Projektbereich A

Discussion Paper No. A-512

Economic Possibilities for the Grandchildern of John Maynard Keynes

Peter Funk

January 1996

I would like to thank Tilman Börgers, Giacomo Corneo, Ulrich Kamecke, Urs Schweizer and Reinhard Selten for their comments. Financial support by the Deutsche Forschungsgemeinschaft, SFB 303 at Bonn University, is gratefully acknowledged.

Abstract

This paper explores the impact of insatiable needs on the sustainability and the direction of technological change and economic growth. In a simple framework it is shown that growth can only be sustained if either the opportunity costs of research are small at low levels of research or if some needs are insatiable. The first source of sustained growth (low opportunity cost of research) also enhances an efficient spread of growth over different technologies and commodities, while the second (insatiable needs) typically induces the 'wrong kind' of growth. In connexion with Keynes' essay 'On the Economic Possibilities of our Grandchildren' [1931], we consider relative needs as the main source of insatiability of needs.

Journal of Economic Literature Classification Numbers: D60, D62, O12, O30.

Keywords: Endogenous growth, sources of sustained growth, direction of growth, relative needs.

Wirtschaftspolitische Abteilung, Universität Bonn, Adenauerallee 24-26, D 53113 Bonn

Tel.: 49-228-739230, Fax: 49-228-739221, e-mail: or106@unitas.or.uni-bonn.de

1 Introduction

This paper explores the impact of **insatiable needs** on the **sustainability** and the **direction** of technological change and economic growth.

(1) We first introduce a simple **model** of growth in which the intensity and the direction of growth are determined endogenously. As in Schumpeter's theory of economic development (Schumpeter [1911]), change in the present article is modelled as a sequence of temporary economies that are perfectly competitive in all traditional markets and monopolistically competitive in new markets. Which new markets are opened or which new technologies are intorduced depends on the set of perceived potential innovations and on the expected profitability of these potential innovations. All resources – including the resources that are necessary for technological change – flow to those activities that generate sufficiently high profits.

(2) The core question of the present paper is a normative question. How well does actual (equilibrium) development perform as compared to potential development? Our **welfare criterium** is much weaker than the standard concept of Pareto-efficiency. In a world with many different industries, technologies, and commodities, with knowledge externalites, and with agents that are short lived and – possibly – short-sighted, one can hardly expect that the 'right' innovations are chosen in every period. In fact, in Funk [1996] we show in a framework similar to that of the present paper, that generically the wrong innovations will be carried through in many periods (even if all agents are perfectly rational). The assumptions on public knowledge about the evolution of innovation-possibilities, on forward looking rationality, on the coordination of beliefs, and on inter-generational transfers that are so very strong and unrealistic that we prefer to use an alternative welfare concept which is sufficiently strong to capture a popular notion of dynamic efficiency and which is sufficiently weak to be aimed at by people that are less than clair-voyant. For us, the relevant question is whether development can *persistently* go into wrong directions, can persistently neglect direction that could and should be further develop, or can even completely peter out before potential gains are exhausted. In accordance our welfare criterium (which we call 'solving the long-run economic problem' using Keynes' terminology) only excludes *persistent* suboptimalites.

(3) In the present article we are concerned with persistent suboptimalities that arise due

to *insatiable needs*. We hold that a major source for insatiability of needs are externalities in consumers' tastes. In accordance with Keynes [1931] we will say that needs are 'relative' if the case of consumption externalities. **Relative needs** are present whenever some individuals care about what others own, earn, do, think, or consume. While it may be reasonable to assume that absolute needs (those needs of an individual that are independent of other individuals) are weakly (i.e. asymptotically) satiable, relative needs, almost by definition, cannot be simultaneously satiated by all members of a society. No doubt, relative needs are omnipresent in our society. And no doubt too, they strongly influence economic variables. Where they do so, they should matter to economists. In fact, they have mattered to economists. The importance of relative needs has not only been stressed by heterodox economists like Veblen [1899]. It also has been emphasized by many mainstream economists. One can find evidence in the writings of almost all great economists from Adam Smith to John Maynard Keynes.

Keynes' Essay on the Long-run Economic Problem

For the sake of presentation we loosely link this paper to Keynes' essay 'Economic Possibilities for our Grandchildren', which he "first presented in 1928 as a talk to several small societies, including the Essay Society of Winchester College and the Political Club at Cambridge"[1]. We do so not because Keynes stands latest in the list of great economist who explicitly referred to relative needs, but

(1) because in his essay Keynes refers to relative needs in the context of the long run economic problem (the satiation of absolute needs), which is at the center of the paper,

(2) because Keynes explicitly argues that absolute needs are satiable, while relative needs are insatiable (which, somewhat weakened, is assumed in one of the main propositions of the present paper), and

(3) because Keynes fails to realize that exactly this scenario of satiable absolute needs and insatiable relative needs is unfavorable for the solution of the economic problem – the satiation of absolute needs – even if these *are* satiable.

Before we make precise all this, we take a closer look at Keynes' essay. The essay makes clear why Keynes, the economist, was not much worried about the long-run. The reason

[1] Keynes [1931].

for this indifference was not his well documented opinion that in the long-run we are all dead. It was rather his firm belief that in the long-run mankind would have solved its economic problem. In fact, Keynes believed that our absolute needs (those needs that do not depend on what others consume or own) are in principle satiable and that they would in fact be satisfied for the grandchildren of his generation if *per capita real income* would continue to rise as it did in the past 200 years. He draws

> " the conclusion that, assuming no important wars and no important increase in population, the *economic problem* may be solved, or at least be in sight of solution, within a hundred years. This means that the economic problem is not – if we look into the future – *the permanent problem of the human race*."[2]

Although there has been at least one important war since Keynes first gave his talk at Cambridge and although there has been a tremendous increase in population since then, per capita real input *has* risen at least as fast since 1928 as it did in the two preceding centuries. It seems natural for us – the grandchildren of Keynes generation – to check how far his prediction has already come true.

Most importantly for the present paper which is concerned not only with the sustainability of growth, but also with its direction, one has to realize that the economic problem – the satisfaction of our needs – is a problem with many dimensions. One can not necessarily say much about the solution of this problem by simply referring to continuous growth of per capita real income, which aggregates over many different individuals and over many different needs:

First, there are many different types of *individuals*. Some people manage to provide themselves with plenty of good food, housing and clothing, performing a pleasant job for a moderate amount of time. However, while some no longer sweat for their daily bread, many more remain poor in absolute terms by any standard. Following a world bank report[3] there are almost one billion of people that – far from being satiated – "suffer from undernourishment in terms of having not enough calories for an active working life". This *is* an economic problem and a solution to the problem is not much closer in sight today

---

[2]Keynes [1931], p. 326.

[3]World Bank [1986]

than it was in 1928. We will speak of *persistent inequalities* in such cases of persistent (absolute) poverty in a world of continuously growing average real income (since in such a case an alternative feasible development could make some individuals drastically better off without making anybody else drastically worse off).

Second, there are different needs even for a single individual. While some individuals may come close to satisfy *many* of their basic needs, virtually nobody manages to satisfy *all* of his most basic material needs. The need for clean air or clean and chloride free water for instance is not satisfied for some of the most wealthiest individuals. And many people still work much for a job they don't like enough -unlike Keynes has predicted. In these examples development quite persistently neglects some type of feasible and desirable improvements. We will speak of *persistent inefficiencies* in such cases.

Thus, if we take into account the fact that the economic problem has many dimensions we can hardly claim that we have come close to its solution. Empirically Keynes' prophecy has not yet been fulfilled. Possibly we only have to wait for longer – about this we can only speculate and theorize. This brings us to the essential theoretical questions of this paper: Are there inherent forces in a laissez-faire market economy guaranteeing that continuous growth of *per capita real* income (aggregated over persons and commodities) induces progress in the satisfaction of *all* needs for *all* mankind? *Does laissez-faire development (asymptotically) exhaust all potential gains form feasible development?* If yes, we have said that laissez-faire development solves the long run economic problem. Development solves the long-run economic problem, if both persistent inefficiencies and persistent inequalities are excluded.

Note that sustained growth, if feasible and desired, is a necessary condition for solving the economic problem. Since sustained per capita growth (world-wide) may be considered as an empirical fact, while the solution of the economic problem can't, we need a theory that does not identify the two, but rather allows to identify conditions that possibly enhances sustained growth, but that enhance the wrong kind of growth.

Relative needs. Critical voices in the audience of Keynes' first presentation in Cambridge may already have objected that his informal definition of the economic problem makes little sense if our needs are insatiable. Keynes in fact admits the possibility of insatiable needs. In order to rescue his vision about the solution of the economic problem he distinguishes between absolute needs and relative needs:

> "··· Now it is true that the needs of human beings may seem to be insatiable. But they fall in two classes – those needs which are absolute in the sense that we feel them whatever the situation of our fellow human beings may be and those which are relative in the sense that we feel them only if their satisfaction lifts us above, makes us feel superior to, our fellows. Needs of the second class, those which satisfy the desire for superiority, may indeed be insatiable; for the higher the general level, the higher still they are. But this is not so true of the absolute needs – a point may soon be reached, much sooner perhaps than we all of us are aware of, when these needs are satisfied in the sense that we prefer to devote our further energies to non-economic purposes."[4]

While Keynes makes clear that by "solving the economic problem of mankind" he means satisfying everybody's *absolute* needs, he is not very precise about *when* relative needs become relevant. Implicitly he seems to assume that relative needs only gain momentum after absolute needs have been satiated. Keeping in mind the multi-dimensionality of the economic problem it seems clear that some relative needs (and the relative needs of some) become relevant before all absolute needs (and the absolute needs of all) are satiated. In this case the answer to our question of how apt the laissez-faire is in exploiting the potentials of feasible development strongly depends on the presence of relative needs.

The remaining of this article is organized as follows. In section 2 we introduce the framework of development and define our welfare criterium. This criterium excludes both persistent inefficiencies and persistent inequalities. In section 3 we first show that if all needs are weakly satiable, then the profits in terms of own inputs from potential innovations tend to zero if growth is sustained. As a corollary it follows that growth cannot be sustained if needs are weakly satiable and if the opportunity costs of research are high. If, on the other hand the opportunity cost of research are sufficiently low, then we show that equilibrium development solves the economic problem (in particular, growth is sustained if this is feasible). If research is sufficiently pleasant at low levels of research this not only enhances growth, it also enhances the 'right kind' of growth. We then introduce insatiable needs.

[4] Keynes [1931], p. 326.

Since we think that the most prominent case for insatiable needs are 'relative needs' we assume (weakly) satiable absolute needs and insatiable relative needs. This class of of preferences also reflects Keynes' view about satiable absolute needs and insatiable relative needs. We show that laissez-faire development *always* fails to solve the economic problem, although growth may be sustained.

Section 4 gives examples in which development fails to solve the economic problem in the long-run due to insatiable needs. The examples show that the relevant question is not whether needs are relative or absolute, but whether they are satiable or insatiable. The persistent inequality of example A1 and the persistent inefficiency of example A2 are due to insatiable absolute needs. We reconsider these examples in the presence of relativ needs and show how easily persistent long-run market failures arise in the presence of relative needs. Section 5 concludes with a discussion of different potential sources of sustained growth.

2 The Model

We first introduce a very simple framework of development in which both the intensity and the direction of growth are determined endogenously. While the model allows for the presence of relative needs, neither the model nor the definition of the long-run economic problem depend on the presence or the absence of relative needs.

Keynes identified the solution of the economic problem with the satiation of consumers' absolute needs. The solution of this problem (in finite times) would implicitly require that absolute needs are satiable. The requirement of (strongly) satiable wants contradicts standard assumptions of non-satiability of most of modern economic theory. Fortunately, strongly satiable absolute needs are not necessary to give a precise sense to Keynes informal idea of the long-run economic problem. We will therefore allow for insatiable needs and say that laissez-faire (i.e. equilibrium) development solves the long-run economic problem if the satisfaction of no need which could further be enhanced is persistently neglected in the course of development. We do not require that a bliss point is reached (or asymptotically approached) if this is technically infeasible.

As we have noted in the introduction there are many ways in which the tastes of one consumer may depend on other consumers. Here we will simply assume that the utility of one consumer not only depends on what he consumes himself, but also on what other consumers consume.

Also note that one may whish to investigate into the causes of relative needs. Some argue, for instance, that consumption externalities arise because consumers want to signal some feature of their personality which can not be directly observed. Consumers may want to convey information about their wealth by the consumption of conspicuous commodities (see Corneo and Jeanne [1995] or Bagwell and Bernheim [1994]). In contrast, we simply *postulate* the existence of relative needs. There may be many different reasons for why a person cares for the type of shirt others wear or cars they drive. We do not try to uncover here the social and psychological causes for relative needs, as little as we try to explain the biological or psychological causes for absolute needs. We simply take them as given, without however, assuming much about their particular shape.

Commodities and preferences. There are finitely many types of individuals $i \in I$ and finitely many groups of commodities $h \in H$. The commodities y_{hn} within one group h are produced with a common input x_h and vertically product differentiated with quality $q_n \in I\!R_+, n \in I\!N$. In general a consumption plan of consumer i in a given period is a vector of inputs $x^i = (x^i_1, \cdots, x^i_H)$ and a vector $\tilde{y}^i = (\tilde{y}^i_1, \cdots, \tilde{y}^i_H)$ of consumption functions, where $\tilde{y}^i_h$ describes the list of the quality differentiated commodities of group h consumed by i. We assume that the utility of a consumer depends on the qualities consumed by the other consumers. To fix ideas we assume that it depends on the vector of *average* qualities, $\bar{q}$, consumed in the economy, where $\bar{q}_h = ([\sum_0^\infty q_{hn}\hat{y}_{hn}]/[\sum_0^\infty \hat{y}_{hn}])$, and where $\hat{y}_{hn}$ is the mean consumption of commodity h with quality n in the economy. Preferences are represented by a utility function $u^i : I\!R^H_+ \times I\!R^{H\infty}_+ \times I\!R^H \to I\!R$, $(x_1, \cdots, x_H, \tilde{y}_1, \cdots, \tilde{y}_H, \bar{q}_1, \cdots, \bar{q}_H) \mapsto u^i(x, \tilde{y}, \bar{q})$, where $x = (x_1, \cdots, x_H)$ is a vector of input sales, $\tilde{y} = (\tilde{y}_1, \cdots, \tilde{y}_H)$ is a vector of consumption functions for all groups of commodities,

Note that although we have allowed for externalities with respect to quality, we do not include externalities with respect to quantity. We could as well allow for such externalities. For our propositions it only matters whether or not externalities with respect to quantities of, say, commodity h peter out if the quantities of single commodity of group h become very large (Hundreds of cakes per day per person). Neglecting externalities with respect to quantity amounts to assume that they in fact do peter out. We think that this is not unrealistic. In contrast, we will (in general) not assume that the externalities with respect to quality peter out if quality growth without bound.

State of knowledge and competitive equilibria. The **state of knowledge** in a given period t is defined as the (finite) set of technologies $\mathcal{Y}^t = \{Y_1^t, \cdots, Y_{\#\mathcal{Y}^t}^t\}$, $\#\mathcal{Y}^t < \infty$, that individual firms can use at t. A technology produces a single output with single input (although this can easily be generalized). The **aggregate technology**, $\hat{Y}_k$, corresponding to a given individual technology Y_k^t in $\mathcal{Y}^t$, is the smallest cone containing that individual technology. In the considered case of single-input-single-output technologies, $\hat{Y}_k$ is simply a linear technology that has as slope the minimal average productivity of Y_k^t.

Temporary equilibria given the state of knowledge in a given period are defined as (temporary) competitive equilibria of the economy that operates with the aggregate technologies. The idea is that in a sufficiently large economy with free entry to all traditional technologies, aggregates behave as if the economy were perfectly competitive given the appropriate cone technologies. In order to guarantee that individual firms are 'small' compared to the rest of the economy, it is assumed that the asymptotic cones of all individual technologies do not contain strictly positive elements,[5] or, with single-input-single-output technologies, that the average product tends to zero if the amount of input tends to infinity. We denote $\alpha(Y_k)$ the maximal average productivity of technology Y_k.

Innovation possibilites. Given the state of knowledge $\mathcal{Y}^t$ at t there is a set of perceived potential innovations. More generally, there is an exogenous **innovation-function** $\mathcal{I}$ that defines a set of potential innovations $\mathcal{I}(\mathcal{Y})$ for each state of knowledge $\mathcal{Y}$. Potential innovations have the same general properties as traditional technologies (i.e. they are 'small').

Research and choosen innovations. There are scarce resources that can be used to improve upon existing technologies. We call these resources 'research'. For simplicity, it is assumed that the maximal supply of research just suffices to implement any of the potential innovation of that period. Thus, at most one potential innovation in $\mathcal{I}(\mathcal{Y})$ is chosen in each period.

[5] For a non-cooperative foundation of this equilibrium concept (given the state of knowledge) in a general equilibrium Cournot framework see Novshek and Sonnenschein [1980] and in a general equilibrium Bertrand framework see Funk [1995].

Innovators can use their innovations as monopolists. Since technologies are small compared to the aggregate technology of the competitive sector, the innovator of a given period (if there is an innovation in that period) takes as given all prices of the competitive sector and chooses the innovation which guarantees the highest profits. After the period of innovation, there is free entry to the new technology, as well, and profits are reduced to zero. Thus innovators choose the innovation that generates highest short-run profits given current prices.

We assume that there is free entry to the innovative activity. Therefore the full profits of the innovator are paid to researches. If R is the maximal amount of research hours in the economy and π_I^t is the profit of a potential innovation I in t, then the 'wage' for one hour of research is $w_R^t = (\pi_I^t / R)$. Given w_R^t and given the other prices at t a researcher can decide whether to do research or not. To fix our attention we consider two very stylized polar scenarios:

Scenario 1: 'Research has low opportunity cost'. The full amount of research is supplied, irrespectively of what w_R^t and other prices are. In a more general framework this can be replaced by the assumption that the marginal disutility from doing little research is very low (see Funk [1996]). We therefore say in this case that research has low opportunity cost.

Scenario 2: 'Research has high opportunity cost'. A researcher supplies all his research to innovators if and only if $w_R^t \geq w_h^t$ for all $h \in H$. We are in this Scenario 2 for instance, if each input is the labor of a certain skill-level and if a researcher can do the tasks of any skill level, being completely indifferent as to which task he performs.

With Scenario 1 the potential innovation generating highest profits is implemented in each period. With Scenario 2 this innovation is implemented if it generates sufficiently high profits. Otherwise no innovation is carried through in that period.

Note that one of the simplifications of the present model is that individual technologies are small. Since this also is the case for the innovator his profits will be small as compared to aggregate quantities. This is why we can neglect research incomes when defining temporary equilibrium. In Funk [1996] we allow for a non-negligable research sector. This is more realistic but does not much alter the insights of the simpler present framework.

Equilibrium development. The new aggregate technology in the industry in which an innovation occurs is the smallest cone containing the improved technology used by the innovator. Depending on the new state of knowledge there is a new set of potential innovations (defined by the innovation function), of which none or the most profitable one is carried through, which in turn defines the state of knowledge of the next period. Thus, given the innovation function and given an initial state of knowledge, we get a sequence of states of knowledges, which we call **equilibrium development of knowledge**, and a correspoding sequence of temporary competitive allocations and prices, which we call **equilibrium development**.

In Funk [1996] we explain how features of the innovation-function can cause inefficient dead locks of equilibrium development. Here we we want to concentrate on problems that arise due to properties of preferences. We therefore assume that the innovation-function is nicely behaved: Firstly we will always assume that there is no loss of opportunities. This essentially amounts to exclude negative external effects of the research in one direction on the productivity of research into another direction. Formally, if $I \in \mathcal{I}(\mathcal{Y})$ and $\mathcal{Y} \subset \mathcal{Y}'$, then $I \in \mathcal{I}(\mathcal{Y}')$ (We call this assumption **No loss of opportunities**). Secondly, we assume that it should be possible to at least slightly improve upon the initial knowledge in just one period if there is a path from the initial state of knowledge $\mathcal{Y}$ to a 'better' state of knowledge $\mathcal{Y}'$. The necessary improvement may be a very minor one (i.e. it has to be bounded away from zero). Formally, we assume that given the innovation function $\mathcal{I}$ there exists a $\beta \in [0, 1)$ such that for all states of knowledge $\mathcal{Y}^0$, $\mathcal{Y}$, all qualities $q^0, q \in I\!R_+$, any class of commodities $h \in H$ and any natural number $\tau \geq 1$ with $\mathcal{Y} \in \mathcal{P}^\tau(\mathcal{I}, \mathcal{Y}^0)$ there exists a $\mathcal{Y}' \in \mathcal{P}^1(\mathcal{I}, \mathcal{Y}^0)$ and a $q' \in I\!R_+$ (We call this assumption **'Convexity'**). Thirdly, we assume that the average products of the technologies cooresponding to the limiting potential state of knowledge (when time tends to infinity) tend to zero if the amount of input employed tends to infinity (we call this assumption **'No explosion of quantities'**).

For the sake of exposition, we assume that in each industry h and in each period there is one process-innovation, improving upon the technology producing the highest quality in industry h, and one product innovation, increasing the highest quality $q_{hn_h^t}$ to $q_{hn_h^t+1}$. Thus, in equilibrium development, in any given period (t) and any industry (h) only one quality is consumed. At any point in time the relevant aggregate technologies are completely described by the vector $(q_h^t, \alpha_h^t)_{h \in H}$, where q_h^t is the highest quality producable in industry

h at t and where α_h^t is the slope of the aggregate technology for this quality at t.

Potential development. We call **potential development of knowledge** the hypothetical sequence of states of knowledge that one would get if, in every period all potential innovations were carried through. The sequence of aggregate technologies corresponding to potential development is fully described by the sequence $(q_{\mathcal{P}h}^t, \alpha_{\mathcal{P}h}^t)_{h \in H}$, where $q_{\mathcal{P}h}^t$ is the highest quality producable in industry h at t in potential development and where $\alpha_{\mathcal{P}h}^t$ is the slope of the aggregate technology for this quality (given potential development). Let $q_{h\mathcal{P}}$ and $\alpha_{h\mathcal{P}}$ be the limits of these values if time is taken to infinity (note that these 'limits' may be infinite) .

We say that laissez-faire **development solves the long-run economic problem** if for any equilibrium development all potential gains from development are exhausted asymptotically, i.e., if the price in terms of own inputs of the highest quality q_h^t tends to $(1/\alpha_{h\mathcal{P}})$ and if the highest known quality q_h^t tends to $q_{h\mathcal{P}}$, for all commodities h in which some consumer is insatiated (strictly positive marginal utility) at the limit of development.

One may hold that this definition is overly strong in that it compares equilibrium development with *potential* development which is not restricted by the resource constraints of the private sector. One may therefore prefer to define the welfare criterium only in terms of feasible developments. A feasible development of knowledge simply is a sequence of states of knowledges $(\mathcal{Y}^t)_t$, such that for all t the state of knowledge at t, $\mathcal{Y}^t$, can be reached by a potential innovation given the previous state of knowledge, $\mathcal{Y}^{t-1}$, i.e. if $\mathcal{Y}^t \setminus \mathcal{Y}^{t-1} \in \mathcal{I}(\mathcal{Y}^{t-1})$ for all $t > 0$. Because of our assumptions on the innovation function an alternative definition, comparing equilibrium development with *feasible* developments only, is equivalent to the chosen definition, since the limit of the potential development of knowledge also is the limit of some feasible development of knowledge.

3 Satiable and Insatiable Needs

3.1 Weakly Satiable Absolute Needs

We say that consumer i's **absolute needs are weakly satiable with respect to quantity** if for each inputs h and any consumed quality n of output h

(1) $u_{hn}^i \not\to 0$ for any sequence of consumption plans, for which $y_{hn}^i \not\to \infty$ and for which the

vector $\bar{q}_h - q_h$ is constant (not strongly satiable);
(2) $u^i_{x_h}$ is bounded;
(3) $u^i_{hn} \to 0$ for any sequence of consumption plans with $y^i_{hn} \to \infty$ and with the vector vector $\bar{q} - q$ being constant (asymptotically satiable).

Note that part (1) and (2) is all what is needed for the efficiency result, below. For better comparability of the efficiency result with the inefficiency result (in the presence of relative needs), we also require asymptotic satiability (part (3)).

When defining satiable absolute needs we are interested in a consumer's marginal utility from quality in group h if the vector of economy-wide average qualities $\bar{q}$ always equals the vector of average qualities q consumed by the consumer. To this end we define a reduced utility function. The reduced utility function of consumer i is v^i, defined by $v^i(x, \tilde{y}) = u^i(x, \tilde{y}, \bar{q} = q^i)$, where $\bar{q}$ is the vector of average qualities corresponding to $\tilde{y}$.

We say that consumer i's **absolute needs are weakly satiable with respect to quality** if the marginal rates of substitution between two commodities of the same group h, q^τ and $q^\tau + \Delta q^\tau$, $\frac{v^i_{y_{hq^\tau}}}{v^i_{y_{h(q^\tau+\Delta q^\tau)}}}$, tend to 1 for all sequences $(q^\tau, \Delta q^\tau)_\tau$ such that $q^\tau \to_\tau \infty$ and such that Δq^τ is a feasible quality increment (with respect to $\mathcal{I}$) given q^τ for all τ.

Consumer i's **absolute needs are weakly satiable** if they are weakly satiable with respect to quantity and with respect to quality.

We say that **there are no relative needs** if all the individual utility functions u^i are invariant in the vector of average consumed qualities $\bar{q}$.

Finally, we say that **growth is sustained** if the sequence of innovations never comes to a halt.

Lemma 1 *If there are no relative needs, if all consumers' absolute needs are weakly satiable, and if growth is sustained, then the potential profits form innovations in terms of own inputs tend to zero in any industry.*

Proof: We only sketch the proof here.
(1) From 'Weak satiability of absolute needs' follows that the maximal profits in terms of own inputs π^t_{hq}/w^t_h ($\pi^t_{h\alpha}/w^t_h$) from a quality (process) innovation in industry h tend to zero if the number of quality (process) improvments in that industry tends to infinity. Note that monopoly quantities cannot tend to infinity because of the assumption 'no explosion of quantities' (This is why we need not assume asymptotic satiability with respect to

quantities).

(2) Because there are only finitely many groups of commodities, the number of quality or process innovations in at least one industry, say industry 1 has to tend to infinity if growth is sustained.

(3) From (1) and (2) it follows that, if growth is sustained, then π^t_{1q}/w^t_1 or $\pi^t_{1\alpha}/w^t_1$ tends to zero. Therefore π^t_{hq}/w^t_1 and $\pi^t_{h\alpha}/w^t_1$ must tend to zero for all h, otherwise the infinite sequence of innovations in industry 1 could not occur.

(4) From (3) and the assumption that absolute needs are weakly satiable, it follows that π^t_{1q}/w^t_h and $\pi^t_{1\alpha}/w^t_h$ tend to zero in the course of development. ■

Corollary 2 *If there are high opportunity costs of research, if there are no relative needs and if all consumers' absolute needs are weakly satiable, then growth is not sustained in laissez-faire development. Therefore, laissez-faire development fails to solve the long-run economic problem.*

Proof: Suppose growth were sustained. Then, because of lemma 1, in the long-run, the wage for one hour of research that the most profitable potential innovator can offer to researchers, is smaller than the wage of usual workers in the industry of this innovator. Because we assume Scenario 2 (high opportunity cost of research) potential researchers would therefore prefer not to do research. Therefore, growth cannot be sustained. ■

Proposition 3 *If there are low opportunity costs of research, if there are no relative needs and if all consumers' absolute needs are weakly satiable, then laissez-faire development solves the economic problem.*

Proof: Suppose that $(q^t_h, (p^t_h/w^t_h))$ does not tend to the limit of $(q^t_{h\mathcal{P}}, (1/\alpha^t_{h\mathcal{P}}))$ in the course of development. Then, introducing the potential technology corresponding to $(q^{\bar{t}}_{h\mathcal{P}}, (1/\alpha^{\bar{t}}_{h\mathcal{P}}))$ is profitable in terms of input h at sufficiently large $\bar{t}$. However, no single step innovation may lead to that potential technology. It follows from the convexity assumption on the innovation-function that, at $\bar{t}$, there is a feasible single innovation which is profitable too. From the 'no loss of opportunities' assumption it follows that this innovation remains feasible at any $t > \bar{t}$. As a consequence its profit in terms of input h does not tend to zero. This contradicts lemma 1. (A more detailed proof is given in Funk (1996)). ■

Thus, if there are low opportunity costs of research, if there are no relative needs and if all consumers' absolute needs are weakly satiable, then the satisfaction of *no* need of *anybody* is persistently neglected (relative to potential development). Knowledge externalities allone do not cause persistent inefficiencies in this case.

We will now introduce an additional externality (a consumption externality) and again only ask for persistent inefficiencies.

3.2 Insatiable Relative Needs.

We now show that the conclusion of Proposition 3 is reversed for a simple but plausible class of preferences if consumers also have relative needs. This class of preferences reflects Keynes' informal view on satiability of our needs: Satiable absolute needs together with insatiable relative needs. We show that laissez-faire development systematically fails to solve the economic problem if consumers have relative needs. Insatiable relative needs are a potential source of sustained growth, but they typically induce the wrong kind of growth.

We have already defined weakly satiable absolute needs. We now define **insatiable relative needs.** Once one accepts the presence of relative needs, it is most natural to assume that they are insatiable.

Relative needs for commodities of industry h are insatiable if there exists a $\kappa > 1$ such that given any vector of economy-wide average qualities $\bar{q}$, the marginal rates of substitution of $i \in I$ between two commodities of the same group h, q^τ and $q^\tau + \Delta q^\tau$, $\frac{u^i_{y_{hq^\tau}}}{u^i_{y_{h(q^\tau+\Delta q^\tau)}}}$ are greater than κ if $\bar{q} \geq q$, (where $(q^\tau, \Delta q^\tau)_\tau$ is any sequence of feasible quality increments).

Lemma 4 *The profits in terms of own inputs from product innovations in industries satisfying relative needs are bounded away from zero.*

Proof.

Take an industries satisfying relative needs, say industry h. For all t, at least on type of consumers, say i, consumes no more than average quality of this commodity, i.e. $q^{it}_h \leq \bar{q}^t_h$. Thus $\frac{u^{it}_{y_{hq^{it}_h}}}{u^i_{y_{h\hat{q}^t_h}}}$ is greater than κ, for some innovation $\hat{q}^t_h$ and some $\kappa > 0$. Thus, the price increment a process innovator can get for a better product is bounded below by κ. As we have assumed that both quality innovations and process innovations in an industry always improve upon the best known technology, the corresponding monopoly quantities do not tend to zero. Therefore the corresponding profits are bounded away from zero. ■

Proposition 5 *If absolute needs are weakly satiable and if relative needs are insatiable in some industries, then laissez-faire development fails to solve the long-run economic problem. Resource and labor saving innovations in all industries as well as product innovations of commodities without insatiable relative needs come to a complete halt before all potential gains of development in these directions have been exhausted.*

Proof.

(1) *The profits from process innovations and the profits from product innovations in industries not satisfying relative needs tend to zero if these innovations are chosen infinitely often.* At equilibrium of any period $q^{it} = \bar{q}^t$ for all types of consumers.[6] Therefore, in industries that do not satisfy relative needs, weak satiability of absolute needs with respect to quality can be used as in the proof of Lemma 1.

(2) From (1) and Lemma 4 follows that either growth peters out or at least one of the industries satisfying relative needs is continuously product improved, while the process innovations in all industries and all innovations in industries not satisfying relative needs are persistently neglected after a certain stage of development. ■

It follows from Lemma 4 that research wages that can be paid by the most profitable innovation have a strictly positive lower bound. Whether or not this lower bound is higher than the maximal industrial wages, depends on the specifics of the innovation-function. In section 4 we give examples in which research wages always remain above industrial wages. Then, relative needs are a source of sustained growth.

Corollary 6 *If there are low opportunity costs of research, if absolute needs are weakly satiable and if relative needs are insatiable in some industries, then growth is sustained under laissez faire-development but the economic problem is not solved. Some commodities with insatiable relative needs are perpetually product improved, although all consumers persistently prefer other feasible developments.*

[6] Note that for an innovator it is optimal not to ration, such that the better quality he may produce is spread over all the members of at least on type of consumers. Since he is small, his presence does not influence economy-wide averages.

Proof: Corollary to Proposition 5.

If the principle determining which commodities satisfy relative needs is that relative needs are strong for commodities which are consumed in public, visible to others, for instance, then development is persistently biasd in favor of conspicuous consumption.

4 Examples

In section 1 we have explained that development may fail to 'solve the long-run economic problem' due to 'persistent inefficiencies' or due to 'persistent inequalities'. In the present section we give formal examples for both kinds of suboptimalities. They arise because of insatiability of needs. In the first two examples (A1 and A2) there are no relative needs (absolute needs are insatiable). The two next examples (B1 and B2) differ from the first examples in that we assume the presence of relative needs. The examples make clear that for the long-run suboptimality of growth, the crucial question is whether needs are satiable or insatiable, and not whether needs are absolute or relative. However, as we have explained we believe that a major source for insatiability are relative needs. The examples with relative needs are more natural and much more straightforward then the examples without relative needs.

4.1 Insatiable Absolute Needs

4.1.1 Example A1: Persistent Underdevelopment.

In the first example laissez-faire development fails to solve the economic problem because of a persistent inefficiency. One part of the population enjoys perpetual growth of wealth, another part of the population remains poor in absolute terms. Asymptotically the first group of individuals is fully satiated, the second group remains hungry. We call this persistently unequal only because there is an alternative feasible development under which, asymptotically, *everybody* becomes satiated. The example is reduced to the minimal elements that are necessary to generate the persisitent inequality. We therefore deviate from the setting of section 3 in assuming that there are only process-innovations. The example should only illustrate the impact of a violation of 'weak satiability of absolute needs'. In Funk [1996] the example is elaborated to a plausible story of satiation and underdevelopment.

Consumers and commodities. There are only two produced commodities: one low quality commodity, y_1, and one high quality commodity, y_2. Low quality commodites are produced with unskilled labour, x_1, and high quality commodities are produced with skilled labour, x_2. Simplifying the model of section 2 there are no product-innovations in the example. All innovations are process-innovations that increase the productivity of the most advanced technology in one of the two industries. There are two types of individuals, differing with respect to endowments (i.e. skills). Some (indexed by 's') own 'skilled' labour only ($e_1^s = 0$, $e_2^s > 0$) and the others (indexed by 'u') own 'unskilled' labour only ($e_1^u > 0$, $e_2^u = 0$).

All individuals have the same continuous and quasi-concave utility function $u(x = x_1 + x_2, y_1, y_2)$ that satisfies the following condition. There exist $\underline{y}, \overline{y} \in I\!R_+$, with $0 < \underline{y} < \overline{y}$, such that $u(x, y_1, y_2) < u(x, y_1', y_2')$ if $[y_1 + y_2 < y_1' + y_2' < \underline{y}]$ or if $[y_1' + y_2' \geq \overline{y}$ and $y_2 < y_2']$. One might think of y_1 as low quality food and of y_2 of high quality food, both being identical in nutritional value. If a person has less than $\underline{y}$ (the hunger-line) he is hungry and does not care for taste. If he has more to eat than $\overline{y}$ (the satiation-line) he does not care for low quality food any more.

Note that these preferences violate part (1) 'Weakly satiability absolute needs w.r.t. quantity'. Absolute needs for the the low quality commodity are strongly satiable in the example.

State of knowledge and temporary equilibria. There are two types of technologies (two industries), the first producing y_1 with x_1, the second producing y_2 with x_1.

Innovation possibilities. There is a potential innovation, I_k^t, for each known technology Y_k^t given the state of knowledge $\mathcal{Y}^t$. It is purely process-improving and increases the efficient scale productivity α_k^t by a factor $\gamma > 1$. To keep matters as simple as possible we assume that there are low opportunity costs of research (scenario 1). To guarantee sustained growth with high opportunity cost of research we would have to assume more about the innovation function.

Note that in the example γ does not depend on how often which innovation has been chosen. The constancy of γ corresponds to the assumption of exponential growth of research productivity, standard in the endogenous growth literature. In the examples with relative we will no longer need to assume exponential growth of research productivity.

Choosen innovations. In the example the potential state of knowledge at t is represented by the potential productivities $(\alpha^t_{\mathcal{P}1}, \alpha^t_{\mathcal{P}2})$ that would be realized if in every period before t all potential innovations would have been carried through. Laissez-faire development solves the long-run economic problem in the example if the real prices in both industries $(p^t_1/w^t_1, p^t_2/w^t_2)$ tend to the limit of $(1/\alpha^t_{\mathcal{P}1}, 1/\alpha^t_{\mathcal{P}2})$.

In the simple example it is clear that only one technology will be active in a given period in each industry. This is the technology with maximal efficient scale productivity in the corresponding industry. We denote these productivities by α^t_1 and α^t_2. Innovators will always choose to improve upon one of these two technologies. We will now argue that innovations only occure in the high quality industry.

Suppose that e^s_2 is sufficiently large (given the utility function and given α^0_2) to ensure that $x^s_2\alpha^s_2 > \overline{y}$. Then, the s do not buy y_1 if the price for y_1 is strictly positive. Therefore, at (temporary) equilibrium the u cannot purchase y_2 (this follows from the budget condition of the s and the fact that $\alpha^0_2 = (w^0_2/p^0_2))$. If we make sure that the u are really hungry by setting $e^u_1\alpha^0_1 < \underline{y}$ then they will certainly not demand y_2 if $p^0_2 > p^0_1$. Thus any price vector with $(w^0_1/p^0_1) = \alpha^0_1, (w^0_2/p^0_2) = \alpha^0_2, p^0_2 > p^0_1$ is an equilibrium (if e^s_2 is sufficiently large and e^u_1 sufficiently small). If α^t_2 grows and $\alpha^t_1 = \alpha^0_1 \forall t$, then any price vector with $(w^t_1/p^t_1) = \alpha^0_1, (w^t_2/p^t_2) = \alpha^t_2$, $p^t_2 > p^t_1 > 0$ is an equilibrium. In order to guarantee that $\pi^t_2 > \pi^t_1$ in each period we simply have to choose (p^t_2/p^t_1) sufficiently large, given the exact form of the innovation-function. Then the innovator in each period will in fact choose the second technology. Thus, $\alpha^t_2 \to \infty$, $\alpha^t_1 \not\to \infty$ while it would be feasible for both productivities to tend to infinity. Allthough the low quality commoditiy could in principle (asymptotically) become a free commoditiy for everybody, the unskilled remain hungry in absolute terms. This is a persistent inequality. Laissez-faire developement fails to solve the long-run economic problem.

4.1.2 Example A2: Too much work for to high quality.

In the second example laissez-faire development failes to solve the long-run economic problem because of a persistent inefficiency. Asymptotically consumers work more than the would like to (as compared with other feasible and prefered developments).

As in the first example development in Example A2 has two possible dimensions. In the present example these are quality-improvements on the one hand and productivity growth on the other hand. Unbounded quality growth and unbounded productivity growth are feasible and efficient (in a strong sense). Actual development, while continuously improving upon existing qualities, does not much rise the productivity of labor. Process-innovations are *persistently* neglected. We will show that this is strongly inefficient.

There is only one type of consumer. All consumers have identical endowments e and identical preferences. There is a single input x (labor) and a quality differentiated output y_n, with quality $q_n \in I\!R_+, n \in I\!N$. Preferences are represented by a utility function $u : I\!R_+ \times I\!R_+^\infty \to I\!R$, $(x, \tilde{y}) \to [(\sum_0^\infty y_n)(e-x)]^{1/2} + [\sum_0^\infty q_n y_n]^{1/2}$, where $\tilde{y}$ describes the list of the quality differentiated commodities consumed.

To complete the description of the economy we have to specify the innovation-function. There are two potential innovations $(I_{qk}^t, I_{\alpha k}^t)$ for each known technology Y_k^t given the state of knowledge $\mathcal{Y}^t$. The first, $I_{\alpha k}^t$, is purely process-improving. It increases the efficient scale productivity a_k^t by a factor $\gamma > 1$. The second, I_{qk}^t, allows to produce a higher quality with I_{qk}^t identical to Y_k^t (except for the quality of the output). It increases the quality q_k^t produced by Y_k^t by a factor $\xi > 1$.

As in the underdevelopment example γ and ξ do not depend on how often which innovation has been chosen. We may say that the constancy of these factors corresponds to the assumption of exponential growth of research productivity, standard in the endogenous growth literature. In addition, it excludes the presence of spill-overs from actual research on one type of innovations on the productivity of research in another direction.

The chosen innovation is always strictly better than the old technology it improves upon, independently of how the innovator decides. If, for simplicity, we assume that the initial state of knowledge $\mathcal{Y}^0$ consists of a single technology Y^0 only, then the state of knowledge at 1, $\mathcal{Y}^1 = \{Y^0, I^0\}$ can be replaced by $\{I^0\}$ without losing any relevant information. Similarly, for each $t > 0$ the innovator can either choose a process-improving innovation, I_α^t, or a quality-improving innovation, I_q^t. Thus $\mathcal{Y}^t$ can be represented as a singleton for all t. Given t let $n(t)$ (resp. $m(t)$) be the numbers of times the quality-improving (resp. process-improving) innovation has been chosen, with $n(t) + m(t) = t$. Then, $q^t = \xi^{n(t)} q^0$ and $\alpha(q^t) = \gamma^{m(t)} \alpha^0$, where $\xi > 1$ and $\gamma > 1$.

In the example the quality can tend to infinity in the course of development and, at the same time, the amount of labor needed to produce one unit of the highest known quality can tend to zero. In the limit of potential development $(\alpha^t_{\mathcal{P}}(q^t_{\mathcal{P}}), q^t_{\mathcal{P}})$ the commodities of all qualities are free goods. Development is long-run efficient or solves the long-run economic problem in the example if these potentials of development are asymptotically exhausted in equilibrium development, i.e. if the consumed qualities q^t tend to infinity (the limit of $q^t_{\mathcal{P}}$)) and the labor costs $p^t(q^t)/w^t$ per unit of consumed output tend to zero (the limit of $1/\alpha^t_{\mathcal{P}}(q^t_{\mathcal{P}})$).

In the appendix we show that in equilibrium development growth is sustained even if there are opportunity costs of research (scenario 2). However, actual development , while continuously improving upon existing qualities, does not raise the productivity (in terms of quantity) of labor. Process-innovations are *persistently* neglected. Instead, consumers would prefer to choose process-innovations in each period. Thus it is Pareto-efficient always to choose the process-improving innovation, whereas in equilibrium development quantities are never chosen. In particular, laissez-faire development fails to solve the economic problem.

When evaluating utilities at temporary equilibrium we can without loss of generality replace the utility function u by the function $\tilde{u} : I\!R^3_+ \to I\!R$, $(x, y, q) \to [y(e-x)]^{1/2} + [qy]^{1/2}$, where y is the non-negligible consumed quantity of the latest publicly known quality q. All marginal rates of substitution in the example are 'well behaved' including those involving qualities, i.e. $\frac{\tilde{u}_q}{\tilde{u}_x}$ and $\frac{\tilde{u}_q}{\tilde{u}_y}$ both tend to 0 if q tends to infinity. Clearly, what is needed in addition to the assumption of (relative) satiation of absolute needs in order to exclude long-run inefficiencies like that of the example is an assumption that limits the feasible quality increments. Accordingly, the assumption of 'Weakly satiable absolute needs with respect to quality', which in section 3 excludes examples like the present one, is a *joint* assumption on preferences and the innovation-function.

4.2 Relative Needs

We now reconsider the two previous examples, altering only preferences and (in the second example) being less demanding concerning the innovation-function.

4.2.1 Example B1. Persistent Underdevelopment.

The economy is exactly identical to the economy of Example A1 , except for preferences. Consumer's utility now also depends on $\hat{y}_2$, the average high-quality consumption in the economy. All individuals have the same utility function $u(x, y_1, y_2, \hat{y}_2)$ that satisfies the following condition:

There exist $\underline{y}, \overline{y} \in I\!R_+$, with $0 < \underline{y} < \overline{y}$, such that

$u(x, y_1, y_2, \hat{y}_2) < u(x, y_1', y_2', \hat{y}_2')$ if $[y_1 + y_2 < y_1' + y_2' < \underline{y}]$ or if $[y_1' + y_2' \geq \overline{y}$ and $(y_2 - \hat{y}_2) < (y_2' - \hat{y}_2')]$.

Thus in the present version of the example a consumer is interested in 'quality', only because others consume high quality commodities. However, they do care for their neighbors consumption only when they have satiated their most basic needs. As long as they are hungry they go for quantity only, no matter how rich or poor they are. Note that in this example there are externalities with respect to quantities. It is obvious that the example can be rewritten with product innovation and externalities with respect to quality (similar to example B2).

Formally, laissez-faire development behaves exactly as in Example A1. Development completely neglects the improvement of the low quality technology. Profits from innovating in the high quality industry do not peter out because of the consumption externalities among the consumption of the rich. As before, these profits remain higher than the potential profits from potential innovations in the low-skill industry. The unskilled, therefore, remain hungry.

4.2.2 Example B2. Too Much Work for Too High Quality.

The economy mimics that of Example A2. Only preferences differ and we do not need to postulate exploding research productivity.

There is a single input x and a quality differentiated output y_n, with quality $q_n \in I\!R_+, n \in I\!N$. All consumers have identical endowments e and identical preferences. The utility of one consumer depends on the qualities consumed by other consumers. More precisely it depends on the difference $q - \bar{q}$ of the average quality $q = ([\sum_0^\infty q_n y_n]/[\sum_0^\infty y_n])$ consumed by the consumer himself and the average quality $\bar{q} = ([\sum_0^\infty q_n \bar{y}_n]/[\sum_0^\infty \bar{y}_n])$ consumed by other consumers, where $\bar{y}_n$ is the mean consumption of quality n in the economy. Given mean consumption in the economy utilities are $u(x, \tilde{y}, \bar{q}) = [(\sum_0^\infty y_n)(e - x)] + v(q - \bar{q})$ where v is a strictly increasing function.

Technologies and innovation-function are defined as in example A2, except that we do not assume that quality growth and productivity growth are exponential. Without relative needs exponential quality or productivity growth are necessary to sustain growth if research has high opportunity cost. Without exponential growth of productivity profits in terms of inputs tend to zero in the absence of relative needs. Allowing for relative needs makes it much easier to generate sustained growth. Exploding productivity or quality is no longer needed even if research has high opportunity cost.

As Example A2 we want to make sure that quality is chosen in each period. This is the case if $(\hat{p}_q^{(0,n)}/p^{(0,n)})$ is bounded away from one (and efficient scales of the innovations are appropriately specified), where $p^{(0,n)}$ is the temporary equilibrium price after the quality innovation has been chosen n times and the process-innovation has not yet been chosen, and where $\hat{p}_q^{(0,n)}$ is the monopolistic price if quality has been chosen n times and cost-reduction has not yet been chosen (This is sufficient to guarantee that the quality innovation is chosen, even if there are opportunity costs of research). Note that temporary equilibrium quantities and prices (always for the highest existing quality) as well as utilities are constant over time (if process-innovations are never chosen).

Given $\bar{q}^t$, which does not depend on the choices of a single consumer, we have

$$\frac{\hat{p}_q^{(0,n)}}{p^{(0,n)}} \geq \frac{u_{y_{n+1}}}{u_{y_n}} \,|_{(x_n, \tilde{y}_n)} = \frac{(e-x) + \frac{q_{n+1}-q_n}{y_n} v'}{e-x} = 1 + \frac{q_{n+1}-q_n}{y_n(e-x)} v' = 1 + \frac{q_{n+1}-q_n}{\bar{u}} v',$$

where $\bar{u}$ is the constant utility at temporary equilibrium.
(a) $v(q - \bar{q}) = q - \bar{q}$. Then, we can make sure that quality is always chosen even if there is no exponential growth of research productivity in quality. For instance, set $q_n = q_0 + n\xi$. Then, $\frac{\hat{p}_q^{(0,n)}}{p^{(0,n)}} \geq 1 + (\xi/\bar{u}) > 1$.
(b) $v(q - \bar{q}) = (q - \bar{q})^{1/3}$. Quality is always chosen even if the quality increments $q_{n+1} - q_n$ are very small or tend to zero with increasing n.

Obviously, it would be efficient to choose the process-innovation in each period, since the utility increment from product innovations is always nill. Of course this extreme inefficiency is due to the fact that there is no absolute need for quality at all in the example.

5 Concluding Remarks

We have analysed a simple model of endogenous change with two potential sources of sustained growth: Low opportunity cost of research and insatiable needs. In the absence of both growth cannot be sustained in the present framework. However, while laissez-faire development solves the long-run economic problem in the absence of insatiable needs if there are low opportunity cost of research (Proposition 3), it typically fails to do so if insatiable needs are the engine of growth (Proposition 5).

Most people would rather agree with Keynes about the existence of insatiable relative needs. We think that, empirically, relativ needs are much more important than their minor role in economic theory suggests. An extreme view even holds that observed sustained growth in the developed and 'satiated' part of the world is mainly due to unsatiable relative needs. However, from a theoretical point of view their are several potential engines of sustained growth, only one of which are insatiable relative needs and from an empirical point of view it may be difficult to distinguish them.

(1) The sustainability of growth in the endogenous growth literature (see Romer [1990], Aghion and Howitt [1992], Grossman and Helpman[1990]) depends on **exponential growth of the research productivity** (in the simplest case growth of the stock of knowledge equals the amount of research times the stock of knowledge).[7]

In examples A1 and A2 we have explicitly assumed exponential growth of research productivity: Given the same amount of research, the size of a potential innovation in a given direction increases by a constant factor (γ or ξ). Example A2 depends on the fact that ξ does not decrease over time. Without this assumption growth would either peter out (if research has opportunity cost) or eventually shift to process-innovations. In contrast, the assumption of exponential growth of research productivity was not needed to sustain growth in the examples with relative needs.

In section 3 we did not need to explicitly deal with exponential growth of research

[7] However, even with exponential growth of the research productivity, growth in this literature can only be sustained because workers do not reduce their supply of labor when their consumption increases, not even when real wages tend to infinity (see Durantant [1995]). In Funk [1996] we show how the usual assumption of exponential productivity growth of research can be strenghtend, so that growth is sustained even with high opportunity costs of research, with flexible labor supply, and in the absence of relative needs.

productivity as a separate source of sustained growth. Assuming that absolute needs are weakly satiable with respect to quality is a joint assumption on preferences and on the innovation-function. It allows for exploding productivity of research on quality, but requires that this does not offset decreasing marginal utility from additional quality. In examples A1 and A2 the violation of the assumption is due to exponetial growth of research productivity. In examples B1 and B2 the violation of the assumption is due to non-decreasing marginal utilities (there research productivity can remain constant or even tend to zero). In fact, since q is a non-observable parameter, not much can be learned from dividing the source of growth (and of biased growth) that we have labeled 'insatiable needs' into a utility-component and a technology component. This is also reflected in the fact that by only observing individual or aggregate demands and prices, the equilibrium developments in the examples with relative needs (A1 and A2) and without relative needs (B1 and B2) are not distinguishable.

(2) While it may prove difficult to distinguish empirically between insatiable absolute needs, insatiable realtive needs, and exploding productivity from research as engines of growth, 'low opportunity cost of research', as an engine of growth, implies rahter distinct predictions. A testable prediction for the low opportunity of research scenario would be that in the long-run research wages relative to (other) industrial wages should decrease (in the absence of the other sources of growth). This is implied by Lemma 1. Casual observation does at least not contradict the conclusions from Scenario 1. Research wages in terms of output have been growing in the past, but they have grown less fast than non-research wages In fact, it is not unreasonable to view research as a pleasant task which some like to perform even if they can get higher pay in a non-research sector. Is it not, for some of us, a luxury good that we are willing to pay for (in terms of forgone higher non-research wages), once our basic needs have been satisfied? Once growth has made potential researchers sufficiently rich, research may again become an *exogenous* engine of growth. Thus, if we were to take seriously 'low opportunity cost of research' as a sources of sustained growth, then part of the newly achieved competence of 'endogenously' explaining growth slips out of economists' controle again. However, first, low disutility from research at low levels of research and high levels of income, only guarantees that research cannot *completely* peter out. In our simplified framework, the intensity of growth either is zero or

is the size of one innovation. In a more general framework, even if we can conclude from the assumption that growth does not peter out, we cannot determine the intensity of growth indepently of the profitability of innovations. Second, even if there are low opportunity costs of research, so that growth is *sustained* almost exogenously, the direction of change is still determined endogenously (within the width of the innovation-possibilities) by the profitability of innovations.

6 Appendix

In this appendix we completely work out example A2. We first show that for γ sufficiently large given ξ, and for sufficiently small efficient scale production plans of I^0_α equilibrium development induces people to work more and more. Second, we will show that in optimal development hours worked remain constant at a moderate level. In the example $\alpha^t(q^t)$ remains bounded in equilibrium development though it is possible for q^t as well as for $\alpha^t(q^t)$ to grow without bound (i.e. $\alpha_{\mathcal{P}}(\infty) = \infty$). Laissez-faire development fails to solve the long-run economic problem.

Step 1. We first make sure that the quality-improving innovation is chosen in each period. Let $\hat{p}_q^{(m,n)}$ (resp. $\hat{p}_\alpha^{(m,n)}$) be the monopolistic price if quality (resp. quantity) has been chosen n (resp. m) times till t and if the innovator in t chooses quality (resp. quantity) (for convenience $m(t)$ and $n(t)$ have been replaced by m and n). Let $p^{(m,n)} = (1/\alpha^t(q^t))$ be the competitive price at t of the best public known quality q^t. Similarly, denote the innovations $I_q^{(m,n)}$ and $I_a^{(m,n))}$ and innovators' monopoly profits by $\pi_q^{(m,n)}$ and $\pi_\alpha^{(m,n)}$. The quality-improvement is chosen in *each* period if $(\pi_q^{(0,t)}/w^{(0,t)}) > \max\{R, (\pi_\alpha^{(0,t)}/w^{(0,t)}\})$ for all t. On the one hand it will be shown that $\pi_q^{(0,t)}$ is bounded away from zero, say by $\bar{\pi}$. Since, on the other hand, the real prices a process-innovator can realize are constant (i.e. $1/\alpha^0$) one can easily choose $I_a^{(0,t)} = I_a^{(0,0)}$ such that $(\pi_a^{(0,t)}/w^{(0,t)}) < \bar{\pi}$ (the profit-maximizing quantities given $\hat{p}_\alpha^{(0,t)}$ have to be sufficiently small). Similarly, since we have not fixed R yet, we can choose $R < \bar{\pi}$. In order to show that $(\pi_q^{(0,n)}/w^{(0,t)})$ is bounded away from zero, it is sufficient to show that $(\hat{p}_q^{(0,n)}/p^{(0,n)})$ is bounded away from one ($\max_x\{[\hat{p}_q^{(0,n)}f^{(0,n)}(x) - w^{(0,n)}x]/w^{(0,t)}\} \geq [\hat{p}_q^{(0,n)}\alpha^{(0,n)}\bar{x} - p^{(0,n)}\alpha^{(0,n)}\bar{x}]//w^{(0,t)} = [(\hat{p}_q^{(0,n)} - p^{(0,n)})/w^{(0,t)})]\alpha^{(0,n)}\bar{x}$, where $\bar{x}$ is the efficient scale input of Y^0). We give a lower bound on $(\hat{p}_q^{(0,n)}/p^{(0,n)})$ which is increasing in n and greater than one for $n = 0$. We know that

$$\frac{\hat{p}_q^{(0,n)}}{p^{(0,n)}} \geq \frac{u_{y_{n+1}}}{u_{y_n}} \mid_{(x_n,\tilde{y}_n)} = \frac{\frac{1}{2}(y_n)^{-(1/2)}(e-x)^{(1/2)} + \frac{1}{2}(q_n y_n)^{-(1/2)} q_{n+1}}{\frac{1}{2}(y_n)^{-(1/2)}(e-x)^{(1/2)} + \frac{1}{2}(q_n y_n)^{-(1/2)} q_n} =$$

$$\frac{(e-x)^{1/2} + \frac{q_{n+1}}{q_n^{1/2}}}{(e-x)^{1/2} + \frac{q_n}{q_n^{1/2}}},$$

where $(x_n, \tilde{y}_n)$ is the Walras-consumption at t, i.e. given prices $(1, 1/\alpha^0)$ for labor and output of quality n and with $y_{n'} = 0$ for all $n' \neq n$. Setting $a_n := (e - x_n)^{1/2}$ and $b_n := (q^0 \xi^n)^{1/2}$, the RHS reduces to $\frac{a_n + b_n \xi}{a_n + b_n}$. Clearly, this expression is larger than one for $n = 0$. The expression is increasing in n, since a_n is decreasing and b_n increasing in n. This proves Step 1.

Step 2. Second, it is made sure that for all t any development in which quantities are chosen $m+1$ times Pareto-dominates any development in which quantities are chosen m times only. The inefficiency in the example is stronger than a mere failure of solving the long-run economic problem (the innovation which should always be chosen is never chosen in equilibrium development).

As we have noted, when evaluating utilities at temporary equilibrium we can without loss of generality replace the utility function u by the function $\tilde{u} : \mathbb{R}_+^3 \to \mathbb{R}$, $(x, y, q) \to [y(e-x)]^{1/2} + [qy]^{1/2}$, where y is the non-negligible consumed quantity of the latest publicly known quality q. Given the technologically determined Walras prices $p^t = 1/\alpha(q^t)$ and using the budget constraint the utility function of consumers at t can be replaced by a function from $\mathbb{R}_+$ to $\mathbb{R}$, $x \to \alpha(q^t)^{1/2} x^{1/2} [(e-x)^{1/2} + q^{t(1/2)}]$. This function is strictly concave.

Denote by $u(m, n)$ the utility a consumer realizes at the Walras equilibrium of period $t = m + n$ if α (resp. q) has been chosen m (resp. n) times. Let x_n be the corresponding hours worked (they do not depend on m as can be seen from the utility function). We show that $u(m', n') > u(m, n)$ for all m, n, m', n', with $m + n = m' + n', m' > m$. Then

$$\frac{u(m', n')}{u(m, n)} = \frac{\gamma^{m'(1/2)} x_{n'}^{(1/2)} \{(e - x_{n'})^{1/2} + q_0^{1/2} \xi^{n'(1/2)}\}}{\gamma^{m(1/2)} x_n^{(1/2)} \{(e - x_n)^{1/2} + q_0^{1/2} \xi^{n(1/2)}\}}$$

Since x_n and $x_{n'}$ lie in the interval $[(e/2), e]$ it follows that

$$\frac{u(m', n')}{u(m, n)} > \frac{\gamma^{(m'-m)(1/2)} (e/2)^{(1/2)} \{q_0^{1/2} \xi^{n'(1/2)}\}}{e^{(1/2)} \{(e/2)^{1/2} + q_0^{1/2} \xi^{n(1/2)}\}} = \frac{\gamma^{(m'-m)(1/2)} q_0^{1/2} \xi^{n'(1/2)}}{e^{(1/2)} + (2q_0)^{1/2} \xi^{n(1/2)}}.$$

Thus $u(m',n') > u(m,n)$ if $\gamma^{\frac{m'-m}{2}} q_0^{1/2} \xi^{n'(1/2)} > e^{(1/2)} + (2q_0)^{1/2}\xi^{n(1/2)}$ or if $\gamma^{\frac{m'-m}{2}} > (e/q_0)^{(1/2)}\xi^{-n'/2} + 2^{1/2}\xi^{\frac{n-n'}{2}}$ or if $\gamma^{\frac{m'-m}{2}} > (e/q_0)^{(1/2)}\xi^{-n'/2} + 2^{1/2}\xi^{\frac{m'-m}{2}}$ since $n - n' = m' - m$. For $m' - m = 1, n' = 0$, this reduces to $\gamma^{1/2} > (e/q_0)^{(1/2)} + (2\xi)^{1/2}$, which is satisfied if γ is sufficiently large given (e/q^0) and ξ. Furthermore, for γ sufficiently large, the LHS grows faster in $m' - m$ than the RHS, and the RHS is decreasing in n'. Therefore the inequality holds for all m, n, m', n', with $m + n = m' + n', m' > m$. This proves Step 2.

Thus it is Pareto-efficient always to choose the process-improving innovation, whereas in equilibrium development quantities are never chosen. In particular, laissez-faire development fails to solve the economic problem. Note that the number of hours worked increases in equilibrium development, since it does not depend on m.

7 References

Aghion, P. and Howitt, P. [1992]: A Model of Growth through Creative Destruction. *Econometrica* 60 No. 2: 32-351.

Bagwell, L.S. and D.B. Bernheim [1994]: Veblen Effects in a Theory of Conspicuous Consumption. Mimeo, University of Stanford.

Corneo, G. and O. Jeanne [1995]: Conspicuous Consumption and the Existence of Upward Sloping Demand Curves. Mimeo, University of Bonn.

Duranton, G. [1995]: Endogenous Labor Supply, Growth and Overlapping Generations, Mimeo, London School of Economics.

Funk, P. [1995]: Bertrand and Walras Equilibria in Large Economies. *J. Econ. Theory* 67: 436-466.

Funk, P. [1996]: *The Direction of Technological Change*, unpublished Habilitationsschrift, University of Bonn.

Grossman, G. M. and Helpman E. [1991]: *Innovation and Growth in the Global Economy*, MIT Press.

Keynes, J.M. [1931]: *Essays in Persuasion*, Macmillan, London, page numbers refer to the edition of 1972.

Romer, P. M. [1990]: Endogenous Technological Change. *Journal of Political Economy* 98: 71-102.

Schumpeter J. A. [1911]: *Theorie der wirtschaftlichen Entwicklung*. München, Leipzig, Duncker & Humblot.

World Bank [1986]: *Poverty and Hunger: Issues and Options for Food Security in Developing Countries*, Washington, DC: World Bank.