# Choice of Maturity and Financial Intermediation ${ }^{1}$ 

Joachim Schuhmacher<br>University of Bonn<br>Department of Business Administration ${ }^{2}$

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${ }^{2}$ Joachim Schuhmacher, BWL I, Adenauerallee 24-42, 53113 Bonn, Germany, email: schuhmacher@uni-bonn.de.

## Choice of Maturity and Financial Intermediation


#### Abstract

Firms face the problem of choosing a debt maturity structure when financing an investment project. In addition, they have to decide which financing source to take. The aim of this article is firstly to give an explanation for the differing maturity choices by firms. As is shown below, the maturity choice depends mainly on the probability of realization of the cash-flow after each period. This means that firms prefer financing congruent to the realizations of the cash-flow. Secondly, this article explains the advantage of using a financial intermediary. It is shown that the financing source depends on the maturity choice. If a firm finances short-term it prefers bank loans whereas public debt is chosen by firms financing long-term.


Keywords Maturity Structure, Financial Intermediation

JEL classification D82, G21, G32, G33.

## 1. Introduction

The motivation for this article is the observation that firms prefer short-term financing with a bank whereas in contrast to this the financing source for long-term financing is the capital market. This statement is not new. Greenbaum and Thakor (1995) claim that bank loans are usually of short maturity. ${ }^{1}$ Therefore they have to be renewed quite often and through this process the bank learns more about the quality of a firm. Even though the bank has to spend some capital during these renewals, in the long run the bank has an advantage compared with the capital market because it knows in general the true quality of a firm. This means that the bank benefits from spending some money in early periods. James (1987) showed in his article that bank loans are significantly of much shorter maturity than public debt. The average maturity of bank loans is 5.6 years whereas the average maturity of public debt is nearly 18 years. Gertner and Scharfstein (1991) also claim that bank debt has shorter maturity than public debt.

This article has two aims. First, it motivates different maturity choices by firms. Thereafter it is shown which financing source fits best to which maturity. The framework in our model is the typical financial relationship. It is assumed that the owner of the firm just has the technology for the investment project but he does not have the financial resources for conducting it. Therefore the owner has to think about the financing source for his project. Here, he can choose between the capital market and a financial intermediary. ${ }^{2}$ Due to this financing situation there exists an agency-problem, which arises by financing investment projects with outside capital. This problem is caused by the fact that creditors and debtors have different interests. ${ }^{3}$ The owner of the firm always wants to continue the firm ${ }^{4}$ whereas the creditors are mainly interested in the repayment of their capital. To receive outside capital the firm has to promise the investor that its repayments lead to an expected utility level which is at least as high as his reservation utility level. If one assumes risk neutral individuals the expected repayments

[^0]are larger or equal to the level the invested capital has after investing it at the riskless interest rate. The level of the riskless interest rate depends on the length of the investment period. We assume an increasing interest rate structure, so that we have higher interest payments with longer maturity. This structure is rarely used while dealing with maturity choices. ${ }^{5}$ Normally the choice of maturity is seen as a signal which gives some information about a firm's quality. ${ }^{6}$ In contrast to this, the aim of our model is to give an explanation why firms choose different maturities due to potential inefficient investment, inefficient liquidation and varying capital costs dependent on the maturity choice. It is especially focused on the former two points.

The framework is a two-period model. The realization of the investment project's cash-flow is uncertain concerning the point of time of realization and its level. Firms realize their cash-flow either at date $t=1$ or at $t=2$. They differ in their respective probabilities of receiving the cash-flow from the investment project. If the cash-flow is not realized at date $t=1$, one can observe a signal about the second period cash-flow. After we have motivated the different maturity choices for the case of symmetric information we deal with the case of asymmetric information about the signal. It is shown how the firms can be made better off by the existence of the financial intermediary.

The resulting inefficiencies with long-term financing are independent of the financing source. There is inefficient investment if the cash-flow arises at $t=1$. This point is clarified below. Inefficiencies from the viewpoint of the owner of the firm arise with short-term financing and insolvency because in this case many creditors liquidate too often. The financial intermediary tries to resolve this problem since it observes the signal and makes a nearly efficient firm policy from the owner's point of view. This means that the decisions are not overall efficient but lead to better results than many creditors would achieve. Each firm faces the trade-off between inefficient liquidation and inefficient investment independent of the information structure. With short-term financing they could finance cheaper if the cash-flow arises after one period and invest the surplus in another project. In case the cash-flow does not arise, the firm risks to be liquidated or it has to refinance which will be shown below to be more expensive than just

[^1]long-term financing. If the firm finances long-term the owner consumes the private benefit for sure but has higher expected capital costs.

In both cases, symmetric and asymmetric information, we get a threshold probability $p^{*}$ or $p^{*}$ for the first period's cash-flow. The 'good' firms, i.e. firms who have a high probability of receiving their cash-flow at $t=1$, are going to finance short-term. The 'bad' firms prefer longterm financing. They are willing to carry higher expected capital costs, but their probability of being liquidated or refinancing is too high to risk short-term financing.

The advantage of financial intermediaries in the context of control costs is generally known, ${ }^{7}$ whereas the motivation in our model is the same as in the one by Chemmanur and Fulghieri (1994). The function of the financial intermediary is to prevent inefficient liquidations caused by financing with many creditors. The financial intermediary is more willing to observe a costly signal about the firm's quality under certain conditions. Therefore, given these conditions it chooses a better firm policy from the owner of the firm's point of view. The reason is that the financial intermediary itself carries the observation costs alone whereas with many creditors each of them has to carry these costs. Due to the multiplying of these costs observation with many creditors will never occur. One could also think about a situation in which just one investor as a representative of the other ones observes the signal. But in this case we get the classical free-rider problem because no creditor is willing to observe the signal. The social benefit from such a situation is much larger than the private benefit of the respective creditor. Since he does not take the social benefit into his account observation does not occur.

In the following model we just deal with debt contracts. This constrains the model of course and it could be true that firms are better off by choosing another financing contract. The restriction seems to be appropriate for two reasons. Debt contracts can cause inefficiencies, but other financing choices can also result in inefficiencies or agency cost, which could be larger than the ones by debt contracts. ${ }^{8}$ The other reason is that we focus on an explanation for maturity choices which is most natural in a debt context. For the same reason of concentrating

[^2]on special problems this restriction is quite often made in the financing literature. ${ }^{9}$

As said above there are articles that concentrate on other issues in the context of maturity structure choices. Diamond (1991b), (1993a) deals with the quality of a firm. He presents a model in which we have two different typs of firms. Good firms choose a maturity structure that enables them to benefit from new information at $t=1$ and to distinguish themselves from bad firms. Sharpe (1991) introduces different maturity choices putting weight on working incentives for managers. The maturity of a debt contract is chosen so that the manager has the best incentives to work hard. Our model is different to these articles. Neither do we have an adverse selection problem nor a hidden action problem. ${ }^{10}$ Houston and Venkataraman (1994) concentrate on the question how a firm can reduce its capital costs by choosing an optimal maturity mix, whereas due to the discrete structure of our model a maturity mix does not improve anything here. The model which comes closest to the one presented here is outlined by Diamond (1993b). In this article he claims that banks are the short-term lenders whereas the public can act as long-term lenders. This result is not derived explicitly in the two-period model but assumed. He makes the assumption that only banks can implement liquidation after the first period. Thus, to have liquidation we need banks to be the short-term lender. In contrast to this it is shown below that banks actually have an advantage as short-term lenders compared to bondholders, and that firms benefit from this fact.

Our model fits in a whole sequence of articles which look at decisions about financing sources of firms. Rajan (1992) shows in his article that firms with good future perspectives finance via the capital market, whereas firms with poor future expectations choose a financial intermediary as a financing source. Berlin and Mester (1992) describe a model in which firms with a high probability of insolvency prefer financial intermediaries. In contrast to this, firms with a low probability of insolvency finance via the capital market. Diamond (1991a) demonstrates that firms without reputation get better rates while financing via a financial intermediary, whereas firms with reputation prefer the capital market as their financing source. The last but one article to be mentioned in this context is the one by Chemmanur and Fulghieri (1994). In

[^3]their model the financial intermediary is acquiring reputation that in case of financial distress, the firm can rely on the financial intermediary to make a better decision than many creditors. Finally, one has to emphasize an article by Bolton and Freixas (1996). They reach as a result that especially starting-up firms prefer bank financing because bank lending prevents inefficient liquidation. Accordingly, mature firms are the ones that issue outside equity and bonds. Each result points in the same direction. It is shown that bad, young, or more generally speaking firms which are possibly going to have financial problems in the future prefer financing with just one large creditor to financing with many small ones. The following model gives an explanation why firms prefer financial intermediaries while financing short-term due to minimizing the probability of inefficient liquidation. With a long-term debt contract it is better to choose the capital market as the financing source.

The problem which is common to almost all articles is the fact, ${ }^{11}$ that they all talk about banks even though the models would not be changed by talking about one big creditor. Gertner and Scharfstein (1991) point into this direction. They claim that private debt is bank debt but could also be held by a large investor. ${ }^{12}$ The reason is that one never looks at the refinancing part of the financial intermediary. Therefore it is possible to equalize financial intermediaries and big creditors. If one wants to write an article about financial intermediation one should also deal with the refinancing problem of financial intermediaries. The following model does not focus on the refinancing problem either, thus the critique just mentioned fits here, as well. The reason for not dealing with the refinancing part is that there are only two closed-model approaches that can help to explain this fact. The first approach is the well known one by Diamond (1984). He uses a one-period time structure. Furthermore, there exists another model by Breuer (1995) which explains the refinancing part of financial intermediaries. He focuses on intertemporal aspects. Young firms with a low probability of continuation prefer financial intermediaries as a financing source. In contrast to this old firms, this means firms with a high probability of being on the market in the following period, get their capital from the capital market. Financial intermediaries are supposed to be on the capital market in the next period for sure. Thus, they have an incentive to implement the firm's first best policy, never default

[^4]and refinance through the capital market. A difference to Diamond is that Breuer does not have to use the Law of Large Numbers because it is sufficient but not necessary that each financial intermediary finances just one firm.

This article is organized as follows. In section 2 the basic model is presented. Thereafter we deal with financing with many creditors or a financial intermediary in the case of symmetric information. The optimal maturity choice is derived. In section 4 we introduce asymmetric information about the signal in the initial model. As before, we outline the two financing choices. After we have calculated the optimal maturity choice and the optimal financing source of the firm, the two optimal choices are compared to each other. Section 5 is an extension of the model. The article finishes with a summary of the main results in section 6 .

## 2. The Model

In the following section the underlying two-period model is presented. The time structure is as follows:

There are three points of time, $t=0,1,2$. At $t=0$ the firm is raising capital on the capital market and carries out its investment project. Only at this time the firm is able to get external funding of its investment project. We assume strip financing so that the capital can only be used for the project and the project's cash-flow cannot be used for anything before the outstanding debt is satisfied. We will come back to this problem later. Thereafter firms can only rely on internal funds in case they get the opportunity to conduct a further profitable investment project. To finance the project at date $t=0$ the firm needs an amount $I_{0}$. Each investor posseses one unit of capital. Thus the firm has to borrow capital from $I_{0}$ individuals. As said above, the cash-flow of the project is uncertain. This uncertainty is due to the fact that it is not known when the cash-flow arises and how high it is going to be. We assume that each firm that is trying to get capital owns an investment project that is worth to be undertaken. ${ }^{13}$ At point of time $t=0$ we have symmetric information. Both, the owner and the creditors know the probability densities $f\left(x_{1}\right)$ and $g\left(x_{2}\right)$ of the cash-flow, where $x_{i}$ is the cash-flow of period $i$. This is valid for each firm. As it is said above, the cash-flow can vary

[^5]with respect to the point of realization, but occurs only at one point of time. We assume that with probability $p$ the cash-flow is realized at $t=1$, and with the counter probability $(1-p)$ the cash-flow arrives at $t=2$. The information about the realization of the cash-flow is a public good, so that each individual knows the level of the cash-flow. ${ }^{14}$ If the cash-flow is zero at $t=1$, a signal $s$ about the future cash-flow can be observed. Furthermore, at the end of the second period a cash-flow is realized for sure. What is generally known at $t=0$ are the probability distributions of $x_{1}, x_{2}$ and $s$.

The agents in this model are risk-neutral and consume after each period. This means that the owner of the firm maximizes his expected utility, where the intertemporal utility function looks like $U(x)=G+y_{1} k_{1}+y_{2} k_{2}$. The variable $G$ represents the net gain (the cash-flow minus the capital cost) from the investment project(s) at date $t=2$. We discuss the variable $G$ in more detail in the following sections. The owner has a private benefit of $k_{1}>0$ if the firm continues until date $t=2 .{ }^{1516}$ Thus $y_{1}$ is a binary variable, which is zero if the firm is liquidated at $t=1$ and one otherwise. This benefit is independent of the cash-flow at $t=2$. Therefore the owner always wants to continue the firm until $t=2$. Furthermore, he earns an exogenous control rent $k_{2}>0$ at $t=2$ only in the case when there is no liquidation at $t=1$ and $t=2$. Again, $y_{2}$ is a binary variable, which is one if the firm can repay its entire face value of debt either after period one or two and zero otherwise. This means, the owner consumes $k_{1}$ if he gets $k_{2}$ but not vice versa. The investors expect an interest rate payment that is equal to the riskless interest rate.

[^6]Next, we define the level of the cash-flows and the liquidation values. It is assumed that if the cash-flow is not realized at $t=1$ one can observe a signal $s$. This signal is stochastic as viewed from $t=0$. No realization of cash-flow at $t=1$ is generally seen as a bad indication for the cash-flow of the following period. Thus, the possible realization of the cash-flow at $t=1$ is higher than the expected value of it at $t=2, E_{0}\left[x_{1} \mid x_{1}>0\right]>E_{0}\left[x_{2} \mid x_{1}=0\right]$. There is a one to one relation between the signal and the realized cash-flow in $t=2, \operatorname{prob}\left[x_{2}=s \mid s\right]=1$. Thus $x_{2}$ and $s$ have the same density function $g(\cdot)$ viewed from $t=0$. Even though the expected cash-flow is lower at $t=2$, there still can be a good signal about the cash-flow. It is important to mention that the cash-flow at $t=2$ does not necessarily have to be low. If the firm is insolvent it can be liquidated. The liquidation value at date $t=1$ from the viewpoint of $t=0$ is stochastic. What is generally known is the probability distribution of $L_{1}$. The level of $L_{1}$ is independent of the realization of the cash-flow which is quite a strong assumption. ${ }^{17}$ For simplicity we set the liquidation value at date $t=2$ equal to zero, $L_{2}=0$. We constrain the expected value of $L_{1}$, so that $\bar{L}_{1}$ is lower than the invested amount of capital, $I_{0}$, times one plus the riskless interest rate, $E_{0}\left[L_{1}\right]<I_{0}\left(1+\hat{r}_{01}\right)$. The variable $\hat{r}_{01}$ stands for the one period riskless interest rate, where the indices stand for the maturity of the debt contract. There is also full information about the future riskless interest rate $\hat{r}_{12} \cdot{ }^{18}$ Since the agents in our model are rational the following equation must hold, $\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)=\left(1+\hat{r}_{02}\right)$. Otherwise there would be the opportunity to realize gains from arbitrage. Another assumption concerning the interest rate structure is that there is no interest payment for the long-term debt contract before the expiration date. Thus, at date $t=2$ we have the entire repayment of this debt contract.

Before we comment on the capital costs for each maturity structure we want to hint that the expected capital costs are in general independent of the maturity. Differences in these costs are due to agency problems. The difference in expected financing costs in our model is caused by the existence of different firm policies, dependent on the maturity choice. The

[^7]capital cost structure one gets below shows that long-term refinancing is more expensive than just short-term financing. In particular, the capital cost structure looks as follows:
a) the capital costs are low if one just needs the capital for one period and finances short-term,
b) it is more expensive to choose a long-term debt contract,
c) the most expensive way is the case in which one has to refinance.

To be more precise, it can be outlined in the appendix that $\left(1+r_{01}\right)<\left(1+r_{02}\right)$ and $(1+$ $\left.r_{01}\right)\left(1+r_{12}\right)>\left(1+r_{02}\right)$. The $r_{i j}$ are risk-adjusted interest rates. The calculations of the riskadjusted interest rates demand too much information which we do not have at this point of time. Thus we postpone the calculations of the exact values of them for later.

For simplicity we use a two-point distribution for the realizations of the variables. Next, we specify the cash-flows, the liquidation values and finally the signal.

$$
x_{1}=\left\{\begin{array}{lll}
a & \text { with } & \text { prob. } p \\
0 & \text { with } & \text { prob. }(1-p)
\end{array}\right.
$$

We do not want to contrain the cash-flow with endogenous variables such as $I_{0}\left(1+r_{01}\right)$ whose level we derive below but one can think about the size of $a$ as being sufficient high to repay any outstanding debt.

$$
x_{2}=\left\{\begin{array}{lll}
0 & \text { with } & x_{1}=a \\
b & \text { with } & \text { prob. } q \text { and } x_{1}=0 \\
c & \text { with } & \text { prob. }(1-q) \text { and } x_{1}=0
\end{array}\right.
$$

As above, we just want to give a rough contraint for the different levels of the cash-flow. The cash-flow $c$ is as the cash-flow $a$ high enough to repay any debt. Only in case that the low cash-flow $b$ is realized the firm is not able to satisfy its debt and becomes insolvent.

$$
L_{1}=\left\{\begin{array}{lll}
d & \text { with } & \text { prob. } r \\
0 & \text { with } & \text { prob. }(1-r),
\end{array}\right.
$$

where $r d<I_{0}\left(1+\hat{r}_{01}\right)$. Again, we assume that $d$ is large enough to satisfy any debt. Nothing changes if we lower $d$, the only relation that has to hold is $d>q b+(1-q) c$.

$$
L_{2}=0
$$

Since we know that we only deal with firms which have a good investment project it is clear that

$$
\frac{p a}{\left(1+\hat{r}_{01}\right)}+\frac{(1-p)[q b+(1-q) c]}{\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)} \geq I_{0} .
$$

This equation gives us the net present value of the investment project.

To sum up, we have six different scenarios one has to look at:

$$
z=\left\{\begin{array}{rlll}
\left(x_{1}, x_{2}, s, L_{1}\right) & \text { with } & \text { prob. } p_{i} & \\
(\mathrm{a}, 0,0,0) & \text { with } & \text { prob. } p_{1} & (=p \cdot(1-r)) \\
(\mathrm{a}, 0,0, \mathrm{~d}) & \text { with } & \text { prob. } p_{2} & (=p \cdot r) \\
(0, \mathrm{~b}, \mathrm{~b}, 0) & \text { with } & \text { prob. } p_{3} & (=(1-p) \cdot q \cdot(1-r)) \\
(0, \mathrm{~b}, \mathrm{~b}, \mathrm{~d}) & \text { with } & \text { prob. } p_{4} & (=(1-p) \cdot q \cdot r) \\
(0, \mathrm{c}, \mathrm{c}, 0) & \text { with } & \text { prob. } p_{5} & (=(1-p) \cdot(1-q) \cdot(1-r)) \\
(0, \mathrm{c}, \mathrm{c}, \mathrm{~d}) & \text { with } & \text { prob. } p_{6} & (=(1-p) \cdot(1-q) \cdot r)
\end{array}\right.
$$

A further very important assumption arises from the difference between short-term and longterm contracts. The main difference between these two types is the flexibility of the contracts. A short-term contract can be renegotiated at each point of time. This can be advantageous for the owner if he benefits from new information. But the receiving of new information can also harm the owner. Thus, there is a trade-off between the consequences of new information
and its respective probabilities. ${ }^{19}$ In contrast to this, the long-term contract gives safetiness to both sides. This contract cannot be renegotiated in our model. In other models it can be in the interest of both parties to renegotiate the terms of a long-term contract. For example, in Sharpe (1991) the parties renegotiate because negotiation mitigates moral hazard effects. In our model the owner cannot influence the outcome of the project and as a consequence there is no reason for the creditors for renegotiation. Furthermore, there are different interests in renegotiation. The owner would like to renegotiate if $a$ is realized whereas the creditors would like to renegotiate if a bad signal arises.

The difference in these contract types means the following: If there exists a new project at date $t=1$ and the initial project is financed short-term with a realization of the cash-flow at date $t=1$, the owner of the firm has an option to invest this earnings in a new project. The expected rate of return of the new project is $\bar{r}_{g}$. This rate of return is high so that there is no doubt that it exceeds the risk-free interest rate, $\bar{r}_{g} \geq \hat{r}_{12}$. We get a different situation if the initial project is financed long-term and the cash-flow arises at $t=1$. As was assumed earlier the owner is constrained to rely on internal funds as the only financing source. He cannot decide on his own how to use the project's cash-flow. But due to the fact that there is no renegotiation it is not possible to undertake the new project. With long-term financing it is only allowed to invest the cash-flow at the riskless interest rate until the firm has to repay its debt. In addition to this, since the firm cannot get outside capital as well, by choosing long-term financing it foregoes the opportunity to invest in a further good project. Thus, the owner cannot have both, on ongoing firm until date $t=2$ for sure and the possibility of conducting a new project. ${ }^{20}$

Finally, one could motivate the assumption of excluding external funding from the viewpoint of the owner. If one assumes that $k_{2}$ can be arbitrarily large, the owner would never risk of losing this control rent. Since $r_{g}$ is uncertain the firm might be liquidated if the following investment project runs badly. As a consequence, the owner rather does not conduct the new

[^8]project than invest in the project with external funds and possibly lose the control rent.
This difference between the two financing maturities has to exist. Otherwise there would not be an incentive to finance short-term if the private benefit $k_{1}$ of the owner is sufficient high. What is meant by this is the fact that by financing long-term the firm can consume $k_{1}$ for sure. In contrast to this the owner must have an advantage in financing short-term besides the possibility of having lower expected capital costs. This means, the owner has to be rewarded for financing short-term and realizing the cash-flow after one period. In this way short-term financing becomes attractive so that the owner of the firm is willing to risk losing his private benefit $k_{1}$ and the control rent $k_{2}$.

In the third section we assume full information at each point of time. This assumption will be dropped in the fourth section. Thus, in section 4 we assume that the realization of the signal is only private information of the owner. The financing party is able to observe the signal by incurring some cost $l$. Only in this context it makes sense to introduce the financial intermediary. Otherwise the debtor cannot benefit from its existence since many investors are going to make efficient decisions about the firm's policy as well, if they have no information costs. Later on we get the commonly known result that the interest rates for borrowing from financial intermediaries are more expensive than borrowing from capital markets. In case of asymmetric information financial intermediaries observe the signal if the cash-flow is zero and endogenize these costs in the interest rate they charge. We go into more detail later.

The following decision tree (Figure 1) clarifies the time structure of the model:

## 3. Symmetric Information about the signal

What is shown in this section is the first aim of the whole article, namely to motivate different maturity choices by firms. It should become clear why firms choose a maturity structure that depends mainly on their probability $p$ of receiving the cash-flow after the first period. Therefore we assume that both parties, the owner and the $I$ creditors possess the same information at each point of time. This is enough to get our results. We start by discussing in detail the different outcomes of the financing decision given a certain maturity structure.

### 3.1 Symmetric Information between owner and many creditors

### 3.1.1 Long-term financing

As a starting point it is taken as given that the owner chooses a long-term contract. With long-term financing there is no decision to be made by the creditors at $t=1$. The owner of the firm decides about the company's policy. There are two scenarios that can occur: first, the cash-flow is realized at $t=1$ and secondly, the cash-flow arises at the end of the second period. We start with the former.

The cash-flow appears at $t=1$. The only thing the owner is allowed to do with the cash-flow is to invest it at the riskless interest rate $\hat{r}_{12}$. At the end of the second period he repays the entire amount of debt. The creditors receive $I_{0}\left(1+r_{02}\right)$ whereas the owner gets the rest, $\left[a\left(1+\hat{r}_{12}\right)-I_{0}\left(1+r_{02}\right)\right]$. Since the first period cash-flow is high enough, the owner is able to consume the private benefit $k_{1}$ and his control rent $k_{2}$. If the cash-flow did not arise at $t=1$ both parties observe a signal about the future cash-flow. We assumed earlier that there is no renegotiation about the firm's future policy. Thus, independent of the realization of the signal the firm is ongoing until $t=2$ and the predicted cash-flow is realized for sure. The payments to the creditors depend on the cash-flow. In any case the owner consumes the private benefit $k_{1}$. If $b$ is realized the firm is liquidated and the owner loses the control rent $k_{2}$, otherwise the creditors receive full repayment and the owner consumes the surplus and the control rent.

The risk-adjusted interest rate $r_{02}$ can be calculated in the following way,

$$
p \cdot I_{0}\left(1+r_{02}\right)+(1-p) \cdot q \cdot b+(1-p) \cdot(1-q) \cdot I_{0}\left(1+r_{02}\right) \geq I_{0}\left(1+\hat{r}_{02}\right) .
$$

### 3.1.2 Short-term financing

Here, we assume that the owner of the firm prefers short-term financing. In this case the control rights of the firm depend on the realization of the cash-flow at $t=1$.

The easiest case occurs if the cash-flow is realized at $t=1$. In this case we already know that $x_{1}=a$. The owner repays the debt and invests the rest in a future project where the expected return equals $\bar{r}_{g}$. Thus, the payments to the creditors are $I\left(1+r_{01}\right)$. The level of financial means the owner possesses at $t=2$ is $\left[a-I\left(1+r_{01}\right)\right]\left(1+r_{g}\right)$. Since the debt is entirely repaid he is also able to consume the private benefit $k_{1}$ plus his control rent $k_{2}$. Remember that the second project is financed with internal funds. This means that even in bad states of this project the firm is not liquidated so that the owner consumes the control rent $k_{2}$ for sure.

If the cash-flow is not realized at $t=1$ the creditors gain the control rights of the firm. Now they are in the power to decide whether to liquidate or to continue the firm. Since we assume in this section full symmetric information at each point in time, the creditors know the realization of the signal. We give the creditors the full bargaining power. ${ }^{21}$ They make a take-it-or-leave-it offer to the owner which he can accept or reject. We model this offer in the following way. The creditors demand the entire future cash-flow. If the owner accepts this offer ('take-it') the firm is ongoing until $t=2$. In this case the owner is able to consume the private benefit $k_{1}$ and furthermore, if the signal is good he is able to consume his control rent $k_{2}$. The owner also has the choice to reject the offer ('leave-it') and if this happens the firm is liquidated at date $t=1$. The owner loses both, the private benefit $k_{1}$ and the control rent $k_{2}$. One can claim that liquidating the firm cannot be an equilibrium, especially if $L_{1}=0$ and the signal is good, so that the owner should always reject. The question is how the creditors can

[^9]credibly commit to liqudating the firm. This is not too difficult in this setting, whereas this problem becomes more severe in case of financing with the financial intermediary. It is very unlikely that many creditors bargain with the owner of the firm about the future cash-flow, for example, there are too high transaction costs involved. It is more realistic that the initial contract between the two parties contains the covenant that the firm is continued only if many creditors receive the entire cash-flow. As such, in times of financial distress it seems very credible that the firm is being liquidated if the owner rejects the offer. Thus, due to the credible commitment the owner accepts the offer.

There are four different situations one has to look at:
1)

$$
L_{1}=d \quad \text { and the signal is good, so that } s=c .
$$

This situation occurs with probability $p_{6}$. As just said, the creditors make a take-it-or-leave-it offer to the owner. They demand the full second period cash-flow and the owner accepts. As a consequence, the creditors receive $c$ at the end of the second period. Since the second period cash-flow is high enough the firm is not liquidated. Thus, the owner is able to consume both, the private benefit $k_{1}$ and his control rent $k_{2}$. The level of $r_{12}$ reaches it maximum because the owner loses the entire second-period cash-flow. Thus, $r_{12}$ can be calculated the following way,

$$
c=I\left(1+r_{01}\right)\left(1+r_{12}\right) .
$$

2) 

$$
L_{1}=d \quad \text { and the signal is bad, so that } s=b .
$$

With probability $p_{4}$ does this case happen. Since the repayment at $t=2$ can only be less than liquidating the firm right away the creditors are going to liquidate the firm in this situation. The payments to the creditors at $t=1$ are $I_{0}\left(1+r_{01}\right)$, whereas the owner receives only the remaining part, $\left[d-I_{0}\left(1+r_{01}\right)\right] .^{22}$ Thus, at $t=2$ he owns $\left[d-I_{0}\left(1+r_{01}\right)\right]\left(1+\hat{r}_{12}\right)$ but lost the private benefit $k_{1}$ and the control rent $k_{2}$ due to the early liquidation.
3)

$$
L_{1}=0 \quad \text { and the signal is good, so that } s=c .
$$

The probability of this situation equals $p_{5}$. In this situation the creditors are not going to receive any payments at $t=1$. But this is no problem since they know that the second period

[^10]cash-flow is going to be high. As in situation 1 the creditors demand the full second period cash-flow $c$. The owner accepts the offer so that he is able to receive his private benefit $k_{1}$ and consumes his control rent $k_{2}$. The level of the risk adjusted interest rate $r_{12}$ is again at its maximum value.
4)
$$
L_{1}=0 \quad \text { and the signal is bad, so that } s=b .
$$

Finally, the probability of this last case is $p_{3}$. As in the preceding situation the liquidation value is zero so that there cannot be any repayment of the debt at $t=1$. Even though the signal is bad the creditors would never liquidate the firm. They continue the firm to recover some of their losses. The only party receiving at least some cash-flow are the creditors, and this amount equals $b$ at $t=2$. This situation is different to the two preceding ones with continuation. In those situation the creditors received more than enough to be compensated for the continuation. Therefore, the firm was not liquidated and the owner was able to consume the control rent $k_{2}$. Furthermore, one can also look at the calculation of $r_{12}$ to realize that the firm is not liquidated in the two situations. In contrast to the situation with a low cash-flow the owner is able to satisfy the outstanding debt with the high second period cash-flow. This is different in this situation. Since the cash-flow is so low the owner is never able to repay the entire borrowed amount of capital. The creditors lose partly their capital. As such, the only punishment the creditors have is to liquidate the firm. Otherwise the owner would be as well off as in the other situations and the creditors would be the only ones that suffer from the low second-period cash-flow. If there is a difference for the creditors there also should be a difference for the owner. Furthermore, we do not want to let the owner benefit from this low cash-flow due to a low liquidation value while financing short-term. By financing long-term he would also be liquidated in this situation. With long-term financing and $x_{2}=b$ the firm would have lost the entire cash-flow and the control rent $k_{2}$, too. Thus, this outcome is not very critical. To conclude, the firm is liquidated at date $t=2$. The owner receives the private benefit $k_{1}$ but loses his control rent for sure. The level of $r_{12}$ is not interesting because even if $r_{12}=\hat{r}_{12}$ there is no full repayment.

We can summarize that in three situations the firm is going to be continued and in the other one the creditors liquidate the firm. The owner loses the entire second period cash-flow for sure but depending on the situation he can at least consume his private benefit $k_{1}$ and the control rent $k_{2}$. One can derive the risk-adjusted interest rate $r_{01}$ as follows,

$$
\begin{gathered}
p \cdot I_{0}\left(1+r_{01}\right)+(1-p) \cdot q \cdot r \cdot I \cdot\left(1+r_{01}\right)+(1-p) \cdot(1-q) \cdot c+ \\
+(1-p) \cdot q \cdot(1-r) \cdot \frac{b}{1+\hat{r}_{12}} \geq I \cdot\left(1+\hat{r}_{01}\right)
\end{gathered}
$$

### 3.2 Symmetric information between owner and financial intermediary

The case where owner and financial intermediary (in short (FI)) have equal information is the same as if the owner deals with many creditors. Right now there is no difference between the interest rates charged by the FI and by many creditors. In the following section we will assume that the FI observes in any case if the cash-flow is not realized. This is independent of the maturity choice and occurs with probability $(1-p)$, so that the FI has to charge a higher interest rate. If this occurs with asymmetric information there is a difference in the financing source. As said above, in this section the signal is public, therefore it does not really matter which financing source is chosen by the owner. The only thing that matters is maturity.

Covenants in long-term contracts could make a difference between a FI and the capital market. If the FI wants to be involved in the firm's policy in case that the signal is bad the owner would never choose the FI as a financing source, for example, the FI wants to liquidate the firm with a bad signal and a high liquidation value. In such a situation he risks being liquidated at $t=1 .{ }^{23} \mathrm{He}$ will never have this risk by financing via the capital market. Nevertheless, this scenario contradicts our meaning of a long-term contract. Thus, the only reason for mentioning this idea is to show where differences between financing sources could occur. In our model there are no differences between the financing sources.

[^11]Finally, one has to comment on the credibility of the take-it-or-leave-it-offer by a FI. In the case with many creditors this was not very difficult. It was credible that it would be too costly to bargain with many creditors so that liquidating to firm after rejecting the offer was the only outcome. By financing with a FI this situation is different. One only has to bargain with just one creditor. As a consequence one cannot use the same argument as before. There are two ways a FI can credibly precommit to liquidate the firm if the owner rejects the offer. First, one has to recognize that the FI does not just finance one firm but it finances many firms. So, what happens if it does not liquidate the firm even though this was the 'leave-it' part of the offer. The FI would lose its reputation for being tough. As a consequence other firms observe this outcome and in the future they would also reject this offer. We do not want to go to much in detail but sketch the reason why a FI has to liquidate even though this seems to be no subgame perfect equilibrium. The setting is commonly known as the chain-store paradox. If the FI does not fight for the last firm there is no reason to liquidate the second but last firm and so on. At the end the FI does not liquidate any firm. The situation changes if the type of the FI is not known, this means it is uncertain if the FI is a tough one that liquidates always or if it is weak. In this environment it can be a subgame perfect equlibrium that the FI liquidates. Thus, liquidation can be credible if the firm rejects the offer.

Secondly, the FI can employ a manager who is known to be very strict. By tying its hands the FI commits to a liquidation even though this harms both parties, the firm and the FI itself. But this commitment is credible if the manager possesses this reputation that is needed in this context. This is the same reasoning used in monetary politics. By choosing a conservative central banker the government tries to commit credibly to a low inflation policy. ${ }^{24}$

### 3.3 The decision problem of the owner

### 3.3.1 The general maximization problem

As outlined above the owner of the firm maximizes his expected utility. He can choose the maturity of the debt contract and as a constraint of his maximization problem the creditors get an expected interest yield equal to the riskless interest rate of the respective maturity. We can now look at the maximization problem which is as follows:

[^12]$$
\max \left\{U^{1}, U^{2}\right\}
$$
where $U^{1}$ is the expected utility from choosing short-term financing and $U^{2}$ is the respective utility with long-term financing. In particular,
\[

$$
\begin{gathered}
U^{1}=\max _{r_{01}} E\left[G+y_{1} k_{1}+y_{2} k_{2}\right] \\
\text { s.t. } p \cdot I_{0} \cdot\left(1+r_{01}\right)+(1-p) \cdot q \cdot r \cdot I_{0} \cdot\left(1+r_{01}\right)+(1-p) \cdot(1-q) \cdot c+ \\
+(1-p) \cdot q \cdot(1-r) \cdot \frac{b}{1+\hat{r}_{12}} \geq I_{0} \cdot\left(1+\hat{r}_{01}\right)
\end{gathered}
$$
\]

and

$$
\begin{gathered}
U^{2}=\max _{r_{02}} E\left[G+y_{1} k_{1}+y_{2} k_{2}\right] \\
\text { s.t. } p \cdot I_{0}\left(1+r_{02}\right)+(1-p) \cdot q \cdot b+(1-p) \cdot(1-q) \cdot I_{0}\left(1+r_{02}\right) \geq I_{0}\left(1+\hat{r}_{02}\right) .
\end{gathered}
$$

The utility function is $U(x)=G+y_{1} k_{1}+y_{2} k_{2}$, where $G$ are the payoffs from the investment project(s) with limited liability which was outlined in the previous subsection, $k_{1}$ is the private benefit the owner receives if his firm continues until $t=2$ and $k_{2}$ is the control rent at $t=2$ without liquidation at any point of time. The outcomes of the binary variables with their respective probabilities are specified in the following subsection.

The two inequalities are the individual rationality constraints for the investors. Investors demand an expected repayment of at least the riskless interest rate. In equilibrium these conditions are fulfilled with equality because either one can assume that the owner has full bargaining power at $t=0$ or that the capital market is perfectly competitive. The exact interest rate levels and their comparisons with each other are conducted in the appendix.

The first inequality stands for the short-term risk-adjusted interest rate $r_{01}$. The left part specifies the expected payments to the investors and the right term is their reservation utility. The first part of the inequality represents the case in which the cash-flow occurs at $t=1$. This occurs with probability $p$. The rest of the left hand side stands for the situation with no cash-flow at $t=1$. The second term represents the situation in which the signal is bad but the liquidation value is high enough so that the investors receive full repayment. The two other
terms stand for the creditors receiving the full second period cash-flow. They discount with the riskless interest rate $\hat{r}_{12}$ because the cash-flow is known for sure after the first period. The riskless interest rate is known at date $t=0$. Furthermore, the company has limited liability, so that it cannot pay more than it earns through the investment project.

The second inequality is the calculation of the risk-adjusted long-term interest rate. The structure is the same as in the case for the short-term risk-adjusted interest rate. The left hand side contains the expected payments to the creditors and the right hand side is the reservation utility level. The first term stands for the realization of the cash-flow at date $t=1$. If the cash-flow is realized early, the owner has to invest the cash-flow at the riskless interest rate. He is not allowed to spend the cash-flow on something else, e.g. another investment project. In the second term the cash-flow is too low to repay the debt in full. Therefore the investors get the entire cash-flow. The last term expresses that the owner can satisfy the entire face value of his debt with a high second period cash-flow.

### 3.3.2 The optimal choice of maturity

To decide which of the two maturity contracts is the optimal one the owner has to compare the expected utility of each contract. We use the date $t=2$ as the reference time point.

If the owner chooses a long-term financing contract, his expected utility is determined as follows:

$$
\begin{gathered}
E\left[U^{l}\right]=\left(p_{1}+p_{2}\right) \cdot\left\{\left(a\left(1+\hat{r}_{12}\right)-I_{0}\left(1+r_{02}\right)\right)+k_{1}+k_{2}\right\} \\
+\left(p_{3}+p_{4}\right) \cdot k_{1}+\left(p_{5}+p_{6}\right) \cdot\left\{\left(c-I_{0}\left(1+r_{02}\right)\right)+k_{1}+k_{2}\right\} .
\end{gathered}
$$

The expected utility with short-term financing looks like:

$$
\begin{gathered}
E\left[U^{s}\right]=\left(p_{1}+p_{2}\right) \cdot\left\{\left(a-I_{0}\left(1+r_{01}\right)\right)\left(1+\bar{r}_{g}\right)+k_{1}+k_{2}\right\} \\
p_{3} \cdot k_{1}+p_{4} \cdot\left(d-I_{0}\left(1+r_{01}\right)\right)\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right) \cdot\left(k_{1}+k_{2}\right) .
\end{gathered}
$$

To get the optimal maturity choice as a result one has to compare the two expected utilities. As mentioned above the individual rationality constraints are fulfilled with equality. By equating the two expected utilities one receives a threshold $p^{*} .{ }^{25}$ The threshold is as follows: ${ }^{26}$

$$
p^{*}=\frac{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]}{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]+\left[a-I\left(1+r_{01}\right)\right]\left[\bar{r}_{g}-\hat{r}_{12}\right]}
$$

As usual this probability is only defined if the whole fraction is between zero and one. Let us start with the numerator. We already know that $d\left(1+\hat{r}_{12}\right)$ is larger than $b$, but if one adds $k_{1}$ the term in brackets is positive. Since $q r$ is a probability the whole numerator is positive.

As a consequence the denominator has to be positive, too. Since the first part is the same as in the numerator one only has to look at the last term. We already know that $a>I\left(1+r_{01}\right)$. Furthermore, we assumed that the new project is very lucrative so that $\bar{r}_{g}$ is larger than $\hat{r}_{12}$. To conclude, the last term is positive and because we add this term to the previous one, the denominator becomes larger than the numerator.

## Result

As a result we get that the numerator and the denominator are both positive, whereas the denominator is larger than the numerator. If a company has a probabilitiy of receiving the cash-flow at $t=1$ that is larger than $p^{*}$, it is going to choose a short-term contract. Whereas, if the probability $p$ is too low, the firm prefers long-term financing.

By deciding between these two maturities the owner must weigh up several points. If he finances long-term the firm will never be liquidated at $t=1$. This means that the owner is able to consume $k_{1}$ for sure. He pays for this safetiness with a higher expected interest rate and what is much more important is the fact that he forgoes the opportunity to invest in a

[^13]new project if the cash-flow is realized at date $t=1$. While financing short-term the owner takes two risks. First, he might be liquidated if the cash-flow is not realized at $t=1$ and secondly, his capital costs rise in case of continuation. On the other side he has possibly the opportunity to invest in a following project. Therefore the probability $p$ is the driving force in his decision.

Before we start with the asymmetric information case one has to mention one surprising result. We made the extreme assumption that the creditors have the full bargaining power and make the take-it-or-leave-it offer in case that the firm becomes insolvent. If we would model the bargaining power in a different way so that the owner pays only $\hat{r}_{12}$ as second period interest payments ${ }^{27}$ the maximum the creditors receive at $t=2$ equals max $\left[b, I\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)\right]$. If the high cash-flow is realized the owner gets $c-I\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)$. In this setting the threshold $p^{*}$ does not change. At first glance this result seems to be amazing but it is very intuitive. In the initial situation the owner received no second period cash-flow in case of insolvency. As a consequence the first period risk-adjusted interest rate was as low as possible. If one raises the bargaining power of the owner the creditors just get the minimum for the second period. ${ }^{28}$ As a consequence to this the first period risk-adjusted interest rate has to rise. One can recognize a trade-off between the two different bargaining schedules. In the initial one, the first period risk-adjusted interest rate is low and refinancing is very expensive because the owner loses the entire second-period cash-flow. Otherwise, the risk-adjusted interest rate $r_{01}$ is high and refinancing becomes very cheap. In the end there is no difference for the threshold $p^{*}$. This means that the decision about the maturity does not depend on the bargaining power of each party. This result will be outlined in the appendix.

Finally, there is one last fact which is worth to be commented on. The equation of $p^{*}$ does not contain the parameter $k_{2}$. This is actually no surprise because the owner of the firm cannot influence the probability of consuming $k_{2}$. If the owner eventually consumes this control rent $k_{2}$ does not depend on the maturity choice but on the state of the world. If $k_{2}$ were not part

[^14]of the model the threshold $p^{*}$ does not depend on the bargaining power at all, so that every scenario is possible, e.g. zero payments to the creditors at $t=2$. This means, the maturity choice does not rely on the distribution of the cash-flow at date $t=2$.

## 4. Asymmetric information about the signal

In this section we want to derive the second result of the paper. It should become clear why some firms prefer debt financing with a FI whereas others choose the capital market as the financing source. Furthermore, it is shown that a FI has an advantage compared to the capital market when financing short-term but with long-term financing it is the other way around. Throughout the whole chapter we retain that the creditors have the full bargaining power and that they make the take-it-or-leave-it offer to the owner.

### 4.1 Asymmetric information between owner and many creditors

The case of asymmetric information is only relevant if the cash-flow is zero at $t=1$. Otherwise everything is known to both parties, since we assumed that the realization of the cash-flow is public knowledge. In this section we assume that if $x_{1}=0$ only the owner of the firm is able to observe the signal $s$. Each creditor is also able to observe the signal but he has to spent some capital on this observation. These observation costs are $l$. In this context it is assumed that each creditor has to spent the costs $l$ to learn the signal. ${ }^{29}$

There are several reasons why this assumption may be satisfied. If one investor observes the signal and wants to make it public, he causes more costs due to the publication. He has to carry the entire costs himself since no other creditor is willing to give him some capital after receiving the signal. Perhaps it would be possible to write a contract contingent on the information receipt but to sign contracts with each creditor leads to high transaction costs and is thus not feasible. Therefore we have a free-rider problem, no creditor is willing to carry all the costs. ${ }^{30}$ Another justification could be that no creditor possesses the technology to spread the information. To sum up, each creditor has to spent the costs $l$ for observing the signal. Finally, it is clear that an observation by each creditor cannot be efficient because the

[^15]observation costs do not give the society any benefit. Thus these costs are a loss and should be minimized from a social point of view. Observation makes only sense if the liquidation value is high. Otherwise the creditors do not liquidate the firm in any case. We can give a sufficient condition so that observation does not occur, $I_{0}\left(1+r_{01}\right)>q I_{0}\left(1+r_{01}\right)+\frac{(1-q) c}{1+\hat{r}_{12}}-I_{0} l$. The left hand side is the payment to the creditors for sure and the right hand side reflects the expected payments minus the observation costs.

### 4.1.1 Long-term financing

We start again with the case of long-term financing. In this context the different information structure of the two parties does not matter. The creditors do not have any power to decide about the firm's policy. Therefore we get the same results as in the case of symmetric information and long-term financing. Due to this fact we do not have to present the calculation of the risk-adjusted interest rate $r_{02}$. The owner of the firm continues the firm in any case. This decision is even efficient from the point of view of minimizing expected observation costs and continuing the firm until $t=2$ because $b+k_{1}>d\left(1+\hat{r}_{12}\right) .{ }^{31}$

### 4.1.2 Short-term financing

As before, the easiest case is the one in which the cash-flow appears at $t=1$. Since the information structure does not matter we receive equal results to the corresponding case with symmetric information.

The asymmetric information structure is only of interest if the cash-flow is zero at $t=1$ and the owner chose short-term financing. Therefore, we describe now the situation that the cash-flow is not realized at $t=1$. By assumption the capital market is closed for external funding. As a consequence the owner cannot get ouside capital and the creditors receive the control rights of the firm so that they have the power to decide about the firm's future policy. Since these creditors are not going to observe the signal, their decisions are based on date $t=0$ information. It should be clear that this can lead to inefficiencies. The creditors take as given that no cash-flow at $t=1$ is a bad sign for the future cash-flow. ${ }^{32}$ Therefore they have

[^16]a tendency to liquidate too often or too early from the owner's and the social welfare's point of view. The calculation of the risk-adjusted interest rates $r_{01}$ and $r_{12}$ is given at the end of this subsection.

As before we have to divide this scenario into (two) different situations.
1)

$$
L_{1}=0 \quad \text { and } \quad \frac{\overline{x_{2}}}{\left(1+\hat{r}_{12}\right)}<I\left(1+r_{01}\right) .
$$

In this case the expected value of future cash-flow is too small so that the creditors do not expect that they can be entirely repaid. Nevertheless they have no other choice than continuing the firm. By liquidating they would not get anything, thus they can improve their situation by continuing. We assumed that only the realization of the signal is private information. The realization of the cash-flow is known to everyone. Thus, the creditors demand the entire second period cash-flow. If the level of the second period cash-flow is $b$ the creditors liquidate the firm. The reasoning is the same as before. In this case the owner loses his control rent $k_{2}$ but he is still able to consume the private benefit $k_{1}$. In the second possible situation the creditors get full repayment and receive $c$. Due to the high cash-flow the firm is not liquidated and the owner is able to consume his private benefit $k_{1}$ and the control rent $k_{2}$. In general, since the low liquidation value is realized, $L_{1}=0$, the owner is able to get his private benefit $k_{1}$ for sure. The probabilities are $p_{3}$ and $p_{5}$ respectively.
2)

$$
L_{1}=d \quad \text { and } \quad \frac{\bar{x}_{2}}{\left(1+\hat{r}_{12}\right)}<I\left(1+r_{01}\right) .
$$

Of course, the expected value of the future cash-flow does not change so that the creditors do not expect to be entirely repaid by continuing the firm. Thus, they liquidate the firm since the liquidation value is large enough to satisfy the entire face value of debt. The payments to the creditors at date $t=1$ are $I\left(1+r_{01}\right)$, whereas the owner of the firm receives $\left[d-I\left(1+r_{01}\right)\right]$ whose value is $\left[d-I\left(1+r_{01}\right)\right]\left(1+\hat{r}_{12}\right)$ at $t=2$. Due to the liquidation at $t=1$ he lost the private benefit $k_{1}$ and his control rent $k_{2}$. This situation occurs with a probability of $p_{4}+p_{6}$.

Since we went through all possible situations we can now derive $r_{01}$, which can be calculated from the following individual rationality constraint of the creditors,

$$
\begin{gathered}
p \cdot I\left(1+r_{01}\right)+(1-p) \cdot r \cdot I\left(1+r_{01}\right)+(1-p) \cdot(1-r) \cdot q \cdot \frac{b}{1+\hat{r}_{12}}+ \\
+(1-p) \cdot(1-r) \cdot(1-q) \cdot \frac{c}{1+\hat{r}_{12}} \geq I\left(1+\hat{r}_{01}\right)
\end{gathered}
$$

the calculation of $\left(1+r_{12}\right)$ is conducted as before,

$$
c=I\left(1+r_{01}\right)\left(1+r_{12}\right),
$$

so that the owner defaults if $x_{2}=b$ and is liquidated, whereas if $x_{2}=c$ he is able to fully repay the outstanding debt.

To sum up, in case the cash-flow is not realized at date $t=1$, the creditors will liquidate the firm if the high liquidation value is realized. This liquidation is too early if $s=c .^{33}$ If $L_{1}=0$ the firm is not liquidated because then the creditors can at least cover some of their losses by continuing the firm. With cash-flow at $t=1$ we get the same results as in the previous section with symmetric information. The owner is able to satisfy the demands of creditors and consumes the private benefit $k_{1}$ and his control rent $k_{2}$. Furthermore, he is able to invest in the new projet.

### 4.2 Asymmetric information between the owner and the financial intermediary

### 4.2.1 Long-term financing

As mentioned earlier the FI always wants to observe the signal independent of the maturity of the debt contract. As a consequence to this, the FI is going to observe the signal with probability $(1-p)$. Without this assumption our model is still valid but one does not reach is a strict distinction between many creditors and a FI in case of long-term financing. There

[^17]are three points to mention in the context that a FI charges a higher interest rate and firms are willing to pay this premium.

In general, FI write contracts that entail many covenants. A FI wants to know in detail what its capital is used for and how the firm is doing. This is a big difference to financing with the case of many creditors. Thus, it has to control the firm to get to know if the firm acts in the contracted way. In our model the FI can become informed about the condition of the firm by observing the signal.

Furthermore, by choosing a FI as a financing source firms generally do not just get the capital, but services and advice as well. James (1987) gives evidence that banks provide some special services that are not available from other lenders. As Bolton and Freixas (1996) mention, firms using bank loans receive an additional service which is generally referred to as relationship banking. Due to this fact a FI has to charge higher interest rates. As Fama (1985) said, "there must be something special about bank loans that makes some borrowers willing to pay higher interest rates than those on other securities of equivalent risk ". Apart from this, many articles get as a result that a FI charges higher interest rates. ${ }^{34}$ Debtors are willing to pay a premium for financing via FI. As in our model, the higher interest rates are not given exogenously but they are derived endogenously.

In the outlined model there is no need to pay a premium with long-term financing since a firm does not need advice and can do as well as with many creditors. The above mentioned points mean that there still can be a reason to finance even long-term with a FI. One just has to think about a situation where it is of advantage for a firm if it is controlled by a FI, for example to reduce agency-problems or due to positive checks to increase its reputation as being a good firm. Nevertheless, in our model a firm is not willing to finance long-term with a FI.

Since the firm is never going to take the FI as a financing source while choosing long-term financing we do not have to go through the case of long-term financing, asymmetric information and FI in detail. Therefore we are just going to deal with the case of short-term financing.

[^18]
### 4.2.2 Short-term financing

The much easier case arises if the cash-flow is realized at $t=1$. The owner of the firm pays back his debt with the cash-flow. Since he is able to repay the entire face value the payments to the FI at date $t=1$ are, $I\left(1+r_{01}+f i\right) .{ }^{35}$ The owner possesses $\left[a-I\left(1+r_{01}+f i\right)\right]\left(1+r_{g}\right)$ at $t=2$ and he is furthermore able to consume his private benefit $k_{1}$ and his control rent $k_{2}$. The more difficult and interesting case is the one in which the cash-flow is not realized at date $t=1$. As before with many creditors, the control rights of the firm switch to the FI and it observes the signal. The observation costs are sunk and they are therefore not included in the decision of liquidation or continuation. These costs appear only in the calculation of the risk-adjusted interest rate $r_{01}+f i$.

What the owner is trying to achieve by choosing a FI is that the firm is not liquidated as often as with many creditors. Remember that with probability $\left(p_{4}+p_{6}\right)$ the firm is liquidated by many creditors. Thus the firm is trying to lower this probability of early liquidation. From a social welfare point of view this is also desirable especially because liquidation with a good signal is not efficient.

The four scenarios we are going to get equal the ones in the case of symmetric information with many creditors. Nevertheless we have to go through each situation in detail to sketch differences or similarities to the corresponding situation with many creditors.
1)

$$
L_{1}=d \quad \text { and the signal is good, so that } s=c .
$$

The FI continues the firm because it can be assure of a full repayment of the refinanced debt. As a consequence the FI makes a take-it-or-leave-it offer to the owner which the owner accepts. Thus, the FI gets the entire second period cash-flow $c$. Since the firm is not liquidated, the owner is able to consume the private benefit $k_{1}$ and the control rent $k_{2}$. The continuation of the

[^19]firm is efficient and this situation occurs with probability $p_{6}$. This is actually an improvement compared with the situation of asymmetric information with many creditors. That situation led to liquidation but in contrast to this in the present case we get a continuation of the firm. Therefore, from the viewpoint of the owner it is better to choose the FI instead of many creditors as a financing source. The price the owner pays for this better decision is the higher interest rate $\left(1+r_{01}+f i\right)$ at date $t=1$ which is only of interest in two situations, $(i) x_{1}=a$ and (ii) $L_{1}=d$ combined with $x_{2}=b$. Otherwise the owner loses the entire cash-flow anyway and thus, he does not care about the higher interest rate.

If a firm is willing to pay the premium $f i$ depends on the probability $p_{6}$ which is the probability of this improved situation. We try to give reasons why the owner should pay the premium. First of all, one has to realize that the private benefit $k_{1}$ is very high so that the owner never wants to lose this benefit. Secondly, it seems to be true that if the probabilty of $p_{6}$ is not very high it may not be worth to pay a premium, even a low one, either. But remember that $p_{6}=(1-p)(1-q) r$, so that a low $p_{6}$ might be caused by a low probability $(1-p)$. In general, the probability $(1-p)$ cannot be too high otherwise one would rather finance long-term. This probability influences the level of the risk premium because the lower $(1-p)$ the lower the expected observation costs $(1-p) l$. As a consequence a low $(1-p)$ leads to a low $(1-p) l$ and this finally gives us a low premium $f$ i. This means that if $(1-p)$ is low the risk premium is also neglectably small. In this case it is still worth paying this little amount $f i$ and not risking early liquidation and losing the large private benefit $k_{1}$.

Due to these reasons it seems reasonable to assume that each firm prefers to pay this riskpremium. Our results do not depend on this statement. Without this assumption our results would depend on this probability $p_{6}$, but to get clearer results one should take the statement just made as given.
2)

$$
L_{1}=d \quad \text { and the signal is bad, so that } s=b .
$$

Since the repayment at $t=2$ is less than the value of a liquidation at $t=1$ the FI has to
liquidate the firm. The payment to the FI at $t=1$ is $I\left(1+r_{01}+f i\right)$, whereas the owner receives only the rest, whose value is at date $t=2\left[d-I\left(1+r_{01}+f i\right)\right]\left(1+\hat{r}_{12}\right)$. Nevertheless he lost the private benefit and his control rent with this early liquidation. Thus, in this situation we get the same result as in the previous section. The existence of or the financing with the FI did not lead to a superior solution. We already know that this outcome is not efficient. In this situation the owner is actually better off with many creditors because he can save the premium $f i$ but this saving is nothing compared to the possibility of losing $k_{1}$ in situation 1. To conclude, we receive an early liquidation with probability $p_{4}$.
3)

$$
L_{1}=0 \quad \text { and the signal is good, so that } s=c .
$$

This situation occurs with probability $p_{5}$. Here, the owner is not able to repay any amount of debt at $t=1$. But since the signal is good, the FI is continuing the firm. It gets the entire second period cash-flow but does not liquidate the firm at $t=2$. The owner consumes his private benefit $k_{1}$ and is also able to get his control rent $k_{2}$. This situation equals exactly the one with many creditors. The owner loses the entire cash-flow and therefore the premium charged by the FI does not change anything.
4)

$$
L_{1}=0 \text { and the signal is bad, so that } s=b .
$$

In this situation the liquidation value remains the same. Thus the FI cannot receive any payment from liquidating the firm. Therefore the only thing the FI can do is to continue the firm. Even though the signal is bad, continuation is the only senseful decision because it cannot get worse. The FI is the only one who receives some cash-flow at $t=2$, whose level is equal to $b$. The owner of the firm loses his control rent in any case but due to the continuation he is able to get his private benefit $k_{1}$. The probability of this last case equals $p_{3}$. As in situation 3 the higher interest rate does not matter because the owner loses the entire cash-flow independent of the premium.

We are now able to derive the individual rationality constraint to calculate the risk-adjusted interest rate $1+r_{01}+f i$ that the FI is going to charge,

$$
\begin{gathered}
p \cdot I\left(1+r_{01}+f i\right)+(1-p) \cdot(1-q) \cdot c+(1-p) \cdot q \cdot r \cdot I\left(1+r_{01}+f i\right)+ \\
+(1-p) \cdot q \cdot(1-r) \cdot \frac{b}{1+\hat{r}_{12}} \geq I\left(1+\hat{r}_{01}\right)+(1-p) l .
\end{gathered}
$$

To sum up, with asymmetric information the FI continues in three situations whereas many creditors continue in only two situations. The FI liquidates only if the signal is bad combined with a high liquidation value, and many creditors liquidate if the liquidation value is high. If one compares the two scenarios one recognizes that the oucome of situation 3 and 4 of this section is identical to situation one of the previous section. Furthermore, situation 1 and 2 cover the second situation with many creditors and the only differences between the two financing sources occur here. Thus, the FI leads to an improvement in situation one, which occurs with probability $p_{6}$. In contrast to this, the owner could have saved $f i$ in situation 2 of this section by financing with many creditors but referring to the reasons given above this amount should not be too high. It seems reasonable that for a sufficient high $k_{1}$ the owner is willing to pay the premium $f i$. We can conclude that the owner of the firm chooses the FI as the financing source if he prefers short-term financing.

### 4.3 The optimal choice of maturity

As should be accepted by now from the previous discussion, the owner has to compare only two choices to find the optimal maturity. If he finances long-term, the financing source is going to be the capital market with its many investors. In the other case the owner chooses short-term financing with the FI as the financing source. To get the optimal maturity choice the owner has to compare the expected utilities of each financing decision.

If he chooses a long-term debt contract his expected utility equals

$$
\begin{aligned}
& E\left[U^{l}\right]=\left(p_{1}+p_{2}\right) \cdot\left\{\left(a\left(1+\hat{r}_{12}\right)-I\left(1+r_{02}\right)\right)+k_{1}+k_{2}\right\} \\
& +\left(p_{3}+p_{4}\right) \cdot k_{1}+\left(p_{5}+p_{6}\right) \cdot\left\{\left(c-I\left(1+r_{02}\right)\right)+k_{1}+k_{2}\right\} .
\end{aligned}
$$

The expected utility of a short-term debt contract can be written as

$$
\begin{gathered}
E\left[U^{s}\right]=\left(p_{1}+p_{2}\right) \cdot\left\{\left(a-I\left(1+r_{01}+f i\right)\right)\left(1+r_{g}\right)+k_{1}+k_{2}\right\} \\
p_{3} \cdot k_{1}+p_{4} \cdot\left(d-I\left(1+r_{01}+f i\right)\right)\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right) \cdot\left(k_{1}+k_{2}\right) .
\end{gathered}
$$

Again we get a threshold $p^{*}$ which is equal to

$$
p^{* *}=\frac{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]}{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]+\left[a-I\left(1+r_{01}+f i\right)\right]\left[\bar{r}_{g}-\hat{r}_{12}\right]} .
$$

The result of $p^{*}$ is close to the one with symmetric information. The only difference between these results is the risk-adjusted interest rate $1+r_{01}+f i$. Now the question arises how did $p^{*}$ develop compared with $p^{*}$. What we see is that the numerator remains constant. In contrast to this the denominator decreased. This occurs because the last term of the denominator decreases whereas all the other terms do not change. Nevertheless the last term is still positive so that the whole fraction is positive and between zero and one. The reason is the same as in the previous section. The cash-flow $a$ was assumed to be very large, $a>I\left(1+r_{01}+f i\right)$, so that any outstanding debt can be satisfied.

## Result

Thus the probability $p^{*}$ moves up towards one. This means that some firms which financed short-term in case with symmetric information switch to long-term financing with asymmetric information.

## 5. Extension

In this section an extension of the model is presented. If one does not like the assumption about the inability of renegotiating long-term contracts we present another reason why a firm which finances long-term cannot conduct the following project at $t=1$. In this case we allow for renegotiation and drop the assumption of observable size of invested means and assume that the creditors cannot control the level of the investment. Thus, it is not clear at any point of time if the owner invests the borrowed capital in the project or consumes it right away. We try to show by introducing this assumption that the renegotiation process will fail, this
means that the creditors do not allow the owner to invest the initial project's cash flow in the new project. This information structure is no problem at date $t=0$. We can set the net present value and the control rent $k_{1}$ high enough so that the owner always wants to conduct the initial project because by not investing he will lose these two things for sure. The problem arises at $t=1$ with long-term financing and a realized cash-flow of $a$. In this case the owner would like to benefit from the following project. For simplicity we assume that this project has a limited investment size $I_{1}$ which is roughly $\left(a-\frac{I_{0}\left(1+r_{02}\right)}{1+\hat{r}_{12}}\right)$. Since $r_{02}$ is derived endogenously for each firm this is a very loose relation which fits to the main focus of this section, namely to sketch a setting in which renegotiation fails. Thus, our used relations do not have to match specise but they should make sense in quantitative measures. The owner could benefit without harming the creditors if he invests only $I_{1} \approx a-\frac{I\left(1+r_{02}\right)}{1+\hat{r}_{12}}$. In this case he could conduct the project and the creditors are being repaid for sure. But since the investment level is unobservable the creditors cannot be sure that the owner uses just $I_{1}$. If the creditors allow the owner to use the cash-flow $a$ it is possible that the owner invests $I_{1}$ and consumes the rest. There is no punishment mechanism at the end except that the owner can lose the private benefit $k_{2}$ what he anticipates in his behavior. Since the creditors can be assure that the owner consumes the rest with a sufficient low $k_{2}$ they demand a very high interest rate so that they receive an expected repayment of $I\left(1+r_{02}\right)$. But this repayment could be so high that it seems beneficial to consume the entire cash-flow $a$ at $t=1$ and to not invest at all. Moreover, since the creditors will expect and anticipate such a behavior they will not allow to use the cash-flow at first place. Thus, renegotiation fails and the owner is not able to invest in the following project. The presented setting can also be outlined graphically (Figure 2).

We assume a uniform distribution of $r_{g}$ and rely on the assumption that the owner will consume $a-I_{1}$ for sure. First, one can notice that $I_{1}\left(1+\bar{r}_{g}\right)$ is above $I\left(1+r_{02}\right)$. If this were not the case renegotiation would fail right away because the creditors do not expect to be entirely repaid. To allow the owner using the cash-flow the creditors demand a debt contract with face value $R_{2}$ that gives them an expected repayment of $I\left(1+r_{02}\right)$. The expected gain for the owner from conducting the project is the distance from $o$ to $m$. This expected gain
equals the line between $o^{6}$ and $m^{6}$. Again, the exact relationships between the parameters are not important. What is important is the fact that the expected gain is below the investment costs $I_{1}$. As such, the owner can improve his situation by consuming the entire cash-flow $a$ at date $t=1$ than investing in the project. ${ }^{36}$

To sum up, what we get as a result is the fact that if we allow for renegotiation between the parties at date $t=1$ with a long-term contract renegotiation will fail. This means, that the creditors do not let the owner invest in the new project because they do not expect that the owner invests in the project at all. They suspect the owner to consume the entire cash-flow of $a$ and after all they do not have a real punishment scheme for the harming behavior of the owner at $t=2$.

As just said, he cannot decide on his own how to use the initial project's cash-flow he needs the consent of the creditors in case that he wants this cash-flow as the financing source for another project. In this context the assumption about $r_{g}$ is important. If the return $r_{g}$ were safe there would be no problem because the creditors do not lose anything and thus, they could agree to invest in the new project. The problem arises with an uncertain return of $r_{g}$. Now, the creditors can actually lose something if the project is conducted.

To make this point clearer let us go one step deeper. Jensen and Meckling (1976) compare the payoff schemata of equity with a call option. Due to this convex course firm owners are willing to undertake risky projects. Thus, they behave risk loving even though they are supposed to be risk neutral. Myers (1977) describes an underinvestment problem in context with debt. He claims that a firm could refuse to take some profitable investment projects if it would benefit only the creditors and not the owner of the firm. In our model the situation is the other way around. The payoff schemata of creditors is concave so that they prefer safe projects. ${ }^{37}$ Even though the project is worth to be undertaken there is no agreement with the owner. One can claim that the creditors behave risk avers. Thus we get another underinvestment problem. The project is not conducted because only the owner of the firm has a profit by conducting

[^20]this new project. With long-term financing it is only allowed to invest the cash-flow at the riskless interest rate until the firm has to repay its debt. In addition to this, since the firm cannot get outside capital as well, by choosing long-term financing it foregoes the opportunity to invest in a further good project.

If we allow for renegotiation one also has to think about what happens if the owner finances long-term and no cash-flow is realized at date $t=1$. The question is if there can be voluntary liquidation by the owner because he is in control of the firm. Both parties want a continuing firm if the liquidation value is zero but if $L_{1}=d$ liquidation could become a superior choice at least from the creditors‘ point of view. Again, for $s=c$ there are similar opinions about the firm's policy but if the signal is bad, liquidation can be benefical for the creditors because $d\left(1+\hat{r}_{12}\right)>b$. The question arises if the owner could credibly precommit to liquidate the firm with a bad signal and a high liquidation value. If he were able to, he would lose the private benefit $k_{1}$ in any case. One can split up this setting into two different parts: $(i) I\left(1+r_{02}\right)>$ $d\left(1+\hat{r}_{12}\right)>b$ and $(i i) d\left(1+\hat{r}_{12}\right)>I\left(1+r_{02}\right)>b$. In the first part the owner loses the entire liquidation value. In contrast to this, in the second he gains $d\left(1+\hat{r}_{12}\right)-I\left(1+r_{02}\right)$. But since $k_{1}>d\left(1+\hat{r}_{12}\right)-b$ the private benefit is also larger than $d\left(1+\hat{r}_{12}\right)-I\left(1+r_{02}\right)$. Thus, the owner does not benefit from an early liquidation. Furthermore, such a precommitment is not time consistent and the owner cannot credibly commit to this policy.

To sum up, this extension described a setting in which renegotation with a long-term contract fails and that the owner never liquidates voluntarily the firm at $t=1$.

## 6. Summary

At first, the article motivated different maturity choices by firms. Secondly, it was shown why firms choose different financing sources combined with different maturity choices. Section three was dedicated to the previous point. Even though we had full symmetric information at each point of time firms choose different maturity structures. The driving force of this result was the probability of receiving the cash-flow after period one. If a firm is sure to realize its cash-flow early it is going to finance short-term and takes the risk of early liquidation or more expensive refinancing. On the other hand, a firm that does not expect its cash-flow to arise so early, finances more safely by choosing long-term debt. This result fits to the tendency
of financing congruent to the realizations of cash-flows of a firm's investment project. This congruent financing is a general guideline of how someone should finance its projects. Hart and Moore (1994) claim that the faster the returns arrive the shorter the maturity of the debt will be. Moreover, they find an explanation why assets should be matched with liabilities. ${ }^{38}$ In addition, we got the surprising result that the maturity choice was independent of the bargaining power of each party.

In the fourth section we introduced asymmetric information about the signal. First of all one has to realize that in our model it was quite difficult for the FI to be better than many creditors. Generally, many articles use the assumption that there is always liquidation in case of insolvency with many creditors. With this assumption the FI is more often superior to many creditors in case of short-term financing. Even though we did not assume this fact it was still possible to show that by choosing a FI as a financing source with short-term financing a firm can improve its situation.

Finally, one could have made the cash-flows, the liquidation value and the signal continuous in their realizations. But this would make things more difficult without further gain. The idea that was presented is much easier to understand within the discrete structure of the variables.

[^21]
## Appendix

Below we show the relation between the different risk adjusted interest rates.
Market equilibrium conditions:

- short-term interest rate

$$
\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)+p_{3} \frac{b}{1+\hat{r}_{12}}+p_{4} I_{0}\left(1+r_{01}\right)+\left(p_{5}+p_{6}\right) \frac{c}{1+\hat{r}_{12}}=I_{0}\left(1+\hat{r}_{01}\right)
$$

- long-term interest rate

$$
\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{02}\right)+\left(p_{3}+p_{4}\right) b+\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{02}\right)=I_{0}\left(1+\hat{r}_{02}\right)
$$

- capital market

$$
I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)=I_{0}\left(1+\hat{r}_{02}\right)
$$

By assumption, since we have a normal interest rate structure we know that $I_{0}\left(1+\hat{r}_{01}\right)<$ $I_{0}\left(1+\hat{r}_{02}\right)$. As a consequence the payments from the long-term contract have to be higher than the ones for a short-term contract. Let us assume right now that $I_{0}\left(1+r_{02}\right)-I_{0}\left(1+r_{01}\right)<0$. In this case the payments from the long-term contract are larger only in the situation which occurs with probability $p_{3}$. But this difference is nothing compared with the difference which happens with probability $\left(p_{5}+p_{6}\right)$. Thus, it seems intuitively clear that the relation mentioned above cannot hold. We can give a very strong threshold for $b$ so that long-term financing $I_{0}\left(1+r_{02}\right)$ has to be larger than short-term financing $I_{0}\left(1+r_{01}\right)$,

$$
\left(p_{3}+p_{4}\right) b<p_{3} \frac{b}{1+\hat{r}_{12}}+p_{4} I_{0}\left(1+r_{01}\right) .
$$

We already know from the equilibrium condition for the short-term interest rate that

$$
I_{0}\left(1+r_{01}\right)=\frac{I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)-p_{3} b-\left(p_{5}+p_{6}\right) c}{\left(1+\hat{r}_{12}\right)\left(p_{1}+p_{2}+p_{4}\right)} .
$$

Thus,

$$
\begin{gathered}
\left(p_{3}+p_{4}\right) b\left(1+\hat{r}_{12}\right)<p_{3} b+p_{4} \frac{I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)-p_{3} b-\left(p_{5}+p_{6}\right) c}{p_{1}+p_{2}+p_{4}} \\
\left(p_{3}+p_{4}\right)\left(p_{1}+p_{2}+p_{4}\right) b\left(1+\hat{r}_{12}\right)<\left(p_{1}+p_{2}+p_{4}\right) p_{3} b+p_{4} I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)-p_{3} p_{4} b-p_{4}\left(p_{5}+p_{6}\right) c \\
p_{3}\left(p_{1}+p_{2}+p_{4}\right) b \hat{r}_{12}+p_{4}\left(p_{1}+p_{2}+p_{4}\right) b\left(1+\hat{r}_{12}\right)+p_{3} p_{4} b<p_{4} I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)-p_{4}\left(p_{5}+p_{6}\right) c \\
b<\frac{p_{4} I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)}{p_{3}\left(p_{1}+p_{2}+p_{4}\right) \hat{r}_{12}+p_{3} p_{4}+p_{4}\left(p_{1}+p_{2}+p_{4}\right)\left(1+\hat{r}_{12}\right)}
\end{gathered}
$$

If $b$ is lower than the term on the right hand side the capital costs for long-term financing are higher than the ones for short-term financing.

It should be clear that refinancing is more expensive than long-term financing. The owner loses the entire cash-flow if he has to refinance. Thus the costs of refinancing reach the maximum the owner can give to the creditors.
vspace 1 cm
Now we want to derive the threshold $p^{*}$. The expected utilities look as follows:

- short-term

$$
\begin{gathered}
E\left[U^{s}\right]=\left(p_{1}+p_{2}\right) a\left(1+\bar{r}_{g}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+p_{3} k_{1}+ \\
+p_{4} d\left(1+\hat{r}_{12}\right)-p_{4} I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right)\left(k_{1}+k_{2}\right)
\end{gathered}
$$

Using the market equilibrium condition with
$-p_{4} I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)=\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)+p_{3} b+\left(p_{5}+p_{6}\right) c-I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)$
leads to

$$
\begin{gathered}
=\left(p_{1}+p_{2}\right) a\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+ \\
+p_{3}\left(b+k_{1}\right)+p_{4} d\left(1+\hat{r}_{12}\right)-I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right)\left(k_{1}+k_{2}\right)+\left(p_{5}+p_{6}\right) c
\end{gathered}
$$

- long-term

$$
\begin{gathered}
E\left[U^{l}\right]=\left(p_{1}+p_{2}\right) a\left(1+\hat{r}_{12}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{02}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+\left(p_{3}+p_{4}\right) k_{1}+ \\
+\left(p_{5}+p_{6}\right)\left(c+k_{1}+k_{2}\right)-\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{02}\right)
\end{gathered}
$$

Using the market equlibrium condition with

$$
-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{02}\right)-\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{02}\right)=\left(p_{3}+p_{4}\right) b-I_{0}\left(1+\hat{r}_{02}\right)
$$

gives us

$$
\begin{gathered}
=\left(p_{1}+p_{2}\right) a\left(1+\hat{r}_{12}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+\left(p_{3}+p_{4}\right)\left(k_{1}+b\right)+ \\
+\left(p_{5}+p_{6}\right)\left(c+k_{1}+k_{2}\right)-I_{0}\left(1+\hat{r}_{02}\right)
\end{gathered}
$$

Equating the two expected utilities

$$
\begin{gathered}
\left(p_{1}+p_{2}\right) a\left(1+\hat{r}_{12}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+\left(p_{3}+p_{4}\right)\left(b+k_{1}\right)+\left(p_{5}+p_{6}\right)\left(k_{1}+k_{2}+c\right)-I_{0}\left(1+\hat{r}_{02}\right)= \\
\left(p_{1}+p_{2}\right) a\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+ \\
p_{3}\left(b+k_{1}\right)+p_{4} d\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right)\left(k_{1}+k_{2}+c\right)-I_{0}\left(1+\hat{r}_{02}\right) \\
\left(p_{1}+p_{2}\right) a\left(1+\hat{r}_{12}\right)+p_{4}\left(b+k_{1}\right)=\left(p_{1}+p_{2}\right) a\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)- \\
-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)+p_{4} d\left(1+\hat{r}_{12}\right)
\end{gathered}
$$

Since $p_{1}+p_{2}=p$ and $p_{4}=(1-p) q r$ we get

$$
\begin{gathered}
p a\left[\left(1+\hat{r}_{12}\right)-\left(1+\bar{r}_{g}\right)\right]+p I_{0}\left[\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)-\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)\right]+(1-p) q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]=0 \\
\Leftrightarrow \quad p^{*}=\frac{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]}{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]+\left[a-I_{0}\left(1+r_{01}\right)\right]\left[\bar{r}_{g}-\hat{r}_{12}\right]},
\end{gathered}
$$

and with a financial intermediary

$$
p^{*}=\frac{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]}{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]+\left[a-I_{0}\left(1+r_{01}\right)+f i\right]\left[\bar{r}_{g}-\hat{r}_{12}\right]} .
$$

Second but last it is proven that even if the owner has the bargaining power so that the creditors receive only $\hat{r}_{12}$ as the risk-adjusted interest rate the threshold does not change.

Market equilibrium conditions:

- short-term interest rate

$$
\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)+p_{3} \frac{b}{1+\hat{r}_{12}}+p_{4} I_{0}\left(1+r_{01}\right)+\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{01}\right)=I_{0}\left(1+\hat{r}_{01}\right)
$$

- long-term interest rate

$$
\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{02}\right)+\left(p_{3}+p_{4}\right) b+\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{02}\right)=I_{0}\left(1+\hat{r}_{02}\right)
$$

- capital market

$$
I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)=I_{0}\left(1+\hat{r}_{02}\right)
$$

The expected utilities change as follows,

- short-term

$$
\begin{gathered}
E\left[U^{s}\right]=\left(p_{1}+p_{2}\right) a\left(1+\bar{r}_{g}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+p_{3} k_{1}+ \\
+p_{4} d\left(1+\hat{r}_{12}\right)-p_{4} I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right)\left(c+k_{1}+k_{2}\right)-\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)
\end{gathered}
$$

Using the 'new'market equilibrium condition leads to

$$
\begin{gathered}
=\left(p_{1}+p_{2}\right) a\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)\left(1+\bar{r}_{g}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+ \\
+p_{3}\left(b+k_{1}\right)+p_{4} d\left(1+\hat{r}_{12}\right)-I_{0}\left(1+\hat{r}_{01}\right)\left(1+\hat{r}_{12}\right)+\left(p_{5}+p_{6}\right)\left(c+k_{1}+k_{2}\right)
\end{gathered}
$$

- long-term

$$
\begin{gathered}
E\left[U^{l}\right]=\left(p_{1}+p_{2}\right) a\left(1+\hat{r}_{12}\right)-\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{02}\right)+\left(p_{1}+p_{2}\right)\left(k_{1}+k_{2}\right)+\left(p_{3}+p_{4}\right) k_{1}+ \\
+\left(p_{5}+p_{6}\right)\left(c+k_{1}+k_{2}\right)-\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{02}\right)
\end{gathered}
$$

These two expected utilities are the same ones as in the initial situation. Thus, the result is identical.

Finally, we give the owner of the firm the full bargaining power so that the creditors receive just their reservation utility viewed from $t=1$ at the end of the second period. Their reservation utility is zero if $L_{1}=0$ and $I_{0}\left(1+r_{01}\right)$ otherwise. Since they have the control rights of the firm they can demand $I_{0}\left(1+r_{01}\right)$ if the high liquidation value is realized.

- short-term interest rate

$$
\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{01}\right)+\left(p_{4}+p_{6}\right) I_{0}\left(1+r_{01}\right)=I_{0}\left(1+\hat{r}_{01}\right)
$$

- long-term interest rate

$$
\left(p_{1}+p_{2}\right) I_{0}\left(1+r_{02}\right)+\left(p_{3}+p_{4}\right) b+\left(p_{5}+p_{6}\right) I_{0}\left(1+r_{02}\right)=I_{0}\left(1+\hat{r}_{02}\right)
$$

The expected utilities change as follows,

- short-term

$$
\begin{gathered}
E\left[U^{s}\right]=\left(p_{1}+p_{2}\right)\left[\left(a-I_{0}\left(1+r_{01}\right)\right)\left(1+\bar{r}_{g}\right)+k_{1}+k_{2}\right]+p_{3}\left(k_{1}+k_{2}+b\right)+p_{4}\left[d-I_{0}\left(1+r_{01}\right)\right]\left(1+\hat{r}_{12}\right)+ \\
+p_{5}\left(k_{1}+k_{2}+c\right)+p_{6}\left[c-I_{0}\left(1+r_{01}\right)\left(1+\hat{r}_{12}\right)+k_{1}+k_{2}\right]
\end{gathered}
$$

- long-term

$$
E\left[U^{l}\right]=\left(p_{1}+p_{2}\right)\left[a\left(1+\hat{r}_{12}\right)-I_{0}\left(1+r_{02}\right)+k_{1}+k_{2}\right]+\left(p_{3}+p_{4}\right) k_{1}+\left(p_{5}+p_{6}\right)\left(c-I_{0}\left(1+r_{02}\right)+k_{1}+k_{2}\right)
$$

Using the same proceeding as before we get

$$
\Leftrightarrow \quad p^{* *}=\frac{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]+q(1-r) k_{2}}{q r\left[b+k_{1}-d\left(1+\hat{r}_{12}\right)\right]+q(1-r) k_{2}+\left[a-I_{0}\left(1+r_{01}\right)\right]\left[\bar{r}_{g}-\hat{r}_{12}\right]} .
$$

This equation is the same as before with one exception which is $q(1-r) k_{2}$. This is no surprise because modelling the bargaining power as done the owner is able to consume the control rent $k_{2}$ even though the second period cash-flow is low. As a consequence, he can influence the probability of consuming $k_{2}$ with his maturity choice. Thus, the control rent has to appear in the relevant equation, as well. Thus, if we neglect $k_{2}$ everything remains the same and we get the result that the distribution of the second period cash-flow has no effect on the maturity choice.

## References

Aghion and Patrick Bolton (1992), An "Incomplete Contracts" Approach to Financial Contracting, Review of Economic Studies, 59(3), pp. 473-494.

Anderson, Ronald and Suresh Sundaresan (1996), Design and Valuation of Debt Contracts, Review of Financial Studies, 9(1), pp. 37-68.

Berlin, Mitchell and Loretta J. Mester (1992), Debt Covenants and Renegotiation, Journal of Financial Intermediation, 2(2), pp. 95-133.

Berglf, Erik and Ernst-Ludwig von Thadden (1994), Short-Term Versus Long-Term Interests: Capital Structure With Multiple Investors, Quarterly Journal of Economics, 109(4), pp. 10551084.

Bolton, Patrick and Xavier Freixas (1996), A Dilution Cost Approach to Financial Intermediation, mimeo, London School of Economics.

Breuer, Wolfgang (1993), Finanzintermediation im Kapitalmarktgleichgewicht, Wiesbaden.
Breuer, Wolfgang (1994), Finanzintermediation und Wiederverhandlungen, Kredit und Kapital, 27(2), pp. 291-309.

Breuer, Wolfgang (1995), Finanzintermediation und Reputationseffekte, Kredit und Kapital, 28(4), pp. 516-534.

Chemmanur, Thomas J. and Paolo Fulghieri (1994), Reputation, Renegotiation, and the Choice between Bank Loans and Publicly Traded Debt, Review of Financial Studies, 7(3), pp. 475-506.

Dewatripont, Mathias and Jean Tirole (1994a), The Prudential Regulation of Banks, Cambridge and London: The MIT Press.

Dewatripont, Mathias and Jean Tirole (1994b), A Theory of debt and Equity: Diversity of Securities and Manager-Shareholder Congruence, Quarterly Journal of Economics, 109(4), pp. 1027-1054.

Diamond, Douglas (1984), Financial Intermediation and Delegated Monitoring, Review of Economic Studies, 51(3), pp. 393-414.

Diamond, Douglas (1991a), Monitoring and Reputation: The Choice between Bank Loans and Directly Placed Debt, Journal of Political Economy, 99(4), pp. 689-721.

Diamond, Douglas (1991b), Debt Maturity Structure and Liquidity Risk, Quarterly Journal of Economics, 106(3), pp. 709-737.

Diamond, Douglas (1993a), Seniority and Maturity of Debt Contracts, Journal of Financial Economics, 33(3), pp. 341-368.

Diamond, Douglas (1993b), Bank Loan Maturity and Priority when Borrowers can refinance, in: Mayer and Vives (ed.), Capital Markets and Financial Intermediation, pp. 46-68.

Fama, Eugene F. (1985), What's different about Banks?, Journal of Monetary Economics, 15(1), pp. 29-39.

Flannery, Mark J. (1986), Asymmetric Information and Risky Debt Maturity Choice, Journal of Finance, 41(1), pp. 19-37.

Gale, Douglas and Martin Hellwig (1985), Incentive-Compatible Debt Contracts: The OnePeriod Problem, Review of Economic Studies, 52(4), pp. 647-663.

Gertner, Robert and David Scharfstein (1991), A Theory of Workouts and the Effects of Reorganization Law, Journal of Finance, 46(4), pp. 1189-1222.

Greenbaum, Stuart I. and Anjan V. Thakor (1995), Contemporary Financial Intermediation, Fort Worth.

Harris, Milton and Artur Raviv (1990), Capital Structure and the Informational Role of Debt, Journal of Finance, 45(2), pp. 321-350.

Hart, Oliver and Bengt Holmstrm (1987), The Theory of Contracts, in: T. Bewley (ed.), Advances in Economic Theory, Fifth World Congress of the Econometric Society, Cambridge: Cambridge University Press.

Hart, Oliver and John Moore (1994), A Theory of Debt Based on the Inalienability of Human Capital, Quarterly Journal of Economics, 109(4), pp. 841-879.

Hart, Oliver and John Moore (1995), Debt and Seniority: An Analysis of the Role of hard Claims in Constraining Management, American Economic Review, 85(3), pp. 567-585.

Houston, Joel F. and Sudipto Venkataraman (1994), Optimal Maturity Structure with Multiple Debt Claims, Journal of Financial and Quantitative Analysis, 29(2), pp. 179-197.

James, Christopher (1987), Some Evidence on the Uniqueness of Bank Loans, Journal of Financial Economics, 19(2), pp. 217-235.

Jensen, Michael und William Meckling (1976), Theory of the Firm: Managerial Behavior, Agency costs and Ownership Structure, Journal of Financial Economics, 3(4), pp. 305-360.

Jensen, Michael (1986), Agency Costs of free Cash-Flow, Corporate Finance, and Takeovers, American Economic Review, 76(2), pp. 323-329.

Myers, Stewart C. (1977), Determinants of Corporate Borrowing, Journal of Financial Economics, 5(2), pp. 147-175.

Rajan, Raghuram G. (1992), Insiders and Outsiders: The Choice between Relationship and Arms Length Debt, Journal of Finance, 47(4), pp. 1367-1399.

Rogoff, Kenneth (1985), The Optimal Degree of Commitment to an Intermediate Monetary Target, Quarterly Journal of Economics, 100(4), pp. 1169-1189.

Sharpe, Steven A. (1991), Credit Rationing, Concessionary Lending, and Debt Maturity, Journal of Banking and Finance, 15(3), pp. 581-604.

Stulz, Rene (1990), Managerial Discretion and Optimal Financing Policies, Journal of Financial Economics, 26(1), pp. 3-27.

Williamson, Stephen D. (1986), Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing, Journal of Monetary Economics, 18(2), pp. 159-179.

Winton, Andrew (1995), Costly State Verification and Multiple Investors: The Role of Seniority, Review of Financial Studies, 8(1), pp. 91-123.


[^0]:    ${ }^{1}$ Greenbaum and Thakor (1995) p. 118.
    ${ }^{2}$ For a definition of financial intermediaries see Breuer (1993) and Greenbaum and Thakor (1995), with special focus on banks see Dewatripont and Tirole (1994a).
    ${ }^{3}$ See Jensen and Meckling (1976), Jensen (1986) and Stulz (1990).
    ${ }^{4}$ This is a commonly made assumption. See Harris and Raviv (1990).

[^1]:    ${ }^{5}$ See Sharpe (1991), Diamond (1991b), Diamond (1993a) and Hart and Moore (1995) who assume a zero interest rate structure.
    ${ }^{6}$ Compare with Flannery (1986) and Diamond (1991b).

[^2]:    ${ }^{7}$ See Diamond (1984) and Williamson (1986).
    ${ }^{8}$ See Anderson and Sundaresan (1996).

[^3]:    ${ }^{9}$ See e.g. Diamond (1991b), Hart and Moore (1995) and Breuer (1994) or (1995).
    ${ }^{10}$ See for a characterization Hart and Holmstrm (1987).

[^4]:    ${ }^{11}$ The only exception is Bolton and Freixas (1996).
    ${ }^{12}$ See Gertner and Scharfstein (1991) p. 1192.

[^5]:    ${ }^{13}$ Otherwise we can drop unprofitable firms since we are not presenting a signaling model. See for example Diamond (1991b) and (1993a).

[^6]:    ${ }^{14}$ This assumption seems to be very critical because the optimality of the standard debt contract was derived in a setting with observation costs for getting to know the realization of the cash-flow, e.g. Gale and Hellwig (1985) and Williamson (1986). We still use the standard debt contract for two reasons. First, one could introduce observation costs but this would just complicate the whole model. But more important, if the firm borrows capital from the capital market it is forced by law to publish its balance so that each investor can receive information about the firm's cash-flows. Therefore it is very questionable if the cash-flow of the firm is not a public good.
    ${ }^{15}$ Diamond (1993a) makes implicitly the same assumption because he assumes in his model that there is a private benefit from being an ongoing firm within the first period. Later we will constrain this private benefit $k_{1}$ so that the owner never liquidates the firm voluntarily.
    ${ }^{16}$ To simplify calculations we assume that the time preference for consumption of $k_{1}$ is one, so that one does not have to adjust $k_{1}$ for later comparisons.

[^7]:    ${ }^{17}$ Houston and Venkataraman (1994) make a similar assumption. In their model the liquidation value at date $t=1$ is independent of beliefs about the future cash-flow. They also admit that in order to be more realistic, one would have to make the liquidation value dependent on future cash-flows.
    ${ }^{18}$ One could make $\hat{r}_{12}$ stochastic, but this makes things more difficult without any further insights.

[^8]:    ${ }^{19}$ See for example Diamond (1991b) and (1993a).
    ${ }^{20}$ See section 5 for an extension of the model. In the extension we allow for renegotiation of the long-term contract but still get the same result as without allowing renegotiation of this contract type.

[^9]:    ${ }^{21}$ The assumption that the creditors get all the bargaining power seems to be unrealistic. This assumption is extreme, but our results do not depend on it. This will be proven in the appendix. The main reason for giving many creditors the full bargaining power is to simplify calculations.

[^10]:    ${ }^{22}$ Compare Harris and Raviv (1990), who have the same sharing rule, p. 328 .

[^11]:    ${ }^{23}$ This situation refers to Diamond (1993b) who mentioned that "Because bank loans often have strict covenants that allow even long-term lenders to exercise control, banks may always have the 'right to liquidate" ".

[^12]:    ${ }^{24}$ Compare with Rogoff (1985).

[^13]:    ${ }^{25}$ The whole calculation of the threshold $p^{*}$ is shown in the appendix.
    ${ }^{26}$ In an independent work Bolton and Freixas (1996), who focused on dilution costs for firms and financial intermediaries by raising equity, derived a threshold $\hat{p}_{1}$ which can be interpreted in a similar way as our threshold. At first, their threshold was the cut-off probability between choosing equity in combination with riskless bonds and risky bonds alone. In a next step, they calculated thresholds for financing with banks, outside equity and risky bonds. As in our model the probability $p$ of receiving first period cash-flow is the important parameter.

[^14]:    ${ }^{27}$ One might think that the risk-adjusted interest rate is $\hat{r}_{12}$ due to the fact that the payoffs are known for sure at date $t=1$.
    ${ }^{28}$ Actually, the minimum would be a payment slightly above their reservation utility, which is zero if $L_{1}=0$ and $I_{0}\left(1+r_{01}\right)$ otherwise. Again, we show the outcome of this scenario in the appendix.

[^15]:    ${ }^{29}$ This assumption is not critical, see e.g. Winton (1995).
    ${ }^{30}$ See Hart (1995), p. 127, who claims that due to free-rider problems monitoring by mutiple investors does not occur.

[^16]:    ${ }^{31}$ This inequality is not time consistent because $k_{1}$ is a paramter at date $t=1$ but we assumed that the time preference for private consumption is one.
    ${ }^{32}$ See the assumptions about the level of the variables.

[^17]:    ${ }^{33}$ Aghion and Bolton (1992), Dewatripont and Tirole (1994b) and Berglf and von Thadden (1994) try to resolve this problem by allocating the control rights efficiently dependent on the state of nature, so that too early liquidation does not arise.

[^18]:    ${ }^{34}$ See for example Chemmanur and Fulghieri (1994).

[^19]:    ${ }^{35}$ To retain the same notation and to make clear that financial intermediaries charge higher interest rates, we add a constant $f i$ to the 'old' risk-adjusted interest rate, so that the bank interest rate in this case would be $\left(1+r_{01}+f i\right)$. The calculation of $\left(1+r_{01}+f i\right)$ is shown below.

[^20]:    ${ }^{36}$ Actually, the expected gain could also be located slightly above $I_{1}$ because one has to discount this gain to compare the level with $I_{1}$.
    ${ }^{37}$ This course fits to a put option.

[^21]:    ${ }^{38}$ See Hart and Moore (1994) p. 864/5.

