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Endogenous Growth, Temporary Equilibrium, and the Direction of Change

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Abstract

This article studies the long-run direction of technological change in an endogenous growth model. Development is modeled as a sequence of temporary equilibria in an overlapping generations framework. We introduce a concept of 'long-run efficient development' which excludes *persistent* inefficiencies. The concept is much weaker than short-run or long-run Pareto-efficiency and does not depend on our particular model. The main theorem of the article gives conditions on agents' expectations and preferences and on the evolution of innovation possibilities under which equilibrium development, guided by current prices and profit expectations, is long-run efficient. *Journal of Economic Literature* Classification Numbers: D50, D60, O12, O30, O33.
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1 Introduction

TRADITIONAL AS WELL AS MODERN growth theories are mainly concerned with the intensity of change, with the growth rate of *aggregate* variables such as gross national product, aggregate consumption, aggregate stock of capital or the aggregate stock of knowledge. In contrast, this article concentrates on the long-run *direction* of change.

The question. Suppose that, given the state of technological knowledge in an economy with many commodities and different industries, there is a large set of conceivable improvements of this knowledge. In order to implement some of these improvements, costly innovations have to be carried through. Furthermore, assume that the choice among the potential innovations and imitations is determined by profit seeking investors. Then, the state of knowledge will typically not be widened simultaneously in all possible directions. Only sufficiently profitable potential innovations and imitations will be chosen in each period. Thus, the allocation of the resources necessary for change not only determines the *rate* of growth, as analysed in the literature on endogenous technological change. It also determines the *direction* of change. The aim of this article is to precisely formulate and answer the following question: What are conditions guaranteeing that the direction of change be efficient in the long-run in the sense that development, guided by profit expectations, eventually exhausts all feasible gains from development?

The model. Development is modeled as a sequence of temporary equilibria in an overlapping generations framework without bequests. In each period the state of knowledge is defined by the set of different technologies already known and by the age structure of this knowledge. Depending on the current state of knowledge, new technological possibilities emerge. More generally, there is a mapping, the 'innovation function', which defines a set of potential innovations for each state of knowledge and the amount of resources ('research') required for each of these innovations. The function is given exogenously and is not subject to economic explanation.

Given the state of knowledge and the incumbent industrial structure, temporary equilibrium (TE) determines prices and quantities on all markets including those for research, old assets (ownership rights for incumbent firms) and new assets (innovations and imitations). The kind and amount of new assets produced at TE, together with the old state of knowledge and the incumbent industrial structure determine the state of knowledge and the industrial structure of the following period. This in turn determines a new horizon of perception, a new TE (not necessarily uniquely), and a new state of knowledge. Thus, given an initial state of knowledge, we get a sequence of TE and a corresponding sequence of states of knowledge (equilibrium development of knowledge). In the path of development several technologies of different efficiency and age may temporarily coexist in the same industry.

The criterium. The innovation function, together with the initial state of knowledge, also defines 'potential development'. Roughly speaking, this is the hypothetical path of development that would arise if in each period *all* potential innovations of that period — irrespective of their profitability — would be carried through without bearing the corresponding cost and if the results of these innovations were made freely available to the whole economy. Equilibrium development is called long-run efficient if in the long term *all* industries are developed as efficiently in equilibrium development as in potential development. The criterium does not depend on intertemporal optimization and its applicability does not force us to work with the 'infinite horizon intertemporally optimizing representative agent' of most of normative growth theory, which, from a descriptive point of view "adds little or nothing to the story anyway, while encumbering it with unnecessary implausibilities and complexities".[1] Nevertheless, the criterium is sufficiently strong to exclude essential long-run market failures that are by no means excluded automatically by equilibrium development.

The result. The main theorem of this article gives conditions on agents' preferences and expectations and on the innovation function, guaranteeing that equilibrium development is long-run efficient. In a sense the theorem is a straightforward formalization of a popular claim about the dynamic efficiency of the free market system: It gives conditions under which current prices and short-run profits are sufficient private incentives to make sure that the right set of technologies is introduced in the long term. However, the theorem also makes more precise the limits to the claim of dynamic efficiency of the market system. All assumptions of the theorem are 'tight' in the sense that a class of counter examples can be given for the violation of any one of them.

Departures from the literature. The nature of our question accounts for two relevant deviations from most of the literature on endogenous growth theory (see Romer [1990], Grossman and Helpman [1991a, 1991b], Segerstrom [1991], Aghion and Howitt [1992]): We do not rely on the rational-expectation hypothesis and allow for perfect competition on all layers of the model.

TEMPORARY EQUILIBRIUM AND EXPECTATIONS. Most models of the new literature on endogenous growth assume that all decision takers have perfect foresight about all relevant endogenous future data. This assumption seems problematic already in the stationary world of these models. In our model, with many different industries that may develop at different and non-stationary speeds, perfect foresight or rational expectations would be quite forbidding assumptions. The innovation function merely describes the current generation of potential innovations as perceived by the agents with a certain previous knowledge. Nobody can foresee the future unfolding of the function. We, the

[1] Solow [1994], p 49.

theorist, will assume some properties of the innovation functions, and the agents in the economy may observe past properties and form expectations about future properties. But the specific function itself needs neither to be known to us, the theorist, nor to the agents in the economy. Also note that in the General Equilibrium framework of the present article one cannot hope for uniqueness of Rational Expectation Equilibria. Thus, even assuming perfect rationality and full knowledge of the innovation function would not suffice to justify Rational Expectation Equilibria (see Guesnerie [1992]). Clearly, when *framing* our question we do not want to rely on clear-voyant agents or on rational expectations. And when *answering* the question we want to find out *how* rational their expectations should be in order not to cause long-run market failures.

PERFECT COMPETITION. Almost all the models in the literature on edogenous growth are models of monopolistic competition. In contrast, the present paper provides a simple model of endogenous growth with perfect competition on all layers of the model, where by 'perfect competition' we mean price-taking firms and consumers, together with market-clearing prices and where by 'endogenous growth' we mean intentional production of new knowledge by profit-seeking private agents. Endogenous growth in that sense requires that agents that once have invested in the production or the purchase of new knowledge are able to recover these costs by non-negative profits (or 'quasi-rents') in later periods. This is perfectly possible for price-taking firms (in the standard Arrow-Debreu world frims typically make non-zero profits). Contrary to what is sometimes asserted, there is no conflict between *price-taking* behavior and endogenous growth in the above sense. There *is* a conflict between endogenous growth and free entry to new technologies and there also is a conflict between endogeneous growth and price-taking perfect competition if individual firms' technologies are linear homogeneous in all rival inputs. [2] In our model entry to new technologies (innovations) and entry to relatively 'young' technologies (imitations) is costly in general. Instantaneous replication of technologies by individual incumbent firms is excluded (individual technologies are 'small'). This suffices both to allow for endogenous growth and to give a foundation to the assumption of price-taking firms. Imitation of a given technology may become cheaper the longer the technology is known (so that we allow for free entry to sufficiently old technologies) and *'time-consuming'* replication of individual technologies need not be excluded.

We will later argue that our result could be derived in monopolistic competitive models and explain why we choose the perfectly competitive version.

In section 2 the TE model is introduced. Existence of TE is shown and equilibrium development is defined. In section 3 assumptions on preferences, expectations and the innovation function are

[2] For a detailed exposition of this conflict, see Romer [1990].

introduced, the efficiency theorem is stated and the main arguments of the proof are outlined. To simplify the exposition the structure of model presented in section 2 has been restricted in several ways. Extensions are outlined in section 4. Concerning the commodity set, the most important restriction is that there are only finitely many commodities. Thus, the only possible innovations, once all commodities have been introduced, are process innovations. This allows to concentrate on the assumptions on expectations and the innovation function. The interesting addition necessary to exclude long-run market failures with richer commodity sets is a generalization of an assumption on preferences, discussed in section 4. A second simplification in this respect is that firms produce a single output with a single input. Allowing for multi-input individual firms would naturally lead to the labor-saving versus capital-saving discussion in the literature on induced technical change of the early sixties (Fellner [1961], Kennedy [1964], Samuelson [1965], von Weizsäcker [1966]). Concerning the innovation/imitation process the most relevant simplification is one about investors' uncertainty. In section 4 we will argue that the TE approach easily allows for more complex uncertainty and that the results do not much depend on the simplification. Section 4 also sketches the necessary changes required for a monopolistic competitive interpretation of the model. Furthermore, section 4 discusses the relation between sustained growth and the direction of growth. The proofs are given in an appendix.

2 The Model

Development is modeled as a sequence of temporary equilibria in an overlapping generations model. There are finitely many types of individuals $i \in I$ and there is a continuum of members $a \in A_i$ of each type i, with $\cup_{i \in I} A_i = A$. An individual may simultaneously be a consumer (purchasing consumption commodities), a worker or researcher (supplying labor), an owner (owning primary inputs and owning shares of firms), an investor (trading shares). We will generally call them consumers. Every consumer lives two periods. For each old consumer in each period a young consumer of the same type is born.

There are K different industries, K (possibly different) perishable consumption commodities and K (possibly different) primary inputs. In each period $t \in I\!N$ and each industry $k \in \{1, \cdots, K\}$ incumbent firms produce one consumption good k (sold to consumers) with one primary input k (bought from consumers). Incumbent firms are owned by consumers who can trade their ownership rights (assets). The age structure of the set of technologies known to incumbent firms in period t defines the state of knowledge at t, and the vector of numbers of incumbent firms using each of these technologies determines the industrial structure of period t. Given the state of knowledge of t, innovators can produce new assets (claims to new firms) by producing new knowledge (innovations) and imitators can produce new assets by copying old knowledge (imitations). There is free entry to this activity, which requires 'research' as an input. How much research exactly is needed to carry through different innovations and imitations given the state of knowledge at t is determined by the innovation function. The number and the types of new assets, produced by innovators and imitators and sold to consumers, together with the incumbent industrial structure of period t, determine the new industrial structure and the new state of knowledge of the following period.

Since the model combines elements of standard temporary equilibrium theory (for an overview see Grandmont [1977]) with elements of endogenous growth theory, we indicate in advance where the present model deviates from standard temporary equilibrium models to incorporate the endogenous growth elements.

The markets on which temporary demand and supply are expressed are the markets for inputs (including that for research), for consumption commodities, and for ownership-rights ('assets'). In the standard general equilibrium model the ownership structure is exogenously given just as the distribution of endowments in primary inputs is exogenously given (see Debreu [1959]). In contrast, in the present framework 'ownership rights' or 'claims to firms', or, as we call them, 'assets', are traded. New assets (claims to new firms) too are traded on temporary markets. The temporary equilibrium quantities on the markets for new assets determine the direction of change. New assets are 'produced' by innovators and imitators (and supplied to the market). Old assets (claims to existing firms), in turn, are supplied by old consumers. Young consumers demand both new and old assets. Which old assets and which new assets they demand depends on their expectations about future prices and dividends of these assets.

Neither incumbents nor innovators and imitators need to look ahead. Only young consumers (as investors) have incentives to think about the future. They do this in a way which is standard in temporary equilibrium theory: They observe signals about past and present data of the economy and form expectations about future variables.

We successively introduce the incumbent sector, the innovation/imitation sector and the consumer sector. We deviate from the standard order of exposition (which puts consumers first), so that, when we come to consumers, the reader already knows the interpretation of the signals cosumers observe, and of the variables about which they form expectations.

For each sector temporary aggregate demand and supply will be defined: supply by incumbent firms (supply for the K outputs and demand for the K inputs), temporary aggregate supply by innovators and imitators (supply for *new* assets and demand for research) and temporary aggregate demand (demand for K outputs, supply of K inputs and of research, demand for new and old assets by young consumers and supply of *old* assets by old consumers). From there temporary excess demand and temporary equilibrium are defined as usual. The type and the number of new assets introduced at temporary equilibrium of a given period determines the direction of change of that period. An evolving sequence of temporary equilibrium (given an initial industrial state) will be called an equilibrium development.

2.1 Technologies and Incumbent Firms

In each industry k there is a set of potential types of technologies $n \in N_k$. Therefore, $\cup_{k=1}^{K} N_k = N$ is the set of all potential types of technologies, with $N_k \cap N_h = \emptyset$ for $k \neq h$.

The technologies of type n are represented by the 'unit' technology of type n, $Y_n \subset I\!R_- \times I\!R_+$, which is *not* the technology used by *individual* firms. An *individual* firm with technology of type n uses a 'small replication', ϵY_n, of the unit technology, with ϵ very small relative to aggregate endowments. The 'aggregate' technology of the sector using technology of type n is a 'large replication' of the unit technology, with the interpretation that it is the 'sum' of many small individual technologies. If there is free entry to technology of type n, then the aggregate technology of type n simply is the smallest cone containing the unit technology (Y_n), or, equivalently, the smallest cone containing the individual technology ϵY_n. In this case the aggregate technology of type n is a standard usual constant returns to scale technology. However, in a given period, there need not be free entry to the uses of technology of type n. In any given period t, the 'size' of the incumbent sector is the 'number' of incumbent firms knowing technology n. This 'size' is exogenously given and denoted by λ_n^{t-1}. For the time being it is not important how this size was determined. In period t is given. When we come to innovators and imitators we will see that it is determined by the accumulated innovation and imitation activity of previous periods. And when we come to consumers we will see that λ_n^{t-1} also is the amount of assets of type n (claims to firms using technology of type n) in the hands of consumers, since all firms are owned by consumers (owners). This assets have been purchased by young consumers at $t-1$ and are endowments of old consumers at t.

The aggregate technology $\hat{Y}_n^t$ of the sector using technology n at time t is the $\lambda_n^{t-1}-$times replication of the unit technology of type n (Y_n):

$$\hat{Y}_n^t = Ch(\lambda_n^{t-1} Y_n) \in I\!R_- \times I\!R_+,$$

where $Ch(\lambda_n^{t-1} Y_n)$ denotes the convex hull of $\lambda_n^{t-1} Y_n$. Note that the convex hull has to be taken, because a fraction of the small individual firms (with technology ϵY_n) may remain inactive.

We will assume that the aggregate sector using type n technology behaves perfectly competitive. This assumption is warranted if ϵ is small given the aggregate vector of endowment and can be formally justified along the lines of the Cournot model in Novshek and Sonnenschein [1980] or the Bertrand model in Funk [1995a], if the average production of Y_n tends to zero if the amount of input employed tends to infinity. Therefore, there is no need to explicitly modeling individual firms, who are the true actors in the incumbent sectors.

To make sure that the unit technology is of the kind that allows for a noncooperative foundations of the assumption of perfect competition we make some assumptions. The unit technology of type n, has a unique efficient scale and $0 \in Y_n$. Efficient scale inputs of Y_n are denoted a_n and the corresponding efficient scale productivity (maximal output per unit of input) is denoted α_n. We assume that there exists $\underline{a} > 0, \bar{a} < \infty$ such that $\underline{a} \leq a_n \leq \bar{a}$ $\forall n \in N$.

Given current prices $q^t = (p^t_k, w^t_k)_{k \in \{1, \cdots K\}} \in I\!R^{2K}$, the sector using the aggregate technology of type n chooses input demand and output supply $(-x, y) \in \hat{Y}^t_n$ that maximize profits $\pi_n(q^t) = p^t_k y - w^t_k x$, where (p^t_k, w^t_k) are the current output and input prices in industry k, with $n \in N_k$.

We denote by $\pi_n(q^t)$ the 'unit profits', i.e., the profits of the unit technology of type n if prices q^t. Note that these profits are zero if the aggregate technology produces on its linear segment. In this case individual firms produce at efficient scales. Locally, the sector behaves as if there were free entry. These profits are strictly positive if the aggregate technology produce above efficient scales. The profits are then distributed to the owners of type n assets in relation to their shares. Without the expectations of such profits no change would arise in the present model.

2.2 Innovations and Imitations

The industrial structure at period t is determined by the size of the (aggregate) incumbent technologies $\lambda^{t-1} = (\lambda^{t-1}_n)_{n \in N}$. In each period this structure is changed by innovators and imitators introducing technologies that were previously not used ($\lambda^{t-1}_n = 0$) and augmenting the size of already active type of firms ($\lambda^{t-1}_n > 0$). Innovations and also imitations bear a cost (in terms of research needed) that depends on the past development of knowledge. We assume that, of the full history of industrial structure, only the ages by which different types of technologies are known matter for the productivity of research in producing further knowledge. The age, θ^t_n, of technology n is defined by

$$\theta^t_n = \begin{cases} \max\{\theta | \lambda^{t-\theta}_n > 0\} & \text{if } \lambda^{t-1}_n > 0 \\ 0 & \text{if } \lambda^{t-1}_n = 0, \end{cases} \tag{1}$$

where $\theta_n^t = 0$ means that n is not known yet. The *state of knowledge* at t is defined as $\theta^t = (\theta_n^t)_{n \in N}$. The amount of research needed to produce new knowledge or to replicate old knowledge is given by the *innovation function* $x_{R\cdot}(\cdot)$ which defines the amount $x_{R\tilde{n}}(\theta^t)$ of research [3] needed to implement innovation $\tilde{n}$ if the current state of knowledge is θ^t. Concerning the uncertainty of the success of innovations we choose the simplest framework that allows to address our issue. We assume that investors know in advance the success of an innovation, i.e. know with certainty which technology will result (and can be used in $t+1$) from a given innovation in t.[4] Thus, in this simplified framework the set N of types of possible technologies coincides with the set of possible innovations. Some types of technologies may be chosen without doing any research given the state of knowledge θ^t (namely if $x_{Rn}(\theta^t) = 0$). Other types of technologies cannot be chosen at all given the state of knowledge θ^t, (namely if $x_{Rn}(\theta^t) = \infty$). The set of technologies that are active already or that can be activated at t is $N^t = \{n \in N | \lambda_n^{t-1} > 0 \text{ or } x_{Rn}(\theta^t) < \infty\}$. The set of new potential innovations given the state of knowledge θ^t is $\{n \in N | x_{Rn}(\theta^t) < \infty, x_{Rn}(\theta^\tau) = \infty \forall \tau < t\}$. We assume that there is a uniform upper bound on the number of new potential technologies per period.

Typically, innovation of technologies that are different and better than known technologies ($\theta_n^t = 0$) will be more expensive than small improvements and pure imitations of known technologies ($\theta_n^t > 0$) and imitation of quite recent knowledge ($\theta_n^t > 0$ but small) will be more expensive than imitation of old knowledge (θ_n^t large). The sooner the imitation of an innovation becomes cheap (and the cheaper it becomes) the harder it is for the innovator to internalize future gains from the innovation. Since we want to raise the question about *long-run* efficiency even in a world in which the profits that determine the direction of change are *short-run*, we assume that all technologies that are known for a sufficiently long period of time can be imitated free of cost. The condition does not exclude patenting, since the period of costly imitations can be large. Without the assumption the efficiency theorem of the next section would need a stronger condition to exclude distortionary bubbles, than it presently needs.

ASSUMPTION (FIOT) (FREE IMITATION OF SUFFICIENTLY OLD TECHNOLOGIES): *There exists a $T < \infty$ such that for all $t \in I\!N$, all k and all $n \in N_k$, with $\theta_n^{t-T} \geq T$ there is an $n' \in N_k$ with $x_{Rn'}^t = 0$ and $\alpha_{n'} \geq \alpha_n$.*

Given the state of knowledge θ^t, the innovation or imitation technologies are publicly known

[3] In accordance to the notation for other inputs $(x_1, \cdots, x_K)$, we use the notation x_R for research inputs.

[4] From a conceptual point of view not much has to be added if, for each type of investor (i.e. consumer), an innovation induces a probability distribution over the set of potential technologies N. Investors' uncertainty about the success of an innovation they have invested in, naturally fits into the very approach of the present article (although it would imply additional notation). This will become clear when we come to consumers.

or, in other words, there is free entry to these activities (i.e., producing 'claims to new firms' at the costs specified by the innovation-function). An individual innovator can produce the assets corresponding to a single individual technology. Since all individual technologies are small we assume that innovators behave competitively. Because of free entry, we can assume for all n a perfectly competitive innovation/imitation sector with linear aggregate technology, producing $\Delta\lambda_n$ new unit assets of type n with $\Delta\lambda_n x_{Rn}(\theta^t)$ units of research: Given the innovation function, the current state of knowledge and current prices (current price A_n^t of asset n, current price w_R^t of research), he chooses $\Delta\lambda_n$ to maximize

$$A_n^t \Delta\lambda_n - w_R^t \Delta\lambda_n x_{Rn}(\theta^t).$$

As usual with linear homogeneous aggregate technologies, the supplied quantity either is zero, is infinite, or is indeterminate (if $A_n^t = w_R^t x_{Rn}(\theta^t)$). Also note that although innovators produce assets that become productive only in future periods, they do not have to form expectations about the future. They simply take as given today's asset prices. Of course the *demand* for these assets will depend on investors' expectations about future dividends and asset prices.

If $(\Delta\lambda_n)_{n\in N}$ is the supply of new assets at t, then $x_R^t = \sum_{n\in N} \Delta\lambda_n x_{Rn}(\theta^t)$ is the demand for research at t.

2.3 Consumers

For all $k \in \{1, \cdots, K\}$ consumer $a \in A_i$ is endowed with e_k^{i1} and e_k^{i2} of input k when young and old respectively, and with e_R^{i1} and e_R^{i2} of research inputs when young and old respectively. Denote $e = (e^i)_{i\in I}$, $e^i = (e_R^{i1}, e_1^{i1}, \cdots, e_K^{i1}, e_R^{i2}, e_1^{i2}, \cdots, e_K^{i2})$. In addition, if a is old in period t, he owns assets $(\bar{\lambda}_n^a)_{n\in N^t}$, where $\bar{\lambda}_n^a$ is the number of unit assets of type n he has purchased in the previous period and where $N^t \subset N$ is the set of types of assets that can in principle be traded in period t (see above). Note that, although the number $\#N$ of potential assets may be (countable) infinite, we have made sure in the previous section, that the number $\#N^t$ of assets that can be traded at t is finite (only finitely many new assets can be introduced in each period).

A trading plan of consumer $a \in A_i$ is a vector $z^a = (z^{a1}, z^{a2}) = (-x^{a1}, y^{a1}, \lambda^a, -x^{a2}, y^{a2}, -\lambda^a)$, where $x^{a1} = (x_R^{a1}, x_1^{a1}, \cdots, x_K^{a1})$ and $x^{a2} = (x_R^{a2}, x_1^{a2}, \cdots, x_K^{a2})$ are a's input sales, $y^{a1} = (y_1^{a1}, \cdots, y_K^{a1})$ and $y^{a2} = (y_1^{a2}, \cdots, y_K^{a2})$ are a's output purchases and $\lambda^a = (\lambda_n^a)_{n\in N^t}$ are a's asset purchases when young and asset sales when old. The corresponding consumption plan is $v^a = (-x^{a1}, y^{a1}, -x^{a2}, y^{a2})$. Utility functions are defined with respect to such consumption plans. The utility function $u^i : (\mathbb{R}_-^{K+1} \times \mathbb{R}^K)^2 \to \mathbb{R}_+$ is assumed to be continuous, strictly quasi-concave and strictly increasing.

In each period consumers receive signals $\overleftarrow{s}^t = (\cdots, -s^{-1}, s^0, \cdots, s^{t-1}, s^t)$ about past and present data of the economy. The information about period $\tau \leq t$, $s^\tau = (q^\tau, \lambda^\tau)$ contains all prices $q^\tau = (w^\tau, p^\tau, A^\tau) \in S$ and the industrial structure of period τ, $\lambda^\tau = (\lambda_n^\tau)_{n \in N}$, where S is the unit simplex of $I\!R_+^{2(K+1)+\#N^t}$, $w^\tau = (w_R^\tau, w_1^\tau, \cdots, w_K^\tau)$ are the input prices, $p^\tau = (p_1^\tau, \cdots, p_K^\tau)$ the output prices and $A^\tau = (A_n^\tau)_{n \in N}$ the asset prices at τ, and where $\lambda_n^\tau = \lambda_n^{\tau-1} + \Delta\lambda_n^\tau$ is the total (i.e. mean) number of unit assets of type n, supplied by old consumers ($\lambda_n^{\tau-1}$), innovators and imitators ($\Delta\lambda_n^t$).

Depending on the signal $\overleftarrow{s}^t$, a young consumer $a \in A_i$ observes today, he forms expectations $\psi^i(\overleftarrow{s}^t) = \psi^{ti}(s^t)$ about tomorrow's prices q^{t+1}. The function ψ^i is assumed to be continuous. We make two assumptions that greatly simplify the exposition. Firstly, we assume that the expectations about prices are point expectations, i.e. $\psi^i(\overleftarrow{s}^t) \in I\!R^{2K+1+\#N^t}$ is a vector of expected prices. Secondly, as we have already noted, we assume that consumers know the technologies that correspond to new assets.[5] Given any sequence of past data, $\overleftarrow{s}^{t-1}$ the function $\psi^{ti} : \Delta \times I\!R^{\#N^t} \to I\!R_+^{2K+1+\#N^t}$ is assumed to be continuous.

A young consumer $a \in A_i$ chooses $z^{a1} \in I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#N^t}$ and plans $z^{a2} \in I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#N^t}$ subject to $(x^{a1}, x^{a2}) \leq e^i$, $q^t z^{a1} \leq 0$ and $\psi^i(\overleftarrow{s}^t) z^{a2} \leq \lambda^a \pi(\psi^i(\overleftarrow{s}^t))$, with $\pi(\psi^i(\overleftarrow{s}^t)) = (\pi_n(\psi^i(\overleftarrow{s}^t)))_{n \in N}$, where $\pi_n(\psi^i(\overleftarrow{s}^t))$ are the profits of the unit technology of type n if prices are $\psi^i(\overleftarrow{s}^t)$ (see section 2.2). Similarly, an old consumer $a \in A_i$ chooses $z^{a2} \in I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#N^t}$ subject to $x^{a2} \leq e^{i2}$, $q^t z^{a2} \leq \bar{\lambda}^a \pi(q^t)$, with $\pi(q^t) = (\pi_n(q^t))_{n \in N}$, where $\pi_n(q^t)$ are the profits of the unit technology of type n if prices are q^t. Let $\xi^{at}(s^t)$ be the set of utility maximizing choices of a consumer $a \in A_i$ subject to the above budget conditions given $\overleftarrow{s}$ and, if a is an old consumer, given the portfolio of assets, $\bar{\lambda}^a$, he acquired when he was young. Note that the optimal consumption plan of a consumer is unique, since utilities are strictly quasi-concave. The corresponding set of optimal portfolios need not be a singleton. Whenever the expected return (price plus dividend) of a dollar invested into different assets are identical for a young consumer he will be indifferent between these assets.

Solving the consumer problem for all types of consumer and aggregating over all types defines the temporary aggregate *demand correspondence* of the economy at t given history and incumbent asset ownership, $(\overleftarrow{s}^{t-1}, (\bar{\lambda}^a)_{a \in A})$. The correspondence is denoted $\xi^t : S \times I\!R^{\#N^t} \to I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#N^t}$.

Similarly, solving all aggregate firms' problems defines the temporary aggregate *supply correspondence* of the economy at t given the history $\overleftarrow{s}^{t-1}$. The correspondence is denoted η^t :

[5] In the more general case an innovation induces a subjective probability distribution over the set of potential technologies. In the case one has to capture the additional uncertainty in consumers' expectation functions $(\psi^i)_{i \in I}$ and in profit expectations $\pi(\psi^i(\overleftarrow{s}^t))$.

$S \times I\!R^{\#N^t} \to I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#N^t}$. In other words, the vector $(-x_R^t, (-x_k^t)_k, (y_k^t)_k, (\Delta\lambda_n)_n) \in \eta^t(s^t)$, if and only if

(1) $(x_k, y_k) = (\sum_{n \in N_k} x_n^t, \sum_{n \in N_k} y_n^t)$ for all k, where for all n $(-x_n^t, y_n^t)$ solves the type n (aggregate) incumbent firm's problem and

(2) $\Delta\lambda_n$ solves the type n aggregate innovator's problem given q^t and θ^t for all $n \in N$,

(3) $x_R^t = \sum_{n \in N} \Delta\lambda_n x_{Rn}(\theta^t)$.

Temporary *excess demand* given $(\overleftarrow{s}^{t-1}, (\bar{\lambda}^a)_{a \in A})$ is defined by $\zeta^t(s^t) = \xi^t(s^t) - \eta^t(s^t)$.

2.4 Temporary Equilibrium

Given industrial history and ownership structure $(\overleftarrow{s}^{t-1}, (\bar{\lambda}^a)_{a \in A})$, a vector of aggregate quantities $z^t = (-x_R^t, (-x_k^t)_k, (y_k^t)_k, (\lambda_n)_n)$ together with a vector of current data, s^t is called *temporary equilibrium* (TE), if $z^t \in \xi^t(s^t) \cap \eta^t(s^t)$. Note that a temporary equilibrium exists whenever there exists a vector of current data, s^t, with $0 \in \zeta^t(s^t)$.

Existence of temporary equilibrium given the history will be shown (in the appendix) along the line of the existence proof in Debreu [1959]. Note that the supply of all current inputs (including research) is bounded. Limited resource availability allows the use of a standard compactification argument for the input and outputs of all incumbent industries as well as for all costly innovations $(x_{Rn}(\theta^t) > 0)$. This is not true for 'free' imitations $(x_{Rn}(\theta^t) = 0)$. We make a simple convention that solves the problem. Note that if $x_{Rn}(\theta^t) = 0$ and $A_n^t = 0$ for asset n, then the innovator of type n is indifferent about which amount of new assets of type n he 'produces' and supplies. Producing infinite amounts of new assets of type n is feasible and (weakly) optimal. Furthermore, at price $A_n^t = 0$ consumers either are indifferent (if their expectations about $\pi_n(q^{it+1})$ and A_n^{it+1} are zero) or demand infinite amounts of asset n. Therefore, if $x_{Rn}(\theta^t) = 0$, we can always clear the market for asset n by treating the asset as if it were a free good. We make the convention that infinite amounts of such assets are allocated to each type of consumer.

Proposition 1 *For all industrial histories $\overleftarrow{\lambda}^{t-1}$ and asset ownership structures $(\bar{\lambda}^a)_{a \in A}$, there exist a temporary equilibrium (i.e. there exists an s^t with $0 \in \zeta^t(s^t)$).*

2.5 Development

Given the incumbent industrial structure λ^{t-1} and the temporary equilibrium supply of new assets $\Delta\lambda^t$, the new industrial structure is $\lambda^t = \lambda^{t-1} + \Delta\lambda^t$. The corresponding new state of knowledge is θ^{t+1} (which is derived as in equation 1). Given the innovation function $x_{R\cdot}(\cdot)$ and the initial industrial history $\overleftarrow{s}^0$ we thus get a sequence of temporary equilibrium, which is called an *equilibrium development* and a sequence of states of knowledge, $(\theta^t)_{t\in I\!N}$, which is called an *equilibrium development of knowledge* given $x_{R\cdot}(\cdot)$ and $\overleftarrow{s}^0$.

Note that, given an initial industrial history, there may be multiple equilibrium developments (and equilibrium developments of knowledge) since temporary equilibrium need not be unique.

We now define *potential development of knowledge* given the innovation function $x_{R\cdot}(\cdot)$ and the initial industrial history $\overleftarrow{s}^0$. The potential development of knowledge is the hypothetical path of knowledge which one would get, if in every period all potential innovations were carried through (all innovations that are not infinitely costly). Given $\theta_{\mathcal{P}}^{t-1} = (\theta_{\mathcal{P}n}^{t-1})_{n\in N}$ define

$$\theta_{\mathcal{P}n}^t = \begin{cases} \theta_{\mathcal{P}n}^{t-1} + 1 & \text{if } \theta_{\mathcal{P}n}^{t-1} > 0 \\ 1 & \text{if } \theta_{\mathcal{P}n}^{t-1} = 0, x_{Rn}(\theta_{\mathcal{P}n}^{t-1}) < \infty \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Given the initial industrial history let the corresponding state of knowledge also be the potential state of knowledge, i.e. $\theta_{\mathcal{P}}^0 = \theta^0$. Then, the sequence is called *potential development of knowledge* given the innovation function $x_{R\cdot}(\cdot)$ and the initial industrial history $\overleftarrow{s}^0$.

3 Long-Run Efficient Development

We can now turn to the main question of this article. Under which conditions on the innovation function, on preferences and the expectation function does equilibrium development exhaust all potential gains from development? We say that the development of knowledge exhaust all potential gains from development or is long-run efficient if all technologies actually used in the long term are as good, or almost as good, as the best potential technologies and if all firms sell their products at marginal cost prices of these most efficient technologies. Formally, let $\alpha_{\mathcal{P}k}^t = \max_{n\in N_k, \theta_{\mathcal{P}n}^t > 0} \alpha_n$ denote the maximal efficient scale of all potential technologies at t. Then **equilibrium development is long-run efficient** if $\lim_{t\to\infty}(p_k^t/w_k^t) = \lim_t(1/\alpha_{\mathcal{P}k}^t)$ in all industries k.

The following assumptions are crucial conditions for long-run efficiency of equilibrium development in the sense that examples of persistently inefficient development can be given if any of them is not satisfied (see Funk (1995b)). Thus the conditions offer a taxonomy of potential causes for long-run market failures.

Long-run efficiency is an asymptotic condition, and the present article does not say anything about the relative speed of convergence in different industries. This is the price we pay in order to work in a very general setting concerning preferences and the innovation function. However, the taxonomy of persistent *in*efficiencies directly leads to specific results about the relative speed of development if one is sufficiently specific about the functional forms of preferenes and of the innovation function.

3.1 Assumptions on the Innovation Function

(1) Innovations that once could have been chosen, but have not been chosen, should remain potential innovations later, and should not need more research later than earlier:

ASSUMPTION (NLO) (NO LOSS OF OPPORTUNITIES): *For all* $n \in N$, *with* $\theta_n^t = 0$, *there exists an* $n' \in N$, *with* $Y_{n'} = Y_n$, *such that for all* t, $x_{Rn'}^t \leq x_{Rn}^{t-1}$.

Assumption (NLO) is restricted to opportunities that have not been implemented ($\theta_n^t = 0$) yet. A potential innovation, once it *has* been implemented, does not necessarily remain a potential innovation thereafter. If a successful innovation is patented, for instance, it is excluded form the set of potential innovation for a certain time.

(2) Given a state of knowledge at t and a technology that could in principle be reached in finitely many periods starting with the knowledge of t, there should be a potential innovation at t that —at least slightly— improves the state of knowledge of t into the direction of the given technology.

ASSUMPTION (CONV) (CONVEXITY): *For all* $\tau < \infty$ *there is a* $\beta \in [0, 1)$ *such that for all* θ^0, *all* k, *and all* $n_1, n_2 \in N_k$ *with* $\theta_{n_1}^0 > 0, \theta_{\mathcal{P}n_2}^\tau > 0$, *there exists an* $n \in N_k$ *with* $\theta_{\mathcal{P}n}^1 > 0$ *such that* $\alpha_n \geq \beta\alpha_{n_1} + (1 - \beta)\alpha_{n_2}$.

This assumption says nothing about the cost of the potential innovations (except that, by definition, the costs must be finite).

(3) If the research productivity in an industry grows over time, this may be interpreted as an external effect of the previous production of knowledge on the productivity of further research. Such external effects may cause 'exploding' productivity of research in some industries. This can in principle be harmful for the development of other sectors (because profits in such industries will be high relative to the profits in industries without exploding productivity of research). Certainly, we do not want to exclude exponential growth of research productivity which is the (more or less directly assumed) engine of sustained growth in most of the 'endogenous growth literature'. In order to exclude that the productivity growth of research in one sector hinders the development of other sectors we will assume that the external effects of research on the productivity of further research are not fully industry specific.

The *productivity of research* on innovation n, given the state of knowledge θ is defined as $\delta_n(\theta) = (\alpha_n / x_{Rn}(\theta))$. The growth of research productivity is defined with respect to sequences of potential innovations.[6] Given an initial state of knowledge, an innovation function and a feasible development, let $(\hat{n}_k^t)_{t\in I\!N}$, $\hat{n}_k^t \in N_k^t$, be a sequence of potential innovations in industry k. Given an initial state of knowledge, an innovation function and a feasible development, the *growth rate of research productivity* with respect to the sequence $(\hat{n}_k^t)_{t\in I\!N}$ is defined as $\dot{\delta}^t_{\hat{n}_k^t} = (\delta^t_{\hat{n}_k^t} - \delta^{t-1}_{\hat{n}_k^{t-1}})/\delta^{t-1}_{\hat{n}_k^{t-1}}$. Note that exploding productivity of research either means that the increase of the α's or the reduction of the corresponding research becomes big or both.

ASSUMPTION (MINSPILL) (MINIMAL NON SPECIFIC SPILLOVERS): *Given any feasible development of knowledge, given any h and any sequence $(\hat{n}_h^t)_{t\in I\!N}$, $\hat{n}_h^t \in N_h^t$, of potential innovations, there exists an $\epsilon > 0$ and a $\bar{t} < \infty$, such that for all k and for all $t > \bar{t}$: $(\dot{\delta}^t_{n_k}/\dot{\delta}^t_{\hat{n}_h}) > \epsilon$ if $\dot{\delta}^t_{\hat{n}_h} > 0$ and $\alpha_k^t < \lim_t \alpha^t_{\mathcal{P}k}$, where $(n_k^t)_t$ is the most efficient potential innovation in k at t, i.e. $\alpha_{n_k^t} = \max_{n\in N_k^t} \alpha_n$ and where α_k^t is the most efficient known technology in k at t, i.e. $\alpha_k^t = \max_{n\in N_k^t, \theta_n^t > 0} \alpha_n$.*

This is an assumption on the innovation function and the initial state of knowledge (since it should hold for any feasible development of knowledge).

We also assume that $x^t_{Rn_k^t}$ does not grow exponentially, i.e. $\forall\mu > 0, \exists\bar{t} < \infty : (x^t_{Rn_k^t}/x^{t-1}_{Rn_k^{t-1}}) < 1 + \mu\forall t > \bar{t}$.

(4) The above assumptions on the innovation function would not have sufficient impact on the evolution of asset prices if technologies that are identical but are introduced in different periods or that simply have different names were traded on different markets. Old technologies could remain valuable without ever producing dividends and divert consumers from investing in *profitable* innovations. The following condition simply assumes that identical technologies are not distinguishable on the asset market. This will guarantee that the value of already existing assets is limited by the production cost of new assets of the same type (i.e. that $A_n^t \le w_R^t x^t_{Rn'}$ for any n' with $Y_n = Y_{n'}$). Together with Assumption (FIOT) this allows to make sure that the value of old assets tends to zero.

ASSUMPTION (IA) (IDENTICAL ASSETS): *If $Y_n = Y_{n'}$, then $A_n^t = A_{n'}^t$ for all t.*

Note that the assumption is not as innocuous as it may appear. Even if the only way of distinguishing an old asset from a new asset that can be reproduced free of cost, is its name or date of issue (and not the productivity of the corresponding technology), these old assets remain valuable

[6] Note that the under the term 'productivity of research' one may prefere to understand the 'relative' productivity increase from some known technology, $\bar{n}$, in the industry of the innovation to the productivity of the potential innovation in question. However, in the present framework it is natural to focus on the 'absolute' productivity α_n rather than on the difference of productivity of a potential innovation n and some already known technology $\bar{n}$, since the choice of the old technology $\bar{n}$ would inevitably be arbitrary. In general, several old technologies are in use simultaneously in some industries and – depending on the innovation function – the most profitable innovations need not improve upon the most efficient of them.

and thus continuously attract investors, even if all investors were fully rational. In principle such rational bubbles may prevent investment into assets which are more productive.

3.2 Assumptions on Preferences and Endowments

About utilities we only assume that the marginal rates of substitutions are well behaved on the boundaries of consumption sets.About endowments we make the extreme assumption that $e^i >> 0$ for all $i \in I$. We have studied elsewhere what can happen if these assumptions are not satisfied (see Funk (1995c)).

Assumption (MRS) (Marginal rates of substitution): *Consider any sequence of consumption plans* $(v^\tau)_\tau = (((-x_k^{\tau 1})_k, (y_k^{\tau 1})_k, (-x_k^{\tau 2})_k, (y_k^{\tau 2})_k))_\tau$ *and let* $u^{i\tau}_{y_k^\kappa}$ *(resp.* $u^{i\tau}_{x_k^\kappa}$*) be the derivative of the utility of a type* i *consumer at consumption plan* v^τ *with respect to the output (resp. input, possibly research)* k *when young if* $\kappa = 1$*, and when old if* $\kappa = 2$*. Then, (1)* $u^{i\tau}_{y_h^{\kappa_1}} / u^{i\tau}_{y_{h'}^{\kappa_2}} \to \infty$ *if* $(y_h^{\kappa_1 \tau} / y_{h'}^{\kappa_2 \tau}) \to 0$ *for* $\kappa_1, \kappa_2 \in \{1,2\}$*; (2)* $u^{i\tau}_{y_h^{\kappa_1}} / u^{i\tau}_{x_{h'}^{\kappa_2}} \to \infty$ *if* $y_h^{\kappa_1 \tau} \not\to \infty$ *and* $x_{h'}^{\kappa_2 \tau} \to 0$ *for* $\kappa_1, \kappa_2 \in \{1,2\}$*; (3)* $u^{i\tau}_{x_h^{\kappa_1}} / u^{i\tau}_{x_{h'}^{\kappa_2}} \to 0$ *if and only if* $(x_h^{\kappa_1 \tau} / x_{h'}^{\kappa_2 \tau}) \to 0$ *for* $\kappa_1, \kappa_2 \in \{1,2\}$*.*

The assumtption is satisfied for the following class of utility functions:
$u^i(v) = \sum_{\kappa=1}^2 -\gamma_{x_R^\kappa} (x_R^\kappa)^{\rho_{x_R^\kappa}} - \sum_{k=1}^K \gamma_{x_k^\kappa} (x_k^\kappa)^{\rho_{x_k^\kappa}} + \sum_{k=1}^K \gamma_{y_k^\kappa} (y_k^\kappa)^{\rho_{y_k^\kappa}}$, where $\gamma_{x_k^\kappa} \rho_{x_k^\kappa} > 0$ and $\rho_{x_k^\kappa} > 1$ for $k \in \{R, 1 \cdots K\}$ and $\gamma_{y_k^\kappa} \rho_{y_k^\kappa} > 0$ and $\rho_{y_k^\kappa} < 1$ for $k \in \{1 \cdots K\}$.

3.3 Assumptions on Expectations

Up to now we have imposed almost no restrictions on consumers' expectation functions. Clearly, we cannot do without *any* further restriction on expectations. One can say nothing about the direction of change if there is no predictable link between today's price of asset n and expected future profits of firms of type n. In order to prove an efficiency theorem we have to assume something about this link.

As an informal illustration of what is required consider the following scenario. Suppose that in all periods t consumers of type i_1^t invest a substantial amount of resources in asset n^t, knowing or believing that the corresponding technology is too inefficient to produce high dividends. Then, they must believe that the *value* of asset n^t at $t+1$ ($A_{n^t}^{t+1}$) is high, otherwise they would not spend much for the asset at t. Their expectations can only be fulfilled at $t+1$, if consumers of some type i_2^t at $t+1$ believe that asset n^t will either produce sufficient profits or be sufficiently valuable at $t+2$. This belief in turn will be disappointed if their are no consumers in $t+2$ who have similar beliefs for the following period, and so on. Because of Assumptions (FIOT) and (IA) some consumer in each period (at latest consumer of type i_T^t buying asset n^t at $t+T$) in this chain will certainly be disappointed. In principle these consumers could have known so in advance, be it by reasoning about what has to happen in the future or by experiencing what has happened in the past or both. Thus assumption (FIOT) and (IA) exclude bubbles in asset prices if investors are sufficiently 'rational'. The main task of the assumptions on rationality or learning is to make sure that bubbles do not survive due to expectations that always and persistently have been disappointed in the past and that *have to* be disappointed in the future.

For the sake of concreteness we will give an example of a simple version of adaptive expectations which nevertheless is adequately sophisticated to exclude irrational bubbles. However, there are many such examples, and none seems more natural than others. To avoid ad hoc specifications of the expectation function we use a more elegant assumption about consumers' knowledge of the world they are living in and about their 'eductive' capacity. Although consumers are not living in a stationary world (technologies are continuously changing), we only require them to know features of their world that *are* stationary and that *can* be learned. Thus, in contrast to the concept of 'Rational Expectation Equilibrium' the eductive assumption made here can in principle be rephrased in evolutionary terms. As in many other models the eductive reasoning of individuals does not need to converge (compare Guesnerie [1992] for the relation between an eductive approach and the Rational-Expectation Hypothesis). In our model it merely excludes some extreme believes. Nevertheless, we are well aware of the fact that *any* requirement on peoples rationality may fail empirical tests. The requirement on rationality is therefore stated as an explicit assumption of the efficiency theorem. It is not needed for the existence of temporary equilibrium or that of equilibrium development.

The eductive version of the assumption is as follows:

ASSUMPTION (NIRBE) (NO IRRATIONAL BUBBLES, EDUCTIVE EXPECTATIONS): *All consumers know the structure of the model as well as Assumptions (FIOT), (MRS) and (IA) and this is common knowledge. Their expectations are consistent with this knowledge.*

Even in the eductive version consumers need not know the preferences of others or the innovation function or Assumptions (NLO), (CONV), (MinSpill). Of course, by the very definition of the innovation function, they know the potential innovations of their own period.

The expression 'Consumers know Assumption (FIOT)' means that there is an upper bound $T < \infty$ which is known to all types of consumers such that the imitation of a technology older than T periods is free. This being common knowledge means that all consumers know that all others know this upper bound and also know that others know this, and so on (actually, we only need 'mutual knowledge of T layers'). Similarly, 'all consumers know that $u^{i\tau}_{y^{\kappa_1}_h}/u^{i\tau}_{y^{\kappa_2}_{h'}} \to \infty$ if

$(y_h^{\kappa_1\tau}/y_{h'}^{\kappa_2\tau}) \to 0$' means that there exists a lower bound on $u^{i\tau}_{y_h^{\kappa_1}}/u^{i\tau}_{y_{h'}^{\kappa_2}}$ which tends to infinity if it is known to all consumers that $(y_h^{\kappa_1\tau}/y_{h'}^{\kappa_2\tau})$ tends to zero and this is common knowledge (i.e., all know that all know these upper bounds, and so on. Again, only finitely many (T) iterations suffice).

We now give an example of 'adaptive' expectations that can replace Assumption (NIRBE). Note that, due to continuous improvements of incumbent technologies there may be strong trends in the evolution of 'real' prices (p_k^τ/w_k^τ) in some industries, which even the most casual observer could hardly ignore. We therefore let the expectations for these prices for all type of consumers i be adaptive with respect to the *rate of change*, $\Delta^i(p_k^\tau/w_k^\tau) = \left([(p_k^{i\tau+1}/w_k^{i\tau+1}) - (p_k^\tau/w_k^\tau)]/(p_k^\tau/w_k^\tau)\right)$. Concerning the expectations of asset prices, note that 'adaptive' expectations of the prices for *new* assets can at best be defined in terms of prices for these assets in the current period but not in terms of the earlier periods, simply because these assets were not traded then. However, in our model there is a correlation between the price of an asset and its age. Realizing this fact is about the minimal amount of rationality or learning capacity we have to require from investors. For the example of adaptive expectations, this means that the expected 'real' price of an asset $n \in N_k$ of age θ at t should depend on the previous value of assets of the same age in the same industry, i.e. on the value at τ of assets in $N_k^\tau(\theta) = \{n \in N_k | \theta_n^\tau = \theta\}$ for some some previous periods $\tau < t$. Our simple form of adaptive expectations assumes that price expectations are convex combinations of these previous values:

Assumption (NIRBA) (No Irrational Bubbles, adaptive expectations): *For all $i \in I$ and all histories $\overleftarrow{s}^t$ the expectations $\psi^i(\overleftarrow{s}^t) = q^{it+1}$ satisfy the following conditions: (1) For all $k \in \{1, \cdots K\}$, $\Delta^i(p_k^t/w_k^t)$ is a convex combination of elements in the set $\{\Delta(p_k^\tau/w_k^\tau)\}_{\tau \in \{\bar{t}, \cdots, t-1\}}$ for some $\bar{t} < t$. (2) For all $h \in \{R, 1, \cdots K\}, k \in \{1, \cdots K\}$, $\Delta^i(w_h^t/w_k^t)$ is a convex combination of elements in the set $\{\Delta(w_h^\tau/w_k^\tau)\}_{\tau \in \{\bar{t}, \cdots, t-1\}}$ for some $\bar{t} < t$. (3) For all $n \in N_k^t$, (A_n^{it+1}/w_k^{it+1}) is a convex combination of elements in the set $\cup_{\tau=\bar{t}}^t \{A_{n'}^\tau/w_k^\tau\}_{n' \in N_k^\tau(\theta_n^t)}$ for some $\bar{t} < t$.*

3.4 The Welfare Theorem

We have now introduced all assumptions and can state the efficiency theorem. The theorem is stated with the eductive version of expectations. As has been noted this can be replaced by the adaptive version.

Theorem 1 *Assume (NLO), (CONV), (MinSpill), (IA) (MRS), (NIRBE). Then any equilibrium development is long-run efficient.*

The theorem is proven in the Appendix 5. Here, we only sketch the main line of the argument and point out which assumptions are needed for which purpose.

First, it is shown that the expected returns of investing one unit of research in an industry that is permanently neglected (i.e. an industry with $\alpha_k < \alpha_{\mathcal{P}k}$), are bounded away from zero (Claim (1); the main assumptions being (NLO) and (CONV)). This allows to show that the total amount of research is bounded away from zero, whenever any industry remains underdeveloped in the long-run (Claim (2); here the main assumption is that the disutility of doing research is small if only little research is performed; see the comment on sustained growth in section 4).

Thus, if any industry is continuously neglected, then there must be at least one industry, say industry 1, to which a bounded away from zero amount of research is allocated infinitely often (Claim (3)). Of course, this follows so easily only since we have assumed that there are only finitely many commodities ('industries'). With a more general commodity set, one would mainly have to generalize Assumption (MRS) (see the comment on the commoditiy set in section 4).

Next, it is shown that either the expected return from innovating in industry 1 tends to zero, or the research productivity in industry 1 explodes (Claim (4)). It is mainly for this Claim (4) that Assumption (NIRBE) (or (NIRBA)) is required, since only here an inductive argument is involved. With (NIRBE), consumers at period $t+T-1$ realize that the price at $t+T$ of an asset that is new at t will be zero (knowing (FIOT) and (IA)). Therefore the value of the same asset at $t+T-1$ will be bounded by their maximal profit expectations of this asset. This in turn is known by consumers living at $t+T-2$, who will be able to derive an upper bound on the value of the asset at $t+T-1$. Iterating this argument $T-1$ times we derive an upper bound on the expectation of consumers living at t about the value of the asset at $t+1$. These expectations tend to zero in industry 1 if the productivity of new assets in industry 1 does not explode. With (NIRBA) the corresponding induction argument roughly goes as follows. The actual price of all assets aged T (or more) is zero. Therefore the *expected* price of assets aged T (or more) will also be zero. Hence, the value of an asset of age $T-1$ must be bounded by consumer's profit expectations for that asset. In industry 1 these expectations tend to zero, if the productivity does not explode. Thus the actual price of an asset of age $T-1$ tends to zero. Hence the *expected* price of assets of age $T-1$ tends to zero. By iterating this argument we again can conclude that the expected returns on innovations in industry 1 tend to zero (if the productivity does not explode).

If the research productivity in industry 1 does not explode, then it follows from Claim (1) and Claim (4) that investors should spend their money in an industry that is permanently neglected (if there is such an industry) rather than invest in industry 1. If, on the other hand the research productivity in industry 1 explodes, then we show that potential gains of innovating in any permanently neglected industry ($\alpha_k < \alpha_{\mathcal{P}k}$) accumulate over time and eventually cross any given bound. (Claim (6); assumptions as in Claim (1) plus Assumption (MinSpill)). In this case too, investors should turn away from industry 1 and shift towards the neglected industry. Therefore, in the long run no industry that can further be developed can remain neglected for too long (Claim (9)).

Note that the 'returns' in the above arguments are generally expressed in terms of the inputs of the corresponding industries. Thus one also has to make sure that the relative price of input 1 in terms of other inputs remains bounded (Claim (8); Assumption (MRS)). What remains to be shown is that development of the most advanced technologies in an industry involves the development of the whole industry (Claim (10), Assumption (FIOT)).

4 Comments and Extensions

4.1 Sources of sustained growth

Although our interest in the present paper is mainly the direction rather than the rate of growth, the question about the 'right' direction of growth cannot be completely disentangeled from considerations about the amount of growth. Sustained growth, if it is feasible and desired, is by definition a necessary condition for long-run efficiency of development. We have implicitly made sure that equilibrium growth does not cease in the long-run, if potential growth doesn't. Because different potential engines of sustained growth may have different influences on the direction of growth, we now identify such potential sources of sustained growth in our framework.

4.1.1 Low Opportunity Cost of Research

Growth in section 3 is *always* sustained if this is feasible because innovative activity does not cease even if the profits of innovation become very small. We did not exclude the possibility that the profits from innovations in terms of non-research wages tend to zero. Even for this case, Claim (1) and Claim (2) of the proof of Theorem 1 make sure that growth continues as long as the limit (possibly infinite) of potential development is not reached. These claims rest on the assumption that the disutility of doing very little research is very small (Assumption (MRS)). Thus, the first source of sustained growth is the low disutility of doing a little bit of research or, more generally, *the low opportunity cost for the resources for change.*

This assumption was our way to separate the discussion of the direction of change from that of the amount of change. However, the assumption may be challenged: The inputs called 'research' may be substitutes for inputs in some incumbent industries. As soon as the inputs for innovative activity and those for non-innovating activity overlap (as they should do) our claims about long-run efficinecy are no longer warranted if we do not make out a second potential source of growth.

4.1.2 Exploding Productivity of Research

Although sustained growth in section 3 does not depend on exploding productivity of research, we did not exclude the possibility of exploding research productivity. We now show that growth can be sustainable by *exploding productivity of research*, even if research wages are bounded below by industrial wages. If the research productivity explodes the profits from one sequence of innovations in the same direction do not necessarily peter out in the long-run. In order to assure that exploding productivity of research in one direction cannot persistently prevent innovations in other directions, we had to assume that there are minimal knowledge spillovers to research in one direction to that in other directions.

In order to make more precise the claim that exploding productivity of research is a potential engine of sustained growth we first change our assumption about the opportunity cost of the resources for change, so that the first source of growth is eliminated. Suppose that these resources are the labor of researchers and that researches may either produce new knowledge (i.e. innovations) or may do a non-research job in some of the H industries.[7] Consider the model of the present chapter with the only difference, that for each industry k 'researchers' can also do the job of the 'usual labor' of industry k (in which case $(w_R^t/w_k^t) \geq 1$). Then, showing that the sequence of expected returns to a unit of research in industry k $((\underline{V}_{n_k^t}^{t+1})_{t \in I\!N})$ is bounded away from zero (Claim (1)) is no longer sufficient to guarantee sustained research. The following assumption guarantees sustained research and growth even in the changed setup. It assumes the maximal research productivity over all industries to grow exponentially. Remember that given an initial state of knowledge, an innovation function and a feasible development, the *growth rate of research productivity* with respect to the sequence $(n^t)_{t \in I\!N}$ is defined as $\dot{\delta}_{n^t}^t = (\delta_{n^t}^t - \delta_{n^{t-1}}^{t-1})/\delta_{n^{t-1}}^{t-1}$.

Assumption (Explosion): For all feasible paths of development there is a sequence of innovations $(n^t)_t$ with $(\dot{\delta}_{n^t}^t)_t$ bounded away from zero.

Claim 1': *With Assumption (Explosion), the sequence of expected returns to a unit of research in industry k $(\underline{V}_{n_k^t}^{t+1})_{t \in I\!N}$ is unbounded if industry k is continuously neglected.* This follows as in Claim (6) using Assumptions (MinSpill) and (CONV).

As we have seen, exploding research productivity may not only stimulate perpetual growth, but may at the same time persistently hinder the solution of the (multi-dimensional) economic problem. In the framwork of the present article this possibility is excluded by Assumption (MinSpill), which gains more prominence even, once Assumption (Explosion) is postulated.

[7]This is the usual scenario in endogeneous growth models (see Romer [1990], Agion and Howitt [1992], Grossman and Helpman [1990]).

In order to show Theorem 1 in the changed framework (in which $(w_R^t/w_k^t) \geq 1)$), replace Claim (1) in the proof of the theorem with Claim (1'), and continue with Claim (2) as before.

Note that the assumption guarantees growing productivity from potential research, even in periods in which no research is actually carried through. The assumptions makes sure that if there is no endogenous growth of research productivity, then there is exogenous growth of research productivity. And it *accumulates* if it is not used. In this sense the assumption is stronger than the standard assumption of exponential growth of research productivity in the endogenous growth literatur (which is 'Knowledge increment = Knowledge stock times Research'). As for all other assumptions which we have explicitely marked as such, we do not claim that Assumption (Explosion) is a weak requirement. The two other sources of sustained growth may be more important.

In contrast to the present article most articles in the literature on endogenous growth assume that the supply of labor is fixed. Without fixed supply of labor, the usual assumption about exponential growth is not sufficiently strong to generate sustained growth in standard models of endogenous growth. Duranton [1995] shows that if the income elasticity of leisure is positive in the long run, then growth in standard *endogenous* growth models peters out in the long run. Referring to the empirical study of Pencavel [1986] he suggest, that the income elasticity of leisure *is* positive in the long run, indeed. Note that our boundary assumptions on preferences are consistent with such long-run behaviour of consumer-workers.

4.1.3 Insatiable Needs

Finally, in Funk [1995d] we show how insatiable needs may be the cause of sustained growth. We argue that 'relative' needs, i.e. needs that depend on the consumption of other consumers, are typically insatiable. If we allow for insatiable relative needs we can sustain growth without Assumption (Explosion) even if research wages are bounded below by the wages of industrial labor. However, as is shown in Funk [1995d], this naturally goes hand in hand with long-run market failures concerning the *direction* of change.

Thus, there are at least three potential engines of growth: low disutility of research, exploding research productivity, and insatiable relativ needs. Sustained growth in reality may be caused by a combination of these. Since the welfare implications and correspondingly policy implications differ considerably in the different cases it is important to identify the causes of actual sustained growth.

4.2 Monopolistic competition.

4.2.1 Monopolistic versus perfect competition

Most new growth models build on monopolistic competition assuming that individual incumbent firms are large relative to the size of the relevant market. As we have mentioned in the introduction, the dependence of the new growth literature on monopolistic competition hinges on the presumption that all rival inputs can be costlessly and instantaneously replicated, so that all individual firms' technologies are homogenous of degree one in the rival inputs. Thus, concerning the compatibility of price-taking perfect competition with endogeneous grwoth, the relevant question seems to be whether one accepts the possibility that incumbent individual firms cannot replicate their technologies free of costs in the very short term.

Firstly, note that to be able to give a non-cooperative foundation to our model, only *instantaneous* replicability of individual technologies needs to be excluded. Though the assumption of small individual firms yet lacks a good theoretical foundation (contract theoretical or other), it is not less plausible on empirical grounds, than the assumption that short-run individual firms' technologies are linear homogeneous.

Secondly, if one rejects the possibility of small individual firms, holding that *individual* technologies should be linear homogeneous, then one may have to reject the possibility of perfect competition altogether. Constant returns to scale on the level of individual firms poses heavy problems for the foundation of perfect competition, even in the absence of non-rival inputs. If individual firms' technologies exhibit constant returns to scale, then any single firm in a given industry can profitably deviate from perfect competition by monopolizing all industry specific inputs (i.e. slightly overbidding input prices) and thus becoming a monopolist for its outputs (see Yanelle [1988]).[8] It is difficult to get around this argument unless one gives up the assumption of *instantaneous* replicability of individual technologies. In fact, small efficient scales on the level of the individual firm are a crucial ingredient both of the traditional intuitive case for perfect competition as well as for modern non-cooperative foundations of perfect competition in general equilibrium setups (see Novshek and Sonnenschein [1980] for a Cournot model and Funk [1995a] for a Bertrand model). Even the most ardent advocate of linear homogeneous technologies on the macro level argues that we should "consider the constant returns to scale as the truly general competitive model, approximating to

[8]Note that both the model of Romer [1990] and that of Aghion and Howitt [1992] suffer from the strategic instability due to constant returns to scale technologies of individual firms in the final good sectors. However, the essence of these models does not depend on this aspect (see the versions of these models in Grossman and Helpman [1991c], for instance).

the situation with freedom of entry and firms which are small compared to their industries."[9]

As we will argue below, our result could be derived in monopolistic competitive models. However, a foundation of the monopolistically competitive model rests on very specific and extreme assumptions on preferences or on production functions. Adapted to the needs of the question of the present article, the corresponding model would be much more complex than the perfectly competitive model. The behavior of firms in the monopolistically competitive models of the literature (Bertrand on output markets, price taking on input markets) is natural only if there are many different firms, all producing different outputs with the same inputs. Consistently, Judd [1985], Romer [1990] and Grossman and Helpman [1991a, 1991b] assume that there is a *continuum* of industries, all of which use the same inputs and produce different outputs. In order to guarantee at the same time that firms earn non-zero rents, they assume Dixit-Stiglitz type preferences for outputs (or Dixit-Stiglitz type production functions for a single final consumption good, Dixit and Stiglitz [1977] and Ethier [1982]). The relevant feature of these functions is that none of the continuum of commodities are close substitutes. Romer[10] notes that "(a)lthough it greatly simplifies the analysis, this is not a realistic feature of the model". To address our question about the direction of change in a similar framework we would have to assume the existence of several groups of industries each of which would produce a different Dixit-Stiglitz type group of commodities. This would add little realism (from the point of view of preferences and production functions) and much complexity to the perfectly competitive framework.

4.2.2 A monopolisticly competitive version

Although our analysis is simplified by the perfectly competitive approach, its essential elements do not depend on the assumption of perfect competition. We briefly sketch what changes are necessary to transform our model into a simple monopolisticly competitive model.

First, let technology $\hat{Y}_n^t$ of section 2 be the technology of a single individual firm rather then the aggregate technology of many small firms. Second, to allow for a better comparison with the literature, assume that $\hat{Y}_n^t$ is linear homogenous (i.e. imitation of one period old knowledge is free already). Finally, assume that incumbent firms set prices strategically on output markets while they take prices as given on input markets. All output in an industry will be produced by the best incumbent firm (i.e., with maximal α_j) and sold at a (real) price equal to the inverse of the productivity of the second best firm in the industry (If several incumbent firms in an industry have

[9] McKenzie [1959], p 55.

[10] Romer [1990], p S85.

maximal productivity they all set marginal cost prices and make zero profits). Innovators 'produce' the knowledge of the aggregate technology $\hat{Y}_n$ with an amount of research $x_{Rn}(\theta^t)$, given by the innovation function. Consumers are modeled as before and temporary equilibrium and equilibrium development are defined correspondingly. The efficiency theorem remains valid without any major change in the assumptions (In order to get Claim 2 of the proof of the theorem in a easy way, we assume now that$x_{Rn}(\theta^t) < e_R$, whenever we had $x_{Rn}(\theta^t) < \infty$ in the main framework and we assume that doing research causes no disutility. Furthermore, in order to get Claims 4 and 10 without additional assumptionson expectations we have to make sure, that at least two firms enter to technology n if $x_{Rn}(\theta^t) = 0$, which is no problem because they are indifferent).

Note that the behaviour of incumbent firms in this version of the model (price-taking on input markets, Bertrand on output markets) is ill-founded, unless there are many firms producing different outputs with the same inputs.

4.2.3 Strategic competition for inputs.

Both the monopolistically competitive models of the literature as well as the perfectly competitive model assume that the markets for the resources for change are perfectly competitive. However, if there are some inputs that are used by only few firms, the possibility of strategic competition on input *and* output markets can hardly be ignored. In particular, firms would have non-negligible power on those markets that determine change. Allowing for strategic competition only on output markets of incumbent firms does not much influence our results. The same is not true if individual firms have strong impact on input markets or on the market for research. Large incumbent firms can prevent entry to their industry by partly monopolizing the resources necessary to innovate. This adds an important potential source of long-run inefficiency of the direction of change, which is neglected altogether in the present article.

4.3 Innovations and Imitations

The innovation/imitation process of section 2 takes a very simplistic form. The only uncertainty investors face is that about prices. This kept the model as simple as possible. The temporary equilibrium approach naturally copes with additional uncertainty in a similar way. As has already been remarked, it is not difficult to incorporate uncertainty about the purely technical success of innovations. An innovation $\tilde{n} \in \tilde{N}$ then induces a subjective probability distribution $f^i_{\tilde{n}}$ over the set of potential technologies, N. Furthermore, this probability distribution may depend on the activities of other innovators. Note that this dependence is no esential addition to the simple model, since already the price expectations in section 2 depend on the vector of innovative activity. Incorporating these modifications would require vector of innovative activity at and the expected profits to depend on $f^i_{\tilde{n}}(\overline{s}^t)$ and on $\psi^i(\overleftarrow{s}^t)$ (rather than on $\psi^i(\overleftarrow{s}^t)$ alone). The long-run efficiency theorem would not be much affected by these changes, although some assumptions would need to be reformulated in terms of the supports of the expectations.

The possibility that also the success of an innovation depends on the activity of other innovators allows to model patent rights in a more satisfactory way than was possible in the original version. In the original version a patent could only be given simultaneously to all firms that choose the same innovation at the same time. Now, single firms can be chosen randomly. In the perfectly competitive version with such patents, a successful innovator has only a small impact on the prices and output levels of the economy. As long as his patent is not expired his (possibly strong) influence will mainly be that of changing the horizon of perception of competitors. The possibility of similar innovations may occur to competitors. The cheaper these innovations are, the stronger is the indirect influence of the prior innovation and the faster old technologies will be replaced. In the monopolistically competitive interpretation with patents a successful innovator will serve the whole market for the commodity he produces and directly replace the old incumbent.

4.4 The commodity set and preferences

The set of commodities assumed in the present paper (the fixed number of commodities, mainly) is the very simplest that allows to formulate an efficiency theorem about the direction of change. The introduction of improved qualities or new products is excluded. Progress in the long term is restricted to process innovations. This simplification allows to separate the assumptions on the innovation function from those on preferences. A next step toward a more general set of commodities is to allow for vertical quality differentiation of all types of commodities. This has been done in Funk [1995b] (in an exceedingly simplified model, in which generations do not overlap). The principal addition for an analogous efficiency theorem is that Assumption (MRS) has to be extended to include the marginal rates of substitution between qualities and quantities.

5 Appendix

Proof of Proposition 1:

We suppress the time index where this does not cause ambiguities. First, we allocate infinite amounts of asset n to all types of consumers for if $x^t_{Rn} = 0$. We restrict our attention to assets in $\hat{N}^t = \{n | 0 < x^t_{Rn} < \infty\}$ and, by abuse of notation, exclude from ξ, η, ζ and D those components corresponding to assets not in $\hat{N}^t$. The supply correspondence η and the demand correspondence ξ are upper hemi continuous (uhc) and convex valued (ξ is uhc also on the boundary of S since the budget sets are lower hemi continuous even on the boundary of S. This is so since $x^a \leq e^a$.) For every vector of excess demand z, define $\mu(z) = \arg\max_{q\in\Delta} qz$. The corresponding correspondence μ is uhc and convex valued. For each pair of excess demand and current data (z, s), define $\varphi(z, s) = \zeta(s) \times \mu(z) \times \eta_{\Delta\lambda}(s)$, where $\eta_{\Delta\lambda}(s)$ are the components of $\eta(s)$ describing innovators optimal choices of $\Delta\lambda$. The correspondence $\varphi : I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#\hat{N}^t} \times S \times I\!R^{\#\hat{N}^t} \to I\!R_-^{K+1} \times I\!R_+^K \times I\!R^{\#\hat{N}^t} \times S \times I\!R^{\#\hat{N}^t}$ is uhc and convex valued. We now compactify the domain and the image set of this correspondence.

Let $B = \max\{e_R, e_1, \cdots e_K, \alpha_{k_1} e_{k_1}, \cdots, \alpha_{\#\hat{N}^t} e_{k_{\#\hat{N}^t}}, e_R/x_{R\#\hat{N}^t}, \cdots, e_R/x_{R\#\hat{N}^t}\} < \infty$, where $\alpha_k^t = \max_{n\in N_k, \theta_n^t > 0} \alpha_n$. Clearly, B is an upper bound to the amount of research conducted, to the amount of input k sold and output k produced, and to the amount of new assets in $\hat{N}^t$ produced. Define $D = [-2B, 2B]^{1+2K+\#\hat{N}^t} \times S \times [-2B, 2B]^{\#\hat{N}^t}$. Let $\tilde{\varphi} : D \to D$ be the compactification of φ (i.e. for $(z, s) \in D$ with $\varphi(z, s) \notin D$, set those components of $\varphi(z, s)$ that lie outside of the interval $[-2B, 2B]$ equal to the bound that is closest). The correspondence $\tilde{\varphi}$ is uhc and convex valued and has a fixed point, say $(z^*, s^*) \in \varphi(z^*, s^*)$. Therefore, for all $q \in S$ we have $qz^* \leq q^*z^*$. From Walras Identity follows $q^*z^* \leq 0$ and therefore for all $q \in S$, $qz^* \leq 0$.

Thus, $z^* \leq 0$. If q^* belongs to the interior of S, then it follows that $z^* = 0$. However, we have not excluded that q may belong to the boundary of D.

Consider the vector $\tilde{z}$, for which $\tilde{z}_{\lambda_n} = 0$, if $A_n^* = 0$, $z_n^* < 0$ and for which $\tilde{z}_R = 0$ if $w_R^* = 0, A_n^* = 0 \forall n, z_R^* < 0$ and which equals z^* in all other components. We first show, that there is a corresponding vector of demands and supplies that solves all consumers' and firms' decision problems:

1) Choose all n with $A_n^* = 0$. Since $A_n^* = 0$, young consumers either are indifferent about how much assets of type n to demand or their demand is unbounded. Since $z_R^* \leq 0$ and $x_{Rn} < \infty$ the demand for asset n cannot be unbounded. Thus, young consumers must be indifferent. In this case we can set $\tilde{z}_{\lambda_n} = 0$ without changing the *supply* of assets (and, therefore, without affecting consumers expectations) such that the new trading plans still solve consumers' and innovators' decision problem.

2) Suppose $w_R^* = 0, A_n^* = 0 \forall n \in \hat{N}^t$. Then, innovators are indifferent about their demand for research. Since $z_R^* \leq 0$, we can set $\tilde{z}_R = 0$, without violating innovators optimal decision.

We now show that q^* does not belong to the boundary $\partial\tilde{S}$, of $\tilde{S}$, where $\tilde{S} = \{q \in S | A_n = 0$ if $A_n^* = 0$ and $w_R = 0$ if $w_r^* = 0, A_n^* = 0 \forall n \in N^t\}$. Suppose that $p_k^* = 0$ and $w_k^* \neq 0$. Then, because utilities are strictly increasing in final consumption, $\xi_{y_k} = \{\infty\}$, while $\eta_{y_k} = \{0\}$. Thus, $\zeta_{y_k} = \{\infty\}$. This contradicts $\tilde{z}_{y_k} = z_{y_k}^* \leq 0$. Next, suppose that $w_k^* \neq 0$ and $p_k^* = 0$. Then, $\eta_{x_k} = \{-\infty\}$. Since

$|\xi_{x_k}| \leq e_k$ we have $\zeta_{x_k} = \{\infty\}$. This contradicts $\tilde{z}_{x_k} = z^*_{x_k} \leq 0$. Suppose $p^*_k = 0$ and $w^*_k = 0$. Then, $\xi_{y_k} = \{\infty\}$ and $|\xi_{x_k}| \leq e_k$. If input demand is infinite, then there is excess demand for inputs, which contradicts $z^*_{x_k} \leq 0$. If input demand is finite, then output supply is finite as well, in which case there is excess demand for output, which contradicts $z^*_{y_k} \leq 0$. Finally, suppose $w^*_R = 0$ and $\exists n : A^*_n > 0$. Then $\zeta_R = \{\infty\}$, which contradicts $\tilde{z}_R = z^*_R \leq 0$. Thus, $q^* \notin \partial\tilde{S}$.

Since $q\tilde{z} = qz^* \forall q \in \tilde{S} \subset S$ and $qz^* \leq 0 \ \forall q \in S \subset \tilde{S}$, we also have $q\tilde{z} \leq 0$ for all $q \in \tilde{S}$. Furthermore, $q^*\tilde{z} = 0$ since $q^*z^* = 0$ and since $\tilde{z}$ differs from z^* only in components in which q^* is zero. But q^* belongs to the relative interior of $\tilde{S}$. Therefore, $\tilde{z} = 0$. Since $\tilde{z} \in \zeta(\tilde{s})$, we have an equilibrium. ■

Proof of Theorem 1:

CLAIM 1: *If $\alpha_k < \alpha_{\mathcal{P}k}$, then the sequence $(\underline{V}^{t+1}_{n^t_k})_{t\in I\!N}$ of lowest consumer expectations at t about the return (dividends plus value, in terms of input k) of the amount of assets of type n^t_k that can be produced with one unit of research at t is bounded away from zero, where $n^t_k \in \arg\max_{n\in N^t_k} \alpha_n$.*

Suppose $\alpha_k < \alpha_{\mathcal{P}k}$. By (CONV') there exists a $\beta_1 \in [0,1)$ such that for all t, $\alpha_{n^t_k} \geq \beta_1\alpha^t_k + (1-\beta_1)\alpha_{\mathcal{P}k}$, where $\alpha^t_k = \max_{n\in N_k, \theta^t_n>0} \alpha_n$. Since $\alpha^t_k \to \alpha_k < \alpha_{\mathcal{P}k}$, there exists a $\beta_2 \in [0,1)$ and a $\bar{t} < \infty$ such that for all $t > \bar{t}$, $\alpha_{n^t_k} \geq \beta_2\alpha_k + (1-\beta_2)\alpha_{\mathcal{P}k}$, or such that for all $t > \bar{t}$, $\alpha_{n^t_k} \geq (1+\beta)\alpha_k$, where $\beta = (1-\beta_2)(\alpha_{\mathcal{P}k}/\alpha_k) - 1 > 0$. Thus, there exists a $\beta > 0$ and a $\bar{t} < \infty$ such that $(1/\alpha_k) \geq \left((1+\beta)/\alpha_{n^{\bar{t}}_k}\right)$. Since $\alpha^t_k \leq \alpha_k$, we must have $\sum_{n\in N_k, \alpha_n>\alpha_k} \Delta\lambda^t_n = 0$ and therefore $(p^{t+1}_k/w^{t+1}_k) \geq (1/\alpha_k)$. Note, that this inequality is known to consumers, since they observe the incumbent industrial structure of $t+1$ already at t. Thus, for all type of consumers $(p^{it+1}_k/w^{it+1}_k) \geq (1/\alpha_k) \geq \left((1+\beta)/\alpha_{n^{\bar{t}}_k}\right)$.

Because of (NLO') $x^t_{Rn^{\bar{t}}_k} \leq x^{\bar{t}}_{Rn^{\bar{t}}_k} \leq \infty$ for $t > \bar{t}$. Therefore, the expected dividend in terms of input k of one unit of research spent on innovation $n^{\bar{t}}_k$ at t is at least

$$\left(\frac{p^{it+1}_k}{w^{it+1}_k}\alpha_{n^{\bar{t}}_k} - 1\right)\frac{a_{n^{\bar{t}}_k}}{x^t_{Rn^{\bar{t}}_k}} \geq \left(\frac{1+\beta}{\alpha_{n^{\bar{t}}_k}}\alpha_{n^{\bar{t}}_k} - 1\right)\frac{a_{n^{\bar{t}}_k}}{x^{\bar{t}}_{Rn^{\bar{t}}_k}} = \beta\frac{a_{n^{\bar{t}}_k}}{x^{\bar{t}}_{Rn^{\bar{t}}_k}} > 0,$$

where $a_{n^{\bar{t}}_k}$ are the efficient scale inputs of the unit technology $n^{\bar{t}}_k$, (the LHS of the inequality are the profits per unit of research, that a firm using $n^{\bar{t}}_k$ would realize given the expected prices when producing at efficient scales.) Thus, $(\underline{V}^{t+1}_{n^t_k})_{t\in I\!N}$ is bounded away from zero.

CLAIM 2: *If $\alpha_k < \alpha_{\mathcal{P}k}$, then the sequence $(x^t_R)_t$ is bounded away from zero.*

Suppose $\alpha_k < \alpha_{\mathcal{P}k}$. Then, because of Claim 1 the sequence $(\underline{V}^{t+1}_{n^t_k})_{t\in I\!N}$ is bounded away from zero. Furthermore, consumers at t know that $(w^{t+1}_k/p^{t+1}_k) \geq \alpha^{t-T}_k \not\to 0$. Therefore, the amount of output k consumers at t expect to be able to buy at $t+1$ with one unit of research of t is bounded away from zero. If $x^{it}_R < e^i_R$, for t sufficiently large, then it follows that $(u^i_{y^{t+1}_k}/u^i_{x^t_R})_t$ is bounded. From

(MRS) it follows that $(x^t_R)_t$ is bounded away from zero.

CLAIM 3: *If $(x^t_R)_t$ is bounded away from zero, then there is an industry, say industry 1 and a sequence of innovations $(\hat{n}^t_1)_t$, $\hat{n}^t_1 \in N_1$ such that for a subsequence of periods (same notation) the sequence $\Delta\lambda^t_{\hat{n}^t_1} x^t_{R\hat{n}^t_1}$ of research spent on $\hat{n}^t_1$ is bounded away from zero.*

No research is allocated to technologies that have been introduced before period $t-T$ for the first time (T from (FIOT)). Furthermore, there is a uniform bound on the number of new potential innovations in each period. Since $(x^t_R)_t$ is bounded away from zero, it follows, that there is an $\epsilon > 0$ and a subsequence of periods such that the amount of research allocated to at least one innovation in each period is at least ϵ. Thus, for at least one industry, say industry 1, there is a subsequence of innovations $(\hat{n}^t_1)_t$, $\hat{n}^t_1 \in N_1$, such that at least ϵ research is allocated to $\hat{n}^t_1$ every $\frac{1}{K}$-th period in average, or, more precisely $\max_k \lim_t \frac{\#\{\tau | \Delta\lambda^\tau_{\hat{n}^\tau_1} x^\tau_{R\hat{n}^\tau_1} \geq \epsilon/K\}}{t} \geq \frac{1}{K}$. Therefore, for a subsequence of periods, $\Delta\lambda^t_{\hat{n}^t_1} x^t_{R\hat{n}^t_1}$ is bounded away from zero.

CLAIM 4: *If $(x^t_R)_t$ is bounded away from zero, then there is an industry, say industry 1 and a sequence of innovations $(\hat{n}^t_1)_t$, $\hat{n}^t_1 \in N_1$ such that for a subsequence of periods (same notation), $(\overline{V}^{t+1}_{\hat{n}^t_1}) \to_t 0$ if $\dot{\delta}^t_{\hat{n}^t_1} \to 0$, where $\overline{V}^{t+1}_{\hat{n}^t_1}$ is an upper bound on consumers' expectations at t about the return (dividends plus value in terms of input 1) of the amount of assets of type $\hat{n}^t_1$ that can be produced with one unit of research at t.*

In what follows we only use arguments that can be made if we only have the knowledge assumed by Assumption (NIRBE). Take the sequence $(\hat{n}^t_1)_t$ $\hat{n}^t_1 \in N_1$ of innovations of Claim 2.).

4.1.) We first derive an upper bound (known to all consumers) on the dividends in $t+\tau$ in terms of input 1 of one unit of research invested in t on innovation $\hat{n}^t_1$. A commonly known upper bound on 'real' prices $(p^{t+\tau}_1/w^{t+\tau}_1)$ is derived as follows. For all $n' \in N_1$ with $\theta^t_{n'} > T$ (T from (FIOT)) there exists an $n \in N_1$ with $\alpha_n \geq \alpha_{n'}$ and $x^{t+\tau}_{Rn} = 0$ for all $\tau > 0$. Thus, for the 'best' active technology at $t-T$, i.e. for some element of $\arg\max_{n \in N_1, \theta^t_n \geq T} \alpha_n$, there is an $\tilde{n}^t_1$ with $\alpha_{\tilde{n}^t_1} \geq \max_{n \in N_1, \theta^t_n \geq T} \alpha_n = \alpha^{t-T}_1$ and $x^{t+\tau}_{R\tilde{n}^t_k} = 0$. Thus, all know that $(p^{t+\tau}_1/w^{t+\tau}_1) \leq (1/\alpha_{\tilde{n}^t_1}) \leq (1/\alpha^{t-T}_1)$

The profits of the total number of shares of type $\hat{n}^t_1$ at $t+\tau$ if *all* input 1 would be used by the corresponding firms alone would be at most $\left(\frac{p^{t+\tau}_1}{w^{t+\tau}_1}\alpha_{\hat{n}^t_1} - 1\right) e_1$. Because of the upper bound on real prices an upper bound on the profits of one asset of type $\hat{n}^t_1$ at $t+\tau$ is $\left(\frac{\alpha_{\hat{n}^t_1}}{\alpha^{t-T}_1} - 1\right) \frac{e_1}{\Delta\lambda^t_{\hat{n}^t_1}}$.

Since innovation n^{t-T-1}_1 is chosen at $t-T-1$, we have $\alpha^{t-T}_1 \geq \alpha^{t-T-1}_{\hat{n}^t_1}$ (with strict inequality if $\alpha^{t-T-1}_1 > \alpha^{t-T-1}_{\hat{n}^t_1}$). We therefore get as an upper bound on the profits in $t+\tau$ of one unit of research invested at t in assets of type $\hat{n}_1^t$:

$$\overline{\pi}_{\hat{n}_1^t}^{t+\tau} = \left(\frac{\alpha_{\hat{n}_1^t}}{\alpha_{\hat{n}_1^{t-T-1}}} - 1 \right) \frac{e_1}{\Delta\lambda_{\hat{n}_1^t}^t x_{R\hat{n}_1^t}}.$$

4.2.) Suppose that $\dot{\delta}_{\hat{n}_1^t}^t \to 0$. Then, for all $\mu > 0$, there exists an $\bar{t}$ such that for all $t > \bar{t}$: $(\alpha_{\hat{n}_1^t}/\alpha_{\hat{n}_1^{t-1}})(x_{R\hat{n}_1^{t-1}}^{t-1}/x_{R\hat{n}_1^t}^t) < 1+\mu$. Therefore, for all $\mu > 0$, there exists an $\bar{t}$ such that for all $t > \bar{t}$: $(\alpha_{\hat{n}_1^t}/\alpha_{\hat{n}_1^{t-T-1}}) < (1+\mu)(x_{R\hat{n}_1^t}^t/x_{R\hat{n}_1^{t-T-1}}^{t-1})$. Since the cost of the 'best' innovation in industry 1, $x_{Rn_1^t}^t$ does not grow exponentially, $x_{R\hat{n}_1^t}^t$ does not grow exponentially, as well. Therefore, for all $\mu > 0$, there exists $\bar{t}$ such that for all $t > \bar{t}$: $(\alpha_{\hat{n}_1^t}/\alpha_{\hat{n}_1^{t-T-1}}) < 1 + \mu$. Thus, by 4.1.), for all $\mu > 0$, there exists a $\bar{t}$ such that $\overline{\pi}_{\hat{n}_1^t}^{t+\tau} \leq \mu(e_1/\Delta\lambda_{\hat{n}_1^t}^t x_{R\hat{n}_1^t})$ for all $t > \bar{t}$.

4.3.) We next derive an upper bound $\overline{V}_{\hat{n}_1^t}^{t+1}$ on $V_{\hat{n}_1^t}^{t+1}$ (the value of asset $\hat{n}_1^t$ t $t+1$)by backward induction, starting with a period in the future at which the asset of type $\hat{n}_1^t$ certainly has no value any more. By (FIOT) there is a $T < \infty$ such that it is common knowledge that $x_{R\hat{n}_1^t}^{t+T} = 0$ and hence, by (IA), $A_{R\hat{n}_1^t}^{t+T} = 0$. Thus, a (commonly known) upper bound of $V_{\hat{n}_1^t}^{t+T}$ is $\overline{V}_{\hat{n}_1^t}^{t+T} = \overline{\pi}_{\hat{n}_1^t}^{t+T}$. For all $\tau \in \{1, \cdots, T\}$, let $\overline{A}_{\hat{n}_1^t}^{t+\tau-1}$ be a commonly known upper bound on the amount of input 1 that consumers would be willing to give up in $\tau - 1$ for $\overline{V}_{\hat{n}_1^t}^{t+\tau}$ units of input 1 in τ, and, for all $\tau \in \{1, \cdots, T\}$, let $\overline{V}_{\hat{n}_1^t}^{t+\tau-1} = \overline{\pi}_{\hat{n}_1^t}^{t+\tau-1} + \overline{A}_{\hat{n}_1^t}^{t+\tau-1}$. Starting with $\overline{V}_{\hat{n}_1^t}^{t+T} = \overline{\pi}_{\hat{n}_1^t}^{t+T}$, this defines $\overline{V}_{\hat{n}_1^t}^{t+1}$. Knowing from 4.2) that $\overline{\pi}_{\hat{n}_1^t}^{t+\tau}$ tends to zero if $t \to \infty$ for all $\tau > 0$, we show that $\overline{V}_{\hat{n}_1^t}^{t+1}$ tends to zero if $t \to \infty$. For this it is sufficient to show that for all $\tau \in \{1, \cdots, T\}$, $\overline{A}_{\hat{n}_1^t}^{t+\tau-1} \to 0$ if $\overline{V}_{\hat{n}_1^t}^{t+\tau} \to 0$. This is done in 4.4.).

4.4.) Suppose that for some τ, $\overline{A}_{\hat{n}_1^t}^{t+\tau-1} \not\to 0$, while $\overline{V}_{\hat{n}_1^t}^{t+\tau} \to 0$. Then, since $\Delta\lambda_{\hat{n}_1^t}^t > 0$, and since at a TE old consumerssell all existing assets (if the price is strictly positive), it follows thatthere must be a sequence of types $(i^t)_t$ of young consumers with $(u_{x_1^1}^{i^t t+\tau-1}/u_{x_1^2}^{i^t t+\tau-1}) \to_t 0$. Then, from (MRS) follows that $(x_1^{i^t t+\tau-1}/x_1^{i^t t+\tau}) \to_t 0$.Take some industry $k \neq 1$, with $\alpha_k^t \not\to \infty$ (ifno such industry exists continue with 9.)). Then $x_k^{i^t t+\tau-1} \not\to 0$ (this follows from $y_k^{i^t t+\tau-1} \not\to_t \infty$, $(u_{y_k^1}^{i^t t+\tau-1}/u_{x_k^1}^{i^t t+\tau-1}) \geq \alpha_k^{t-T} > 0$and (MRS)). Therefore, $(x_1^{i^t t+\tau-1}/x_k^{i^t t+\tau-1}) \to_t 0$,and hence, by (MRS), $(u_{x_1^1}^{i^t t+\tau-1}/u_{x_k^1}^{i^t t+\tau-1}) = (w_1^{t+\tau-1}/w_k^{t+\tau-1}) \to_t 0$. Therefore, $u_{y_k^1}^{jt+\tau-1}/u_{x_k^1}^{jt+\tau-1} \to 0$ for all $j \in I$,and hence $(x_1^{jt+\tau-1}/x_k^{jt+\tau-1}) \to_t 0$. Thus,$x_1^{jt+\tau-1} \to 0$ for all $j \in I$. Since $x_k^{i_1 t+\tau-1} \not\to 0$ it follows that $(x_1^{t+\tau-1}/x_k^{t+\tau-1}) \to_t 0$ and therefore $(y_1^{t+\tau-1}/y_k^{t+\tau-1}) \to_t 0$. Thus, there must be at least one sequence of consumer types $(j^t)_t$, young at $t+\tau-1$, such that $(y_1^{j^t t+\tau-1}/y_k^{j^t t+\tau-1}) \to_t 0$ and therefore, $(u_{y_1^1}^{jt+\tau-1}/u_{y_k^1}^{jt+\tau-1}) \to \infty$. Thus, for consumers j^t, the expected own rate ofinter-est of output 1 tends to infinity with t. At the same time, theirexpected own rate of interest of input 1 tends to zero (since they knowthe upper bound $\overline{V}_{\hat{n}_1^t}^{t+1}$ and sincethey can observe the price of asset $\hat{n}_1^t$ at $t+\tau-1$). This ispossible only if $(p_1^{t+\tau}/w_1^{t+\tau})(w_1^{t+\tau-1}/p_1^{t+\tau-1}) \to_t 0$. A commonly known lower bound to thereal price $(p_1^{t+\tau}/w_1^{t+\tau})$ is $(1/\alpha_1^{t+\tau})$ (which canbe observed at $t+\tau-1$). Furthermore, there is a $T<\infty$ such that, for all t, the real wage $(w_1^{t+\tau-1}/p_1^{t+\tau-1})$ is atleast as high as α_1^{t-T} (T from (FIOT)).Thus, there exists a $T<\infty$, such that for all t, $(p_1^{j^t t+\tau}/w_1^{j^t t+\tau})(w_1^{t+\tau-1}/p_1^{t+\tau-1}) \geq (\alpha_1^{t-T}/\alpha_1^{t+\tau})$. Since the growth *rate* of productivity does not grow exponentially, $(\alpha_1^{t-T}/\alpha_1^{t+\tau})$ does not tend tozero. Therefore, we have a contradiction and it follows that $\overline{A}_{\hat{n}_1^t}^{t+\tau-1} \to 0$, whenever $\overline{V}_{\hat{n}_1^t}^{t+\tau} \to 0$.

Claim 5: $(\overline{V}_{\hat{n}_1^t}^{t+1}) \not\to \infty$.

By assumption the growth *rate* of productivity is bounded, i.e. $(\alpha_{\hat{n}_1^t}/\alpha_{\hat{n}_1^{t-1}}) \not\to \infty$. Thus, the upper bound on profits per unit of research defined in Claim 4, $\bar{\pi}_{\hat{n}_1^t}^{t+\tau}$, is bounded. Analogously to 4.3.) and 4.4.) it follows that $\overline{V}_{\hat{n}_1^t}^{t+1} \not\to \infty$.

Claim 6: *If $\alpha_k < \alpha_{\mathcal{P}k}$ and if $\dot{\delta}_{\hat{n}_1^t}^t \not\to 0$, then the sequence $(\underline{V}_{n_k^t}^{t+1})_{t\in I\!N}$ is unbounded.*

From $\dot{\delta}_{\hat{n}_1^t}^t \not\to 0$ and (MinSpill) follows $\dot{\delta}_{n_k^t}^t \not\to 0$. Therefore, $(\alpha_{n_k^t}/\alpha_{n_k^{t-1}})(x_{Rn_k^{t-1}}^{t-1}/x_{Rn_k^t}^t) \not\to 1$. Thus, there exists an $\delta > 0$ and a $\bar{t} < \infty$ such that, for all $t > \bar{t}$, $(\alpha_{n_k^t}/\alpha_{n_k^{\bar{t}}})(x_{Rn_k^{\bar{t}}}^{\bar{t}}/x_{Rn_k^t}^t) > (1+\delta)^{t-\bar{t}}$. Because of (CONV) and $\alpha_k < \alpha_{\mathcal{P}k}$, we have $\alpha_{n_k^{\bar{t}}} > \alpha_k$ for $\bar{t}$ sufficiently large. Thus, $(\alpha_{n_k^t}/\alpha_k) > (1+\delta)^{t-\bar{t}}(x_{Rn_k^t}^t/x_{Rn_k^{\bar{t}}}^{\bar{t}})$. Analogously to Claim 1 it follows that $\underline{V}_{n_k^t}^{t+1} \geq \left((1+\delta)^{t-\bar{t}}(x_{Rn_k^t}^t/x_{Rn_k^{\bar{t}}}^{\bar{t}}) - 1\right)(a_{n_k^t}/x_{Rn_k^t}^t)$, or $\underline{V}_{n_k^t}^{t+1} \geq (1+\delta)^{t-\bar{t}}(a_{n_k^t}/x_{Rn_k^{\bar{t}}}^{\bar{t}}) - (a_{n_k^t}/x_{Rn_k^t}^t)$. Given t, the RHS tends to infinity with t if $x_{Rn_k^t}^t \not\to_t 0$. The claim then follows from (CONV).

Claim 7: *If $\alpha_k < \alpha_{\mathcal{P}k}$, then $(\underline{V}_{n_k^t}^{t+1}/\overline{V}_{\hat{n}_1^t}^{t+1}) \to \infty$.*

This follows from Claims 1 and 4 if $\dot{\delta}_{\hat{n}_1^t}^t \to 0$ and from Claims 5 and 6 if $\dot{\delta}_{\hat{n}_1^t}^t \not\to 0$.

Claim 8: *If $\alpha_k < \alpha_{\mathcal{P}k}$, then the sequence of consumers' maximal expectations at t about $(w_1^{t+1}/w_k^{t+1})_t$ is bounded.*

Suppose $(w_1^{it+1}/w_k^{it+1}) \to \infty$ for some type of consumers i. Since $(w_k^{it+1}/p_k^{it+1}) \not\to 0$ (the *incumbent* industrial structure of $t+1$ is known already at t), it follows that $(w_1^{it+1}/p_k^{it+1}) \to \infty$. Since all know (MRS), their expectations about the demand for output k must grow without bound. Since they also know α_k^{t+1} at t and since $\alpha_k^{t+1} \not\to \infty$, consumers know at t that the demand for output k does not grow beyond some bound (which is independent of t). This is a contradiction.

Claim 9: $\alpha_k = \alpha_{\mathcal{P}k}$ *for all k.*

From 7.) and 8.) follows that $(\underline{V}_{n_k^t}^{t+1}/\overline{V}_{\hat{n}_1^t}^{t+1})(w_k^{it+1}/w_1^{it+1}) \to \infty$ The return in terms of input k of one unit of research invested in $\hat{n}_1^t$ tends zero relative to the return in terms of input k of one unit of research invested in n_k^t. Then $\hat{n}_1^t$ would not be chosen ($\lambda_{\hat{n}_1^t} = 0$), for t sufficiently large. This is a contradiction.

Claim 10: $\lim_t(p_k^t/w_k^t) = \lim_t(1/\alpha_k^t)$ *for all k.*

This follows directly from $(1/\alpha_k^t) \leq (p_k^t/w_k^t) \leq (1/\alpha_k^{t-T})$ (T from FIOT).

The theorem follows from Claims 9 and 10.

6 References

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