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Cheap Talk and Evolutionary Dynamics*

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by Karl H. Schlag

Abstract

The effect of cheap talk in partnership games on the evolutionary dynamics of homogeneous populations under symmetric and random matching is analyzed. As long as the message set is sufficiently large there exists an Asymptotically Stable Set with payoffs arbitrarily close to the maximal payoff for each player. However this only holds true for each Asymptotically Stable Set if there are no more than two strategies. Our results underline the importance of large message sets and reveal the implicit coordination device that drives the efficiency results in the alternative two type population models.

Keywords: evolutionary stability, cheap talk, efficiency

1. Introduction:

We are interested in the effect of communication on the outcomes of dynamic processes in random matching models. Communication is modelled as cheap talk: before the game is played, the players simultaneously exchange messages from some finite set of messages. There is no cost to exchanging these messages and hence "talk is cheap".

The games in which we embed our analysis are restricted to partnership games (Hofbauer and Sigmund, 1988). These are symmetric games in which each player gets the same payoff. They are also referred to as symmetric pure coordination games (Matsui, 1989). In such games the maximal payoff a player can receive is the same for each player and coincides with the only efficient outcome. It seems that partnership games should be an ideal framework in which communication should be advantageous and perhaps even in a dynamic setup drive payoffs to the maximal payoff.

There are various models of cheap talk and similar models of communication that seem to confirm this intuition.

1.1 The literature

Matsui (1989) analyzes cheap talk in pure coordination games and assumes that the population is divided into two types. When the players are randomly matched one type is always matched against the other type, i.e., an individual is never matched against the same type. Under these assumptions Matsui (1989) shows that cheap talk leads to efficiency in the best response dynamics.

Robson (1990) introduces a model with secret handshakes among the mutants. It is assumed that a mutant can recognize when he is matched against another mutant but the rest of the population cannot distinguish them other than by the strategies they play. Robson (1990) shows for two by two unanimity games (A unanimity game is a partnership game in which the payoffs are positive on the main diagonal and zero otherwise) that an Evolutionarily Stable Strategy (ESS) must achieve the maximal payoff (i.e., achieves the maximal payoff against the same strategy).

Wärneryd (1991) shows that in symmetric two by two unanimity games with cheap talk a pure strategy is a weak ESS if and only if it achieves the maximal payoff.

In section 6 the above literature is examined in more detail.

All these results seem to point out that cheap talk leads to efficiency at least in unanimity games. To follow up this conjecture we will set up a basic model and analyze what cheap talk can achieve in a dynamic framework with as little additional structure as possible. The only previous dynamic analysis of cheap talk was done by Matsui (1989, 1991) for a two type population model.

1.2 The solution concept

Before going into the details of the communication and game structure we would like to present the solution concept.

We will consider a dynamic adjustment process in an infinite population pairwise randomly matched to play a symmetric two person game. In order to eliminate coordination devices other than communication we will assume that the population is homogeneous and that the matching is symmetric. This means that neither in the matching nor in the reproduction process nor when the players are playing the game are there any asymmetries, in particular, there is no distinguishing between row and column players. Each individual in the population will be completely characterized by a pure strategy of the game he plays when he is matched.

Now we will introduce the dynamic adjustment process known as replicator dynamics and the solution concept of asymptotic stability.

Maynard Smith and Price (1973) defined the notion of an Evolutionarily Stable Strategy (ESS) for symmetric two person games. The definition was motivated by some intuition for a strategy with fitness derived from this game to survive in an evolutionary process with infrequent mutations. Later Taylor and Jonker (1978) and Zeeman (1990) showed that all ESS are Asymptotically Stable Strategies of the (continuous) replicator dynamics. The replicator dynamics are the approximation of the following discrete time process defined in an infinite population of individuals, each playing a pure strategy. In each period individuals are pairwise randomly matched to play a game, receive a payoff and then reproduce proportionally to the relative payoff they received. After that they die.

The relevant solution concept in dynamic adjustment processes is that of Asymptotically Stable Sets (AS Sets) and Asymptotically Stable Strategies (ASS). Such an AS Set has the property that it is closed and that trajectories starting in a neighborhood of it stay close to the initial starting point and eventually converge to an element of the set. The element a singleton Asymptotically Stable Set (AS Set) (i.e., it contains one element) is called an Asymptotically Stable Strategy (ASS). A population with mean in such a set can resist a one time mutation of sufficiently small frequency in the sense that the perturbed population will stay close and eventually its mean will converge to an element of the set.1 However, ESS is a fairly demanding concept which often fails to exist. Later Thomas (1985) defined the notion of an Evolutionarily Stable Set (ES Set) that contains the definition of an ESS when the ES Set is a singleton set. He showed that ES Sets are Asymptotically Stable Sets (AS Sets). Although a weaker concept, ES Sets need not exist even when AS Sets do.

In an attempt to avoid confusion of these similar terms we present a diagram at the end of the paper showing their interdependencies.

An Asymptotically Stable Strategy (ASS) fails to exist when strategies are

[1] The restriction to a one time entrance can be slightly generalized. In another paper, Schlag (1990) presents a scenario in which a population with mean in an Asymptotically Stable Set can "resist" repeated mutations. The condition is that each time only a sufficiently small frequency of mutants enters.

duplicated. Asymptotically Stable Sets (AS Sets) on the other hand are independent of so called spurious duplification of strategies. In population models there are a large number of individuals. The partition of these individuals into types seems arbitrary and it is not clear why names should affect the stability concept. In another paper, Schlag (1990) argues this point in more detail outgoing from the sensitivity of the ESS concept to spurious duplification. They resolve the problematic by defining a weaker concept called Equivalent Evolutionarily Stable Strategy (eESS).

We therefore find it most natural to search for Asymptotically Stable Sets (AS Sets) and when they appear as singleton sets to treat this as a special case.

1.3 Partnership games

We will restrict our attention to partnership games (for a definition thereof see above). Hofbauer and Sigmund (1988) prove for partnership games that the Asymptotically Stable Strategies (ASS) of the replicator dynamics are precisely the Evolutionarily Stable Strategies (ESS) of the game. We prove the same identity for the case of sets: in partnership games, Evolutionarily Stable Sets (ES Sets) are Asymptotically Stable Sets (AS Sets) and vice versa. These relations are included into the diagram at the end of the paper. Additionally, we show that AS Sets always exist in partnership games: the set of strategies that achieve the maximal average payoff is an AS Set. Hence our solution concept of determining the AS Sets always generates at least one solution and alternative solution concepts of the dynamic adjustment process must not be developed. The above characterization of Asymptotically Stable Sets (AS Sets) is useful because AS Sets and other features of dynamic adjustment processes are not simple to calculate whereas ES Sets can be calculated without reference to the dynamics.

1.4 Cheap talk

The next stage is to introduce cheap talk to the partnership game. Before the game is played each individual sends a message from a finite set of messages. A strategy of the resulting communication game then consists of a message that is sent and a reaction function that, based on the message received specifies which strategy of the game is played. Individuals now are endowed with such communication strategies and the replicator dynamics based on this enlarged symmetric game will be analyzed. It follows that if the underlying game is a partnership game then so is the enlarged communication game. Following the above remarks on the equivalence of ES Sets and AS Sets in partnership games our goal will be to characterize the ES Sets of the communication game.

As a benchmark case we first investigate how this effects partnership games which have a pure strategy that dominates all other strategies in the game (such a strategy we will call dominant). Games with a dominant pure strategy have very "robust" dynamics: there is a unique Evolutionarily Stable Set (ES Set) and it contains all dominant strategies. It seems intuitive that cheap talk should not change such a stable situation. On closer examination though it turns out that dominant strategies often fail to exist when cheap talk is added to the game. The maximal payoff that can be achieved depends not only on the strategy played but also on the signal sent. Still the dynamic stability is preserved in partnership games that have a dominant pure strategy: there is a unique ES Set in which only strategies that dominate all others in the original game are played.

However outside partnership games the stability of a dominant pure strategy is not necessarily preserved when cheap talk is added. In the appendix we present an example to demonstrate this fact. The game is a symmetric two by two game (that is not a partnership game) with a dominant strategy that yields the efficient outcome. Without cheap talk the dominant strategy as a singleton set is the unique Asymptotically Stable Set (AS Set). However when cheap talk is added to this game, AS Sets no longer exist.

So in general games the addition of cheap talk might be harmful to the stability properties of the dynamics of the game. However this cannot happen in partnership games: with or without cheap talk they will always have an Evolutionarily Stable Set (ES Set). This follows from the existence theorem for partnership games.

Next we consider strategies with excess messages, i.e., ones in which not all messages are sent, and characterize whether they are contained in an ES Set or not. We show that a strategy that does not send each signal with positive probability is contained in an Evolutionarily Stable Set (ES Set) if and only if it achieves the maximal payoff. The intuition is that if a population in an ES Set does not use all signals then a neighboring population in the ES Set will play a slightly different mean strategy when receiving the unused signal. Repeating this argument it follows that there will be a population in the ES Set that plays the strategy that leads to the maximal payoff when receiving the unused message. Consequently a mutant can take advantage of this population and take over.

As a corollary we obtain that a strategy of the communication game in which the same action is played disrespect of the messages received will be contained in an ES Set of the communication game if and only if the action achieves the efficient payoff. The intuition is that since the messages are not used for coordination consequently some element of the ES Set will not send each message. Together with the previous result this implies the statement. So constant play of inefficient actions of the partnership game are ruled out by cheap talk, especially inefficient ES Sets of the original game are eliminated. However we will find out that cheap talk also creates ES Sets, these will be sets in which each element sends each message.

Next we show that cheap talk shifts the range of payoffs in ES Sets upwards. How does the ES Set that obtains the largest payoff look? In a game where the maximal payoff is realized in a symmetric outcome, the largest payoff in an ES Set is the same with or without cheap talk (see existence theorem for ES Sets in partnership games). However in games where this is not the case, the maximal average payoff in the communication game (which is equal to the largest payoff in an ES Set) is larger than in the game without communication. Additionally the larger the message set, the closer this payoff gets to the efficient or maximal one.

The increasing effect cheap talk has on the average payoffs in ES Sets can also be shown for the minimal payoff. We show that unless all payoffs in ES Sets are equal to the maximal payoff, the average payoff in any ES Set of the game with cheap talk is strictly greater than the minimal average payoff in an ES Set of the game without cheap talk.

1.5 Efficiency

It turns out that the above increasing effect of cheap talk on the payoffs in ES Sets is sufficient to guarantee "near" efficiency in partnership games with only two strategies given that the message set is large. We characterize all ES Sets for each possible partnership game with two strategies and show that the minimal average payoff in an ES Set (which increases with the number of signals) converges to the efficient payoff.

However these efficiency results for partnership games with two strategies represent a special case and do not generally extend to games with more than two strategies. We present a simple partnership game (unanimity game) with three strategies that has an ESS with average payoff bounded away from the maximal payoff for any size of the message set. The ESS uses each message with positive probability. Consequently a mutant cannot coordinate with a type in the population to get the efficient payoff without sacrificing payoffs. So cheap talk will generally not drive all payoffs in Evolutionarily Stable Sets (ES Sets) of the replicator dynamics close to the maximal payoff.

To summarize, we analyze the effects of cheap talk in homogeneous populations that are symmetrically and randomly matched and evolve according to the replicator dynamics. Cheap talk need not be advantageous if the game is not a partnership game. It can destroy stability in a symmetric two by two game. However in partnership games it has a positive effect, the more messages the better: the range of payoffs in AS Sets is increased and for two by two games all outcomes in AS Sets become nearly efficient. However cheap talk fails to guarantee efficiency when there are more than two strategies in the game.

In section 6 we present the related literature on cheap talk. We give the intuition that leads to our inefficiency result and point out the structure that is needed in addition to cheap talk to achieve efficiency in other models.

2. Preliminaries:

Consider a symmetric two person game $\Gamma(S,E)$ with the pure strategies $S=\{e^i, i=1,..,N\}$ and the payoff function $E:\Delta S\times\Delta S\rightarrow\mathfrak{R}$ where ΔS is the set of probability distributions on S, i.e., $\Delta S=\{x\in\mathfrak{R}^N \text{ s.t. } x_i\geq 0 \text{ and } \sum_{i=1}^{N} x_i=1\}$. For $x\in\Delta S$ let BR(x) be the set of best replies to the strategy x, i.e., $BR(x)=\{y\in\Delta S \text{ s.t. } E(y,x)\geq E(z,x) \text{ for all } z\in\Delta S\}$. For $x\in\Delta S$ let C(x) be the support of x, i.e., $C(x)=\{e^i\in S \text{ s.t. } x_i>0\}$. For $e^i\in S$ we will sometimes write $x(e^i)$ instead of x_i. To simplify notation we will not distinguish between the pure strategy $e\in S$ and the distribution on S that assigns unit probability to e (i.e., S is embedded in ΔS), especially, $\frac{1}{2}e^1+\frac{1}{2}e^2\in\Delta S$.

The replicator dynamics of $\Gamma(S,E)$ on ΔS for continuous time and pure strategy types is as follows:

$$x^0=\bar{x} \text{ and } \frac{d}{dt}x^t_i=[E(e^i,x^t)-E(x^t,x^t)]x^t_i,\ i=1,..,N;\ t\geq 0,$$

where $\bar{x}\in\Delta S$ is the initial state and x^t_i is the frequency of the type using strategy e^i ($e^i\in S$) at time t. It can be shown that for each $\bar{x}\in\Delta S$ the above differential equation defines a unique function $x:\mathfrak{R}^+\rightarrow\Delta S$. To simplify notation we will drop the parameter t from the expressions (e.g., $x=x^t$).

In this context $L\subseteq\Delta S$ is called an Evolutionarily Stable Set (ES Set, Thomas, 1985) if

i) L is closed,

ii) if $p\in L$ then $p\in BR(p)$ and

iii) for any $p\in L$ there exists an open neighborhood U(p) such that $E(p,x)\geq E(x,x)$ for all $x\in U(p)\cap BR(p)$ and where $E(p,x)=E(x,x)$ implies $x\in L$.

Thomas (1985) shows that the definition is not changed if "BR(p)" is left out of condition iii). Once the "BR(p)" term is left out, condition ii) follows from the alternative condition iii): Assume that $p\in L\backslash BR(p)$. Then there exists $z\in\Delta S$ such that $E(p,p)<E(z,p)$. For $\lambda\in(0,1)$, let $x^\lambda=(1-\lambda)p+\lambda z$. Then for sufficiently small $\lambda>0$,

$E(p,x^\lambda)<E(z,x^\lambda)$ which is equivalent to $E(p,x^\lambda)<E(x^\lambda,x^\lambda)$ and thus contradicts condition iii).

This gives us an alternative definition which contains precisely those properties that will drive the subsequent stability results.

$L \subseteq \Delta S$ is an ES Set if and only if L is closed and for any $p \in L$ there exists an open neighborhood U(p) such that $E(p,x) \geq E(x,x)$ for all $x \in U(p)$ and where $E(p,x)=E(x,x)$ implies $x \in L$.

An Evolutionarily Stable Strategy (ESS) is a special case of an Evolutionarily Stable Set (ES Set): a strategy $p \in \Delta S$ is called an Evolutionarily Stable Strategy (ESS) (Maynard Smith and Price, 1973) if {p} is an Evolutionarily Stable Set (ES Set).

Thomas (1985) shows the following properties of an Evolutionarily Stable Set (ES Set). If a connected ES Set contains an ESS then it contains no other strategies, the union of ES Sets is an ES Set and a maximal connected subset of an ES Set is also an ES Set.

Simple calculation shows that every symmetric game with two strategies has an Evolutionarily Stable Set (ES Set). In comparison, every generic symmetric two by two game has an ESS (van Damme, 1991). However for more than two strategies this must not hold any more. Van Damme (1991) presents an example of a game with a unique symmetric Nash equilibrium where the equilibrium strategy is not an ESS. Because of the uniqueness of the symmetric Nash equilibrium neither can the equilibrium strategy be in an Evolutionarily Stable Set (ES Set).

Additionally it is easy to show that the concept of an ES Set is "independent" of spurious duplification of strategies, i.e., of the addition of pure strategies with payoffs identical to some existing pure strategy. Elsewhere (Schlag, 1990), we formalize the notion of independence with respect to spurious duplifications. The addition of identical strategies expands the strategy space of the game. In this context "independent" essentially means that ES sets of the expanded game are expansions of the ES sets of the original game.

We now come to the most important property that justifies the use of the ES

Set as a solution concept. Thomas (1985) proved the pendant of the Taylor and Jonker (1978) and Zeeman (1980) result on the sufficiency of an ESS for the asymptotic stability of a strategy. We will first give a short review of the dynamic stability concepts.

A strategy $p \in \Delta S$ is called a dynamic equilibrium if it is a fixed point of the replicator dynamics. A set $L \subseteq \Delta S$ is called attracting if there exists an open neighborhood W of L such that each trajectory starting in W converges to L ($W \subseteq \Delta S$). A strategy $p \in \Delta S$ is called stable if for every open neighborhood U of p there exists an open neighborhood V of p s.t. the trajectories starting in V do not leave U ($U,V \subseteq \Delta S$). A set $L \subseteq \Delta S$ is called stable if every $p \in L$ is stable. A set $L \subseteq \Delta S$ is called a (locally) Asymptotically Stable Set (AS Set) if it is closed, attracting and stable. A singleton AS Set is called an Asymptotically Stable Strategy (ASS).

In the following we add some notes on the above definitions. A strategy starting in W converges to L ($L,W \subseteq \Delta S$) if for any $\bar{x} \in W$ and $(t_k)_{k \in \mathbb{N}}$ such that $t_k \to \infty$ when $k \to \infty$ ($t_k \in \Re$) it follows that $\inf\{dist(x^{t_k},z), z \in L\} \to 0$ as $k \to \infty$ where x^t solves the replicator dynamics starting at $x^0=\bar{x}$. The definition of stability is slightly stronger than the classical one (see e.g. Bhatia and Szegö, 1970): in the standard definition the set as a whole must be stable, not necessarily each point. Finally, w.l.o.g. we require additionally to the standard definition (Bhatia and Szegö, 1970) for the asymptotically stable set to be closed. We find it intuitive to include dynamic equilibria on the border into the definition of an AS set.

Notice that the union of finitely many AS Sets is again an AS Set. Now we can state the sufficiency theorem.

THEOREM 2.1 (Thomas, 1985):

If $L \subseteq \Delta S$ is an Evolutionarily Stable Set (ES Set) then L is an Asymptotically Stable Set (AS Set) w.r.t. the (continuous) replicator dynamics. The converse is not true.

We refer to (Thomas, 1985) for the proof of this theorem.

3. Partnership games:

A symmetric game $\Gamma(S,E)$ is called a partnership game (Hofbauer and Sigmund, 1988) if $E(e,e')=E(e',e)$ for all $e,e'\in S$. So a partnership game is a symmetric game in which both players always get the same payoff. These games can also be referred to as pure coordination games (Matsui, 1989) that are symmetric. However in the context of homomorphic populations it does not seem to be appropriate to talk about coordination when there may be strict Nash equilibria that are not necessarily symmetric.

For the case of $|S|=2$, the payoffs in a general partnership game are given in table I where $a,b,c\in\Re$ are parameters.

Table I: A general partnership game with two strategies.

	T	B
T	a,a	b,b
B	b,b	c,c

The payoff structure of partnership games has a particular characteristic that is not necessarily true for more general games, namely that the average payoff in a population is constant on any connected set of symmetric Nash equilibria.

LEMMA 3.1:

If $\Gamma(S,E)$ is a partnership game and $G\subseteq\Delta S$ is connected with the property that $x\in G$ implies $x\in BR(x)$, then $x,y\in N$ implies $E(x,x)=E(y,y)$.

PROOF:

Assume that $G \subseteq \Delta S$ is connected and $x \in G$ implies $x \in BR(x)$. For the proof we will need the following claim:

Claim: w.l.o.g. G is piecewise convex, i.e., for any $x,y \in G$ there exists $j \in \mathbb{N}$ and $z^i \in G$, $i=0,1,..j$ such that $z^0=x$, $z^n=y$ and $½z^i+½z^{i+1} \in G$ for $i=0,1,..j-1$.

We will just sketch the proof of the claim. Assume $\{y, x^k, k \in \mathbb{N}\} \subseteq G$ and $dist(x^k,y) \to 0$ as $k \to \infty$. Then it follows that $C(y) \subseteq C(x^k) \subseteq BR(x^k) \cap BR(y)$ if k is sufficiently large. Furthermore $BR(x^k) \cap BR(y) \subseteq BR(½x^k+½y)$, so if k is sufficiently large, $½x^k+½y \in BR(½x^k+½y)$ and w.l.o.g. $½x^k+½y \in G$. The rest follows quite easily.

Notice that this property can be shown for any symmetric game $\Gamma(S,E)$.

Using the above claim the lemma will be proven if we show that $E(x,x)=E(y,y)$ when x,y and $½x+½y \in G$.

Assume not, i.e., w.l.o.g. $E(x,x)>E(y,y)$ which implies $E(x,½x+½y)-E(y,½x+½y)=½[E(x,x)-E(y,y)]>0$. However $½x+½y \in G$ implies that $C(½x+½y)=C(x) \cup C(y) \subseteq BR(½x+½y)$ and therefore $E(x,½x+½y)-E(y,½x+½y)=0$ which is a contradiction.

□

The following theorem due to Hofbauer and Sigmund (1988) presents some more properties that are specific to partnership games.

THEOREM 3.2: (Hofbauer and Sigmund, 1988)

The following claims hold for the replicator dynamics of a partnership game:

i) A strategy $p \in \Delta S$ is an Asymptotically Stable Strategy (ASS) if and only if it is an ESS.

ii) The average payoff in the population strictly increases over time if the trajectory is not at a rest point of the replicator dynamics.

In the biology literature part ii) of the above theorem is called the fundamental theorem of natural selection (see Hofbauer and Sigmund, 1988). The next theorem generalizes the ideas from the above theorem to sets of strategies.

For a population with the mean $x \in \Delta S$ we will call $E(x,x)$ the <u>average payoff</u> of the population x. We say that $x \in \Delta S$ <u>achieves the maximal payoff</u> if $E(x,x) \geq E(y,z)$ for any $y,z \in \Delta S$ (i.e., (x,x) is efficient). We say that x <u>achieves the maximal average payoff</u> if $E(x,x) \geq E(y,y)$ for all $y \in \Delta S$.

We show that in partnership games all AS Sets of the replicator dynamics are ES Sets and vice versa. Furthermore a connected subset is an AS Set if and only if it maximizes the average payoff in a neighborhood containing the best replies.

THEOREM 3.3:

Let $\Gamma(S,E)$ be a partnership game and $L \subseteq \Delta S$ be connected. Then the following statements are equivalent:

i) L is a Evolutionarily Stable Set (ES Set).

ii) L is a Asymptotically stable set (AS Set).

iii) There exists a neighborhood U(L) such that for $p,z \in L$ and $x \in \{U(p) \cup BR(p)\} \backslash L$, $E(x,x) < E(p,p) = E(z,z)$.

In particular $\{x \in \Delta S \text{ s.t. } E(x,x) \geq E(y,y) \text{ for all } y \in \Delta S\}$ is an Evolutionarily Stable Set (ES Set) of Γ that is not necessarily connected.

PROOF:

i) implies ii) is stated in theorem 2.1

ii) implies iii):

Let $L \subseteq \Delta S$ be a connected AS Set containing p. Van Damme (1991) shows that stable strategies must be symmetric Nash equilibrium strategies. Using lemma 3.1 it follows that $L \subseteq \{x \in \Delta S \text{ s.t. } E(x,x) = E(p,p)\}$. Since L is attracting there exists an open neighborhood U of p such that $U \backslash L$ contains no dynamic equilibria and trajectories starting in U converge to L. By part ii) of theorem 3.2 $E(x,x)$ increases strictly when x is not a dynamic equilibrium and hence $E(x,x) < E(p,p)$ for all $x \in U \backslash L$. This shows that statement iii) is true in a neighborhood of p.

We will now show that w.l.o.g. $BR(p) \subseteq U$. For $x \in BR(p)$ and $\lambda \in [0,1]$ let $p^\lambda = (1-\lambda)p + \lambda x$, then $E(p^\lambda,p^\lambda) = (1-\lambda)^2 E(p,p) + 2\lambda(1-\lambda)E(x,p) + \lambda^2 E(x,x) = (1-\lambda^2)E(p,p) + \lambda^2 E(x,x)$. So $E(p^\lambda,p^\lambda) = E(p,p)$ for all $\lambda \in [0,1]$ if and only if $E(x,x) = E(p,p)$

and either $E(x,x)<E(p,p)$ or $E(x,x)=E(p,p)$ and $p^{\lambda}\in L$ for all $\lambda\in[0,1]$ (especially $x\in L$).

iii) implies i):

Let $L\subseteq\Delta S$ be a connected set and U(p) be a neighborhood of L such that for $p,z\in L$, $x\in U(p)\backslash L$, $E(x,x)<E(p,p)=E(z,z)$. It follows from the continuity of E( ) that L is closed.

We will now show that L is a set of symmetric Nash equilibria. Let $p\in L$. For $\lambda\in(0,1)$ and $y\in\Delta S$ define $p^{\lambda}=(1-\lambda)p+\lambda y$. Then if λ is sufficiently small then $E(p,p)\geq E(p^{\lambda},p^{\lambda})$ which implies $(2-\lambda)E(p,p)\geq 2(1-\lambda)E(y,p)+\lambda E(y,y)$. It follows that $E(p,p)\geq E(y,p)$ which means that $p\in BR(p)$.

Let $y\in BR(p)$. Then $E(p,y)=E(y,p)=E(p,p)\geq E(y,y)$ and $E(p,y)=E(y,y)$ implies $y\in L$. Hence L is an ES Set.

Finally to the "in particular" statement:

$L=\{x\in\Delta S$ s.t. $E(x,x)\geq E(y,y)$ for all $y\in\Delta S\}$ can be split into a union of disjunct connected sets, each trivially satisfying condition iii) of the theorem. Therefore L is an ES Set.

□

Due to part iii) of the above theorem we will sometimes refer to the average payoff of a connected ES Set when we mean the average payoff of one of its elements. Using theorem 3.3 the Asymptotically Stable Sets (AS Sets) of the replicator dynamics for partnership games can now be fully characterized by calculating the Evolutionarily Stable Sets (ES Sets) of the game. This simplifies the analysis substantially.

For example, let us calculate the Asymptotically Stable Sets (AS Sets) for some two by two partnership games. Consider the game in table I. If (a,b,c)=(2,0,1) then the AS Sets are {T} and {B}, if (a,b,c)=(1,2,0) then the unique AS Set is $\{\frac{2}{3}T+\frac{1}{3}B\}$ (Only the symmetric Nash equilibrium strategies must be checked to find the elements of an AS Set).

A special class of partnership games that we will use as a benchmark will be those that have a dominant pure strategy.

We will say that the strategy $x \in \Delta S$ (weakly) dominates the strategy $y \in \Delta S$ if $E(x,z) \geq E(y,z)$ for all $z \in \Delta S$. A strategy that dominates all other strategies will be called dominant. Notice that in partnership games if a strategy $x \in \Delta S$ is dominant then it achieves the maximal average payoff which is equal to the maximal payoff.

THEOREM 3.4:

If $\Gamma(S,E)$ is a partnership game in which $e \in S$ is dominant then $G=\{x \in \Delta S \text{ s.t. } E(x,x)=E(e,e)\}$ is the unique Evolutionarily Stable Set (ES Set).

PROOF:

Since e dominates all other strategies and Γ is a partnership game, e achieves the maximal payoff. From theorem 3.3 it follows that G is an ES Set.

To show uniqueness, let $L \subseteq \Delta S$ be an ES Set. Since a mutant of type e can always enter and do at least as well as each other type it follows that $e \in L$ and therefore $L \subseteq G$. Actually a stronger conclusion follows: each connected disjunct component of L must contain e. This however implies that L is connected and therefore that L=G.

□

Finally we present a purely technical lemma that comes in handy when looking for ES Sets.

LEMMA 3.5:

Let $\Gamma(S,E)$ be a partnership game, let $G \subseteq \Delta S$ be an Evolutionarily Stable Set (ES Set) and let $p \in G$. Then

i) $E(p,p) \geq \min\{E(e,e),\ e \in C(p)\} = \min\{E(e,h),\ e,h \in C(p)\}$.

ii) If $E(h,h)=E(h,e)=E(e,e)$ for some $e,h \in C(p)$, $e \neq h$ then there exists a strategy $q \in G$ with strictly smaller support than p, i.e., $C(q) \subset C(p)$ $(C(q) \neq C(p))$.

PROOF:

Let $G \subseteq \Delta S$ be an ES Set and $p \in G$. Let $\alpha, \beta \geq 0$, $e, h \in C(p)$ and $z \in \Delta S$ be defined such that $p = \alpha e + \beta h + (1-\alpha-\beta)z$. Define $q = (\alpha+\beta)e + (1-\alpha-\beta)z$. Since $C(q) \subseteq C(p)$ which implies $q \in BR(p)$ we conclude by part iii) of theorem 3.3 that $E(q,q) \leq E(p,p)$. We now make a simple calculation.

$E(q,q) = (\alpha+\beta)^2 E(e,e) + 2(\alpha+\beta)(1-\alpha-\beta)E(e,z) + (1-\alpha-\beta)^2 E(z,z)$ and

$E(p,p) = \alpha^2 E(e,e) + 2\alpha\beta E(e,h) + \beta^2 E(h,h) + 2\alpha(1-\alpha-\beta)E(e,z) + 2\beta(1-\alpha-\beta)E(h,z)$
$+ (1-\alpha-\beta)^2 E(z,z)$.

So $E(q,q) \leq E(p,p)$ if and only if

$(\beta+2\alpha)E(e,e) + 2(1-\alpha-\beta)E(e,z) \leq \beta E(h,h) + 2\alpha E(e,h) + 2(1-\alpha-\beta)E(h,z)$ if and only if
$2E(e,p) + \beta E(e,e) - 2\beta E(e,h) \leq 2E(h,p) - \beta E(h,h)$.

Using the fact that $E(e,p) = E(h,p) = E(p,p)$ we obtain that $E(q,q) \leq E(p,p)$ if and only if $E(e,e) + E(h,h) \leq 2E(e,h)$.

Therefore $E(e,h) \geq \min\{E(d,d),\ d \in C(p)\}$ and with $E(p,p) \geq \min\{E(e,h),\ e,h \in C(p)\}$ part i) is proven.

Exchanging the weak inequality by an equality sign in the above calculation it follows that if $E(e,e) = E(h,e) = E(h,h)$ then $E(q,q) = E(p,p)$ and hence $q \in G$. Since $C(q) \subset C(p)$ and $C(q) \neq C(p)$, part ii) is proven.

□

4. Partnership games with cheap talk:

Recall the previous framework, in an infinite homogeneous population, the individuals are pairwise randomly matched without asymmetries to play a partnership game Γ(S,E). Each individual plays a fixed pure strategy. The frequencies of the types playing the same pure strategy adapt according to the replicator dynamics. In this setup we search for Asymptotically Stable Sets (AS Sets).

We will now introduce costless communication before the game is actually played in the form of cheap talk. The enlarged game now consists of two rounds, a signalling round and an action round. In the first or signalling round the two players simultaneously send a message from a finite set of messages $M=\{c^1,..,c^n\}$ to the other player. "Talk is cheap" because there is no cost of sending this message. Then in the second or action round each player chooses a pure strategy of the partnership game Γ(S,E) conditioned on the messages sent in the first round. Finally each player receives his payoff E( ) based on the strategy combination played in the second round. Mixed strategies of the enlarged (or communication) game are just randomizations over the pure strategies described above. W.l.o.g. it is assumed for simplicity that each player only conditions his action on the message received from the other player, not on the message he sent.

Although formally correct, we do not condition the strategy a player plays in the second round on the message he sent in the first round. Each player knows which message he sent and there are no mistakes. Additionally rationalization about why the individual plays a certain strategy does not arise because each individual is endowed with some fixed type. Adding these reactions leads to duplification of the present strategy which does not change the results because the notion of an Evolutionarily Stable Set and that of an asymptotically stable set are independent of spurious duplification.

Following Kim and Sobel (1991), a communication game $\Gamma(S^c,E^c)$ is defined by the set of pure strategies $S^c=M\times S^M$ and the payoff function $E^c:\Delta S^c\times\Delta S^c\rightarrow\Re$ satisfying $E^c((m^1,f^1),(m^2,f^2))=E(f^1(m^2),f^2(m^1))$ for $(m^i,f^i)\in S^c$, i=1,2 and linearity in both components. So the pure strategy $\sigma=(m,f)\in S^c$ has the interpretation that the message $m\in M$ is sent in the signalling round and $f(m')\in S$ is the strategy played in the action round after receiving the message $m' \in M$ from the other player in the signalling round. A mixed strategy is then an element of $\Delta(M \times S^M)$.

Our goal will be to analyze the replicator dynamics of $\Gamma(S^c,E^c)$ for pure strategy types, i.e., each type is endowed with a pure strategy in S^c.

Instead of (m,f) we will also write (m|f) and sometimes we will write $\sigma \in S^c$ in the form $\sum_{i=1}^{n} \alpha_i (c^i|f^i)$ where $\alpha_i \geq 0$, $\sum_{i=1}^{n} \alpha_i = 1$ and $f^i \in \{\Delta S\}^M$. For $\sigma \in \Delta(M \times S^M)$ let $BR^c(\sigma)$ be the set of best replies to σ (in the communication game), i.e., $BR^c(\sigma) = \{\sigma' \in \Delta(M \times S^M)$ s.t. $E^c(\sigma',\sigma) \geq E^c(\sigma°,\sigma)$ for all $\sigma° \in \Delta(M \times S^M)\}$.

Certain properties of symmetric two person games do not change when cheap talk is introduced. Each of the following statements are independent of whether we consider the communication game $\Gamma(S^c,E^c)$ or the underlying game $\Gamma(S,E)$:

i) The game is a partnership game.
ii) The maximal payoff is equal to z for some $z \in \Re$.
iii) The maximal average payoff equals the maximal payoff.

However the following phenomena might occur when cheap talk is introduced:

iv) The maximal average payoff is larger with cheap talk than without.
v) Without cheap talk the game has a dominant strategy, with cheap talk it has no dominant strategy.

Following statement i) and theorem 3.2 we will characterize the ES Sets of the communication game.

Statement iv) is true when the maximal payoff is not achieved in a symmetric outcome. To give an example for statement v), consider the game of table I with (a,b,c)=(2,1,0). Notice that T is a dominant strategy and, as a singleton set, is the unique ES Set (theorem 3.4). Consider the communication game with M={L,R} as the set of messages. It is easy to show that this enlarged game no longer has a dominant strategy. Let (A|C,D) be the strategy, play A in the first round and play C (D) in the second round if the other player played L (R) in the first round where $A\in\{L,R\}$, $C,D\in\{T,B\}$. For example, (L|T,T) is not a dominant strategy because $E^c((L|T,T),(L|B,T))<E^c((R|T,T),(L|B,T))$. Similarly (R|T,T) cannot be a dominant strategy.

In the line of the above example a more general statement for partnership games can easily be proven: unless the best reply of the dominant strategy in the original game is the entire set of strategies ΔS then there is no dominant strategy in the communication game.

Despite the destruction of dominant strategies when cheap talk is introduced, their stability remains as stated in the following theorem (compare to theorem 3.4).

THEOREM 4.1:

If Γ(S,E) is a partnership game in which $e\in S$ is dominant then $\{\sigma\in\Delta(M\times S^M)$ s.t. $E^c(\sigma,\sigma)=E(e,e)\}$ is the unique Evolutionarily Stable Set (ES Set) in the communication game.

PROOF:

Let $G=\{\sigma\in\Delta(M\times S^M)$ s.t. $E^c(\sigma,\sigma)=E(e,e)\}$. The proof is similar to that of theorem 3.4. E(e,e) is the maximal payoff in the original game and hence also in the communication game. Therefore it follows by theorem 3.3 that G is an ES Set.

We now need to show uniqueness.

Let $G'\in\Delta(M\times S^M)$ be an ES Set and let $\sigma\in G'$ where $\sigma=\sum_{i=1}^{n}\alpha_i(c^i|f^i)$ and $f^i\in\{\Delta S\}^M$.

Let $I(e)\in S^M$ be such that $I(e)(c^i)=e$ for all $1\leq i\leq n$ and let $\sigma^e=\sum_{i=1}^{n}\alpha_i(c^i,I(e))$. Then

$\sigma^e\in BR^c(\sigma)$ and $E^c(\sigma^e,\sigma^e)=E(e,e)$. From part iii) of theorem 3.3 it follows that $\sigma^e\in G'$, $E^c(\sigma,\sigma)=E(e,e)$ and therefore $G'\subseteq G$.

We will now show that $(c^1,I(e))\in G'$.

Applying part ii) of lemma 3.5 repeatedly to σ^e it follows that $\{(c^i,I(e))$ s.t. $\alpha_i>0\}\subseteq G'$. If $\alpha_1>0$ then we are finished. Otherwise choose some i such that $\alpha_i>0$. It follows that $(c^1,I(e))\in BR^c((c^i,I(e))$ and $E^c((c^1,I(e)),(c^1,I(e)))=E(e,e)$ and hence applying theorem 3.3 again we get $(c^1,I(e))\in G'$.

Finally, just as in the proof of theorem 3.4, we can show that $(c^1,I(e))\in G'$ for any ES Set G' and therefore G'=G must hold.

☐

So in partnership games with a pure strategy that dominates all others, with or without cheap talk there is a unique Evolutionarily Stable Set (ES Set). It seems generally true that if a game has a dominant pure strategy then cheap talk should have little or no effect on the basic stability properties. However this is not true. In the appendix we present a symmetric two by two game with a pure dominant strategy that achieves the efficient payoff. Without cheap talk there is a unique Asymptotically Stable Set (AS Set) that corresponds to an ESS. However after cheap talk is introduced, there are no more AS Sets. Such a complete destruction of stability by introducing cheap talk cannot happen in partnership games because following theorem 3.3 and statement i) above an AS Set always exists in such games.

We now come to some results that follow the intuition that cheap talk should lead to higher payoffs in the context of a reasonable dynamic adjustment process.

Let us consider the communication game based on the game in table I with the parameters (a,b,c)=(2,0,1) and the set of messages M={L,R}. Notice that the maximal payoff is equal to the maximal average payoff. $S^c=\{(A|B,C), A\in\{L,R\}, B,C\in\{T,B\}\}$ is the set of pure strategies of the communication game, i.e., the set of possible types in the population. Table II shows the payoffs to the row player between the eight pure strategies in the communication game (LBT=(L|B,T)).

Table II: The payoffs to the row player of the partnership game of table I for (a,b,c)=(2,0,1) with cheap talk using the message set {L,R}.

	LTT	LBT	LTB	LBB	RTT	RTB	RBT	RBB
LTT	2	0	2	0	2	2	0	0
LBT	0	1	0	1	2	2	0	0
LTB	2	0	2	0	0	0	1	1
LBB	0	1	0	1	0	0	1	1
RTT	2	2	0	0	2	0	2	0
RTB	2	2	0	0	0	1	0	1
RBT	0	0	1	1	2	0	2	0
RBB	0	0	1	1	0	1	0	1

Let $<(A|B,C),(D|E,F)>=\{\lambda(A|B,C)+(1-\lambda)(D|E,F),\ \lambda\in[0,1]\}$. It follows from theorem 3.3 that $\{x\in\Delta(M\times S^M)$ s.t. $E^c(x,x)\geq E^c(y,y)$ for all $y\in\Delta(M\times S^M)\}$ $=\{x\in\Delta(M\times S^M)$ s.t. $E^c(x,x)=E(T,T)\}$ $=<(L|T,B),(L|T,T)>\cup<(L|T,T),(R|T,T)>\cup<(R|T,T),(R|B,T)>$ is an Evolutionarily Stable Set (ES Set) of the communication game (i.e. of the game in table II).

Playing the strategy that gives the maximal payoff is "stable" with and without communication. This is however not the case with the inefficient symmetric Nash equilibrium strategy of the original game, B. Although {B} is an ES Set in the original game we will see that its evolutionary stability vanishes when cheap talk is added. Consider a strategy that plays B disregard of what message is received. If such a strategy is in an ES Set of the communication game then there will be strategies that do no send each message, especially (L|B,B) will be in the set. In such a population the signal R is not sent and mutants can enter that send R and play T if R is sent, i.e., they are of type (R|B,T). Since the type is in the best reply of (L|B,B) and achieves the maximal payoff, mutants of that type will take over.

In order to prove this result generally we will first present a theorem that characterizes all ES Sets that contain a strategy that does not send each message.

For any $\sigma \in \Delta(M \times S^M)$ we will say that σ <u>does not send each message in M</u> if there exists a message $m \in M$ such that if $m' \in M$, $f \in S^M$ and $(m'|f) \in C(\sigma)$ then $m \neq m'$.

THEOREM 4.2:

Let Γ(S,E) be a partnership game, M be a finite message space and $\sigma^* \in \Delta(M \times S^M)$ be such that σ* does not use each message in M. Then σ* is in an Evolutionarily Stable Set (ES Set) of the communication game if and only if σ* achieves the maximal payoff.

PROOF:

Let $m \in M$ be the message that $\sigma^* \in \Delta(M \times S^M)$ does not send.

Proof of the "only if" statement:

Assume that σ* is in an ES Set G but does not achieve the maximal payoff, i.e., $E(e,h) > E^c(\sigma^*,\sigma^*)$ for some $e,h \in S$. Since σ* does not send m it cannot prevent mutations from occurring in its reactions to receiving m. Let $\sigma^\circ \in \Delta(M \times S^M)$ be identical to σ* up to the fact that σ° always reacts to receiving m by playing h. Then $\sigma^\circ \in BR^c(\sigma^*)$ and $E^c(\sigma^\circ,\sigma^\circ) = E^c(\sigma^*,\sigma^*)$ and by part iii) of theorem 3.3 it follows that $\sigma^\circ \in G$. Now σ° cannot prevent a mutant from entering that sends m and always plays e, i.e., of type (m,I(e)). We obtain $E^c((m,I(e)),\sigma^\circ) = E(e,h) > E^c(\sigma^\circ,\sigma^\circ)$ which contradicts the fact that σ° is in the ES Set G.

Proof of the "if" statement:

If σ* achieves the maximal payoff then by theorem 3.3 it is in an ES Set.

□

It follows that either an ES Set achieves the efficient payoff or each of its elements sends each message.

We now are able to prove that inefficient strategies of the original game will be "eliminated" by cheap talk. For $\sigma^* \in \Delta(M \times S^M)$ and $x \in \Delta S$ we will say that <u>x is played in σ* for sure</u> if x is played with probability one when σ* is matched against itself, i.e., $(m^i|f^i) \in C(\sigma^*)$ for i=1,2 implies $f^1(m^2)=x$.

COROLLARY 4.3:

Let Γ(S,E) be a partnership game, M be a finite message space and $\sigma^* \in \Delta(M \times S^M)$ and $x \in \Delta S$ be such that x is played in σ* for sure. Then σ* is in an Evolutionarily Stable Set (ES Set) of the communication game if and only if x achieves the maximal payoff in Γ(S,E).

PROOF:

Following part ii) of lemma 3.5 it follows that w.l.o.g. σ* does not send each of its messages. Theorem 4.2 then implies the statement.

☐

It follows that in the above example the evolutionary stability of B is destroyed and only the evolutionary stability of T remains with cheap talk. However we will see that cheap talk not only eliminates ES Sets but creates new ones. Theorem 4.2 implies that elements of these "new" sets must send each message. We will see that these sets come in the way of any efficiency result that might be conjectured on the grounds of theorem 4.2.

In the following we will develop some general properties of ES Sets of the communication game. We will need the following lemma. It makes a connection between the ES Sets of the communication game and those of the underlying game.

LEMMA 4.4:

Let Γ(S,E) be a partnership game, let $M=\{c^1,..,c^n\}$ be a set of messages and let $\pi \in \Delta(M \times S^M)$ be written in the form $\pi=\sum_{i=1}^{n} \alpha_i (c^i|f^i)$, where $f^i \in \{\Delta S\}^M$, $\alpha_i \geq 0$, $1 \leq i \leq n$ and $\sum_{i=1}^{n} \alpha_i=1$. If π is in an Evolutionarily Stable Set (ES Set) of the communication game

$\Gamma(S^c,E^c)$ and $\alpha_i>0$ $(1\le i\le n)$ then $f^i(c^i)$ is in an Evolutionarily Stable Set (ES Set) of the underlying game $\Gamma(S,E)$.

PROOF:

Let G^c be an ES Set of $\Gamma(S^c,E^c)$ containing π. W.l.o.g. assume that $\alpha_1>0$. For $y\in\Delta S$ define $\sigma(y)=\sum_{i=1}^{n}\alpha_i(c^i|g^i)$ such that $g^1(c^1)=y$ and $g^i(c^j)=f^i(c^j)$ otherwise

$(1\le i\le n, 1\le j\le n)$. Then for $y,z,u,v\in\Delta S$, $E(y,z)>(=)E(u,v)$ if and only if $E^c(\sigma(y),\sigma(z))>(=)E^c(\sigma(u),\sigma(v))$. Therefore the conditions on the ES Set G^c of the communication ensure that $G=\{x\in\Delta S$ s.t. $\sigma(x)\in G^c\}$ is an ES Set of the original game. Finally G contains $f^1(c^1)$.

□

The following theorem shows that cheap talk shifts the set of payoffs in Evolutionarily Stable Sets (ES Sets) upwards. More specifically, all average payoffs of ES Sets in the communication game are at least as large as the minimal average payoff of an ES Set of the game without cheap talk, and they are strictly larger if the efficient payoff is not equal to the minimal average payoff of an ES Set. Moreover there exists an ES Set of the communication game with average payoff that converges to the maximal payoff as the number of messages tends to infinity.

THEOREM 4.5:

Let $\Gamma(S,E)$ be a partnership game and let M be a finite set of messages with $|M|\ge 2$. Let $\mu=\min\{E(x,x)$ s.t. $x\in G$ and G is an ES Set of $\Gamma\}$, $\eta=\max\{E(x,x), x\in\Delta S\}$ and $\pi=\max\{E(x,y), x,y\in\Delta S\}$. Then

i) if $\sigma\in\Delta(M\times S^M)$ is in an Evolutionarily Stable Set (ES Set) of the communication game $\Gamma(S^c,E^c)$ then $E^c(\sigma,\sigma)\ge\mu$, if additionally $\pi>\mu$ then $E^c(\sigma,\sigma)>\mu$ and

ii) there exists an ES Set that of $\Gamma(S^c,E^c)$ with payoff $(1-\frac{1}{n})\pi+\frac{1}{n}\eta$.

PROOF:

Part i) Let $G \subseteq \Delta S^c$ be an ES Set containing σ. Part i) of lemma 3.5 implies that $E^c(\sigma,\sigma) \geq \min\{E^c((c^i,f),(c^i,f)),\ (c^i,f) \in M \times S^M\}$. Furthermore, it follows from lemma 4.4 that for $(c^i,f) \in C(\sigma)$ ($c^i \in M$ and $f \in S^M$) $f(c^i)$ is in an ES Set of Γ(S,E). Therefore $E^c((c^i,f),(c^i,f))=E(f(c^i),f(c^i)) \geq \mu$.

Assume additionally that $E^c(\sigma,\sigma)=\mu<\pi$. Part i) of lemma 3.5 implies that $E^c(h,e)=\mu$ for all $e,h \in C(\sigma)$. It follows then by part ii) that not all elements of G send all of their messages. Finally, theorem 4.2 implies that $E^c(\sigma,\sigma)=\pi$ and with $\pi>\mu$ we get a contradiction.

Part ii): By theorem 3.3 it is enough to show that $\max\{E^c(\sigma,\sigma),\ \sigma \in \Delta(M \times S^M)\}=(1-\frac{1}{n})\pi+\frac{1}{n}\eta$. If $\pi=\eta$ then the claim follows by definition.

So assume $\pi>\eta$.

Let $q=\sum_{i=1}^{n}\beta_i(c^i|g^i)$, $g^i \in \{\Delta S\}^M$, $\beta_i \geq 0$, $1 \leq i \leq n$ and $\sum_{i=1}^{n}\beta_i=1$. Then

$$E^c(q,q)=\sum_i \beta_i^2 E(g^i(c^i),g^i(c^i))+\sum_{i\neq j}\beta_i\beta_j E(g^i(c^j),g^j(c^i)) \leq \sum_i \beta_i^2\eta+\sum_{i\neq j}\beta_i\beta_j\pi$$

$$=\pi-(\pi-\eta)\sum_i \beta_i^2 \leq \pi-(\pi-\mu)/n.$$

Furthermore let $x,u,v \in \Delta S$ be such that $E(x,x)=\eta$ and $E(u,v)=\pi$. If $\beta_i=\frac{1}{n}$, $f^i(c^i)=x$ and $(f^i(c^j),f^j(c^i))=(u,v)$ for $1 \leq i \leq n$ and $i<j \leq n$ then $E^c(q,q)=\pi-(\pi-\mu)/n$.

□

So cheap talk increases the highest and the lowest average payoff in an Evolutionarily Stable Set (ES Set) of the game.

5. Efficiency in partnership games:

In this section we will address the question whether cheap talk can guarantee efficiency. Using theorem 4.5 we will now characterize the Evolutionarily Stable Sets (ES Sets) for each partnership game with two strategies (see table I). Especially we obtain in this special class of two by two games that cheap talk achieves "near" efficiency, i.e, the average payoffs in Evolutionarily Stable Sets (ES Sets) are arbitrarily close to the maximal payoff if the message set is sufficiently large.

Using theorem 3.3 and the fact that the union of ES Sets is an ES Set it is enough to characterize the connected ES Sets.

THEOREM 5.1:

Let $\Gamma(S,E)$ be a partnership game with the strategies $S=\{T,B\}$ and for some $n\geq 2$ let $M=\{c^1,..,c^n\}$ be a set of messages. Let $G=\{x\in\Delta(M\times S^M)$ s.t. $E^c(x,x)\geq E^c(y,y)$ for all $y\in\Delta S\}$. We will distinguish three cases (see table I):

[$a\geq b$ and $b\geq c$] or [$a=c>b$]: Either $\Gamma(S,E)$ has a dominant strategy or it has two strict symmetric equilibria such that $E(B,B)=E(T,T)$. It follows that G is the only Evolutionarily Stable Set (ES Set) in the communication game.

[$a>c>b$]: $\Gamma(S,E)$ has two strict symmetric equilibria such that $E(B,B)<E(T,T)$. There are exactly two connected Evolutionarily Stable Sets (ES Sets) in the communication game, namely G and $\{p^n\}$ (the latter coincides with an ESS) where

$p^n=\sum_{i=1}^{n}\frac{1}{n}(c^i|f^i)$, $f^i:M\rightarrow S$ such that $f^i(c^i)=B$ and $f^i(c^j)=T$ when $i\neq j$ $(1\leq i\leq n,\ 1\leq j\leq n)$ and

$$E^c(p^n,p^n)=(1-\frac{1}{n})E(T,T)+\frac{1}{n}E(B,B).$$

[$b>a\geq c$]: $\Gamma(S,E)$ has a unique symmetric Nash equilibrium (v,v) that is not pure. There are finitely many connected Evolutionarily Stable Sets (ES Sets) in the communication game, each coincides with an ESS. The set consisting of these ESS's is $\{\sum_{i=1}^{n}\frac{1}{n}(c^i|f^i)$ s.t. $f^i:M\rightarrow\Delta S$, $f^i(c^i)=v$ and $(f^i(c^j),f^j(c^i))\in\{(T,B),(B,T)\}$ when $i\neq j$ $(1\leq i\leq n,$

$1 \le j \le n)\}$. Each of these ESS's achieves the average payoff $(1-\frac{1}{n})E(T,B)+\frac{1}{n}E(v,v)$.

PROOF:

The following claims simplify the calculations.

Let $\sigma=\sum_{i=1}^{n}\alpha_i(c^i|f^i)$ be in an ES Set of $\Gamma(S^c,E^c)$ and assume that $\alpha_i>0$. Following lemma 4.4, $f^i(c^i)$ must be in an ES Set of $\Gamma(S,E)$. If additionally σ does not achieve the maximal payoff then from Lemma 3.5 and theorem 4.2 it follows that $E^c(\sigma,\sigma)>\min\{E(f^i(c^i),f^i(c^i)),\ (c^i|f^i)\in C(\sigma)\}$.

The rest of the proof follows with simple calculation and using theorem 4.5. □

However the "near" efficiency result of theorem 5.1 is generally not true for more than two strategies. We present a partnership game (that is a unanimity game) with three strategies in which the communication game has an Evolutionarily Stable Set (ES Set) with an average payoff that is not close to the maximal average payoff, even when the message space is large. Consider the partnership game in table III.

Table III: A partnership game with three strict equilibria when a>b>c>0.

	T	M	B
T	a,a	0,0	0,0
M	0,0	b,b	0,0
B	0,0	0,0	c,c

Let $n \geq 2$, $M=\{c^1,..,c^n\}$ be the set of messages and let $p=\sum_{i=1}^{n}\frac{1}{n}(c^i|f^i)$, $f^i:M \rightarrow S$ such that $f^i(c^i)=B$ and $f^i(c^j)=M$, $i \neq j$, $1 \leq i \leq n$ and $1 \leq j \leq n$. It is easy to verify that p is an ESS of the game in table III with cheap talk using the set of messages M and that $E^c(p,p)=(1-\frac{1}{n})b+\frac{1}{n}c<b<a$.

Notice that it is quite essential for the inefficiency result that the message set is finite and fixed. If each mutant that wants to enter may introduce a new signal like in the secret handshake model of Robson (1990) then a mutant will be able to enter the above population and as in theorem 4.5 it follows that the average payoff in an ES Set must be the maximal payoff. Besides issues concerning intuition there are existence problems in such a model. Consider a partnership game where the maximal payoff is not generated in a symmetric equilibrium, like in the two by two partnership game with parameters (a,b,c)=(1,2,0). Mutants will constantly enter, causing the number of types to go to infinity.

6. Discussion:

Various models analyzing the effect of communication on the outcomes of evolutionary processes of games loosely described as coordination games have independently come to the conclusion that communication leads to efficiency. We set up a very simple model and obtain among other things that cheap talk leads to outcomes that are close to the efficient one when the partnership game has only two strategies and the message set is large. Although a bit weaker, this result seems to go in the same direction as the previous findings. However, for more than two strategies we show that the outcomes must not even be near to the efficient one. This we will refer to as our inefficiency result. Due to these grave differences in the predictions between our model and previous findings we now wish to discuss the related literature and point out the differences in the models themselves that lead to the different results with respect to the efficiency of the outcomes.

Matsui (1989) applies best response dynamics to two by two pure coordination games - in particular partnership games are symmetric pure coordination games. They show that the set of pareto efficient outcomes is the unique Cyclically Stable Set. This result seems to hold generally for pure coordination games with a finite set of pure strategies. The main difference to our model is that they consider a population with two types (e.g., row and column player): in the random matching process a player of one type is always matched against a player of the other type. This presents a coordination device that is built in the matching process. The aim of our analysis is to see how identical players can coordinate using cheap talk without external "help". Hence we choose a model with symmetric matching.

Later, Matsui (1991) reformulates his results for two by two games of common interest. These are games that have a unique weakly pareto optimal outcome.

Kim and Sobel (1991) analyze cheap talk in a related kind of game of common interest and - without mentioning it - their analysis implicitly assumes a two type population like that of Matsui (1989, 1991). They consider games in which the row player achieves his maximal payoff if and only if the column player achieves his maximal payoff - in this definition partnership games are included. Kim and Sobel (1991) prove efficiency of their static solution concept called Equilibrium Evolutionarily Stable Sets (ES Sets). The result is comparable to that of Matsui (1989, 1991), however they differ in the method. Their static concept is based solely on intuitive considerations, not on any explicit dynamic adjustment process.

Wärneryd (1991) analyzes a static concept in the symmetric setup we refer to as random symmetric matching in a homogeneous population. They characterize the weak Evolutionarily Stable Strategies (weak ESS, the definition is due to Thomas, 1985) that are pure of two by two unanimity games with cheap talk. The definition of a weak ESS is similar to the definition of an ESS just that the strict inequality is replaced by a weak inequality. A unanimity game is a partnership game in which the payoff is positive if and only if a strategy is matched against itself, otherwise the payoffs are zero. The result states that a pure strategy is a weak ESS (Wärneryd (1991) calls it a Neutrally Stable Strategy) if and only if it achieves the efficient payoff.

The basic structure to which Wärneryd (1991) applies the static concept is the same as in our model. However, the analysis in the paper is somewhat incomplete. There is no mentioning of the dynamics although it is apparent: weak ESS's are stable (see Thomas, 1985). The result is limited to the case of two strategies. They state that their result is not true for more than two strategies (e.g., analyze the example in table III in section 5) but do not consider this more general case any further. Additionally the weakening of the ESS condition is motivated by non-existence problems - these however result from the unusual restriction of the ESS condition to pure strategies. As shown in theorem 5.1, (mixed strategy) ESS do exist for unanimity games with cheap talk.

Finally, Robson (1990) considers a model with a slightly different form of communication. They assume secret handshakes among the mutants, i.e., the mutants can recognize other mutants that entered at the same time, all other types cannot distinguish them. Robson (1990) considers two by two unanimity games and restricts the analysis to secret handshake mutants that play a pure strategy that is not equal to the population mean strategy. In this restricted framework the strategy that achieves the efficient outcome is the only one that can resist such mutations. The intuition that leads to efficiency in their model also drives the partial efficiency result of our model for the case where not all messages are sent (see theorem 4.2). There are however some details missing in their analysis to make it a complete. They do not include every possible type of mutant so that the stability of the efficient ESS is not fully justified. When formalizing the model completely some slight difficulties seem to arise. Without some additional assumption, like cost of entry, there is no way to prevent an infinite number of different types of mutants to be present in the population.

As demonstrated above, the previous literature predicts efficiency in models of dynamic adjustment in games with cheap talk. In contrast to this stands our inefficiency result (section 5). We now argue the robustness of our result to changes in the model and alongside present the intuition that leads to our inefficiency result. This will then be compared to the intuition that leads to the above efficiency results.

First of all we will show that our inefficiency results are not due to the difference between the replicator dynamics and the best response dynamics.

What results can be derived when the best response dynamics are applied to our model of cheap talk, i.e., when the asymmetries are dropped in the model of Matsui (1989, 1991)? A strategy is called socially stable if no other strategy is accessible from it in the best response dynamics (see Matsui, 1992). Matsui (1992) mentions that an ESS is a socially stable strategy. It follows that the inefficient ESS that is presented in section 5 is a socially stable strategy. Therefore once the asymmetric coordination device apparent in the form of the two type population is removed from the model, the efficiency result of Matsui (1989) breaks down. So the difference in the dynamic processes does not drive the difference in the results.

Is the fact that an ESS can support inefficient equilibria in the game with cheap talk intuitive or just a mathematical peculiarity of this model? We claim that our results are typical under the assumption of finite message sets, costless communication and no asymmetric coordination device - not due to the choice of cheap talk as a model of communication.

In a more realistic model of communication one might introduce communication in finitely many rounds and perhaps give a meaning to the messages exchanged. However in any model without an asymmetric coordination device the situation can arise in which each strategy gets a higher payoff against a different strategy in the population support than against the same. Consequently an individual wants to avoid being matched against his own type. This we refer to as the meeting dilemma. When restricted to these strategies all possible messages will be sent to maximize the probability of not meeting the same type. Notice that flipping a coin between the two players to determine who starts talking does not resolve the problem.

How can cheap talk lead to efficiency in models of dynamic adjustment? Assume that there is a strategy that is not in the best reply of the present population mean that achieves a higher payoff than any strategy in the support.

Consider the framework presented in this paper. Using cheap talk a mutant can coordinate when matched against another mutant to get this superior payoff. Since only a very small frequency of mutants may enter the population the mutant must play a best response to the average population strategy in order to enter. So the mutant must send a message that no one in the present population uses so that he can tell when he is matched against another mutant and not against a type in the former population. This signal serves as a secret handshake between the mutants and the mutants will slowly take over if they are not punished by the original population for using the different message. If a message is not sent in the population then arbitrary mutations can occur in the way the population reacts to receiving this message. Especially there will be a population that does not punish the entering mutant and hence it will take over.

In best response dynamics the intuition is similar. A type in the population will offer to play the pareto superior strategy if he receives a different signal. Again this is only a best reply if there are excess signals.

To summarize, the above described meeting dilemma can result in all messages being sent, thereby minimizing the probability of meeting the same type.

However, if there are no excess signals a sort of signal jamming occurs for the mutants and there is no possibility for a secret handshake and an inefficient outcome may be stable.

If an asymmetry is introduced exogenously like in the two type population model of Matsui (1989) the individuals can coordinate and there will be no incentive not to meet the same type. Therefore there will be excess signals and secret handshakes are possible.

It is therefore demonstrated that the inefficiency result may be natural and that the efficiency results in the two type model of Matsui (1989) are equally driven by the population structure as by the pregame communication.

What is the role of the messages? In fact only two messages are needed to ensure efficiency in the two type population models of Matsui (1989, 1991) and Kim and Sobel (1991). Thus their result cannot explain the emergence of large message sets. In randomly matched populations without asymmetries in the matching process the players are better off with large message sets (i.e., a rich language) even though they will not necessarily achieve the efficient payoff. So our analysis suggests a cause for the emergence of large message sets.

So communication alone cannot ensure efficiency in a model of dynamic adjustment however large message sets will be preferred.

Appendix - Example in which cheap talk destroys stability:

We present a simple example in which cheap talk "ruins" stability in the replicator dynamics. Consider the game in table IV.

Table IV: A symmetric game with a dominant strategy.

	T	B
T	2,2	1,2
B	2,1	0,0

This game has a unique efficient outcome which is achieved by the dominant strategy T. {T} is the unique Asymptotically Stable Set (AS Set), in fact T is an ESS. The stability of the strategy T seems quite robust. However we will show that the addition of cheap talk will ruin all stability, i.e., there will be no more AS Sets in the communication game.

We will need the following lemma that is generally true for the replicator dynamics, here stated informally. Let G be an AS Set. If a pure strategy e dominates another pure strategy h that is in the support of an element of G then there exists another element of G with e in and h not in its support. Assume that this is not true, i.e., that the minimal relative proportion of the frequency of h to the frequency of e is strictly positive. Since e dominates h it follows that e will always reproduce faster than h and hence the relative proportion of the frequencies of h to e will never increase. Consider the trajectory starting close to the strategy that achieves the minimum with a strictly smaller relative proportion of the frequencies of h to e. Because G is an AS Set, the trajectory must converge to an element of G. However, as stated above, the proportion never increases which contradicts the minimality of the relative proportion.

We are now ready to analyze the example. Consider the communication game $\Gamma(S^c,E^c)$ of the game in table IV with M={L,R} as the set of messages (see section 4 for the definitions).

Assume that $G \subseteq \Delta(M \times S^M)$ is an AS Set of the replicator dynamics of $\Gamma(S^c,E^c)$. Take any $\sigma \in G$. W.l.o.g. $(L|g) \in C(\sigma)$ for some $g \in S^M$. We will apply the above lemma three times to show that G contains a pure strategy and then show that this strategy is unstable.

Since (L|T,T) dominates all strategies (L|f) for each $f \in S^M$, after applying the above lemma repeatedly there exists $\sigma^* \in G$ such that $\sigma^*=(1-\alpha)(L|T,T)+\alpha(R|r)$ for some $\alpha \in [0,1)$ and $r \in \{\Delta S\}^M$.

If $\alpha>0$ then apply the same procedure to the component of σ^* concerning the signal R, and we obtain that there exists $\sigma^\circ \in G$ such that $\sigma^\circ=(1-\beta)(L|T,T)+\beta(R|T,T)$ for some $\beta \in [0,1]$.

W.l.o.g. assume that $\beta<1$. Applying the procedure a third time we obtain that $(L|T,T) \in G$. From the payoff structure it follows that $(R|B,T) \in BR^c((L|T,T))$ and $E^c((L|T,T),(R|B,T))=E(T,B)<E(T,T)=E^c((R|B,T),(R|B,T))$. So it follows that (L|T,T) is not stable in a population consisting of types (L|T,T) and (R|B,T), and therefore it cannot be in an AS Set.

So cheap talk brings "confusion" into the replicator dynamics of such games. However, the fact that T is a weakly dominant strategy is necessary to construct such a counterexample. It is easy to show that when T is the unique best response to any strategy that cheap talk does not affect its stability.

It should be mentioned that best response dynamics behave quite differently in the above game: it can be shown that the set of efficient payoffs is the unique cyclically stable set of the game in table IV with cheap talk using the messages {L,R}.

Diagram of the interdependencies of the solution concepts

The following figure presents an overview of the relations between solution concepts used in the context of evolutionary games and the replicator dynamics.

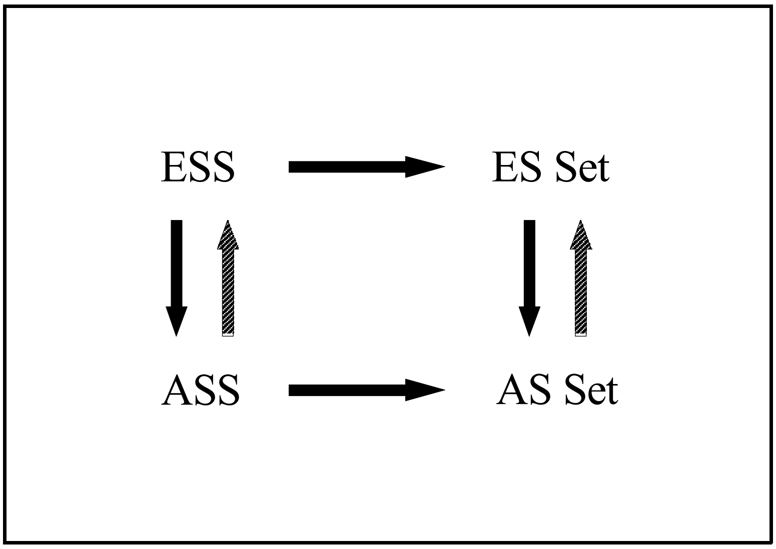

Figure 1: Inclusions go in the direction of the bold arrows. The shaded arrows represent inclusions that hold for partnership games.

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