnings, but does not allow for re-marriage, and shows that the government provides a substantial amount of insurance against the death of a spouse: for young households with children, the loss has declined to 15 years of earnings after taxes and transfers. Finally, the third line, which also allows for remarriage as a kind of informal insurance against loss of a spouse, shows that the resulting income loss is reduced to between 8 and 9 years of annual earnings for young households, with smaller reductions for older households who face lower remarriage rates.[11] Overall, our

[10]Both widows and widowers have lower re-marriage rates than divorcees for each age group (Norton and Miller, 1990, and Wilson and Clarke, 1992).

[11]Our computed income losses without re-marriage are in line with the results in the literature (for example, Burkhauser et al. (2005) and Weaver 2010). This literature, however, has not computed effective income

results suggest that income losses in the case of death of a spouse are substantial, but much less than a simple calculation that does not take into account non-linear taxes, social security survivor benefits, and remarriage, would suggest. Further, human capital losses expressed as a fraction of household human capital decline with age, suggesting that younger households should purchase more life insurance than older households.

Although other sources of informal insurance are possible, we argue that, with one exception, they are likely to be insignificant. For example, it is possible that the surviving spouse increases their own labor supply. However, the substantial empirical literature examining the responses of spouses labor force participation and hours worked to shocks in earnings or disability typically finds little or no effect (Gallipoli and Turner 2011, Heckman and Macurdy 1980, Gruber and Cullen 1996). Similarly, private transfers from outside of the household appear insignificant following the disability of a spouse (Gallipoli and Turner 2009) and we argue are also likely insignificant following death. One possibility that is plausibly significant is that the cost of living for a family is reduced following the death of a spouse, and we return to this issue in Section 4 below.

With our measure of the human capital loss in hand, we can now present our estimates of underinsurance. Figure 2 plots the ratio of life insurance payouts to the size of the loss for married households with children. The first line plots holdings for all households with kids and shows that the median household is insured against only one-tenth of the of the loss expected from the death of a spouse. This rises to roughly 50% by middle age, and to 75% by retirement. The second line depicts the same data for the sample of married households with kids that purchase some life insurance. This figure begins at roughly 30% and rises to close to 100% only as households reach their late 50's. This is our main empirical result: there exists a positive correlation between age and the degree to which households insure against mortality risk by purchasing life insurance. Further, young households are severely under-insured, whereas older households are almost fully insured.[12]

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losses taking into account re-marriage.

[12] The fact that households are underinsured even after conditioning only on those households that purchase life insurance suggests that fixed costs in the purchase of life insurance are not the only reason for the observed underinsurance. However, this does not rule out the possibility that a significant fraction of households do

2.4 Human Capital

Data on earnings and financial wealth are also drawn from the 6 surveys of the Survey of Consumer Finance (SCF) conducted between 1992 and 2007 and life-cycle profiles are constructed in the way described in Section 2.1. We continue to focus on married households with children. The variable "financial wealth" is defined as "net worth" in the SCF, which is the value of all assets (including housing and excluding human capital) minus the value of all debt (including mortgage debt). Labor earnings are defined as wages and salaries plus two-third of business and farm income.

Figure 1 plots the median ratio of net worth to labor earnings for married households with children of each age starting at 23 and ending at 60. As shown in the figure, households in their 20's and early 30's hold almost all of their wealth in human capital with the stock of net financial assets (including housing) less than one years flow of income from their human capital (i.e. labor earnings). By age 45, household net worth is roughly twice labor earnings, and it is not until households reach their 50's that net worth exceeds three time annual labor earnings.

The pattern in figure 1 is driven by the rapid accumulation of net financial assets, as labor earnings are also increasing in the early part of the life cycle. To illustrate this, figure 3 plots the lifetime profile of labor earnings derived from our data for households from age 23, when many households have left college, to age 60.[13] As is well-known, labor earnings rapidly increase until age 35-40, after which they grow more modestly reaching a peak about age 50, and then declining as households approach retirement.

We follow a long tradition by interpreting these earnings profiles as the result of human capital accumulation decisions motivated by high returns to post-college education and on-the-job training (e.g. Becker 1964 and Ben-Porath 1967). There is a large literature estimat-

not buy any life insurance because of fixed costs. We return to this issue in Section 6.1.

[13]The age-earnings profile in figure 1 is computed from cross-sectional data, but a very similar concave life-cycle pattern emerges in studies that use panel data drawn from the PSID (Heathcote et al., 2010). Further, this concave pattern is also observed for the earnings or wages of individual workers, though the household earnings profile lies, of course, strictly above the individual earnings profile (Heathcote et al., 2010, Huggett et al., 2011).

ing the returns to college-education, and some work on the returns to post-college education. Overall, the literature suggests a rate of return in the range of $8\% - 10\%$ (Krueger and Lindhal, 2001), though individual estimates vary considerably and there is a large amount of heterogeneity due to differences in ability (Cunha, Heckman, and Navarro, 2005, and Taber, 2001). Estimates of wage gains from on-the-job training imply rate of returns that are even higher than 10% (Blundell et al., 1999, and Mincer, 1994). Clearly, this evidence suggests that for many young households there is a strong incentive to invest in human capital. Moreover, this incentive exists regardless of whether returns are higher for the young because of decreasing returns to human capital accumulation, as in Ben-Porath (1967) and Huggett et al. (2011), or because human capital investment is less productive for older households, as in the model we describe next.

3. Model

In this section, we develop a general version of the model and discuss two theoretical results. The model structure is similar to the incomplete-market model with human capital developed in Krebs (2003) and the limited-enforcement model analyzed in Wright (2001). Our first theoretical result is a convenient characterization of equilibria (proposition 1 and proposition 2) that highlights the tractability of the model. In particular proposition 1, which shows that the solution to the household decision problem is linear in total wealth,[14] is indispensable for our quantitative analysis of a model with a very large number of types and uncertainty states. Our second theoretical result (proposition 3) is an analytical result showing that, for a special case of the general model, age and insurance are positively correlated in equilibrium. Proofs are relegated to the Appendix.

3.1. Goods Production

Time is discrete and open ended. There is no aggregate risk and we confine attention to stationary (balanced growth) equilibria. We assume that there is one good that can be

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[14] Angeletos (2007) and Moll (2012) develop tractable models of entrepreneurial activity in which individual consumption/saving policies are also linear in wealth. In either approach, tractability is achieved through the assumption that production exhibits constant returns to scale at the household level.

consumed or invested in physical capital. Production of this one good is undertaken by one representative firm (equivalently, a large number of identical firms) that rents physical capital and human capital in competitive markets and uses these input factors to produce output, Y, according to the aggregate production function $Y = F(K, H)$, where K and H denote the aggregate levels of physical capital and human capital, respectively. The production function, F, has constant-returns-to-scale, satisfies a Inada condition, and is continuous, concave, and strictly increasing in each argument. This constant-returns-to-scale assumption in conjunction with the assumption that human capital is produced under constant-returns-to-scale (see below) implies that the model exhibits endogenous growth.

Given these assumptions on F, the derived intensive-form production function, $f(\tilde{K}) = F(\tilde{K}, 1)$, is continuous, strictly increasing, strictly concave, and satisfies a corresponding Inada condition, where we introduced the "capital-to-labor ratio" $\tilde{K} = K/H$. Given the assumption of perfectly competitive labor and capital markets, profit maximization implies

$$\begin{aligned} r_k &= f'(\tilde{K}) \\ r_h &= f(\tilde{K}) + f'(\tilde{K})\tilde{K} \ , \end{aligned} \tag{1}$$

where r_k is the rental rate of physical capital and r_h is the rental rate of human capital. Note that r_h is simply the wage rate per unit of human capital and that we dropped the time index because of our stationarity assumption. Clearly, (1) defines rental rates as functions of the capital to labor ratio: $r_k = r_k(\tilde{K})$ and $r_h = r_h(\tilde{K})$. Finally, physical capital depreciates at a constant rate, δ_k, so that the (risk-free) return to physical capital investment is $r_k - \delta_k$.

3.2. Households

There are a continuum of households of mass one. Households are indexed by their age j, the exogenous state (shock) s_j, their human capital, h_j, and their asset holdings, a_j. In our quantitative analysis, the exogenous state has two components, $s_j = (s_{1j}, s_{2j})$, where s_{1j} refers to the family state of the household and s_{2j} describes idiosyncratic labor market risk. The family state s_{1j} is defined by the marital status (married, widowed, single-not-widowed), the number of kids, and the gender in the case of a single household, for a total of 17 different states. Note that mortality risk corresponds to the transition from married household to widowed household. The process $\{s_j\}$ is Markov with stationary transition probabilities $\pi_j(s_{j+1}|s_j)$. We denote by $s^j = (s_1, \ldots, s_j)$ the history of exogenous states up to age j and let $\pi_j(s^j|s_0) = \pi_j(s_j|s_{j-1}) \ldots \pi_0(s_1|s_0)$ stand for the probability that s^j occurs given s_0. At age $j = 0$, a household begins life in the initial state (a_0, h_0, s_0).

The life of a household is divided into three phases. The first phase runs from age $j = 0, \ldots, J$ and is the focus of our analysis. In the quantitative application, we identify $j = 0$ with age 23 and $j = J$ with age 60. Thus, with 17 different family states and 38 different age-groups, we have $17 * 38 = 646$ different household types in the first phase of life. In this phase, households are working and married households face the (age-dependent) risk that an adult member of the household dies (mortality risk). For simplicity, we assume that the event that both adults die simultaneously has zero probability and, given our focus on married households with children, that single households do not face mortality risk. Married households also face the (age-dependent) risk of divorce. Single households meet with age-dependent probability to form a married household. Some households have children and the number of children in a household can increase or decrease by one. All transition probabilities over family states are exogenous. See Section 4 and the Appendix for more details about the specification of these transition probabilities.

The second phase of life, $j = J + 1$, is the pre-retirement stage. This phase is similar to the first phase, but now households do not age. However, they retire stochastically with fixed probability. The third and final phase of life is retirement. In the retirement phase, households receive no labor income and can only invest in a risk-free asset. Retired households die with constant probability and are then replaced by a household age $j = 0$ (age 23 in Section 4). Given that the focus of our analysis is on married households with kids in the first phase of life (i.e. with a household head not older than 60), we relegate to the Appendix a more detailed discussion of the decision problem in the second and third phase of life.

Households are risk-averse and have identical preferences that allow for a time-additive expected utility representation with logarithmic one-period utility function and pure discount factor β. For a household choosing the consumption plan $\{c_j\}$, expected life-time utility is given by

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\begin{aligned}
&\sum_{j=0}^{J} \beta^j \sum_{s^j} u(c_j, s_j) \pi_j(s^j | s_0) \\
&+ \sum_{s^{J+1}} V_{J+1}(h_{J+1}(s^J), a_{J+1}(s^{J+1}), s_{J+1}) \pi_{J+1}(s^{J+1} | s_0)
\end{aligned} \tag{2}
$$

where $u(c_j, s_j) = \gamma_0(s_j) + \gamma_1(s_j)\, lnc_j$ is the one-period utility function of the household, V_{J+1} is the value function in the pre-retirement stage and γ_0 and γ_1 are preference shocks that depend on the family state. For our baseline quantitative model we use $u(c, s) = ln(c/n(s))$, where n is the number of household members in consumption equivalence units. We then relax this assumption in Section 6. Note that we have abstracted from the labor-leisure choice of households.

Households can invest in physical capital as well as human capital and they can buy and sell a complete set of financial assets (contracts) with state-contingent payoffs. More specifically, there is one asset (Arrow security) for each exogenous state s. We denote by $a_{j+1}(s_{j+1})$ the quantity bought (sold) at age (in period) j of the asset that pays off one unit of the good if s_{j+1} occurs at age $j+1$ (in the next period). Given an initial state, (h_0, a_0, s_0), a household chooses a plan, $\{c_j, h_{j+1}, \vec{a}_{j+1}\}$, where the notation $\vec{a}$ indicates that in each period the household chooses a vector of asset holdings. Further, c_j stands for the function mapping partial histories, s^j, into consumption levels, $c_j(s^j)$, with similar notation used for the other choice variables. A budget-feasible plan has to satisfy the sequential budget constraint, human capital evolution equation, and non-negativity constraints on total wealth (financial plus human), consumption and human capital

$$
\begin{array}{lrcl}
i) & z_j(s_j) r_h h_j + a_j(s_j) & = & c_j + x_{hj} + \sum_{s_{j+1}} a_{j+1}(s_{j+1}) q_j(s_{j+1}) \\
ii) & h_{j+1} & = & (1 - \delta_{hj} + \eta_j(s_j)) h_j + \varphi_j(s_j) h_j + \phi x_{hj} \\
iii) & h_j + \sum_{s_{j+1}} a_{j+1}(s_{j+1}) q_j(s_{j+1}) & \geq & 0 \\
iv) & c_j & \geq 0 & , \quad h_{j+1} \geq 0 \ ,
\end{array} \tag{3}
$$

where $q_j(s_{j+1})$ is the price of a financial contract in period j that pays off if s_{j+1} occurs in $j+1$, which in our Markovian setting only depends on asset type s_{j+1} and current state s_j.

Note that the equations in (3) have to hold in realization; that is, they hold for all j and all sequences s^j. Note also that (3iii) represents a debt constraint, and that (3iv) requires the stock of human capital, h_{j+1}, to be non-negative, which prevents elderly workers from shorting their human capital.

The term z_j in equation (3ii) is a labor productivity shock that captures transitory movements in earnings and we normalize it to one: $E[z_j] = 1$. In the human capital evolution equation, the term η_j measures the loss/gain of household human capital when there is a transition from married household to single households and vice versa (death of spouse, divorce, marriage).[15] The term $\varphi_j h_j$ represents increases in human capital that do not require an active input of resources, including returns to experience and learning-by-doing one's job, which are often referred to as experience capital. Note that this term has a random component so that φ_j also captures any labor market risk that is not part of the productivity shock z_j. The randomness in φ_j might describe variations in the return to experience that often occur when workers switch their employer and/or occupation. Finally, the term ϕx_{hj} captures the active human capital investment made by the household.

Note that the budget constraint (3i) assumes that physical capital and human capital are produced using similar technologies in the sense that one unit of physical capital can be transformed into ϕ units of human capital. Thus, we assume constant returns to scale at the household level. This assumption, also made in Krebs (2003), implies that the household decision problem displays a certain linearity with respect to physical capital investment and human capital investment in the sense that goods invested in either human capital or physical capital generate returns that are independent of household size, where size is measured by total wealth (see below).

The assumptions we make in (3) have the advantage that they keep the model highly tractable, which, as we argued before, is essential for the theoretical and quantitative analysis conducted in this paper. Tractability in the general case requires that we do not impose a restriction on the ability of households to decumulate human capital. However, in the

[15] For notational ease, we expand the family state, s_{1j}, to include last period's marital status (married, single widowed, single not widowed) so that the η-shock only depends on the current s_{1j} and not on $s_{1,j-1}$. See the quantitative section for details.

calibrated model economy used for our quantitative analysis, the restriction $h_{j+1} \geq (1-\delta_{hj})h_j$ is always satisfied in equilibrium; that is, it holds for all household types at all ages and all realizations of uncertainty. Similarly, the restriction that human capital investment is non-negative, $\varphi_j h_j + \phi x_{hj} \geq 0$, is always satisfied in equilibrium. Thus, imposing these restrictions in (3) would not change the conclusions drawn in the quantitative analysis. [16]

In addition to the standard budget constraint, each household has to satisfy a sequential enforcement (participation) constraint, which ensures that at no point in time individual households have an incentive to default on their financial obligations. More precisely, individual consumption plans have to satisfy

$$\begin{aligned} & \sum_{n=0}^{J-j} \beta^n \sum_{s^{j+n}|s^j} u(c_{j+n}, s_{j+n}) \pi_j(s^{j+n}|s_j) \\ + \quad & \beta^{J+1-j} \sum_{s^{J+1}|s^j} V_{J+1}(h_{J+1}(s^J), a_{J+1}(s^{J+1}), s_{J+1}) \pi_{J+1}(s^{J+1}|s_j) \\ \geq \quad & V_d(h_j(s^{j-1}), a_j(s^j), s_j) \end{aligned} \tag{4}$$

for all j and s^j, where V_d is the value function of a household who defaults. Note that the constraint set defined by (4) may not be convex since both the left-hand side and the right-hand side are concave functions of h. This is the non-convexity issue alluded to in the introduction; in proposition 1 we show how this problem is surmounted in the current setting.

The default value function V_d, is defined by the household decision problem in default. In this paper, we allow for different specifications of this default problem. In the baseline version of the model, we model default along the lines of Chapter 7 of the US bankruptcy code. More precisely, we assume that upon default all debts of the household are canceled and all financial assets seized so that $a_j(s_j) = 0$. Following default, households are excluded from purchasing insurance contracts and borrowing (going short), but they can still save in a

[16] Note that in (3) we have explicitly imposed a non-negativity constraint on the stock of human capital, and our general characterization of the household decision rule (proposition 1) holds with this constraint imposed. Of course, for a certain range of parameter values this constraint binds in equilibrium, but for the parameter values used in our quantitative analysis this constraints never binds (does not bind for all households types and uncertainty states).

risk-free asset. We assume that exclusion continues until a stochastically determined future date that occurs with probability $(1-p)$ in each period; that is, the probability of remaining in (financial) autarky is p. Following a default, households retain their human capital and continue to earn the wage rate r_h per unit of human capital. After regaining access to financial markets, the households expected continuation value is $V^e(h,a,s)$, where (h,a,s) is the individual state at the time of regaining access. For the individual household the function V^e is taken as given, but we will close the model and determine this function endogenously by requiring that $V^e = V$, where V is the equilibrium value function associated with the maximization problem of a household who participates in financial markets.[17] Details are found in the Appendix.

3.3 Equilibrium

We assume that insurance markets (financial markets) are perfectly competitive and abstract from transactions costs. Thus, insurance contracts (financial contracts) are priced in an actuarially fair manner (risk neutral pricing):

$$q_j(s_{j+1};s_j) = \frac{\pi_j(s_{j+1}|s_j)}{1+r_f}\ . \tag{5}$$

The pricing equation (5) can be interpreted as a zero-profit condition for financial intermediaries that can invest in physical capital at the risk-free rate of return $r_f = r_k - \delta_k$ and can fully diversify idiosyncratic risk for each insurance contract s_{j+1}.

Below we show that the optimal plan for individual households is recursive; that is, the optimal plan is generated by a policy function, g. This household policy function in conjunction with the transition probabilities, π, define a transition function over states, (h,a,s), in the canonical way. This transition function together with the initial distribution, μ_0, and sequence of distributions for new-born households, $\{\mu_{t,new}\}$, induce a sequence of equilibrium distributions, $\{\mu_t\}$, of households over individual states. We assume that the

[17]The previous literature has usually assumed $p = 1$ (permanent autarky). See, however, Krueger and Uhlig (2006) for a model with $p = 0$ following a similar approach to ours. Note also that the credit (default) history of an individual household is not a state variable affecting the expected value function, V^e. Thus, we assume that the credit (default) history of households is information that cannot be used for contracting purposes.

financial capital of households who die is inherited by new-born households, which imposes a restriction on the mean of the marginal distribution $\mu^m_{t,new}$ over a. Note that we allow the distribution $\{\mu_{t,new}\}$ to have an explicit time-dependence since in our endogenous growth model the mean value of h and a will growth over time, and a stationary distribution over intensive-form or growth-adjusted variables can only be obtained if the mean value of the extensive-form variables also grows for new-born households. In our quantitative analysis, we directly specify the distribution of new-born households over growth-adjusted states.

Assuming a law of large numbers, aggregate variables can be found by taking expectation with respect to the induced equilibrium distribution. For example, the aggregate stock of human capital held by all households in period t is given by $H_t = \int \sum_j h_j d\mu_{tj}(h_j)$. A similar expression holds for the aggregate value of financial wealth. In equilibrium, human capital demanded by the firm must be equal to the corresponding aggregate stock of human capital supplied by households. Similarly, the physical capital demanded by the firm must equal the aggregate net financial wealth supplied by households. Because of the constant-returns-to-scale assumption, only the ratio of physical to human capital is pinned down by this market clearing condition. That is, in equilibrium we must have for all t

$$\tilde{K} = \frac{\int \sum_j \sum_{s_{j+1}} q_j(s_{j+1}; s_j) a_{j+1}(s_{j+1}) d\mu_{jt}(h_j)}{\int \sum_j h_j d\mu_{tj}(h_j)} , \tag{6}$$

where $\tilde{K}$ is the capital-to-labor ratio chosen by the firm.

To sum up, we have the following equilibrium definition:

Definition A stationary recursive equilibrium is a collection of rental rates (r_k, r_h), an aggregate capital-to-labor ratio, $\tilde{K}$, a household value function, V, an expected household value function, V^e, a household policy function, g, and a sequence of distributions, $\{\mu_t\}$, of households over individual states, (h, a, s), such that
i) Utility maximization of households: for each initial state, (h_0, a_0, s_0), and given prices, the household policy function, g, generates a household plan that maximizes expected lifetime utility (2) subject to the sequential budget constraint (3) and the sequential participation constraint (4).
ii) Profit maximization of firms: aggregate capital-to-labor ratio and rental rates satisfy the

first-order conditions (1).
iii) Financial intermediation: financial contracts are priced according to (5)
iv) Aggregate law of motion: the sequence of distributions, $\{\mu_t\}$, is generated by g, π, μ_0, and $\{\mu_{t,new}\}$.
v) Market clearing: equations (6) holds for all t when the expectation is taken with respect to the distribution μ_t.
vi) Expected household value function is identical to the household value function: $V^e = V$.

Note that the equilibrium value of $\tilde{K}$ determines the equilibrium growth rate of the economy (see Appendix for details). Note also that in equilibrium the goods market clearing condition (aggregate resource constraint) automatically holds:

$$C_t + K_{t+1} + \frac{1}{\phi} H_{t+1} = (1-\delta_k) K_t + \frac{1}{\phi} H_t + \frac{1}{\phi} \int \sum_j (\eta_j(s_j) + \varphi_j(s_j) - \delta_{hj}) h_j d\mu_{tj}(h_j) + F(K_t, H_t) \tag{7}$$

3.4. Characterization of Household Problem

We next show that optimal consumption choices are linear in total wealth (human plus financial) and portfolio choices are independent of wealth. This property of the optimal policy function allows us to solve the quantitative model, which has a large number of household types and uncertainty states, without using approximation methods. The property also implies that the household decision problem is convex and the first-order approach can be utilized.

To state the characterization result, denote total wealth (human plus financial) of a household of age j at the beginning of the year by $w_j = h_j/\phi + \sum_{s_j} a_j(s_j) q_{j-1}(s_j)$. Note that ϕ measures the productivity of goods investment in human capital and $1/\phi$ is is the shadow price of one unit of human capital in terms of the consumption/capital good. Denote the portfolio shares by $\theta_{hj} = h_j/(\phi w_j)$ and $\theta_{a,j}(s_j) = a_j(s_j)/w_j$. The sequential budget constraint (3) then reads:

$$\begin{aligned} w_{j+1} &= (1 + r_j(\theta_j, s_j)) w_j - c_j \\ 1 &= \theta_{h,j+1} + \sum_{s_{j+1}} q_j(s_{j+1}|s_j) \theta_{a,j+1}(s_{j+1}) \\ c_j &\geq 0 \ , \ w_{j+1} \geq 0 \ , \ \theta_{j+1} \geq 0 \end{aligned} \tag{8}$$

with

$$1 + r_j(\theta_j, s_j) \doteq [1 + \phi z_j(s_j) r_h - \delta_{hj} + \eta_j(s_j) + \varphi_j(s_j)]\, \theta_{hj} + \theta_{aj}(s_j)$$

Clearly, this is the budget constraint corresponding to an inter-temporal portfolio choice problem with linear investment opportunities and no exogenous source of income. It also shows that (w, θ, s) can be used as individual state variable for the recursive formulation of the utility maximization problem. Using this notation, we have the following result:

Proposition 1. The value function and the optimal policy function are given by

$$\begin{aligned}
V_j(w_j, \theta_j, s_j) &= \tilde{V}_{0j}(s_j) + \tilde{V}_{1j}(s_j)\, [\ln w_j + \ln(1 + r_j(\theta_j, s_j))] \\
c_j(w_j, \theta_j, s_j) &= \tilde{c}_j(s_j)\, (1 + r_j(\theta_j, s_j))\, w_j \\
\theta_{j+1}(w_j, \theta_j, s_j) &= \theta^*_{j+1} \\
w_{j+1}(w_j, \theta_j, s_j) &= (1 - \tilde{c}_j(s_j))\, (1 + r_j(\theta_j, s_j))\, w_j
\end{aligned} \tag{9}$$

where the value function coefficients, $\tilde{V}_{0j}(s_j)$, $\tilde{V}_{d,0j}(s_j)$, and $\tilde{V}_{1j}(s_j)$ as well as the optimal consumption-to-wealth ratio, $\tilde{c}$, and the optimal portfolio choice, θ^*_{j+1} are the solution to a maximization problem with linear constraints (see the Appendix).[18]

Proof: See Appendix.

Proposition 1 provides the foundation for our quantitative analysis since it allows us to solve the household decision problem even with a large number of future states and therefore a large number of choice variables (portfolio choices). Note that proposition 1 also means that the non-convexity problem alluded to in the introduction has been solved in the context of the current model. Intuitively, the value functions of the agent in and out of bankruptcy exhibit the same concavity in h and hence cancel each other out in the enforcement constraint.

3.5. Equilibrium Characterization

Define the share of aggregate total wealth of households of age j and state s_j as

$$\Omega_j(s_j) \doteq \frac{E\,[(1 + r_j) w_j | s_j]\, \pi_j(s_j)}{\sum_j \sum_{s_j} E\,[(1 + r_j) w_j | s_j]\, \pi_j(s_j)}$$

[18]The Appendix also contains the corresponding expressions for the default value function and default consumption policy.

Note that $(1+r_j)w_j$ is total wealth of an individual household after assets have paid off (after production and depreciation has been taken into account). Note also that $\sum_j \sum_{s_{1j}} \Omega(s_{1j}) = 1$. Further, Ω is finite-dimensional, whereas the set of distributions over (w, s) is infinite-dimensional. Using the definition of wealth shares and the property that portfolio choices are wealth-independent, in the Appendix we show the following result:

Proposition 2. Suppose that $(\theta, \tilde{c}, \tilde{V}, \tilde{K}, \Omega)$ solves the fixed-point problem defined by the equations (A4), (A9), and (A11) in the Appendix. Then $(g, \tilde{V}, \tilde{K}, \{\mu_t\})$ is a stationary (balanced growth) equilibrium, where g is the individual policy function induced by $(\tilde{c}, \theta)$ and $\{\mu_t\}$ is the sequence of measures induced by the policy function g, the initial measure, μ_0, and the transition matrix over demographic and labor market states, π.

Proof. See the Appendix.

Proposition 2 shows that the stationary equilibrium can be found without knowledge of the infinite-dimensional wealth distribution – only the lower dimensional distribution Ω matters. Proposition 2 facilitates our quantitative analysis significantly since it implies that there is no need to keep track of the entire wealth distribution when computing equilibria.

3.6. Analytical Results

We now derive analytical results for a special case of the model. We use these results to discuss the main determinants of individual consumption, and to prove that in equilibrium there is a positive relationship between age and insurance.

We focus on the first phase of life, $j = 1, \ldots, J$, and on households with two adult members (married households). We consider the case with only mortality risk so that $s_j \in \{d, n\}$, where $s_j = d$ is the event that the death of an adult household member occurs and $s_j = n$ is the event that death does not occur. Note that after the event "death of an adult household member" the household continues to exist. Mortality risk is an i.i.d. random variable, η, with age-independent probability π that death occurs and age-independent human capital loss $\eta(d)$ in the death event. We normalize the mean of η to zero: $\pi\eta(d) + (1 - \pi)\eta(n) = 0$. Note that $\eta(d)$ is the fraction of household human capital that is lost in the event that an adult member of the household dies. We also assume constant labor productivity $z_j(s_j) = 1$

and no human capital risk beyond mortality risk: $\varphi_j(s_j) = \varphi$.

We assume that young households have lower depreciation rates of human capital than older households, $\delta_{hj} < \delta_{h,j+1}$, which implies that expected human capital returns of young households are larger than the returns of older households. We choose state-independent preference parameters $\gamma_0(s_j) = \gamma_1(s_j) = 1$. Finally, we assume that defaulting households are not excluded form financial markets, $p = 0$, which rules out short positions in financial assets (see Appendix).

Using the policy function (9) of our equilibrium characterization result, we find that in this example consumption growth is given by:

$$\begin{aligned} \frac{c_{j+1}}{c_j} &= \beta(1 + r_{j+1}(\theta_{j+1}, s_{j+1})) \\ &= \beta \left\{ (1 + \phi r_h + \varphi - \delta_{h,j+1} + \eta(s_{j+1}))\, \theta_{h,j+1} + \theta_{a,j+1}(s_{j+1}) \right\} \end{aligned} \tag{10}$$

Consumption growth depends on human capital choice, $\theta_{h,j+1}$, ex-ante human capital returns, $\phi r_h + \varphi - \delta_{hj}$, ex-post shocks, $\eta(s_{j+1})$, and asset payoffs (insurance), $\theta_{a,j+1}(s_{j+1})$. From (10) we immediately conclude that consumption is independent of mortality shocks if $\theta_{a,j+1}(d) - E[\theta_{a,j+1}] = \eta(d)\, \theta_{h,j+1}$, where $E[\theta_{a,j+1}] = \pi\theta_{a,j+1}(d) + (1-\pi)\theta_{a,j+1}(n)$ is the fraction of total wealth the household is holding as financial wealth. This is intuitive since $(\theta_{a,j+1}(d) - E[\theta_{a,j+1}])\, w_{j+1}$ is the insurance pay-out in the case of death and $\eta(d)\, \theta_{h,j+1} w_{j+1}$ is the human capital loss in the case of death, and when the two are equal we have full insurance and therefore deterministic consumption growth.

The above discussion demonstrates that the expression $\eta(d)\, \theta_{h,j+1} w_{j+1}$ is the pay-out necessary to achieve full-insurance and $(\theta_{a,j+1}(d) - E[\theta_{a,j+1}])\, w_{j+1}$ is the actual insurance pay-out. Thus, we can define a measure of insurance (the insurance coefficient) as the ratio of the two:[19]

$$I_{j+1} = \frac{\theta_{a,j+1}(d) - E[\theta_{a,j+1}]}{\eta(d)\, \theta_{h,j+1}} \tag{11}$$

[19] Blundell et al (2008) introduce an insurance coefficient that measures the extent to which consumption responds to income shocks. Clearly, their measure captures consumption insurance through self-insurance and the explicit purchase of insurance contracts, whereas our approach confines attention to the latter channel.

Clearly, the insurance measure I varies between 0 (no insurance) and 1 (full insurance). For our example economy we have the following result:

Proposition 3. Suppose the economy is as described above. In equilibrium, young households are less insured than old households and a larger fraction of their total wealth is invested in human capital:

$$\begin{aligned} \theta_{h,j} &\geq \theta_{h,j+1} \\ I_j &\leq I_{j+1} , \end{aligned}$$

where the inequalities are strict if in equilibrium there is some insurance, but not full insurance.

Proof. See the Appendix.

Next we establish this result in a calibrated version of our general model.

4. Calibrating the Model

We now turn to the quantitative analysis. Section 4.1 lays out the model specification. Section 4.2 discusses our calibration strategy and the relevant empirical literature, while Section 4.3 discusses computation.

4.1 Model Specification

We set the length of a time period to one year and let $j = 23, \ldots, 60$ for the first phase of life. As in Huggett et al. (2011), we restrict attention to households up to age 60 for the following three reasons. First, the number of households for each age-group in our SCF-sample drops rapidly after age 60. Second, labor force participation falls near the traditional retirement age for reasons that are not modeled here. Third, the closer we get to the traditional retirement age, the more important non-negativity constraints on human capital investment become. By fitting the empirical life-cycle of earnings and wealth only up to age 60 and introducing a transition-group of households with stochastic retirement, we can ensure that for the calibrated model economy the rate of decumulation of human capital is bounded by the rate of depreciation over the entire life-cycle.

For our baseline model, we assume a one-period household utility function $u(c, s) = ln(c/n(s))$, where n is the number of household members in consumption equivalence units. In other words, we assume a constant marginal utility of household consumption and normalize the constant to one: $\gamma_1(s) = 1$. Note that for this preference specification we have $\tilde{c}_j = 1 - \beta$ in (9). We choose this specification to focus attention on our mechanism. We return to the possibility of life cycle variation in preferences in our robustness analysis in Section 6.

We assume that the exogenous state variable has two components, $s_j = (s_{j1}, s_{j2})$. The first component describes the family state with $s_{1j} \in \{(m_j, k_j)\}$, where m_j is the marital state and k_j denotes the number of kids. We assume $k_j \in \{0, 1, 2, 3\}$ and $m_j \in \{ma, fw, fn, mw, mn\}$ corresponding to married (ma), female widowed (fm), female not widowed (fn), male widowed (mw), and male not widowed (mn). Thus, we have in total 17 family states (we do not have non-widowed single male households with children since we assume that after divorce children live with their mother). The transition matrix over family states is discussed in more detail in the Appendix. Transitions across family states, including mortality risk, lead to changes in the stock of human capital of households, which is captured by the term $\eta_j = \eta_j(s_{1j})$. We discuss our specification of η_j in the Section 4.2 below.

The second component of the exogenous state describes labor market risk specified by the two variables $z_j = z_j(s_{2j})$ and $\varphi_j = \varphi_j(s_{2j})$. We assume that productivity shocks, z_j, and human capital shocks, $\{\varphi_j\}$, are i.i.d. with a finite, symmetric distribution that approximates a normal distribution. The assumption that human capital shocks are independently and (approximately) normally distributed is also made by Huggett et al. (2011) and Krebs (2003). We assume that z_j has mean 1 and φ_j has mean $\bar{\varphi}$ and denote variances of these two random variables by σ_z^2 and σ_φ^2, respectively.

Households in pre-retirement age (age $J = 61$) work and the duration of this phase of life ends stochastically with retirement. Households age $J = 61$ solve a recursive version of the household decision problem described in Section 3 (see also the Appendix). Upon retirement, the human capital of households becomes unproductive.[20] Retired households

[20]Formally, we assume that the labor productivity of retired households drops to zero. To avoid that

can save in a risk-free asset. Households age $j = 23, \ldots, 61$ do not die and retired households die stochastically, in which case they are replaced by a new-born household of age 23. The financial capital of deceased households is passed on to new-born households. New-born households also receive an initial endowment of human capital. The distribution of new-born households over human capital, physical capital, and family states is discussed in Section 4.2 below.

We assume a Cobb-Douglas aggregate production function, $f(\tilde{k}) = A\tilde{K}^{\alpha}$. The computation of equilibria exploits the characterization results in proposition 1 and proposition 2. See the Appendix for more details on our computational approach.

4.2 Model Calibration

4.2.1 Human Capital Risk

Mortality risk is captured in the model by the transition from the marital state $m_j = ma$ to the marital state $m_{j+1} = mw$ (female spouse dies, producing a male widow or widower) or $m_{j+1} = fw$ (male spouse dies). We choose the probability that a male or female spouse dies to match the year-to-year average survival rates for the period 1991-2000 for the US life-tables for the respective group (see figure A5 in the Appendix). We use the re-marriage rates of divorcees from the SIPP as a proxy for the re-marriage rates of widows/widowers, but introduce an adjustment to take into account of the evidence that indicates lower re-marriage rates for widows and widowers. Specifically, we compute the life-cycle profile of re-marriage rates of female/male divorcees from the SIPP and then scale down this life-cycle profile so that the average marriage rate corresponds to the average re-marriage rate of widows/widowers in the SIPP data. The result is depicted in figure A1 in the Appendix, and is in line with the evidence of re-marriage rates of widows and widowers presented in Norton and Miller (1990) and Wilson and Clarke (1992).

The size of the negative human capital shock in the case of the death of an adult house-

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households sell their human capital upon retirement, we also assume 100 percent depreciation of existing human capital in the first period of retirement. Of course, in equilibrium households are almost fully insured against this retirement-shock. Alternatively, we can assume that upon retirement households receive a lump-sum payment from the government.

hold member, η_j, is equated to the (expected) present value of income losses computed as described in Section 2.3, but here we compute the losses separately for the case of death of the husband and death of the wife. The losses we compute take into account the implicit insurance that arises from social security survivor benefits, progressive income taxation, and re-marriage. In addition, we make an adjustment to the human capital losses to take into account the insurance effect of any reduction in living costs resulting from a smaller family size, which we have ignored so far. We account for this effect by scaling the income losses according to the cost of living adjustment suggested by the consumption equivalence scale of Ruggles (1990). The resulting life-cycle profiles of human capital losses in case of death of the husband, respectively wife, are shown in figures A7 and A8 in the Appendix.

We add the effect of remarriage to the human capital loss, and correspondingly do not incorporate into the model a positive human capital shock upon re-marriage, for two reasons. First, in our complete-market model, single households can smooth their consumption by short-selling the asset that pays off in the event of (re)-marriage, which provides married households with implicit insurance against mortality risk that goes beyond the insurance provided by the simple possibility of re-marriage. Clearly, in reality few households have the opportunity to take advantage of this type of financial hedging strategy to reduce income losses, and we therefore take a modelling approach that downplays the benefits from this strategy. Note that our approach still takes into account the implicit insurance provided by the possibility of re-marriage since re-marriage probabilities enter into our calculation of the expected income losses depicted in figures A7 and A8. Our second reason for our modelling approach is to avoid distorting the human capital accumulation decisions of single households. Specifically, our modeling approach requires that human capital shocks are proportional to the existing stock of human capital, which in the case of a positive shock upon re-marriage would mean that single households have an extra, and somewhat unrealistic, incentive to accumulate human capital.

Divorce risk is captured in the model by the transition from the marital state $m_j = ma$ to the marital state $m_{j+1} = fn$ and $m_{j+1} = mn$. We choose the age-dependent probability of divorce so that we match the corresponding separation rates in the SIPP data (see figure A2 in the Appendix). After divorce, the new single-female household receives x percent of the total household human capital and the new single-male household y percent of the total household human capital. The number x is the ratio of median earnings of a single-female household over median earnings of a married household. We assume that after divorce the financial wealth is split equally between the man and the woman.

We choose the two parameters σ_z^2 and σ_φ^2 so that the implied earnings process is consistent with the estimates of the empirical literature on labor market risk. Specifically, a large literature (Carroll and Samwick 1997, Meghir and Pistaferri 2004, Storesletten et al. 2004) has estimated transitory and permanent labor income risk as follows. Observed labor income, y_{hj}, is decomposed into a transitory component, y_{hj}^T, and a permanent/persistent component, y_{hj}^p, with $lny_{hj} = lny_{hj}^T + lny_{hj}^p$, where the transitory component is an i.i.d. process with $lny_{hj}^T \sim N(0, \sigma_{hT}^2)$ and the permanent component is a logarithmic random walk with innovation term $\epsilon \sim N(0, \sigma_\epsilon^2)$. The two variances $\sigma_{h,T}^2$ and σ_ϵ^2 can then be estimated separately using various moment conditions (Meghir and Pistaferri, 2004). In our model, labor income is given by $y_{hj} = z_j r_h h_j$. Thus, z_j can be identified with the transitory component y_{hj}^T and estimates of $\sigma_{h,T}^2$ provide us with an estimate of σ_z^2. Further, the human capital stock h_j can be identified with the permanent component y_{hj}^p, as can be seen from

$$\begin{aligned} \ln h_{j+1} - \ln h_j &= \ln \theta_{h,j+1} - \ln \theta_{hj} + \ln \beta + \ln(1 + r_j(\theta_h, s_j)) \\ &\approx \ln \theta_{h,j+1} - \ln \theta_{hj} + \ln \beta + z_j \phi r_h - \delta_{hj} + \eta_j + \varphi_j \end{aligned} \tag{12}$$

where we used the equilibrium policy for human capital and the approximation $ln(1+x) \approx x$. Equation (12) shows that conditional on family structure, log-earnings in the model follow a random walk with age-dependent drift and innovation term that is approximately normally distributed with variance $\phi^2 r_h^2 \sigma_z^2 + \sigma_\varphi$.

The discussion shows how estimates of the transitory and permanent component of labor market risk, $\sigma_{h,T}^2$ and $\sigma_{h,p}^2$, provide us with estimates of σ_z^2 and σ_φ^2 (for given values of ϕ and r_h). Estimates of $\sigma_{h,T}$ and $\sigma_{h,p}$ vary considerably, with a midpoint of around 0.3 and 0.15, respectively. For $\sigma_{h,T}^2$ we choose the midpoint of 0.30. For the standard deviation of the permanent component we follow Huggett et al (2011) and choose a somewhat lower value, namely 0.0123. This choice is motivated by the fact that estimates of permanent labor income risk will overstate the true value of the variance if there is earnings profile heterogeneity in addition to stochastic shocks with a permanent component (Baker and Solon 2003 and Guvenen 2007).

4.2.2 Investment Returns

We calibrate an annual risk-free rate of $r_f = 3\%$, in line with Kaplan and Violante (2010) and roughly in line with Huggett et al. (2011) and Krueger and Perri (2006) who use a 4% annual risk-free rate, but also deduct capital income taxes.

We choose the age-dependent human capital depreciation rates δ_{hj} to match the life-cycle profile of earnings of the median household in our sample. Specifically, we assume that the age-dependence is described by an exponential function, $\delta_{hj} = A + Be^{-Cj}$, and choose the coefficients A, B, and C in order to minimize the distance (L2-norm) between the empirical life-cycle of median earnings from age 23 to age 60 and the corresponding model prediction. We impose the restriction that human capital depreciation rates averaged over the life cycle are equal to 5 percent, which determines the value of the parameter $\bar{\varphi}$. The implied life-cycle profile of human capital depreciation rates is shown in figure A9 in the Appendix. For each family type s_1, we endow newborn households with human capital so that their earnings level matches the median value of households age 23 of family type s_1 in the data.

4.2.3 Bankruptcy Code

We calibrate the costs of default to match features of the U.S. bankruptcy code. Specifically, we assume that households forfeit all financial assets, experience no garnishment of labor income, and are unable to borrow or buy insurance products for an average length of 7 years, so that the probability of re-establishing full financial market access is $(1-p) = 1/7$.[21] Households in default may save in the risk-free asset, and may continue to rent their human capital to firms.

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[21] Our parameterization is bracketed by Krueger and Perri (2006), who assume $(1-p) = 0$, Chatterjee et al. (2007), who use $(1-p) = 1/10$, and Livshits et al. (2007), who use $(1-p) = 1$ following the first period of default. The degree of variation in the parameter p reflects the fact that, as in our model, these papers abstract from a number of the costs of consumer default, and hence the calibration of the parameter p in part is a proxy for other default costs. In light of this, some authors have argued that the parameters governing the cost of default should be calibrated to match some aspect of the data, as in the choice of the level of wage garnishment in Livshits et al. (2007).

4.2.4 Preferences, Endowment, and Production

We follow Huggett et al. (2011) and assume a capital share in output, α, of .32, and target an aggregate capital to output ratio of 2.94. We also target an aggregate ratio of physical capital to human capital, $\tilde{K}$, of 0.4. Together with the interest rate target of 3 percent, these requirements pin down the parameter values $\delta_k = 0.0785$ and $A = 0.1818$.

The retirement probability of households is chosen so that retirement occurs on average at age 65 and the death probability of retired households is chosen so that the expected age of death is 75. We choose an annual discount factor $\beta = 0.95$ and the human capital productivity parameter ϕ to match the average of the ratio of financial wealth to labor income, which yields $\phi = 0.23$. We choose the frequency distribution of newborn households (households age 23) over family types s_1 in the model equal to the empirical distribution.

Newborn households inherit the financial capital of deceased retired households and receive an initial endowment of human capital. We assume that all newborn households of a particular family type s_1 have the same endowment of financial and human capital, and assign initial endowments of human capital to different family types so that we i) match the empirical distribution of earnings across family type at age 23 and ii) generate an equilibrium capital-to-labor ratio, $\tilde{K}$, that is equal to the target ratio of 0.4. The second requirement yields an aggregate endowment of human capital of households age 23 that is 5 times their financial capital. This seems plausible given that our newborn households are 23 years old and have accumulated human capital through school and college.

4.3 Computation

The computation of equilibria is based on propositions 1 and 2. More specifically, we start with an aggregate capital-to-labor ratio, $\tilde{K}$, which defines the rental rates r_k and r_h, and solve the intensive-form household problem (proposition 1). Given the solution to the household problem, we compute a stationary relative wealth distribution, Ω, using the law of motion described in the Appendix (proposition 2). We use this Ω to compute a new $\tilde{K}$ and iterate over $\tilde{K}$ until the clearing holds. A detailed description of our solution method can be found in the Appendix.

5. Results

We next present the results of our quantitative analysis of the model. Section 5.1 compares the models implications for insurance to the data, beginning with life insurance holdings and concluding with a description of consumption insurance. Section 5.2 discusses the models implications for wealth and the cross sectional variation in consumption levels. Section 5.3 presents our policy experiment: a reform of the bankruptcy code.

5.1 Insurance

We begin by assessing the ability of the model to reproduce the life-cycle pattern of life insurance holdings. In the baseline model, all households of a certain age and family type make the same insurance choice. In contrast, in the data there is substantial heterogeneity within each age group and family category, with many households not purchasing life insurance at all. Hence, we begin by focusing on the intensive margin of life cycle choice, and then return to this issue in section 6.1 where we consider an extended version of the model with additional heterogeneity that generates household choices along both intensive *and* extensive margins.

Figure 6 shows the empirical life-cycle profile of life insurance holdings of married households with children who hold some life insurance (intensive margin) in the data and as predicted by the model. As discussed before, the data display an inverted u-shape pattern: the median young married household holds around $85 thousand in life insurance, rising quickly to $200 thousand before declining slowly down to $75 thousand in their early 60's. Figure 6 shows that the model is able to match these data both qualitatively and quantitatively.

We next evaluate the extent of underinsurance in the model. Figure 7 depicts the ratio of the insurance payout (face value of policy) to the present value losses in the case of death, where present value losses are computed as described in the previous section, for the same group of households. As in figure 6, the model provides an excellent quantitative account of the data: in both the data and the model, young households are insured against roughly 30% of their potential loss, with the figure rising close to 100% only as households reach their late 50's. In other words, there exists a strong correlation between age and the degree to which households purchase insurance against mortality risk.

Our measure of underinsurance in figure 7 implicitly assumes that present value income losses are a good proxy for insurance needs. An alternative approach is to measure insurance needs by the insurance holdings implied by a frictionless version of our model with non-binding participation constraint. In other words, we can use the insurance coefficient I defined in (11). The result of this computation is depicted in figure 8. Specifically, we show the ratio of actual insurance holdings, both in the data and in the model with frictions, over insurance holdings in a model without frictions (full insurance). Figure 8 confirms the result already shown in figure 7: both in the data and in the model, there exists a strong correlation between age and the degree to which households purchase insurance against mortality risk. Indeed, the correlation between age and insurance (coefficient) is roughly the same in both figures; the only difference is that young households are insured against 30% of the losses implied by the present value of income losses and against 40% of the losses according to the model-based calculation. In other words, in the baseline model the present value of income losses is a very good measure of insurance need.

In the model, the human capital losses in the case of a husband's death are different from the human capital losses in the case of a wife's death. Consequently, the life insurance holdings for the two events differ. The SCF does not provide information about the split of insurance between husband and wife, but the SIPP data provide information about this split. In figure 9 we plot life insurance holdings separately for husband and wife, where we again focus on married households with children who have purchased some life insurance. The data show that in both cases there is an inverted u-shape, but this inverted u-shape is much more pronounced for insurance against the husband's death. Further, life insurance against the husband's death is about twice as much as life insurance against the wife's death. Figure 9 also shows that the model provides a good quantitative account of both life-cycle profiles, though on average the model slightly under-predicts holdings of insurance against the death of the husband, and slightly over-predicts the holdings of insurance against the death of the wife.

Figures 6-9 analyze to what extent households insure against mortality risk. Households in the model also face labor market risk and demographic risk in addition to mortality risk, and we next investigate the extent of insurance against all types of risk. To this end, we consider the model implication for the life-cycle variation of consumption insurance measured as lack of consumption volatility. More precisely, we define an insurance measure $1-\sigma_c/\sigma_{a,c}$, where σ_c is the standard deviation of equilibrium consumption growth for households with full access to financial markets and $\sigma_{a,c}$ is the standard deviation of equilibrium consumption growth for households with no access to credit and insurance markets (but the possibility of saving in a risk-free asset). This insurance measure is similar to the insurance coefficient introduced by Blundell et al (2008). Figure 10 shows that consumption insurance increases substantially with age. For example, the value of our insurance measure begins at 0.24 for households age 23 and increases to 0.81 for households age 60.

5.2 Wealth and Consumption

An essential feature of our mechanism generating under-insurance of young households is that young household have little financial wealth relative to their human wealth. In our model, the portfolio mix between human and financial capital is measured by θ_h, the fraction of total wealth invested in human capital. Empirically, we construct a measure of portfolio holdings by taking the ratio of (net) financial wealth to labor earnings, and compare this to the model generated analog which is given by $\frac{1-\theta_h}{\phi r_h \theta_h}$. Figure 11 shows the life-cycle profile of this ratio in the SCF data and according to the model. Clearly, the model provides a very good account of the this dimension of the data for young households, and matches the observed increase in financial wealth relative to human wealth through age 50, although it over-predicts wealth holdings for the oldest households. We view this as a success of the model, as it has not been calibrated to match this target. In other words, one basic prediction of the theory, namely that households with high expected human capital returns should be heavily invested in human capital, is qualitatively and quantitatively supported by the empirical evidence.

Another important dimension of the data is the consumption dispersion over the life-cycle. Figure 12 compares the variance of log adult-equivalent consumption in the US, estimated using data from the Consumer Expenditure Survey, from three different studies—Aguiar and Hurst (2008), Deaton and Paxson (1994), and Primciceri and van Rens (2009)—to the corresponding variance implied by the model. The figure shows that the model captures the increase in consumption dispersion observed in the data. Indeed, the model matches quite well the estimates of consumption dispersion reported by Aguiar and Hurst (2008), in particular the concave shape of the life-cycle profile of consumption dispersion. Note that these estimates are very similar to the one found in Heathcote et al. (2010).

5.3 The Impact of Reforms of the Consumer Bankruptcy Code

There has been a long-standing debate among academic scholars and policy makers as to the relative merits of alternative consumer bankruptcy codes. Our findings suggest that reforms to bankruptcy codes that work to increase the pledgeability of human capital will not only directly increase the ability of households to borrow to consume and invest in human capital, but may also lead households to purchase more insurance against human capital risk which in turn results in further human capital investment. In this section, we explore the effects of a change in consumer bankruptcy codes on household borrowing, insurance, and welfare.

Our experiment is motivated by the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005, which made it more difficult to file for bankruptcy under chapter 7—where debt is only repaid out of existing assets—and therefore forced more households to file under chapter 13 of the US bankruptcy code, where debts are repaid out of current earnings over a period of 3 to 5 years. See, for example, White (2007) for a detailed account of the US bankruptcy code before and after the reform. We implement this experiment by assuming that after implementation of the BAPCPA, in the event of bankruptcy, 30% of households are randomly assigned to file under chapter 13.[22] In line with the code, we model the consequence of filing for bankruptcy under chapter 13 as an exclusion from borrowing and insurance markets for an average of 4 years and a 25 percent garnishment of labor income during the period of exclusion. The BAPCPA also increased bankruptcy filing costs significantly, and we incorporate this change in legislation by introducing a one-off cost of $2000 that is paid in the year of filing for bankruptcy by all defaulting households. Finally,

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[22] An important change the BAPCPA introduced was the "means test". This means test restricted filing under chapter 7 to those households with income below median income adjusted for family type, which suggest that after the reform 50 percent of all households are forced to file bankruptcy under chapter 13. However, defaulting households differ from non-defaulting households, and we take account of this fact by assuming that only 30 percent defaulting households are forced into chapter 13 after the reform. The number of 30 percent corresponds to the fraction of defaulting households in our SCF 2004 sample who have above median income.

the BAPCPA increased the minimum number of years that have to pass until a consumer can file a second time under chapter 7 from 7 years to 8 years, and we incorporate this change in legislation by assuming that households filing for bankruptcy under chapter 7 are excluded for 8 years after the reform (instead of 7 years before the reform).

We compute the welfare consequences of the reform by comparing the lifetime utility in the two economies, before and after the reform. For this comparison, we compute for each age group j and state s the welfare change $\Delta_j(s)$ as the equivalent variation of the bankruptcy reform, measured in units of lifetime consumption, and then average over s using the fixed stationary distribution over s (this distribution over exogenous shocks is not affected by the policy experiment). Note that the welfare change $\Delta_j(s)$ is independent of the initial wealth level of a household so that in our case there is no need to average over wealth using an endogenous wealth distribution. Note further that we conduct a steady state comparison in the sense that we do not take into account the transition path of the aggregate capital-to-labor ratio $\tilde{K}$ (and the corresponding transition path of investment returns).

Figure 13 plots the results of our welfare analysis. The blue line shows that the equivalent variation of the reform is positive for all age groups: it peaks for the youngest households at 0.43%, falling to 0.2% for households age 40, and then increase to 0.34 for households age 60.[23] A welfare gain of almost half percent of lifetime consumption is substantially larger than any gain that the model of Krueger and Perri (2006) would predict, where households are almost fully insured even before the reform. Our welfare results also differ from Chatterjee et al. (2007) and Livshits et al. (2007). In our model, young households gain because after the reform they are able to borrow greater amounts to invest in faster human capital accumulation *and* buy more insurance. Indeed, as shown in figure 14, which depicts the life-cycle profile of the ratio of net worth over labor income, the youngest households have negative net worth after the reform. In contrast, in Chatterjee et al. (2007) and Livshits et al. (2007) households use the additional debt only to improve consumption smoothing over the life-cycle.

[23] The non-monotonicity of the welfare gains is due to general equilibrium effects that work against young households: more human capital accumulation reduces the aggregate capital-to-labor ratio, $\tilde{K}$, and this in turn increases the interest rate and decreases expected human capital returns.

The fact that households invest more in human capital after the reform implies that the capital-to-labor ratio, $\tilde{K}$, increases. We find that the reform of the bankruptcy code increases $\tilde{K}$ by 1.5 percent of its initial value of 0.4. In our endogenous growth model, any change in $\tilde{K}$ also changes the aggregate growth rate of the economy. Specifically, the equilibrium value $\tilde{K}$ is in general lower than the value of $\tilde{K}$ that maximizes aggregate growth, and an increase in $\tilde{K}$ increases aggregate growth.[24] In our calibrated model economy, the growth gain is relatively modest: the annual growth rate increases ny 0.04 percentage point.

The welfare change of policy reform shown in figure 13 has two components: a gain due to improved consumption insurance and a gain due to more economic growth. To separate these two effects, we also consider the welfare consequences of the policy reform under the assumption that households' human capital allocation decisions are fixed at their levels before the reform. In this case, welfare gains are only due to the increase in consumption insurance and growth effects due to capital reallocation are absent. As expected, figure 13 shows that now the welfare gains are monotonically decreasing in age: they start at around 0.5% for the young and fall to almost zero for the oldest households who have very little exposure to human capital risk. The fundamental difference between the two policy experiments can be seen in figure 15, where we plot the insurance measure $1 - \sigma_c/\sigma_{a,c}$ before and after the reform for the two policy experiments. When households can vary their human capital choice, consumption insurance only increases slightly as the increase in insurance purchases is counteracted by a higher exposure to human capital risk. By contrast, when human capital holdings, and hence the level of human capital risk, are held constant, consumption insurance rises significantly.

6. Extensions and Robustness

Our results turn out to be robust to a number of changes in the specification of the model. In this section, we describe some of these robustness exercises with figures relegated to the Appendix. We also report on two extensions to the model that allow us to capture the non-participation decision of some households, and that explore the effects of cost of living

[24] This growth effect has also been discussed in Krebs (2003).

differences on the life insurance decisions of households.

6.1 Extensive and Intensive Margin

The previous analysis has shown that our theory matches well the observed life-cycle variation of the median level of life insurance holdings for those households who have purchased some life insurance (intensive margin). However, if we consider the data on all married households with children, including those who have not purchased life insurance (extensive margin), then the calibrated model overstates the degree of insurance. This is to be expected: our model does not have any heterogeneity within any group of households with the same age and family status and, since the median household purchases some life insurance, our model generates the median holding of participating households. In this section, we discuss an extension of the model that accounts for both the intensive margin and the extensive margin of the life-cycle variation in life insurance holdings.

The extension we pursue in this section introduces a fixed cost that has to be paid once after entering the life insurance market for the first time. For simplicity, we assume that this fixed cost is an additive utility cost that is normally distributed with an age- and type-independent variance and a mean that is age-independent, but varies with the number of kids. This fixed cost can be thought of as capturing the time cost associated with understanding the life insurance market, the dis-utility that comes from the inconvenience of an invasive medical exam, and the psychological distress associated with planning for the death of a loved one. Households who have paid the fixed cost solve a decision problem identical to the one we have analyzed so far. Households who have not paid the fixed cost solve a decision problem with access to all financial assets except those financial assets that pay off in the case of death of a family member (no access to life insurance).

When we choose the size of the fixed cost to match the average level of participation of each family size over the life cycle, we obtain values for the cost that are on the order of \$400 for families with 1 or 2 kids, falling to under \$50 for families with 3 kids. Figure 16 depicts the life-cycle profile of insurance holdings for all married households with children and those married households with children who have purchased some life insurance. Figure 16 shows that this version of the model provides a good account of the life cycle profile of both dimensions of the life insurance data: the extensive margin and the intensive margin. This is confirmed by figure A10 in the Appendix, which shows the life-cycle profile of the participation rate for married households with children. From figure A10 we conclude that the extended model provides an excellent match of the data on participation rates in the life insurance market.

6.2 Annuities, Life Insurance and Bequests

In our baseline model, prior to retirement all agents can buy a complete set of insurance products, including both life insurance and annuities. However, we constrain retirees to save in a risk free security with any wealth remaining at their death distributed to newborn households. In the absence of this constraint, and without a bequest motive, retirees would only purchase annuities. We briefly discuss a variant of our model in which retirees have a bequest motive and are able to buy annuities.

Suppose that retired households preferences are augmented with a bequest motive in the form of an additive utility term of the form $v(b, s) = \kappa * u(b, s)$ where b are bequests and κ governs the strength of the bequest motive. Note that under this assumption, the homotheticity properties of the model are preserved. If annuities are priced in an actuarially fair manner, and if κ is chosen so that the marginal utility of a unit of bequests equals the marginal utility of a unit of annuity wealth, then retirees will choose a level of bequests that equals their annuity wealth. This may be implemented by a portfolio with equal holdings of annuities and life insurance, which is equivalent to the restriction imposed in our baseline model[25].

This turns out to be a not unreasonable description of the data. Although Johnson, Burman and Kobes (2004) estimate that people aged 65 and older hold on average just 1% of their wealth in *private* annuities, Gustman, Mitchell, Samwick and Steinmeier (1997) estimate that people aged 51-61 hold between one quarter and one half of their wealth in annuity-like pensions and social security. Thus, we conclude that our restriction on retirees

[25]These decisions may also be implemented by holding equal amounts of life insurance and annuities with the remainder of their wealth in a risk free asset. Note that we are abstracting from the fact that life insurance can be used to avoid gift and inheritance taxes.

portfolio choices is relatively innocuous. In the next subsection, we return to this issue in the context of younger households.

6.3 Child- and Health-Dependent Preferences

Two recent papers (Koijen et al. 2012 and Hong and Rios-Rull 2012) have used data on life insurance holdings, and holdings of other assets, to estimate the evolution of household preferences as they age and decline in health, the strength of the bequest motive, as well as the effect of changes in household size on the cost of living. In both of these papers, patterns in life insurance data are assumed to be driven solely by variation in preferences and cost of living parameters, in contrast to our paper, where under-insurance of young households is generated through borrowing constraints.[26] Motivated by these papers, in this section we consider an extension of our model that incorporates household preferences that depend on the number of children and the health status of the household. To simplify the discussion, we consider a model without fixed costs of entering the life insurance market and focus on the intensive margin.

We introduce two changes to our baseline model. First, we parameterize the change in the marginal utility of consumption of a household following the death of a spouse, with the parameter varying with the number of children in the household. This may be interpreted as capturing changes in the cost of living (for example, if it is cheaper to live with a smaller household, the marginal utility should decline) beyond the simple insurance component accounted for in our baseline model (see Section 4.2.1), or as capturing the strength of the bequest motive for younger households. In terms of the model, we allow the marginal utility of consumption, γ_1, to change following the death of an adult household member or divorce, and assume that the size of the change may vary with the number of children. To reduce the number of free parameters, we assume that γ_1 is the same for all married households independently of the number of children and that for single households γ_1 is independent of sex (male/female) and marital status (divorcee/widow). We normalize γ_1 of married

[26] Koijen et al. (2012) study a complete-market model without financial frictions. Hong and Rios-Rull (2012) use an incomplete-market model with ad-hoc borrowing constraint, and in principle households in their model can be borrowing constrained and less than fully insured. However, for all households with positive financial wealth, full insurance obtains in their model. See also section 6.4.

households to one and choose the value of the remaining parameters to match the life-cycle average of life insurance holdings of married households with different number of children separately.

The second change to the model specification is the introduction of a health state. Following Koijen et al (2012), we assume that households can be either in good health or in bad health and that households in bad health have lower marginal utility of consumption, γ_1. Further, a married household in bad health who experiences the death of an adult household member becomes a healthy single household. This assumption captures the idea that it is the sick member of the household who dies, an assumption that seems plausible especially for older households. Finally, we assume that up to age 35 all households are in good health and that starting age 35 the probability of becoming sick (moving into bad health) increases linearly. Thus, we have parameterized the health process by two parameters, and we calibrate these two parameters to match two targets taken from Koijen et al. (2012), namely the relative number of households who move from good health into bad health in the age group $50-60$ and the difference in the demand for life insurance between bad health and good health households ages $50-60$.[27]

Our results can be summarized as follows. First, with the changes added to the model, the basic facts about life insurance and other asset holdings over the life-cycle for all married households with children are unchanged. For example, the models prediction for the median life insurance holdings of households with children is barely affected by this change in model specification. In figure A11 in the Appendix we plot the life-cycle profile of life insurance holdings for all married household with children, and find that it is very close to the plot for our baseline model (figure 6). Second, this extension improves the match between model and data in the sense that the extended model replicates additional cross-sectional facts. Specifically, households in bad health demand more life insurance than households in good health and households with two and three children hold substantially more life insurance

[27] Koijen et al. 2012 report that 20 percent of all households age $50-60$ move from good health to bad health. In a early working paper version, they also report the results of a regression that shows that moving from good health to poor health adds about $50,000$ of life insurance controlling for age and other explanatory variables.

than households with one child. In particular, the extended model implies that, consistent with the data, having one additional child increases the bequest motive by an amount that is equal to $\$25,000$ of life insurance holding for a household with one child and by $\$10,000$ for a household with two children. Third, if we interpret the change in marginal utility following the death of a parent as reflecting the consequent change in the cost of living, the resulting changes are relatively modest and increase in the number of kids: the cost of living falls roughly 4% for households with either no kids or 1 kid, rises 2% of households with 2 kids, and by 3% for households with four kids. Equivalently, this may be interpreted as the strength of the bequest motive for young households rising with the number of kids.

6.4 Comparison With Incomplete Market Model

In this section, we compare the performance of our theory to an incomplete markets model in which agents may borrow and save using a risk-free asset subject to an exogenously imposed borrowing constraint, and may purchase life insurance, but are exogenously prohibited from accessing other financial assets. We continue to assume that the life insurance market is competitive with actuarially fair prices, and that preferences are specified as in our baseline model where the marginal utility of household consumption is not affected by the death of a spouse.

It is straightforward to verify that the incomplete-market model predicts that, if households have positive net financial wealth and are hence not borrowing constrained, they will be fully insured against mortality risk and life insurance holdings cover the entire human capital loss in the case of death. In other words, the incomplete-market model predicts that the ratio of life insurance holding over the human capital loss, the insurance coefficient I, is always 1 independently of age for households with positive net financial wealth. By contrast, in our theory, households may hold positive net financial assets and yet still be constrained in their ability to borrow against some subset of the possible future states of nature. This stark prediction of the incomplete markets model is strongly rejected by the data. Figure A12 in the Appendix plots the insurance coefficient I using SCF data on only those married households with children that have positive net worth, where the human capital loss is based on the present value of income losses as described in Section 2.3. As in figure 2, the under-insurance of young families with positive net worth depicted in figure A12 is severe, while older households are almost fully insured. Indeed, there is almost no difference between the life-cycle profiles of insurance depicted in figure 2 and figure A12.

There are, of course, a continuum of incomplete markets models that differ in the restrictions on financial markets that are exogenously imposed. Indeed, by allowing a complete set of assets and carefully choosing exogenous borrowing constraints, it is possible to construct a variant of the incomplete markets model that exactly replicates the equilibrium of our baseline model. More generally, our findings suggest that for any incomplete-market model to match the data on underinsurance it must allow agents to purchase a sufficiently rich array of financial assets so that they can be constrained in their borrowing against income earned in some state tomorrow, while still holding positive net financial assets on average. Of course, an incomplete-market model with sufficiently many assets is observationally very similar to our model, except that our modeling approach is more tractable and determines borrowing constraints endogenously.

7. Conclusion

In this paper, we developed a tractable macroeconomic model in which households accumulate human capital that is both idiosyncratically risky and non-pledgeable against consumer debt. We used the framework to analyze the possible causes and consequences of under-insurance. The results of this paper suggest three lines of future research.

The first concerns the measurement of the extent of insurance against the various forms of human capital risk. In the paper, we restricted attention to insurance against one form of human capital risk, the death of a family member, that is important (large shock size), readily quantifiable, and most likely not subject to issues of moral hazard or adverse selection. Future research on the observed lack of insurance against human capital risk needs to quantify the extent of under-insurance against other sources of risk.

A second line of research concerns the extent of unobserved heterogeneity in returns to human capital across the population. Heterogeneity of human capital returns due to ability differences has been central to the work by, among others, Guvenen et al. (2011), Hugget et al (2011), and Cunha, Heckman, and Navarro (2005). In the current paper, we restricted attention to differences in returns by age, and argued that this dimension of heterogeneity can go a long way towards explaining a number of empirical facts about human capital choice and under-insurance. An important task for future research is to determine the extent to which additional heterogeneity is important in explaining additional empirical facts about human capital choice, borrowing, and insurance.

Finally, a third line of research would broaden the set of assets available to households. The most important alternative asset is housing, which is also risky and which is, to varying degrees, collateralizable. All else equal, the perceived (utility) rates of return to housing investment are large, so that access to this asset will further strengthen the results of this paper: households would like to borrow to invest in housing and human capital, and these investment opportunities will compete with the need to purchase insurance. To what extent this effect is offset by the fact that some housing wealth can be used as collateral against borrowing remains an open quantitative question.

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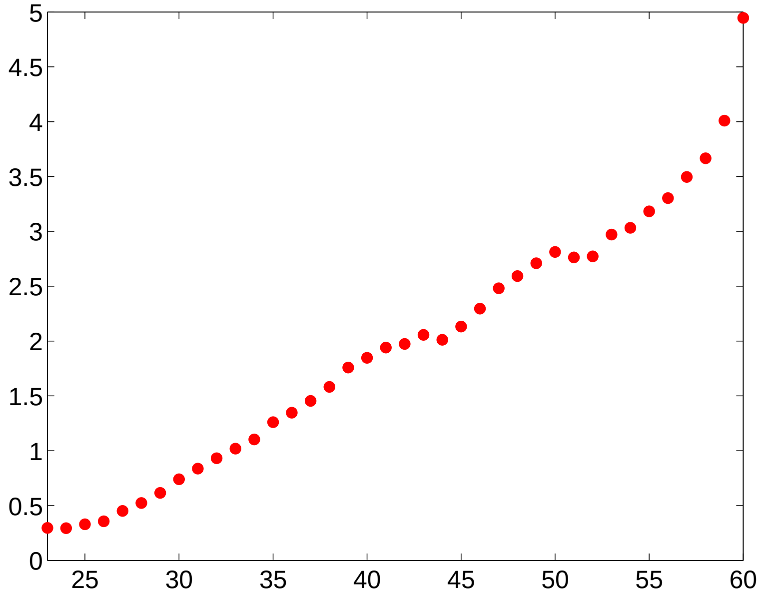
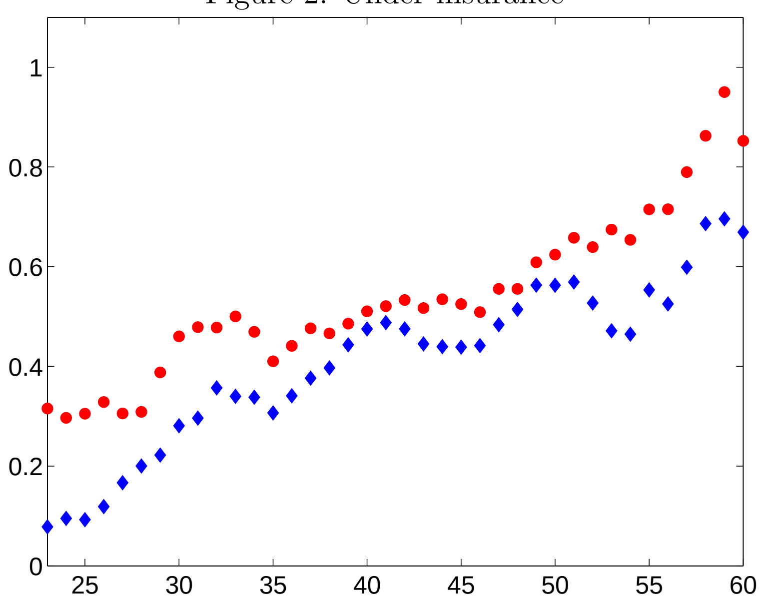
Figure 1: Networth to labor income ratio



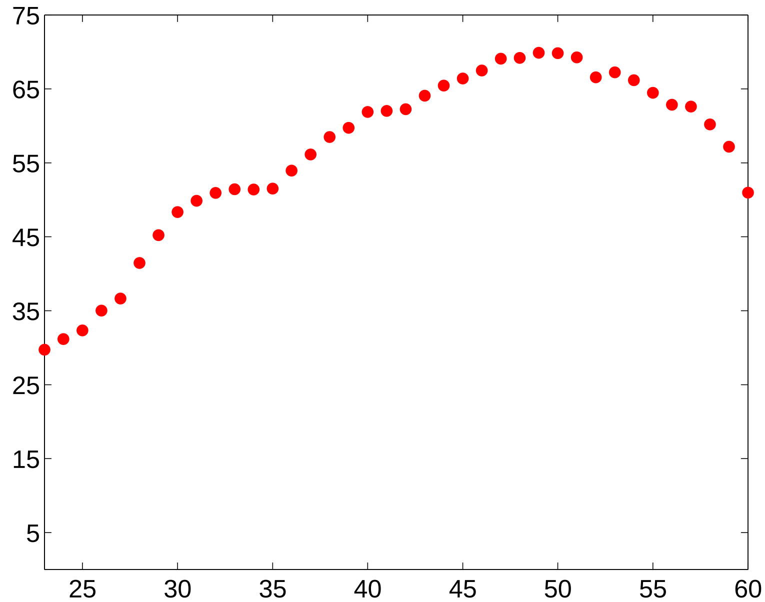
Notes: Life-cycle profile of the median ratio of networth to labor income for married households age 23 - 60 with children from the SCF, surveys 1992 - 2007.

Figure 2: Under-insurance



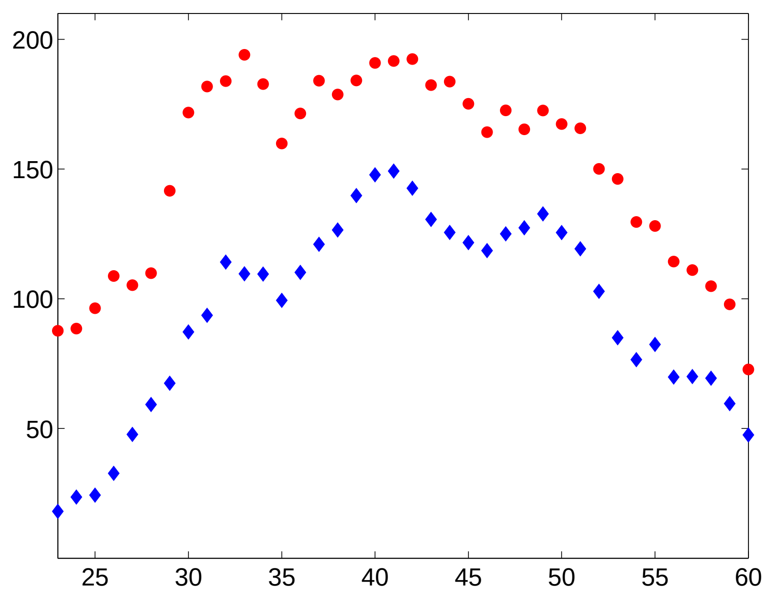
Notes: Ratio of life-insurance holdings to present value income loss in case of death. Red dots are for married households with children that have purchased life insurance. Blue diamonds are for all married households with children. All data are for households age 23 - 60 from the SCF, surveys 1992 - 2007 and show medians of the respective groups. See appendix for calculation of present value loss.

Figure 3: Labor income



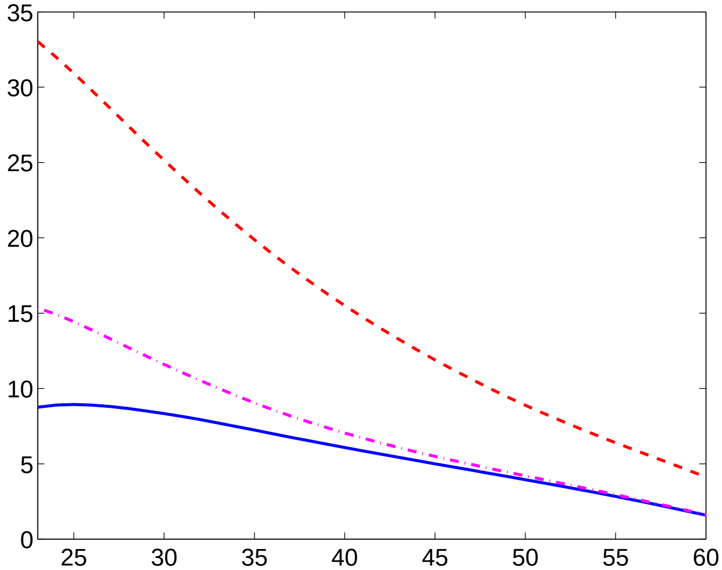
Notes: Life-cycle profile of median labor income for married households age 23 - 60 with children from the SCF, surveys 1992 - 2007 (thousands of year 2000 dollars).

Figure 4: Face value of life insurance



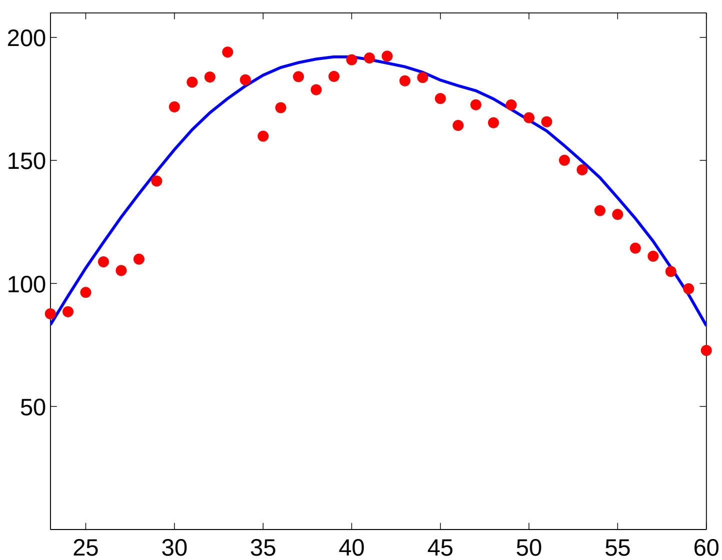
Notes: Life-cycle profile of face value of all life insurance contracts (thousands of year 2000 dollars). Red dots are for married households with children that have purchased life insurance. Blue diamonds are for all married households with children. All data are for households age 23 - 60 from the SCF, surveys 1992 - 2007 and show medians of the respective groups.

Figure 5: Human capital loss



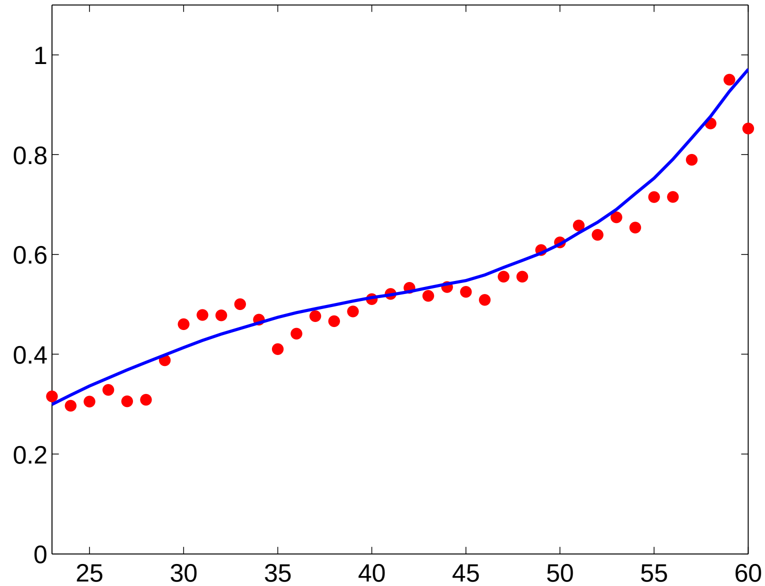
Notes: Life-cycle profile of sum of expected human capital loss in case of husband's and wife's death for all married households with children. Human capital loss is ratio of present value labor income loss over current labor income. Red dashed line: loss before transfers and taxes with zero probability to remarry. Pink dashed-dotted line: loss after transfers and taxes and zero probability to remarry. Blue solid line: loss after transfers and taxes and empirical remarriage rates. All data are for households age 23 - 60 from the SCF, surveys 1992 - 2007. See appendix for further details.

Figure 6: Life insurance



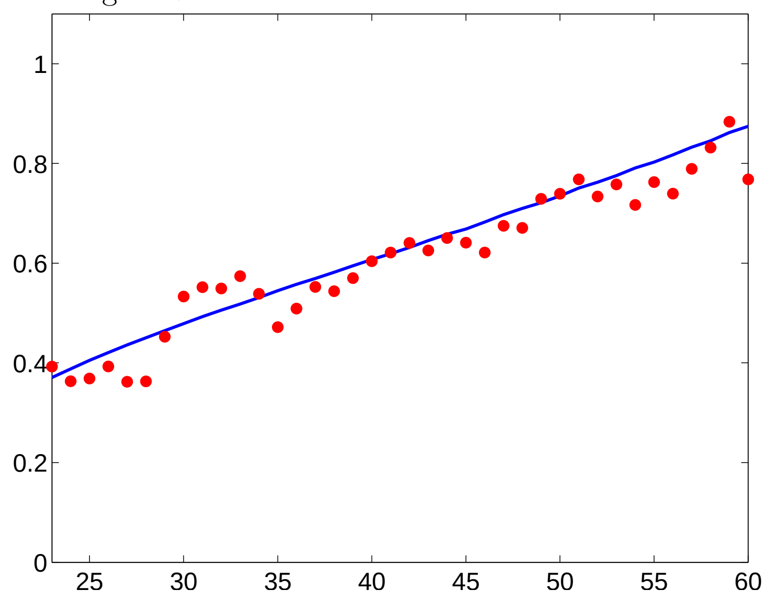
Notes: Life-cycle profile of face value of life insurance contracts (thousands of year 2000 dollars) for married households age 23 - 60 with children. Blue solid line shows model. Red dots show median of all households that have purchased life insurance from the SCF data.

Figure 7: Under-insurance



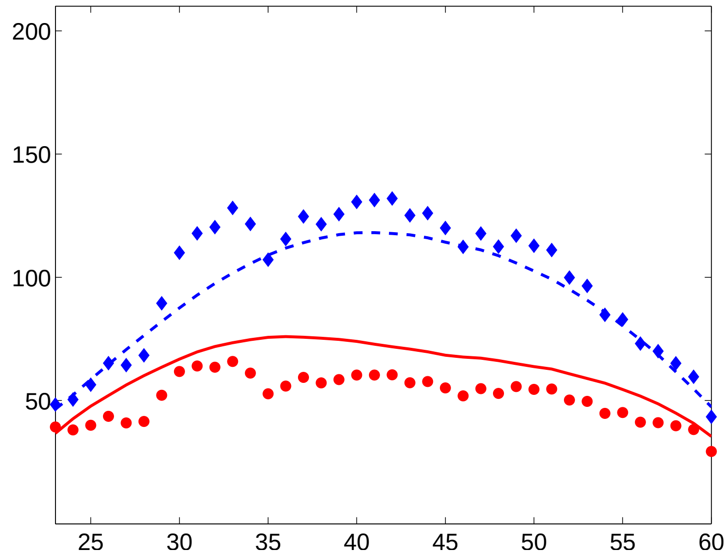
Notes: Ratio of life-insurance holdings to present value income loss for married households age 23 - 60 with children. Blue solid line shows model. Red dots show data for all households that have purchased life insurance.

Figure 8: Life insurance to full insurance ratio



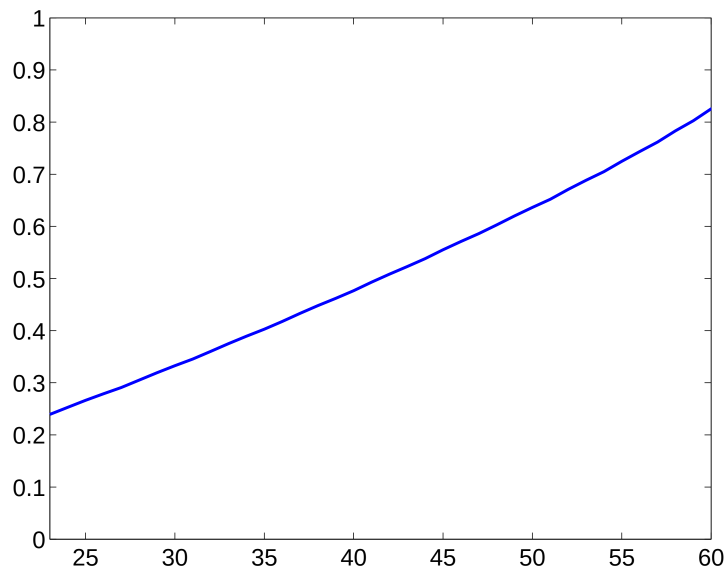
Notes: Ratio of life insurance holdings over full insurance for married households age 23 - 60 with children. Blue solid line shows model. Red dots show data for all households that have purchased life insurance.

Figure 9: Life insurance for husband and wife



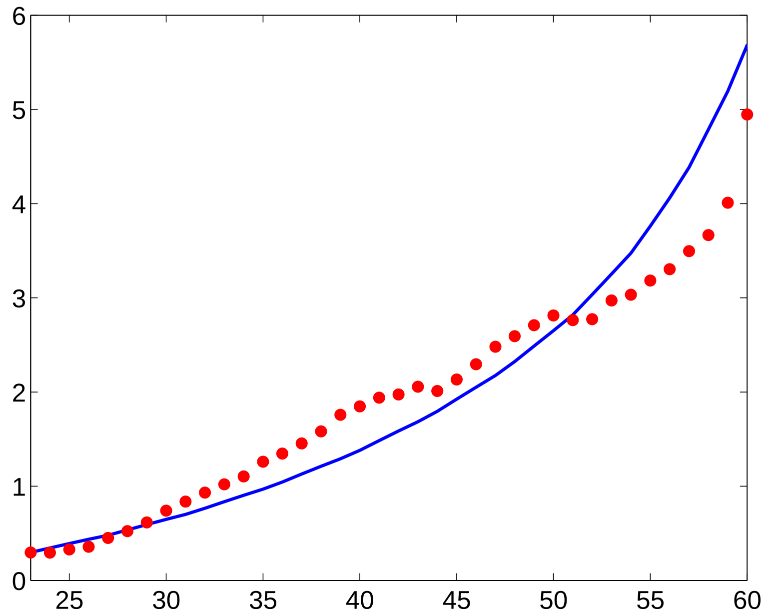
Notes: Life-cycle profile of face value of life-insurance contracts (thousands of year 2000 dollars) for married households age 23 - 60 with children. Red solid line shows face value of life insurance for wife's death from model. Red dots show face value of life insurance for wife's death from data. Blue dashed line shows face value of life insurance for husband's death from model. Blue diamonds show face value of life insurance for husband's death from data. Data are from the SCF and the SIPP. See appendix for details of the construction of data profiles.

Figure 10: Consumption insurance



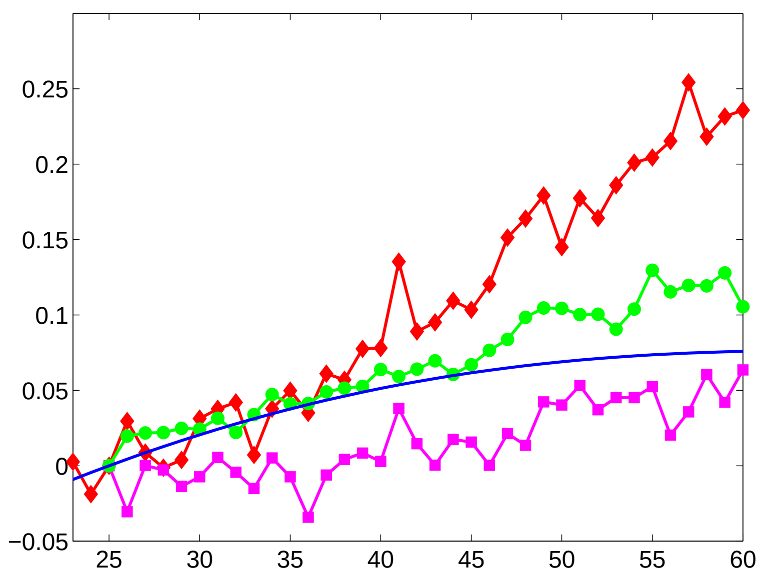
Notes: Consumption insurance in the model for married households age 23 - 60 with children. The insurance measure is one minus the ratio of the standard deviation of consumption in equilibrium relative to the standard deviation of consumption in financial autarky.

Figure 11: Networth to labor income ratio



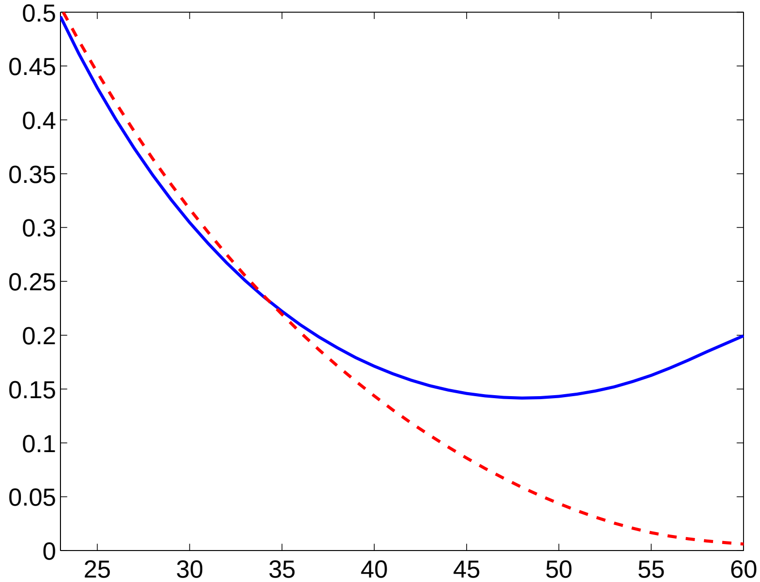
Notes: Life-cycle profile of the median ratio of networth to labor income for married households age 23 - 60 with children. Blue solid line shows model and red dots SCF data.

Figure 12: Consumption inequality



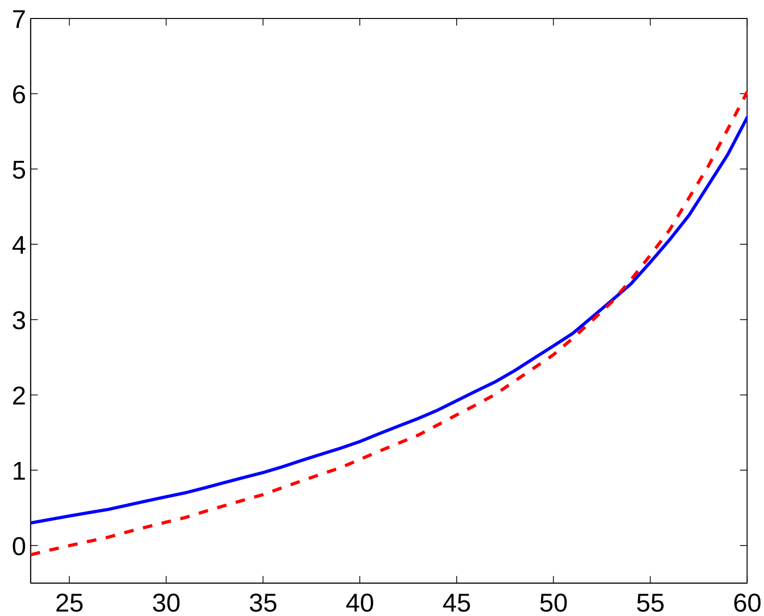
Notes: Life-cycle profile of the cross-sectional variance of consumption. The blue solid line shows the model prediction. The red diamonds show the profile estimated by Deaton and Paxson (1994), the green dots are the estimates of Aguiar and Hurst (2008), and the pink squares are the estimates of Primiceri and van Rens (2009). The data have been normalized to 0 at age 25.

Figure 13: Welfare gains



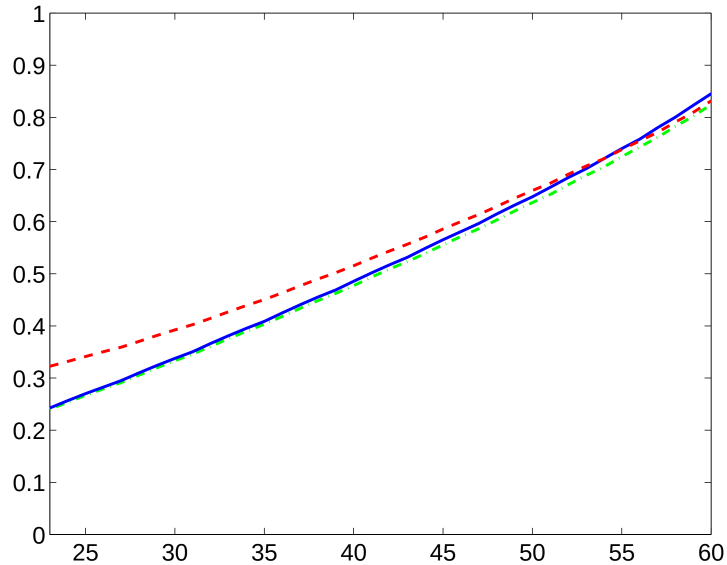
Notes: Life-cycle profile of welfare gain (equivalent variation) of policy experiment for married households age 23 - 60 with children. The blue solid line shows model with endogenous human capital allocation. The red dashed line shows model with fixed human capital allocation.

Figure 14: Networth to labor income ratio



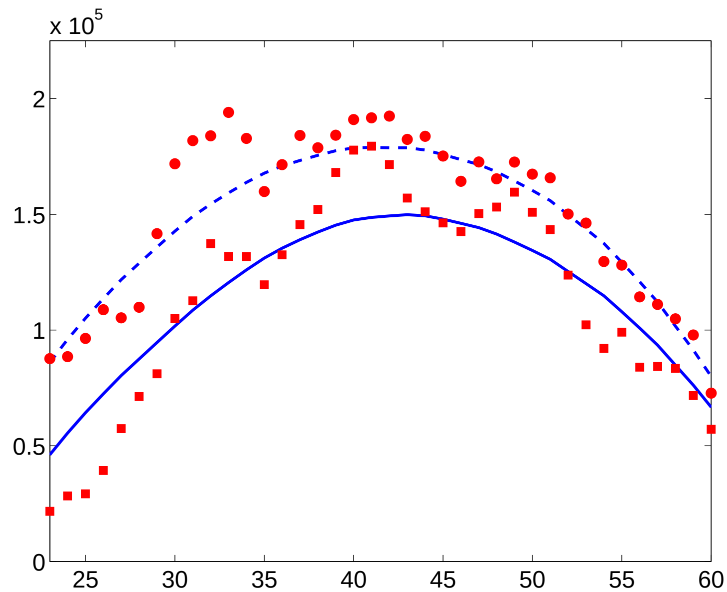
Notes: Life-cycle profile of the median ratio of networth to labor income for married households age 23 - 60 with children. The blue solid line shows model with endogenous human capital allocation. The red dashed line shows benchmark model.

Figure 15: Consumption insurance



Notes: Life-cycle profile for consumption insurance after policy experiment for married households age 23 - 60 with children. The blue solid line shows model with endogenous human capital allocation. The red dashed line shows model with fixed human capital allocation. The green dashed dotted line shows benchmark model.

Figure 16: Life insurance extensive and intensive margin



Notes: Life-cycle profile of face value of life insurance contracts (thousands of year 2000 dollars) for married households age 23 - 60 with children. Blue dashed line shows model prediction for all households that have purchased life-insurance. Red dots show median of all households that have purchased life insurance from the SCF data. Blue solid line shows model prediction for all households. Red squares show median of all households from the SCF data.

Appendix

A1. Proof of proposition 1.

Define total wealth (human plus financial) of a household of age j, w_j, the portfolio choice, θ_j, and the total investment return, r_j as in Section 3.4. Using this notation, the sequential budget constraint is given in (8). For age $j = 1, \ldots, J$, the Bellman equation associated with the household utility maximization problem reads:

$$
\begin{aligned}
V_j(w_j, \theta_j, s_j) \;&=\; \max_{c_j, w_{j+1}, \theta_{j+1}} \left\{ \gamma_0(s_j) + \gamma_1(s_j) ln\, c_j + \beta \sum_{s_{j+1}} V_{j+1}\left(w_{j+1}, \theta_{j+1}, s_{j+1}\right) \pi_j(s_{j+1}|s_j) \right\} \\
s.t. \qquad w_{j+1} \;&=\; (1 + r_j(\theta_j, s_j))w_j - c_j \qquad\qquad (A1) \\
1 \;&=\; \theta_{h,j+1} + \sum_{s_{j+1}} q_j(s_{j+1}|s_j)\theta_{a,j+1}(s_{j+1}) \\
c_j \;&\geq\; 0 \;\;,\;\; w_{j+1} \geq 0 \;\;,\;\; \theta_{h,j+1} \geq 0 \\
V_{j+1}(w_{j+1}, \theta_{j+1}, s_{j+1}) \;&\geq\; V_{d,j+1}(w_{j+1}, \theta_{h,j+1}, s_{j+1}) \;,
\end{aligned}
$$

In default, a household who defaults at age j chooses a continuation plan, $\{c_{j+n}, h_{j+n}\}$, so as to maximize

$$
\begin{aligned}
& \sum_{n=0}^{J-j} (p\beta)^n \sum_{s^{j+n}|s^j} \left[\gamma_0(s_{1,j+n}) + \gamma_1(s_{2,j+n})\, lnc_{j+n}(s^{j+n})\right] \pi(s^{j+n}|s_0) \\
+ & \sum_{n=0}^{\infty} (p\beta)^{J+1-j+n} \sum_{s^{J+1+n}|s^j} V_{J+1}(h_{J+1+n}(s^{J+n}), a_{J+1+n}(s^{J+1+n}), s_{J+1+n})\pi(s^{J+1+n}|s_0) \\
+ & \sum_{n=0}^{J-j} ((1-p)\beta)^n \sum_{s^{j+n}|s^j} V^e_{j+n}(h_{j+n}(s^{j+n-1}), s_{j+n})\pi(s^{j+n}|s_j) \\
+ & \sum_{n=0}^{\infty} (p\beta)^{J+1-j+n} \sum_{s^{J+1+n}|s^j} V^e_{J+1+n}(h_{J+1+n}(s^{J+n}), , s_{J+1+n})\pi(s^{J+1+n}|s_0)
\end{aligned}
$$

where $\{c_{j+n}, h_{j+n}\}$ has to solve the sequential budget constraint (3) with $a_j = 0$. Define the investment return of a household in default as $r_d(\theta_{hj}, s_j) = (1+z_j(s_j)\phi r_h(s_j) - \delta_{hj} + \eta_j(s_j))\theta_{hj}$, which is simply the human capital return times the fraction of wealth invested in human capital. In the period of default, we have in general $\theta_{hj} \neq 1$, but in all periods subsequent to default we have $\theta_{h,j+n} = 1$. In the period of re-gaining access to financial markets, a household in default has no financial assets, and we still have $\theta_{h,j+n} = 1$. The Bellman equation of a household in default reads

$$
V_{dj}(w_j, \theta_{hj}, s_j) \;=\; \max_{c_j, w_{j+1}} \left\{ \gamma_0(s_j) + \gamma_1(s_j) lnc_j + p\beta \sum_{s_{j+1}} V_{d,j+1}\left(w_{j+1}, 1, s_{j+1}\right) \pi_j(s_{j+1}|s_j) \right.
$$

$$
\begin{aligned}
& + \quad (1-p)\beta \sum_{s_{j+1}} V^e_{j+1}\left(w_{j+1}, 1, s_{j+1}\right) \pi_j(s_{j+1}|s_j) \Bigg\} \qquad (A2) \\
s.t. \qquad & w_{j+1} \;=\; (1 + r_{dj}(1, s_j))w_j - c_j \\
& c_j \;\geq\; 0 \;\;,\;\; w_{j+1} \geq 0
\end{aligned}
$$

The Bellman equation (A2) for the default value function together with the Bellman equation (A1) and the condition $V^e = V$ define a Bellman equation determining simultaneously the value function V and V_d. Suppose that the terminal value function V_{J+1} has the functional form (A7). Solving the problem backwards, guess-and-verify shows that the solution to this Bellman equation (A1) and (A2) for all $j = 1, \ldots, J$ is

$$
\begin{aligned}
V_j(w_j, \theta_j, s_j) &= \tilde{V}_{0j}(s_j) + \tilde{V}_{1j}(s_j)\left[\ln w_j + \ln(1 + r_j(\theta_j, s_j))\right] \qquad (A3) \\
c_j(w_j, \theta_j, s_j) &= \tilde{c}_j\left(1 + r_j(\theta_j, s_j)\right) w_j \\
V_{dj}(w_j, \theta_j, s_j) &= \tilde{V}_{d,0j}(s_j) + \tilde{V}_{1j}(s_j)\left[\ln w_j + \ln(1 + r_{dj}(\theta_{hj}, s_j))\right] \\
c_j(w_j, \theta_j, s_j) &= \tilde{c}_j\left(1 + r_{dj}(\theta_{hj}, s_j)\right) w_j
\end{aligned}
$$

with

$$
\tilde{c}_j(s_j) \;=\; \frac{\gamma_1(s_j)}{\tilde{V}_{1j}(s_j)}
$$

The coefficients $\tilde{V}_{1j}$ are determined recursively as the solution to

$$
\tilde{V}_{1j}(s_j) \;=\; \gamma_1(s_j) + \beta \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1})\pi_j(s_{j+1}|s_j)
$$

and the coefficients $\tilde{V}_{0j}$ and $\tilde{V}_{d,0j}$ together with the optimal portfolio choices θ^*_{j+1} are the solutions to the equation

$$
\begin{aligned}
\theta^*_{j+1} &= arg \max_{\theta_{j+1} \in \Gamma_{j+1}} \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \ln\left(1 + r_{j+1}(\theta_{j+1}, s_{j+1})\right) \pi_j(s_{j+1}|s_j) \qquad (A4) \\
\Gamma_{j+1} &\doteq \Bigg\{ \theta_{j+1} \,\Bigg|\, \theta_{h,j+1} + \sum_{s_{j+1}} \frac{\theta_{a,j+1}(s_{j+1})\pi_j(s_{j+1}|s_j)}{1 + r_f} = 1 \;\;,\;\; \theta_{h,j+1} \geq 0 \;\;, \\
& \frac{\tilde{V}_{0,j+1}(s_{j+1}) - \tilde{V}_{0d,j+1}(s_{j+1})}{\tilde{V}_{1,j+1}(s_{j+1})} \geq \left[ln(1 + r_{d,j+1}(\theta_{h,j+1}, s_{j+1})) - ln(1 + r_{j+1}(\theta_{j+1}, s_{j+1}))\right] \Bigg\} \;.
\end{aligned}
$$

and

$$
\tilde{V}_{0j}(s_j) \;=\; \gamma_0(s_j) + \gamma_1(s_j)\ln(\tilde{c}_j(s_j))
$$

$$
\begin{aligned}
&+ \beta \sum_{s_{j+1}} \tilde{V}_{0,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j) \\
&+ \beta \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \ln \Big(1 + r_{j+1}(\theta^*_{j+1}, s_{j+1})\Big) \pi_j(s_{j+1}|s_j) \\
&+ \beta \ln(1 - \tilde{c}_j(s_j)) \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{V}_{d,0j}(s_{1j}) &= \gamma_0(s_{1j}) + \gamma_1(s_j) \ln(\tilde{c}_j(s_j)) \\
&+ p\beta \sum_{s_{j+1}} \tilde{V}_{d,0,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j) \\
&+ (1-p)\beta \sum_{s_{j+1}} \tilde{V}_{0,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j) \\
&+ \beta \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \log\left(1 + r_{d,j+1}(1, s_{j+1})\right) \pi_j(s_{j+1}|s_j) \\
&+ \beta \ln(1 - \tilde{c}_j(s_j)) \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)
\end{aligned}
$$

This completes the proof for the case $j = 1, \ldots, J$.

If $j = J + 1$, the household has entered a transition period from which retirement occurs stochastically at constant probability p_{ret}. In this case, the household problem is an infinite-horizon maximization problem with value function constraint, and the corresponding Bellman equation is a version of (A1) and (A2) in which the age-index is replaced by the constant $J + 1$ (i.e. the index can be dropped) and there is a constant probability p_{ret} that the continuation utility is equal to a given continuation utility V_{ret}:

$$
\begin{aligned}
V_{J+1}(w,\theta,s) &= \max_{c,w',\theta'} \Big\{ \gamma_0(s) + \gamma_1(s) lnc + (1-p_{ret})\beta \sum_{s'} V_{J+1}(w',\theta',s')\, \pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \sum_{s'} V_{ret}(w',\theta',s') \Big\} \qquad (A5) \\
s.t. \quad & w' = (1 + r_{J+1}(\theta,s))w - c \\
& 1 = \theta'_h + \sum_{s'} q_{J+1}(s'|s')\theta'_a(s') \\
& c \geq 0 \ , \ w' \geq 0 \ , \ \theta'_h \geq 0 \\
& V_{J+1}(w',\theta',s') \geq V_{d,J+1}(w',\theta'_h,s') \ ,
\end{aligned}
$$

and

$$
V_{d,J+1}(w,\theta_h,s) = \max_{c,w'} \Big\{ \gamma_0(s) + \gamma_1(s) lnc + p\beta \sum_{s'} V_{d,J+1}(w',1,s')\, \pi_{J+1}(s'|s)
$$

$$
\begin{aligned}
&+ \quad (1-p)\beta \sum_{s'} V^{e}_{J+1}(w', s')\, \pi_{J+1}(s'|s) \Big\} \\
s.t. \quad & w' \;=\; (1 + r_{d,J+1}(1, s))w - c \\
& c \geq 0 \;\;,\;\; w' \geq 0
\end{aligned}
$$

where we assumed that there is no retirement when the household is in default. We first discuss the retirement problem defining V_{ret} and then analyze the household problem in the pre-retirment phase (A5) determining V_{J+1} and $V_{d,J+1}$.

A household in retirement can only invest in the risk-free asset and the only source of income is capital income. Thus, there is no portfolio choice. We assume that retired households die with probability p_{death} and normalize the continuation utility after death to zero. Thus, the retirement value function for a household who retires in the current period has the functional form

$$
V_{ret}(w, \theta, s) = \tilde{V}_{0,ret}(s) + \tilde{V}_{1,ret}(s)\left[\ln w + \ln(1 + r_{J+1}(\theta, s))\right] \tag{A6}
$$

where we assumed that the household still works in the period in which the transition into retirement occurs. The coefficients $\tilde{V}_{0,ret}$ and $\tilde{V}_{1,ret}$ are given by

$$
\tilde{V}_{1,ret}(s) \;=\; \gamma_1(s) + \beta(1 - p_{death}) \sum_{s'} \tilde{V}_{1,ret}(s')\pi_{ret}(s'|s)
$$

and

$$
\begin{aligned}
\tilde{V}_{0,ret}(s) \;\; = \;\; & \gamma_0(s) + \gamma_1(s)\ln(\tilde{c}_{ret}(s)) \\
& + (1 - p_{death})\beta \sum_{s'} \tilde{V}_{0,ret}(s')\pi_{ret}(s'|s) \\
& + (1 - p_{death})\beta \ln(1 + r_f) \sum_{s'} \tilde{V}_{1,ret}(s')\pi_{ret}(s'|s) \\
& + (1 - p_{death})\beta \ln(1 - \tilde{c}_{ret}(s)) \sum_{s'} \tilde{V}_{1,ret}(s')\pi_{ret}(s'|s)
\end{aligned}
$$

where $\tilde{c}_{ret}(s) = \frac{\gamma_1(s)}{\tilde{V}_{1,ret}(s)}$.

For the pre-retirement stage, we conjecture that the solution to (A5) is

$$
\begin{aligned}
V_{J+1}(w, \theta, s) &= \tilde{V}_{0,J+1}(s) + \tilde{V}_{1,J+1}(s)\left[\ln w + \ln(1 + r_{J+1}, \theta_{J+1}, s_{J+1}))\right] \qquad (A7) \\
c_{J+1}(w, \theta, s) &= \tilde{c}_{J+1}\,(1 + r_{J+1}(\theta, s))\, w \\
V_{d,J+1}(w, \theta, s) &= \tilde{V}_{d,0,J+1}(s) + \tilde{V}_{1,J+1}(s)\left[\ln w + \ln(1 + r_{d,J+1}(\theta_h, s))\right] \\
c_{J+1}(w, \theta, s) &= \tilde{c}_{J+1}\,(1 + r_{d,J+1}(\theta_h, s))\, w
\end{aligned}
$$

where the coefficients $\tilde{V}_{J+1}$ are determined by the recursive equation

$$\tilde{V}_{1,J+1}(s) \; = \; \gamma_1(s) + (1-p_{ret})\beta \sum_{s'} \tilde{V}_{1,J+1}(s')\pi_{J+1}(s'|s) \; + \; p_{ret}\beta \sum_{s'} \tilde{V}_{1,ret}(s')\pi_{J+1}(s'|s)$$

and the coefficients $\tilde{V}_{0,J+1}$ and $\tilde{V}_{d,0,J+1}$ together with the optimal portfolio choices θ^*_{J+1} are the solutions to the equation

$$\begin{aligned}
\theta^*_{J+1} \;\; &= \;\; arg \max_{\theta_{J+1}\in\Gamma_{J+1}} \Bigg\{ (1-p_{ret}) \sum_{s'} \tilde{V}_{1,J+1}(s') \ln\left(1 + r_{J+1}(\theta_{J+1}, s')\right) \pi_{J+1}(s'|s) \\
&\quad + \; p_{ret} \sum_{s'} \tilde{V}_{1,ret}(s') \ln\left(1 + r_{J+1}(\theta_{J+1}, s')\right) \pi_{J+1}(s'|s) \Bigg\} \\
\Gamma_{J+1} \;\; &\doteq \;\; \Bigg\{ \theta_{J+1} \Bigg| \theta_{h,J+1} + \sum_{s'} \frac{\theta_{a,J+1}(s')\pi_{J+1}(s'|s)}{1+r_f} = 1 \;\; , \;\; \theta_{h,J+1} \geq 0 \;\; , \\
&\quad \frac{\tilde{V}_{0,J+1}(s') - \tilde{V}_{0d,J+1}(s')}{\tilde{V}_{1,J+1}(s')} \geq [ln(1 + r_{d,J+1}(\theta_{J+1}, s')) - ln(1 + r_{J+1}(\theta_{J+1}, s'))] \\
&\quad \frac{\tilde{V}_{0,ret}(s') - \tilde{V}_{0d,ret}(s')}{\tilde{V}_{1,ret}(s')} \geq [ln(1 + r_{d,ret}(\theta_{J+1}, s')) - ln(1 + r_{ret}(\theta_{J+1}, s'))] \Bigg\}
\end{aligned} \tag{A8}$$

and

$$\begin{aligned}
\tilde{V}_{0,J+1}(s) \;\; &= \;\; \gamma_0(s) + \gamma_1(s)\ln(\tilde{c}_{J+1}(s)) \\
&\quad + (1-p_{ret})\beta \sum_{s'} \tilde{V}_{0,J+1}(s')\pi_{J+1}(s'|s) \\
&\quad + (1-p_{ret})\beta \sum_{s'} \tilde{V}_{1,J+1}(s') \ln\left(1 + r_{J+1}(\theta^*_{J+1}, s')\right) \pi_{J+1}(s'|s) \\
&\quad + (1-p_{ret})\beta \ln(1-\tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,J+1}(s')\pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \sum_{s'} \tilde{V}_{0,ret}(s')\pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \sum_{s'} \tilde{V}_{1,ret}(s') \ln\left(1 + r_{J+1}(\theta^*_{J+1}, s')\right) \pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \ln(1-\tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,ret}(s')\pi_{J+1}(s'|s)
\end{aligned}$$

$$\begin{aligned}
\tilde{V}_{d0,J+1}(s) \;\; &= \;\; \gamma_0(s) + \gamma_1(s)\log(\tilde{c}_{J+1}(s)) \\
&\quad + p\beta \sum_{s'} \tilde{V}_{d0,J+1}(s')\pi_{J+1}(s'|s) \\
&\quad + (1-p)\beta \sum_{s'} \tilde{V}_{0,J+1}(s')\pi_{J+1}(s'|s)
\end{aligned}$$

$$
\begin{aligned}
&+ \beta \sum_{s'} \tilde{V}_{1,J+1}(s') \log\left(1 + r_{d,J+1}(1,s')\right) \pi_{J+1}(s'|s) \\
&+ \beta \ln(1 - \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s)
\end{aligned}
$$

We prove this conjecture as follows.

The sequential problem the household faces at the pre-retirement stage $J+1$ is an infinite horizon problem with value function constraint, and the Bellman operator T associated with equation (A8) is monotone, but in general not a contraction mapping. However, adapting the argument made in Rusticchini (1998), the following result can be shown to hold in our setting:

Lemma Suppose that V_d and V^e are continuous functions. Suppose further that there is a unique continuous solution, V_0, to the Bellman equation without participation constraint. Let T stand for the operator associated with the Bellman equation. Consider the set of continuous functions B_W that are bounded in the weighted sup-norm $||V|| \doteq sup_x |V(x)|/W(x)$, where the weighting function W is given by $W(x) = |L(x)| + |U(x)|$ with U an upper bound and L a lower bound, and endow this function space with the corresponding metric.[1] Then

i) $\lim_{n\to\infty} T^n V_0 = V_\infty$ exists and is the maximal solution to the Bellman equation (9)

ii) V_∞ is the value function, V, of the sequential household maximization problem.

Notice first that a standard argument shows that the Bellman equation (A8) without participation constraint has a unique continuous solution, V_0. Guess-and-verify shows that this solution has the functional form (A7). Define $V_n = T^n V_0$. It is straightforward to show that if V_n has the functional form (A7), then the same is true for $V_{n+1} = TV_n$. From the lemma we know that $V_\infty = \lim_{n\to\infty} T^n V_0$ exists and that it is the maximal solution to the Bellman equation (A8) as well as the value function of the corresponding sequential maximization problem (principle of optimality). Since the set of functions with this functional form is a closed subset of the set of continuous functions, we know that V_∞ has the functional form. This proves that the conjecture is correct.

Finally, suppose that the exogenous state can be decomposed into two components, $s = (s_1, s_2)$, where s_1 defines the family structure and s_2 labor market risk. Assume further that s_2 is i.i.d. It is straightforward to show from (A7) and (A8) that the i.i.d. component s_2

[1] Thus, B_W is the set of all functions, V, with $L(x) \leq V(x) \leq U(x)$ for all $x \in X$. For each particular application of the lemma, it has to be shown that this definition of the set of candidate value functions is without loss of generality for certain lower bound, L, and upper bound, U. In our case, the construction of the lower and upper bound is straightforward.

does not affect choices θ and $\tilde{c}$ or value function coefficients $\tilde{V}_0$ and $\tilde{V}_1$, that is, they are functions only of s_1. This completes the proof of proposition 1.

A.2 Proof of Proposition 2

From proposition 1 we know that individual households maximize utility subject to the budget constraint and participation constraint. Thus, it remains to derive the intensive-form market clearing condition and the stationarity condition determining Ω.

Let $\tilde{w}_j = (1+r_j)w_j$ be the wealth of a household age j after all assets have paid off. The aggregate stock of human capital is

$$\begin{aligned}
H &= \sum_j E[\theta_{h,j+1} w_{j+1}]\pi_j && (A9)\\
&= \sum_j E[\theta_{h,j+1}(1-\tilde{c}_j)(1+r_j)w_j]\\
&= \sum_j \sum_{s_{1j}} E[\theta_{h,j+1}(1-\tilde{c}_j)\tilde{w}_j|s_{1j}]\pi_j(s_{1j})\\
&= \sum_j \sum_{s_{1j}} \theta_{h,j+1}(s_{1j})(1-\tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j})\\
&= \tilde{W} \sum_{s_{1j}} \theta_{h,j+1}(s_{1j})(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j}) \ .
\end{aligned}$$

where $W = \sum_j E[\tilde{w}_j]\pi_j$ is aggregate total wealth after assets have paid off. The second line in (A9) uses the equilibrium law of motion for the individual state variable w, the third line is simply the law of iterated expectations, the fourth line follows from the fact that the portfolio choices only depend on s_1, and the last line is a direct implication of the definition of Ω. A similar expression holds for the aggregate stock of physical capital, K. Dividing the two expressions yields the intensive-form market clearing condition

$$\tilde{K} = \frac{\sum_{s_{1j}}(1-\theta_{h,j+1}(s_{1j}))(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j})}{\sum_{s_{1j}}\theta_{h,j+1}(s_{1j})(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j})} \qquad (A10)$$

Define by $\bar{r}_{j+1}(s_{1j}, s_{1,j+1})$ the expected investment return conditional on age and $(s_{1j}, s_{2,j+1})$ In stationary equilibrium the wealth distribution, Ω, has to satisfy

$$\begin{aligned}
\Omega_{j+1}(s_{1,j+1}) &= \frac{E[\tilde{w}_{j+1}|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1,j+1}} E[\tilde{w}_{j+1}|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})} && (A11)\\
&= \frac{E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1,j+1}} E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}\\
&= \frac{\sum_{s_{1j}} E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1j}, s_{1,j+1}]\pi_j(s_{1j}|s_{1,j+1})\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1j},s_{1,j+1}} E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1j}, s_{1,j+1}]\pi_j(s_{1j}|s_{1,j+1})\pi_{j+1}(s_{1,j+1})}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\sum_{s_{1j}}(1+\bar{r}_{j+1}(s_{1j},s_{1,j+1}))(1-\tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j})}{\sum_j\sum_{s_{1j},s_{1,j+1}}\sum_{s_{1j}}(1+\bar{r}_{j+1}(s_{1j},s_{1,j+1}))(1-\tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j})} \\
&= \frac{\sum_{s_{1j}}(1+\bar{r}_{j+1}(s_{1j},s_{1,j+1}))(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j})}{\sum_j\sum_{s_{1j},s_{1,j+1}}(1+\bar{r}_{j+1}(s_{1j},s_{1,j+1}))(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j})}
\end{aligned}
$$

where the second line uses the equilibrium law of motion for the individual state variable x, the third line is simply the law of iterated expectations, the fourth line follows from the fact that portfolio choices only depend on s_1 in conjunction with the definition of $\bar{r}$, and the last line is a direct implication of the definition of Ω. This completes the proof of proposition 2.

A.3. Proof of proposition 3

For each household age j, the solution of the household maximization problem determines the optimal portfolio choice $\theta_j = (\theta_{hj}, \vec{\theta}_{aj})$. Without loss of generality, assume that all households have some insurance in equilibrium, but not full insurance: $\theta_{aj}(d) \neq \theta_{aj}(n)$ and $\eta(d)\,\theta_{hj} \neq (\theta_{aj}(d) - E[\theta_{aj}])$. In this case, for all age groups j the participation constraint binds if $s = n$ and does not bind if $s = d$. If the participation does not bind, the consumption growth rate must be equal to $1 + r_f$ with log-utility, which given the consumption rule (9) implies that the portfolio return in the bad state is equal to the risk-free rate. Adding the budget constraint, we find that the optimal portfolio choice, θ_j, is determined by the following three equations:

$$
\begin{aligned}
\theta_{hj}\,(1 + r_h - \delta_{hj} - \eta(d)) + \theta_{aj}(d) &= 1 + r_f \qquad (A12) \\
\theta_{hj}\,(1 + r_h - \delta_{hj} - \eta(n)) + \theta_{aj}(n) &= e^{-(1-\beta)(\tilde{V}_j - \tilde{V}_{dj})}\theta_{hj}\,(1 + r_h - \delta_{hj} - \eta(n)) \\
\theta_{hj} + \frac{\pi(d)\theta_{aj}(d)}{1+r_f} + \frac{\pi(n)\theta_{aj}(n)}{1+r_f} &= 1\ .
\end{aligned}
$$

Suppose now that defaulting households keep access to financial markets: $p = 0$. In this case, we have $\tilde{V}_j = \tilde{V}_{dj}$, and from the third equation in (A12) it follows that $\theta_{aj}(n) = 0$. Further, solving for θ_{hj} using $\theta_{aj}(n) = 0$ yields:

$$
\theta_{hj} = \frac{\pi(n)}{1 - \frac{\pi(d)}{1+r_f}(1 + r_h - \delta_{hj} - \eta(d))} \qquad (A13)
$$

Clearly, equation (A13) shows that $\theta_{hj} > \theta_{h,j+1}$ if $\delta_{hj} < \delta_{h,j+1}$. It further follows from equation (A12) that the insurance pay-out is given by:

$$
\theta_{aj}(b) - E[\theta_{aj}] = \pi(n)\,(1 + r_f - \theta_{hj}(1 + r_h - \delta_{hj} - \eta(d)))\ . \qquad (A13)
$$

Using $\theta_{hj} > \theta_{h,j+1}$, it follows that $\theta_{aj}(d) - E[\theta_{aj}] < \theta_{a,j+1}(d) - E[\theta_{a,j+1}]$. This proves the first part of the proposition. A similar argument proves the second part of proposition 3.

A.4. Computation

For ages $j = 1, 2, \ldots, J$, we solve the household problem backwards starting at $j = J$. The solution procedure is as follows:

Step 1: Find $\tilde{V}_{1j}(\cdot)$ and $\tilde{c}_j(\cdot)$ solving (A4)

Step 2: Find the optimal portfolio choice θ_j for given $\tilde{V}_{0,j+1}(\cdot)$ and $\tilde{V}_{d0,j+1}(\cdot)$ using (A5)

1. Pick a current family structure s_{1j}.
2. Pick a human capital choice, $\theta_{h,j+1}$.
3. Pick a future family structure s_{1j+1}.
4. Order the states s_{2j+1} according to the size of the human capital shock η. Pick a partition $S \equiv S_1 \cup S_2$, where $S_1 = \{1, ..., n\}$ and $S_2 = \{n+1, ..., N\}$ with N being the number of states s_{2j+1}.
5. For given $(s_{1j},\ s_{1,j+1})$, and human capital choice $\theta_{h,j+1}$, we find the asset portfolio, $\theta_{a,j+1}(\cdot)$, by

   (a) Use participation constraint for all $s_{2j+1} \in S_1$:

$$\exp\left(\frac{1}{\tilde{V}_{1,j+1}(s_{1j+1})}\left(\tilde{V}_{0,j+1}(s_{1j+1}) - \tilde{V}_{0d,j+1}(s_{1j+1})\right)\right)((1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} + \theta_{a,j+1}(s_{1j+1}, s_{2j+1}))$$
$$= (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1}$$

   (b) Equalization of IMRS for all $s_{2j+1} \in S_2 : \frac{u'(c_j,s_j)}{u'(c_{j+1},s_{j+1})} = \beta(1 + r_f)$.
   Using our utility function this reads $\frac{c_{j+1}}{c_j} = \frac{\gamma_1(s_{1j+1})}{\gamma_1(s_{1j})}\beta(1+r_f)$. Using our consumption policy function, we find $\frac{c_{j+1}}{c_j} = \frac{\tilde{c}_{j+1}}{\tilde{c}_j}(1 - \tilde{c}_j)(1 + r_{j+1})$. Further using $\tilde{c}_j = \frac{\gamma_{1j}}{\tilde{V}_{1j}}$ we arrive at the following condition for all $s_{2j+1} \in S_2$:

$$\frac{\tilde{V}_{1,j}(s_{1j}) - \gamma_{1,j}(s_{1j})}{\tilde{V}_{1j+1}(s_{1,j+1})}((1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} + \theta_{a,j+1}(s_{1j+1}, s_{2j+1})) = \beta(1 + r_f)$$

Thus, we have

$$\theta_{a,j+1}(s_{1j+1}, s_{2j+1}) = -(1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1}\left(1 - \exp\left(\frac{\tilde{V}_{0d,j+1}(s_{1j+1}) - \tilde{V}_{0,j+1}(s_{1j+1})}{\tilde{V}_{1,j+1}(s_{1j+1})}\right)\right)$$
$$\forall s_{2j+1} \in S_1$$

$$\theta_{a,j+1}(s_{1j+1}, s_{2j+1}) = \frac{\tilde{V}_{1,j+1}(s_{1j+1})}{\tilde{V}_{1,j}(s_{1j}) - \gamma_1(s_{1j})}\beta(1 + r_f) - (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1}$$
$$\forall s_{2j+1} \in S_2$$

6. Do this for all s_{1j+1}
7. Do this for all θ_{hj+1}. For given current family structure s_{1j}, find the portfolio vector $(\theta_{h,j+1}, \theta_{a,j+1})$ that "solves" the portfolio constraint. This is our optimal portfolio for given s_{1j}.
8. Do this for all current family structures s_{1j}.

Step 3: Find $\tilde{V}_{0j}(\cdot)$ and $\tilde{V}_{d0,j}(\cdot)$ using (A5)

The household problem for $j = J+1$ we solve as above, but now we drop the j-dependence and solve the corresponding fixed point problem.

A.5. Construction of Family Transition Matrix

We construct the stochastic matrix describing the transition of households over family states s_1 as follows. We proceed in two steps. In the first step, we construct the transition function for marital states and in the second state we construct the transition matrix for the number of kids for each marital state. Age subscripts are dropped for convenience.

A.5.1. Marital States

There are in total 5 marital states: Married (ma), female widowed (fw), female single and not widowed (fn), male widowed (mw), and male single and not widowed (mn). We stack family states in a vector $x = \{ma, fw, fn, mw, mn\}$ and construct transition matrix Π. The transition matrix follows the conventional structure with initial states in rows and terminal states in columns. The order of states is given by the order of x. We set all transition rates between sexes to zero.

$$\Pi = \begin{pmatrix} \pi(ma,ma) & \pi(ma,fw) & \pi(ma,fn) & \pi(ma,mw) & \pi(ma,mn) \\ \pi(fw,ma) & \pi(fw,fw) & 0 & 0 & 0 \\ \pi(fn,ma) & 0 & \pi(fn,fn) & 0 & 0 \\ \pi(mw,ma) & 0 & 0 & \pi(mw,mw) & 0 \\ \pi(mn,ma) & 0 & 0 & 0 & \pi(mn,mn) \end{pmatrix}$$

For a married household, the transition probabilities $\pi(ma, fw)$ and $\pi(ma, mw)$ are computed using the life tables for males, respectively females. We interpret the transition from married household to female single non-widowed, respectively male single non-widowed, as divorce. We assume that the female is the decision maker in a married household and that after divorce the woman does not care about the well-being of the male, which is equivalent to setting transition probability from married to single male non-widowed to zero in the household decision problem: $\pi(ma, mn) = 0$.[2] The probability to stay married is determined as the residual $\pi(ma, ma) = 1 - \pi(ma, fw) - \pi(ma, mw) - \pi(ma, fn)$.

For male and female widowed household, we assume that they either re-marry with probability $\pi(mw, ma)$ and $\pi(fw, ma)$, respectively, or stay widowed with probability $\pi(mw, mw)$, respectively $\pi(fw, fw)$. Similarly, male and female single, non-widowed households can either marry with probability $\pi(mn, ma)$ and $\pi(fn, ma)$, respectively, or stay single with probabilities $\pi(mn, mn)$ and $\pi(fn, fn)$.

A.5.2. Children

We consider 4 different states for the number of kids in the household: no kids, 1 kid, 2, kids, 3 kids (or more). The number of kids increases by one in the case of the birth of a child and decreases by one in the case that a child leaves the household (moves out). The number of children also changes if households marry, in which case the kids of the two marrying households are combined.

We distinguish between the fertility rate of a married woman and th fertility rate of a single woman, but because of data scarcity assume that widowed woman and non-widowed women have the same fertility rates. Similarly, we distinguish between moving-out rates of children for married households and moving-out rates for single households. Denote the

[2] For the law of motion of the model distribution over family states, we adjust these transition probabilities to account for the fact that there are two new households, one fn and one mn.

probability that a married household increases/decreases the number of kids by one by $\pi(ma,+1)$ and $\pi(ma,-1)$ and the corresponding transition probability for a female single household by $\pi(f;+1)$ and $\pi(f,-1)$. For married households, the transition rates for the number of kids are then summarized by the transition matrix

$$T_{ma} = \begin{pmatrix} 1-\pi(ma,+1) & \pi(ma,+1) & 0 & 0 \\ \pi(ma,-1) & 1-\pi(ma,+1)-\pi(ma,-1) & \pi(ma,+1) & 0 \\ 0 & \pi_m(m) & 1-\pi_f(m)-\pi_m(m) & \pi_f(m) \\ 0 & 0 & \pi(ma,-1) & 1-\pi(ma,+1) \end{pmatrix}$$

Similarly, for single female households who do not re-marry the transition rates for the number of kids are summarized by the transition matrix

$$T_f = \begin{pmatrix} 1-\pi(f,+1) & \pi(f,+1) & 0 & 0 \\ \pi(f,-1) & 1-\pi(f,+1)-\pi(f,-1) & \pi(f,+1) & 0 \\ 0 & \pi(f,-1) & 1-\pi(f,+1)-\pi(f,-1) & \pi(f,+1) \\ 0 & 0 & \pi(f,-1) & 1-\pi(f,-1) \end{pmatrix}$$

For male single households who do not marry the number of kids cannot increase, but can decrease by one due to moving out. If we denote the moving out rate by $\pi(m,-1)$, the transition matrix for male single households who do not marry reads:

$$T_m = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \pi(m,-1) & 1-\pi(m,-1) & 0 & 0 \\ 0 & \pi(m,-1) & 1-\pi(m,-1) & 0 \\ 0 & 0 & \pi(m,-1) & 1-\pi(m,-1) \end{pmatrix}$$

Finally, there is the event that a single female household and a single male household get married and the kids are combined. In this case, the transition matrix is for female single households is

$$T_{f,ma} = \begin{pmatrix} \mu_0 & \mu_{1f} & \mu_{2f} & 1-\mu_0-\mu_{1f}-\mu_{2f} \\ 0 & \mu_0 & \mu_1 & 1-\mu_0-\mu_1 \\ 0 & 0 & \mu_{0f} & 1-\mu_{0f} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where μ_{if} denotes the probability that a female and a male marry and the combined number of kids is i. A similar transition matrix describes the transition rates for male single households, which we denote by $T_{m,ma}$

Combining the transition matrices for marital status and the number of kids results in the joint transition matrix for family states:

$$\Pi \otimes T = \begin{pmatrix} \pi(ma,ma)T_1 & \pi(ma,fw)T_1 & \pi(ma,fn)T_1 & \pi(ma,mw)T_1 & \pi(ma,mn)T_0 \\ \pi(fw,ma)T_{f,ma} & \pi(fw,fw)T_2 & \pi(fw,fw)T_2 & 0 & 0 \\ \pi(fn,ma)T_{f,ma} & \pi(fn,fn)T_2 & \pi(fw,fw)T_2 & 0 & 0 \\ \pi(mw,ma)T_{f,ma} & 0 & 0 & \pi(mw,mw)T_3 & \pi(mw,mw)T_0 \\ \pi(mn,ma)T_{f,ma} & 0 & 0 & \pi(mn,mn)T_0 & \pi(mn,mn)T_0 \end{pmatrix}$$

where T_0 is the transition matrix to zero kids in the next period independent of the current number of kids today and $T_{4,f}$ and $T_{4,m}$ denote the respective transition matrices for females and males.

A.6. Calibration of Family Transition Matrix

In this section, we describe how we estimate transition probabilities between family states from the data. The data are the core files of waves 1 to 9 and the wave 2 fertility history topical module from the 2001 panel of the Survey of Income and Program Participation (SIPP). After the National Center for Health Statistics (NCHS) has stopped to publish detailed data on marriage and divorce in 1990, the SIPP has become the primary data source for marital history information (See also Kreider and Field, 2001, for details.). We merge the 9 waves to create a panel of marital status histories. The death probabilities for males and females are constructed using death probabilities from the life tables published by the Human Mortality Database (HMD).

A.6.1 Marital State

Recall that there are 5 marital states: Married (ma), female widowed (fw), female single and not widowed (fn), male widowed (mw), and male single and not widowed (mn). We restrict the sample to reference persons and their spouses to get a sample of household heads comparable to the Survey of Consumer Finances. We label persons as married that report being married with the spouse present or absent.[3] We label persons as widowed following the coding in the data, and label all other single persons as not widowed. This last status includes divorced, separated, and never married. For each individual, we assign an age-specific marital status using the marital status the person had for the longest period of each age. We derive age-specific transition rates by computing the share of individuals who change their marital status with age using the panel dimension of the data. The transition rates are computed for 5-year age bins. The first bin covers $21-25$ and the last bin $58-62$. The mid

[3]The SIPP does not have a marital state "living with partner" as in the Survey of Consumer Finances (SCF).

point of the bin is taken as point in the age profile to which the transition rate is assigned. We regress the raw data on a fourth order polynomial in age. We use the estimated profile as input to our model. If estimated transition rates are negative, we set them to zero.

A.6.2 Remarriage Rates

There is very little data available on the remarriage rates of young widows and widowers, reflecting their rarity in the general population. For example, using data from the SIPP, at young ages there are often very few (one or two) widows in the sample. As a result, the average remarriage rates are very noisy. Some studies resolve this problem by imposing the restriction that the remarriage rates of widows and widowers equal those of divorcees.

However, there is a large amount of evidence suggesting that the remarriage rates of widows and widowers are smaller than those for divorced persons at all ages. For example, Hong and Rios-Rull (2012, online appendix) find that remarriage rates are lower for widows than for divorcees using data from the Panel Study of Income Dynamics (PSID), though they do not provide the exact numbers. However, these data are also subject to small sample concerns. Similar findings using a much richer dataset are found by Wilson and Clarke (1992) using data for 1988 from the NCHS on divorces and marriages. The size of the sample is large: for example, they observe 77,000 widowed women who remarry out of a population of 12.3 million female widows. Using Table 2 from Wilson and Clarke (1992), we find that female widows aged $25-54$ have remarriage rates that are 47% of that of divorcees of the same age. This difference in remarriage rates is not driven by a different age composition of the two samples. Wilson and Clarke (1992) report remarriage rates broken down to smaller age groups and the pattern is very stable across these groups. For age group $25-29$ remarriage rates for widows are 44.9% of the remarriage rates of divorcees, for age group $30-34$ the number is 46.5%, for age group $35-44$ it is 44.4%, and for age group $45-54$ it is 50.7%. Similar results can be found in the report by Norton and Miller (1990), who use the 1985 marriage and fertility history supplement to the Current Population Survey (CPS). They report median duration completed time in divorce and widowhood for persons who remarry. Although this is a selected subsample of widows and divorcees, they report similar differences. The median duration of widows is almost twice as large as for divorcees for persons 45 years and younger.[4]

In light of these concerns, we derive remarriage rates for divorced households and wid-

[4]They only report the median time to remarriage for the pooled group of widows younger than 45 years (approximately 3.9 years). For divorcees of the different subgroups the duration below age 45 is very similar (approximately 2.3 years.)

owed households separately. Exploiting the relative stability of the relationship between remarriage rates of widows and divorcees found above, we start with the rate of remarriage for divorcees and then adjust these numbers down to reflect the above findings. Specifically, using data from the SIPP, we construct a pooled sample of widows from age $30 - 50$ and compare it to a sample of divorceés. We find that remarriage rates for widows are 44% of remarriage rates of divorcees. We use this scaling factor to impute remarriage rates for widows.[5] Our scaling factor of 44% is very close to the data of Wilson and Clarke (1992).

Figure A1 shows remarriage rates calculated using data from the SIPP for both divorcees and widows (male and female). The remarriage rates of widows are depicted by blue diamonds. As shown in the figure, the rates for widows aged less than 30 are missing, due to their absence from the sample. Even after age 30, the rates fluctuate wildly, at around 4% at the beginning of the 30's and rising higher than the levels observed for divorcees in their late 30s. This fluctuation is, however, driven by very few observations. The red dots show the rates for divorcees, and the red dotted line shows the smoothed version of these data that we use in the model. The blue solid line is our adjusted remarriage rate for widows. As can be seen, this line does a good job matching the remarriage rates of people in the 40s, for which we have more data.

A.6.3 Fertility Rates

We use the wave 2 fertility history topical module to derive fertility rates by age. This module has information on the year of birth of the last child.[6] We assign a birth event to a women if there is at most one year difference between the current calender year (2001 in our case) and the year of birth of the child. We adjust the age of the mother by one year if the year of birth was in the previous calender year. The age-specific fertility rate is the share of females at each age that had a birth event. Given that the period during which the child could have been born covers two calender years, we adjust to get rates for a one-year timespan. The fertility rates are computed for 5-year age bins. We regress the transition rate data on a fourth order polynomial in age. We use the estimated profile as input to our model. In line with observed fertility rates, we set fertility from age 45 onwards to zero. We derive separate fertility rates for single $\pi_f(s)$ and married woman $\pi_f(m)$. We use marital status information from the fourth interview of the second wave when the question of the topical module are asked. The results are depicted in figure A3.

[5] If we look over the age range $23-61$ remarriage rates for widows are 42% of the average rate of divorcees and for the a pooled sample from age $40-60$ remarriage rates for widows are 63% of the rates for divorcees.

[6] The month information is suppressed for confidentiality in the public use files.

A.6.4 Moving Out Rates

For the calibration of the probability that children move out of the household, we restrict the sample to those households who have at least one household member who has information at all 9 waves to avoid the underestimation of moving out rates due to sample attrition. We consider as children only children of the reference person that are less than 23 years of age.[7] We assign each child the age and the marital status of its mother. We assign a moving out event if a person that has been a child of the reference person at age j, becomes a reference person, spouse or unmarried partner of a reference person, brother/sister of reference person, other relative of reference person, housemate/roommate, roomer/boarder, or other non-relative of reference person at age $j+1$. We also assign a moving out event if the person turns 23, if the person is 22 at wave 9, or if the person has no further observations before wave 9 and is at least 16 years of age. The moving out rates are computed for 5-year age bins using the age of the mother. We regress the transition rate data on a fourth order polynomial in age. We use the estimated profile as input to our model. [8] We derive separate moving out rates for single $\pi_m(s)$ and married households $\pi_m(m)$. The results are shown in figure A4.

A.6.5 Death probabilities

The probability that a household member dies is taken from the life tables of the Human Mortality Database). We use averages of death probabilities separately for males and females for the period 1990 - 2007. Figure A5 shows the life-cycle profile of the death probabilities for males and females.

A.6.6 Initial Distribution

To derive the initial distribution over family states, we use reference persons and their spouses. We assign each person the family status from the fourth interview in wave 2 (see fertility rates above). The definition for children is as in the case of the moving out rates. We consider all persons age 21 to 25 for the initial distribution (the 5-year bin around age 23).

A.6.7 Consistency

To check the consistency of the estimated family transition matrix with the observed cross-

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[7]In contrast, the SIPP counts as children (variable RFNKIDS) all children in the household under age 18 including grandchildren or children of household members other than the reference person and its spouse.

[8]If estimated transition rates are negative, we set them to zero.

sectional distribution over family states, we have computed various life-cycle profiles derived from the estimated transition matrix and initial distribution. Overall, the deviations between implied cross-sectional distributions and empirical distributions are small.

A.7. Survivor Benefits and Taxes

A.7.1. Survivor Benefits

Suppose death of an adult household member occurs at age j. For each age $k > j$, we can compute a social security survivor benefit for a median-income widowed household, $B_{j,k}$. This benefit also depends on the number of children, n, but for simplicity we suppress this dependence. We compute this benefit as follows:

- **Step 1**: For each j, compute the AIME

  $$AIME_j = \frac{\nu}{j-20} \sum_{i=20}^{j} y_{h,i}^m$$

  where $y_{h,i}^m$ is the median labor income of a married household (with kids) age i and $0 < \nu < 1$ is a weight that measures the fraction of household earnings that has been generated by the deceased household member. We set $\nu = 0.5$ in our baseline. We assume that households' first year of full earnings is at age 20 and further assume $y_{h,20}^m = y_{h,21}^m = y_{h,22}^m = y_{h,23}^m$.

- **Step 2**: Compute PIA_j

  We have

  $$PIA_j = 0.9{*}min\{b_1, AIME_j\} + 0.32{*}min\{b_2, max\{AIME_j - b_1, 0\}\} + 0.15{*}max\{AIME_j - b_2, 0\}$$

  As bend points b_1, b_2, and b_3 we use the official bend points in the year 2000.

- **Step 3**: Compute potential benefits, $\tilde{B}_{j,k}$:

  The amount of benefits the surviving household members can potentially receive is

  $$\tilde{B}_{j,k} = 0.75 * PIA_j - max\{0.5(y_{h,k}^s - \tau), 0\} \; + \; 0.75nPIA_j$$

  where $y_{h,k}^s$ is the labor income of the surviving spouse at age k, n is the number of surviving children, and τ is a fixed threshold. We set the value τ equal to the official threshold for the year 2000.

Marginal Tax Rate	Married Filing Jointly Tax Brackets Over	Below	Single Tax Brackets Over	Below
15.0%	\$0	\$43,850	\$0	\$26,250
28.0%	\$43,850	\$105,950	\$26,250	\$63,550
31.0%	\$105,950	\$161,450	\$63,550	\$132,600
36.0%	\$161,450	\$288,350	\$132,600	\$288,350
39.6%	\$288,350	–	\$288,350	–

Table 1: Tax rates for 2000

- **Step 4**: Compute the maximum family benefit, $B_{j,max}$

  We have

$$B_{j,max} \;=\; 1.5 * min\{b_1^f, PIA_j\} \;+\; 2.72 * min\{b_2^f, max\{PIA_j - b_1^f, 0\}\}$$
$$+\, 1.34 * min\{b_3^f, max\{PIA_j - b_2^f, 0\}\} \;+\; 1.75 * max\{PIA_j - b_3^f, 0\}$$

  As bend points b_1^f, b_2^f, and b_3^f we use again the official bend points in the year 2000.

- **Step 5**: Compute the actual benefit $B_{j,k}$

  The actual benefit paid out to the surviving family members is

$$B_{j,k} = min(\tilde{B}_{j,k}, B_{j,max})$$

A.7.2. Payroll and Social Security Taxes

We compute the average tax rate for a median-income household using estimated earnings profiles for married households and single households. We compute federal taxes with standard deductions taking into account deductions and tax credits for children. We use nominal tax brackets for the year 2000 (which is consistent with using real data in year 2000 dollars) to compute average tax rates.[9] The rates vary according to the filing status of the household. For 2000, the U.S. income tax brackets and marginal tax rates are given in table:

The child tax credit was introduced in 1997 for tax year 1998. In 1998, the basic credit was \$400 per qualifying child (under age 17). After 1998 through 2000, this was increased to \$500 per child. In 2001, the child tax credit was amended, and was supposed to increase

[9] Using data from The Tax Foundation: http://taxfoundation.org/article/us-federal-individual-income-tax-rates-history-1913-2011-nominal-and-inflation-adjusted-brackets

to \$600 per child for 2001-2004, with further increases planned (\$700 for 2005-2008, \$800 for 2009, and \$1000 for 2010). However, in 2003 the tax credit was increased to \$1000 for 2003 and 2004, and this has been extended by successive legislation through 2012. There is a means test for the credit. From the 1997 law, which was in force in 2000, the reduction was \$50 per \$1000 over the threshold of \$110,000 for married filing jointly, and \$75,000 for non married individuals.

The numbers for the personal exemption for married couples, single people, and per dependent for 2000 are $5,600$, $2,800$, and $2,800$. That is, in 2000, a married household filing jointly could claim \$5600 plus an extra \$2800 per dependent.

The social security tax and medicare tax paid by the employee was 6.2% and 1.45%, respectively. We add theses taxes to the federal income tax to arrive at an total average tax rate.

A.8. Survey of Consumer Finance

The data are for the years 1992, 1995, 1998, 2001, 2004, and 2007 drawn from the Survey of Consumer Finances (SCF) provided by the Federal Reserve Board. The Survey collects information on a number of economic and financial variables of individual families through triennial interviews, where the definition of a "family" in the SCF comes close to the concept of a "household" used by the U.S. Census Bureau. See Kennickell and Starr-McCluer (1994) for details about the SCF.

For the sample selection, we follow as closely as possible Heathcote et al. (2010)[10]. We restrict the sample to households where the household head is between 23 and 60 years of age. We drop the wealthiest 1.46% and the poorest 0.5% of households in each year. Heathcote et al. (2010) show that this step makes the sample more comparable to the PSID or CEX data. We drop all households that report negative labor income or that report positive hours worked but have missing labor income or that report positive labor income but zero or negative hours worked. We compute the average wage by dividing labor income by total hours worked, and drop in each year households with a wage that is below half the minimum wage of the respective year. For the data on life-insurance, we restrict the sample further to households that are married or live with a partner.

For the definition of variables we follow Kennickell and Starr-McCluer (1994). We only depart from their variable definitions when considering labor income, where we follow Heathcote et al.(2010) and add two-thirds of the farm and business income as additional labor

[10]We use their Sample B for our analysis.

income. As common in the literature, we associate financial wealth in the model with net worth in the SCF. Households' net worth includes the cash value of life-insurance as in Kennickell and Starr-McCluer (1994), but does not include the face value of insurance contracts. We associate life-insurance in the model with the face value of life-insurance from the data. All data has been deflated using the BLS consumer price index for urban consumers (CPI-U-RS). A detailed description of the relevant variables is as follows:

- **Assets** are the sum of financial and non-financial assets. The main categories of non-financial assets are cars, housing, real estate, and the net value of businesses where the household holds an active interest. Except for businesses all values are gross positions, i.e. before outstanding debt. The main categories of financial assets are liquid assets, CD, mutual funds, stocks, bonds, cash value of life-insurance, other managed investment, and assets in retirement accounts (e.g. IRAs, thrift accounts, and pensions accumulated in accounts.)

- **Debt** is the sum of housing debt (e.g. mortgages, home equity loans, home equity lines of credit), credit card debt, installment loans (e.g. cars, education, others), other residential debt, and other debt (e.g. pension loans).

- **Net-worth** is the sum of all assets minus all debt.

- **Labor income** is wages and salaries plus 2/3 of business and farm income.

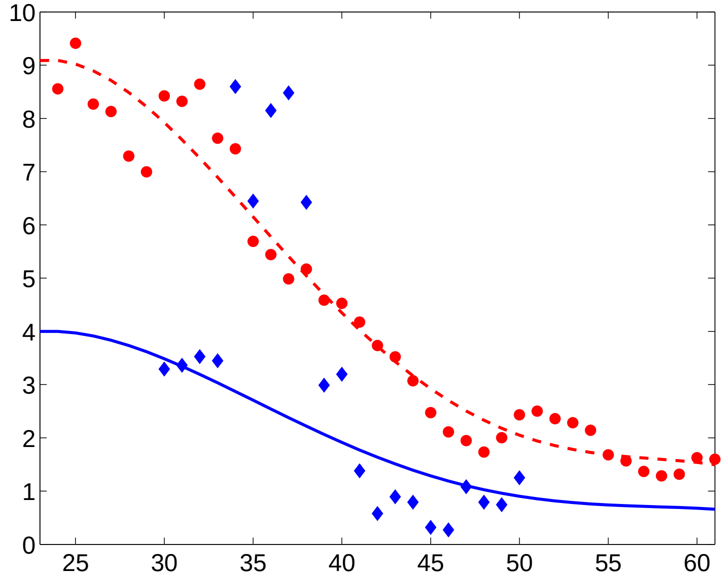
- **Life-insurance** is the face value of all term life policies and the face value of all policies that build up a cash value. The cash value is not part of the life-insurance, but is part of the financial assets of an household.

A.9. Employer-Provided Life Insurance

Here we address the issue to what extent the existence of employer-provided group insurance has the potential to distort our results. If the amount of group insurance offered by the employer exceeds the amount households want to hold, then these households are "involuntarily" over-insured and the insurance holdings observed in the data are not the outcome of the optimal insurance choice by households. Clearly, the phenomenon of involuntary over-insurance can only occur for households who have not purchased any individual life insurance from insurance companies. Although the SCF does not distinguish between group insurance and insurance purchased individually, we can use data on employer provided life insurance from the Survey of Income and Program Participation (SIPP) to analyze this issue. Figure

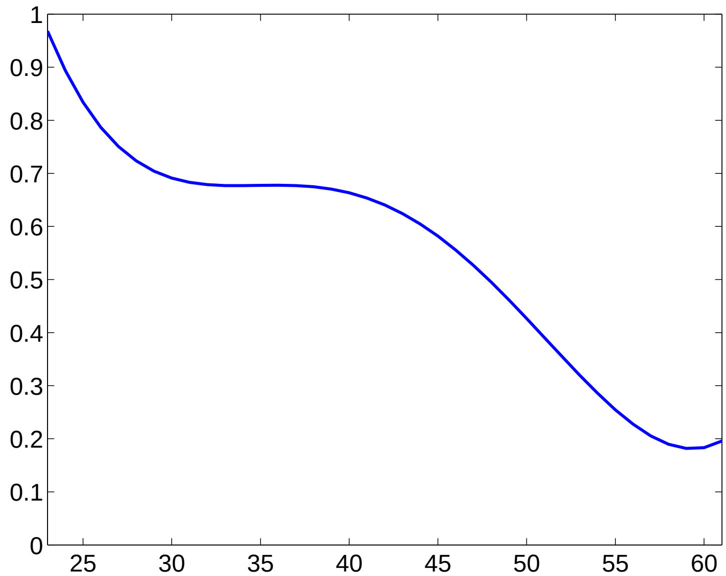
A6 shows the median life insurance holding of married households with children who have purchased some life insurance, and also the holdings of employer-provided life insurance for the same group of households. The figure shows that for each age between 23 and 60, the median household with kids holds substantially more life insurance than the amount of insurance provided by the employer. Further, for the median household the amount of employer-provided life insurance is roughly constant over the life-cycle and the shape of the life-cycle profile of total (group plus individual) life insurance holdings is therefore not much affected by the presence of group life insurance. Thus, we conclude that the consideration of insurance purchases as voluntary is appropriate to a first approximation. Hong and Rios-Rull (2012) come to a similar conclusion after analyzing data drawn from the International Survey of Consumer Financial Decisions.

Figure A1: Remarriage rates



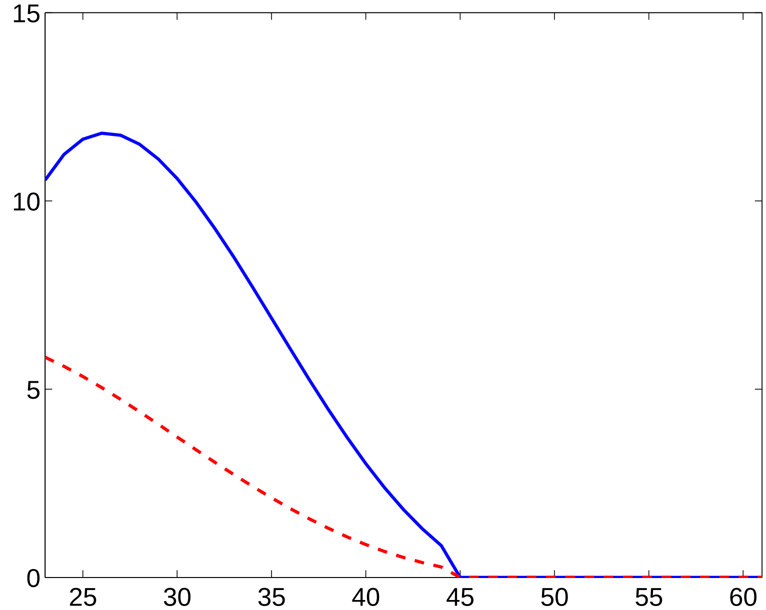
Notes: Life-cycle profile of remarriage rates for households age 23 - 60 (percentage points). Red dots show remarriage rates for divorced singles as measured in the data. The red dashed line shows the smoothed life-cycle profile used in the model as remarriage rates for non-widowed singles. The blue solid line shows the scaled life-cycle profile of remarriage rates used in the model as remarriage rates for widowed singles. The blue diamonds show remarriage rates for widowed singles for age 30-50 as measured in the data. The scaling parameter for the life-cycle profile of remarriage rates is derived comparing the blue diamonds to the red dots over for age 30-50. Remarriage rates are derived using 2001 SIPP data.

Figure A2: Divorce rates



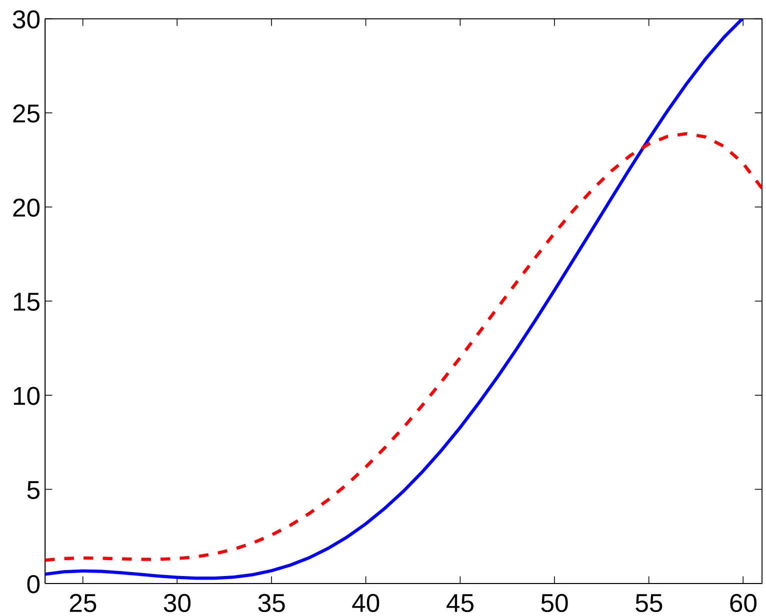
Notes: Smoothed life-cycle profile of divorce rates for households age 23 - 60 (percentage points). Divorce rates are derived for all married households using 2001 SIPP data.

Figure A3: Fertility rates



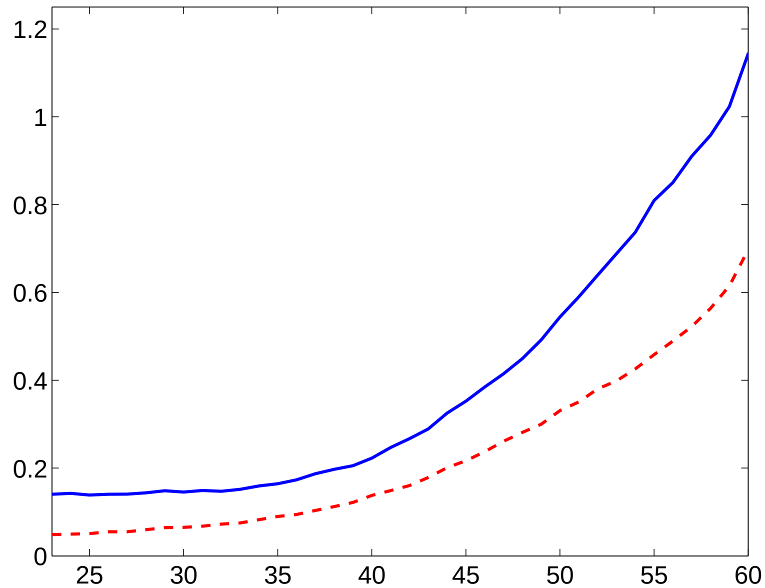
Notes: Smoothed life-cycle profile of fertility rates for households age 23 - 60 (percentage points). Red dashed line shows singles and blue solid line married females. Fertility rates are derived using wave 2 topical module to the 2001 SIPP.

Figure A4: Moving out rates



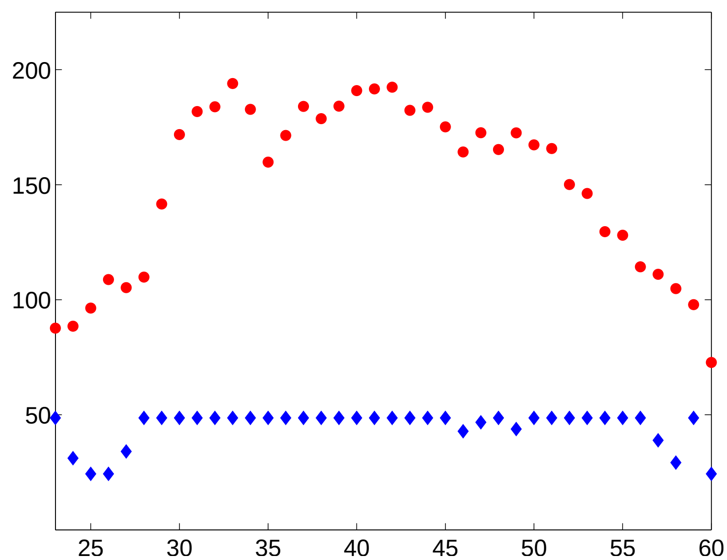
Notes: Smoothed life-cycle profile of moving out rates for single and married households age 23 - 60 (percentage points). Red dashed line shows single parent households. Blue solid line married households. Moving out rates are derived using 2001 SIPP data.

Figure A5: Death probabilities



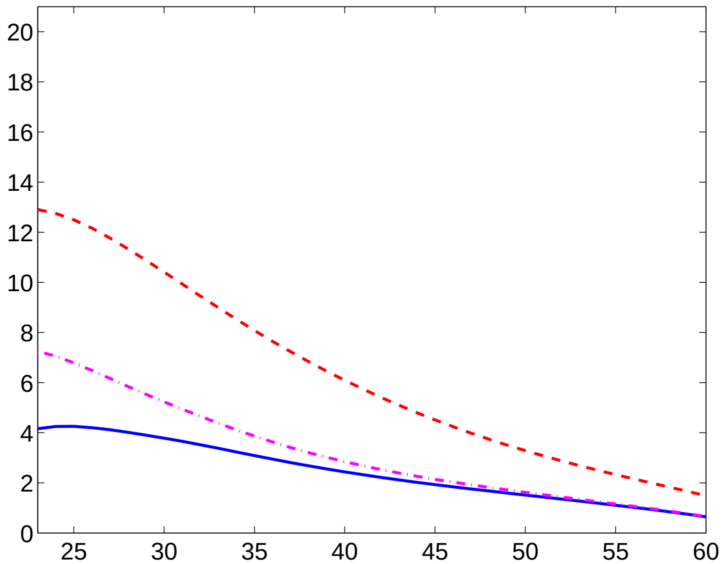
Notes: Death probabilities for males and females age 23 - 60 from the life tables of the Human Mortality Database (percentage points). Blue solid line shows death probability of males. Red dashed line shows death probability of females.

Figure A6: Employer-provided life insurance



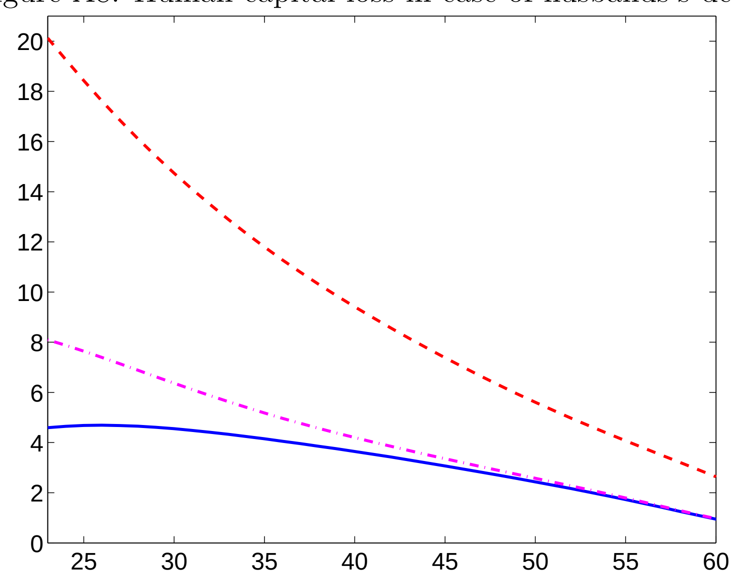
Notes: Life-cycle profile of face value of all life insurance contracts (thousands of year 2000 dollars). Red dots show median face value of all life insurance contracts for married households with children that have purchased life insurance. Blue diamonds show the median face value of employer-provided insurance for this group. All data are for households age 23 - 60 from wave 3 topical module to the 2001 SIPP.

Figure A7: Human capital loss in case of wife's death



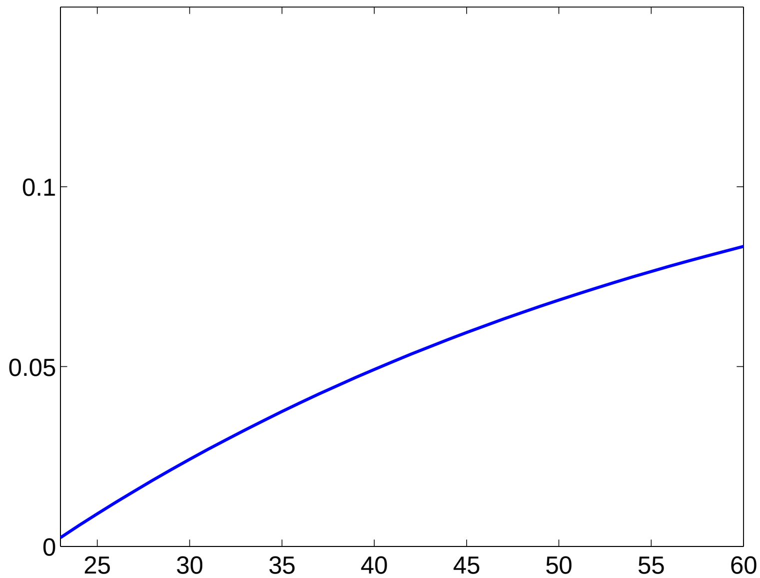
Notes: Life-cycle profile of expected human capital loss in case of wife's death for all married households with children. Human capital loss is ratio of present value labor income loss over current labor income. Red dashed line: loss before transfers and taxes with zero probability to remarry. Pink dashed-dotted line: loss after transfers and taxes and zero probability to remarry. Blue solid line: loss after transfers and taxes and empirical remarriage rates. All data are for households age 23 - 60 from the SCF, surveys 1992 - 2007.

Figure A8: Human capital loss in case of husbands's death



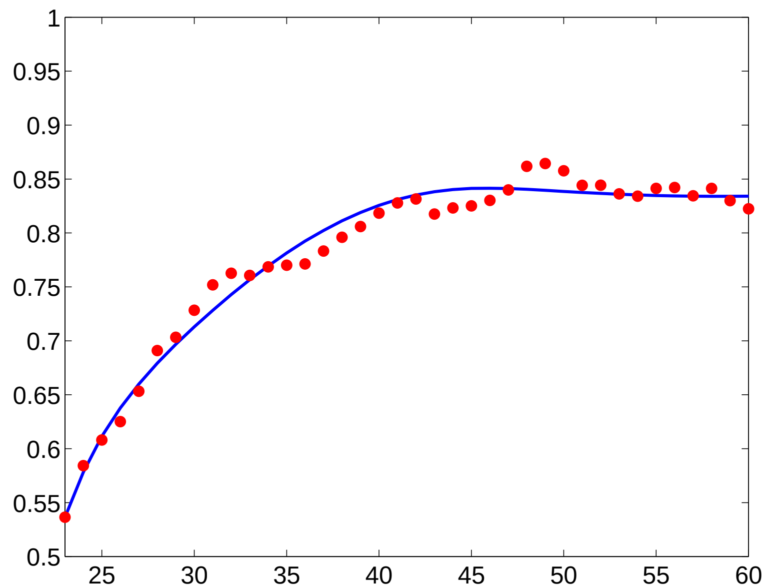
Notes: Life-cycle profile of expected human capital loss in case of husband's death for all married households with children. Human capital loss is ratio of present value labor income loss over current labor income. Red dashed line: loss before transfers and taxes with zero probability to remarry. Pink dashed-dotted line: loss after transfers and taxes and zero probability to remarry. Blue solid line: loss after transfers and taxes and empirical remarriage rates. All data are for households age 23 - 60 from the SCF, surveys 1992 - 2007.

Figure A9: Human capital depreciation rates



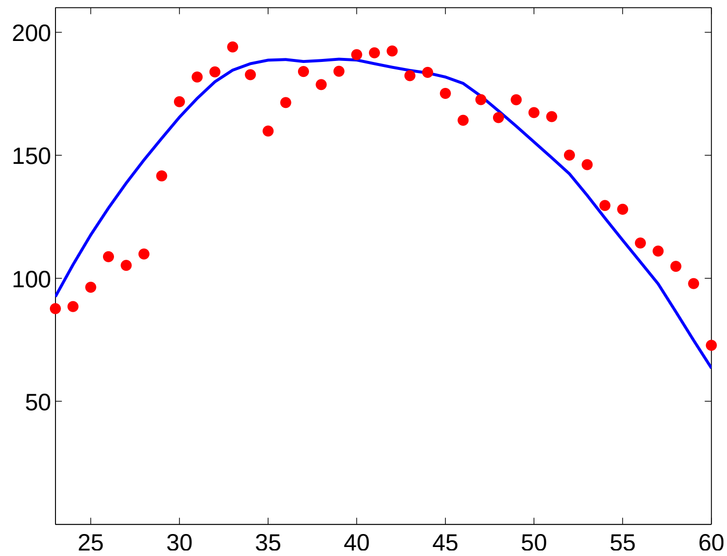
Notes: Life-cycle profiles of human capital depreciation rates δ_{hj}.

Figure A10: Participation rate



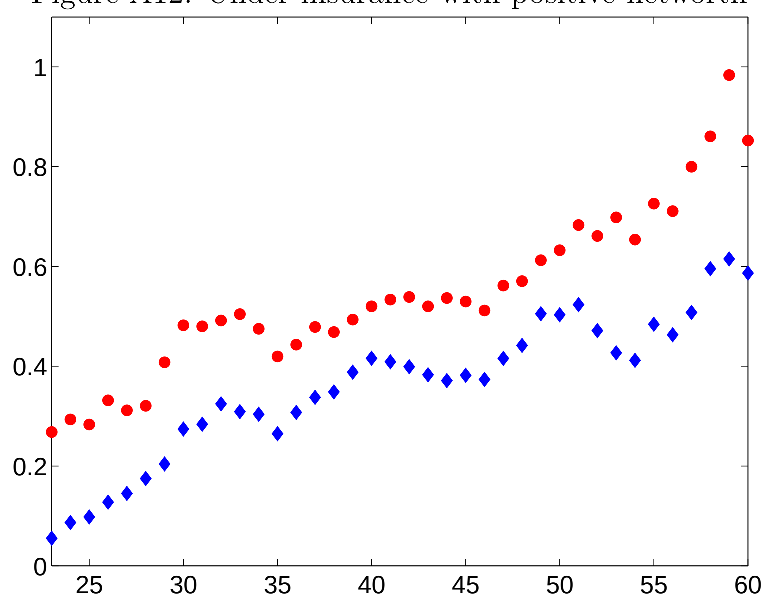
Notes: Life-cycle profile of participation rate in the life-insurance market for married households age 23-60 with children. For each age the red dots show the share of households that report having purchased some life-insurance from the SCF, surveys 1992 - 2007. The blue solid line shows the model prediction.

Figure A11: Extended model with health- and child-dependent preferences



Notes: Life-cycle profile for life insurance holdings for married households age 23 - 60 with children. Blue solid line shows model. Marginal utility from consumption is different for households in poor and good health and differs across single households with different number of kids. Red dots show face value of life insurance contracts (thousands of year 2000 dollars) for married households age 23 - 60 with children that have purchased life insurance from the SCF, surveys 1992-2007.

Figure A12: Under-insurance with positive networth



Notes: Ratio of life-insurance holdings to present value income loss for married households age 23 - 60 with children. Red dots show data for all households that have purchased life insurance and positive networth. Blue diamonds show data for all married households with children and positive networth.

Table A1: Calibration

parameter	value	description
β	0.95	discount factor
ϕ	0.2102	(inverse of) price of human capital
p_{ret}	0.2	probability of retiring
p_{death}	0.1	probability of dying
σ_η	0.1042	standard deviation of permanent shocks
σ_z	0.3	standard deviation of transitory shocks
p	0.8571	probability of remaining in financial autarky
α	0.32	capital share in output
δ_k	0.0785	physical capital depreciation rate
φ	0.1010	learning by doing
A	0.1818	total factor productivity
λ	4.3236	wealth endowment of households with one child