Inequality and Political Consensus*

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INEQUALITY AND POLITICAL CONSENSUS

Abstract This paper develops a model of political consensus in order to explain the missing link between inequality and political redistribution. Political consensus is an implicit agreement not to vote for extreme policy proposals. We show that such an agreement may play an efficiency-enhancing role. Voters anticipate that voting for extremist parties increases policy uncertainty in the future. A political consensus among voters reduces policy uncertainty because power-seeking politicians propose non-discriminatory policies in their own interest. We study how much inequality can be sustained in a democracy and how the limits to redistribution vary with initial inequality. We find that more inequality need not lead to more redistribution. The maximum amount of redistribution decreases with inequality if (and only if) agents are sufficiently patient. In this case inequality is politically self-sustaining.

Keywords: inequality, representative democracy, political consensus, policy uncertainty, comparative statics in political economy.

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1 Introduction

The link between inequality and the extent of political redistribution has been a subject of debate in the recent politico-economic literature. Several authors argued that a more unequal distribution of income or wealth increases the gains from redistribution for poor voters and hence increases the amount of redistribution [Romer, 1975, Roberts, 1977, Meltzer and Richards, 1981, Bertola, 1993, Alesina and Perotti, 1994, Persson and Tabellini, 1994]. These models of political equilibrium predict that more "initial" inequality should lead to more political redistribution. However, this theoretical result is at odds with most of the empirical data on inequality and redistribution [Perotti, 1992, 1994, 1996, Keefer and Knack 1995, Clarke, 1992]. In particular one observes that fiscal variables such as the size of the redistributive government sector are not related to measures of inequality. According to the data, a given unequal distribution may be politically stable even in presence of large inequalities.

In this paper we argue that a political consensus among voters may explain the missing link between inequality and redistribution. By political consensus we mean an implicit agreement not to vote for extreme policy proposals which strongly discriminate against single groups. Such an agreement may be efficient because it protects individuals against the risk of erratic policy changes. The agreement defines a set of decent policies, that can be accepted and another set that is perceived as discriminatory and cannot be accepted. We will show how such a consensus is sustained by individual voting behavior and how it may protect agents against redistribution even in presence of large inequalities.

Our model of consensus is a repeated voting game with two-party competition. It is a well known result in game theory that co-operative outcomes can be sustained in a non-cooperative game if the game is played repeatedly. In a voting game this means that voters can sustain centrist policies when elections are held more than once. Voters anticipate that voting for extremist parties may lead to a breakdown of consensus in the future and hence to policy uncertainty. A political consensus among voters protects them from erratic policy changes because power-seeking politicians propose non-discriminatory policies in their own interest.

In our game there is - besides the government's budget constraint - no restriction on the set of possible political platforms. Political platforms are multidimensional; they can in principle contain individual- or group-specific tax rates and transfers. Agents receive a fixed gross income and income taxation generates an efficiency loss. Our game builds upon a model of repeated elections in Artale and Grüner (1997). This game analyzes political outcomes when parties divide a fixed amount of resources among voters. In their model there are no efficiency losses arising from redistribution, therefore the distribution of initial resources does not play a role for policy outcomes. In the present model efficiency losses are introduced. This enables us to perform our comparative static analysis about the role of initial inequality in the political process and to analyze how much after-tax inequality a democracy tolerates.

Our repeated game has a continuum of redistributive equilibria. Therefore, our comparative static results will refer to the boundaries of the set of equilibria. We study how these boundaries are affected by initial inequality and find that initial amount of inequality may reduce the maximum extent of redistribution from the rich to the poor. Our results fit with three important stylized facts of political life in representative democracies: (i) the absence of political platforms which strongly discriminate against one single groups of society, (ii) the persistence of different political outcomes in otherwise similar countries and (iii) the weak link between inequality and political redistribution.

Besides addressing the relationship of inequality and redistribution, the work in this paper is also of more general interest. Most of the formal analysis of the political process heavily relies upon the assumption that available policies can be ordered along one dimension. It is a well known result that if preferences are single-peaked then there exists a unique majority voting winning platform. Otherwise majority-voting equilibria generally fail to exist. While some authors pessimistically concluded from this that "nearly anything can happen in politics" [Riker 1980, 448] others have made various attempts to theoretically explain why democratic decision processes may lead to stable political outcomes[1]. Most of these attempts consist of imposing restrictions on the distribution of the characteristics of individual preferences. However, majority-voting equilibria can only be established under very restrictive assumptions in one shot political games [see Mc Kelvey, 1987, for a thorough analysis of this issue]. In a repeated voting model the relative stability of political outcomes can be explained. Voters can coordinate on a set of efficient outcomes if they play simple punishment strategies. In our particular example we find that the bounds of the set of political equilibria react in a fundamentally different manner to changes in exogenous variables than do the policy variables in the one-dimensional, one-shot game. A major insight of the present analysis is that with repeated political competition the comparative static results obtained from the one dimensional political problem need no longer hold. This points out that it may be appropriate to check the robustness of other comparative static results in politico-economic games.

The paper is structured as follows. Section 2 briefly reviews the standard view on the link between inequality and redistribution in a one dimensional model of redistributive taxation. Section 3 extends the model to party competition with multidimensional platforms and proves that there only exist equilibria in purely mixed strategies. Section 4 introduces the repeated game and Section 5 defines the concept

[1] It is not our objective to provide a complete survey of this extensive literature here, this has been done elsewhere. The reader may refer to the textbooks by Ordeshook (1984) and Mueller (1990), the discussion in Riker (1980), Ordeshook (1980), and Rae (1980) and also to the survey in chapter 1 of Coughlin (1992).

of political consensus. Section 6 then studies the link between inequality and the extend of redistribution in the repeated game setting. Section 7 concludes.

2 The Standard View

Many politico-economic models postulate efficiency losses from redistribution. These losses may be due to distorted labor supply, tax evasion, costs of bureaucracy etc. In such models voters with less than average income trade off the gains from redistribution against the efficiency losses. Poorer voters face lower opportunity costs of redistribution. For some functional forms of the initial income distributions more inequality is associated with a poorer median voter and therefore leads to more redistribution [Romer, 1975, Roberts, 1977, Meltzer and Richards, 1981, Persson and Tabellini, 1994]. To see this consider the simple redistribution game where agents $i = 1, 2, .., n$ receive an exogenous income Y_i. Throughout the paper we assume that n is odd. There is a general linear income tax. Taxation reduces an agent's own income to $y_i = (1 \Leftrightarrow t)Y_i$ with $t \in [0, 1]$. Many authors such as Perotti (1993) assume that there are convex costs of redistribution. In particular let us assume that tax revenues are given by $\sum_{i=1}^{n} \left(t \Leftrightarrow t^2\right) Y_i$.[2] Tax revenues are distributed equally among agents. Hence, an agent's after tax income is given by: $y_i = (1 \Leftrightarrow t)Y_i + \left(t \Leftrightarrow t^2\right) \bar{Y}$ with $\bar{Y} = 1/n \sum_{i=1}^{n} \left(t \Leftrightarrow t^2\right) Y_i$. Individual preferences over tax rates are single peaked

[2]Perotti does not provide a microfoundation for this quadratic specification but it is straightforward to do so. Suppose e.g. that the tax authority knows the income of all agents. However, agent i can hide his initial income Y_i at a cost C. Hiding income means that, although the tax authority knows the true value of Y_i, it cannot proof this before court. An agent who successfully hides his income pays zero taxes. Suppose now that the costs are proportional to the agent's income: $C = c_i Y_i$. The cost parameter c_i depends positively on burocratic observation effort e_i which is measured in monetary units. In particular assume $c_i = e_i^{1/2}$. Then the burocracy has to fix $e_i \geq t^2$ in order to avoid fiscal fraud. The authority's net revenues are then given by $\left(t - t^2\right) Y_i$.

at

$$t = \max\left[0, \frac{1}{2} \cdot \frac{\bar{Y} \Leftrightarrow Y_i}{\bar{Y}}\right]. \tag{1}$$

Poorer agents prefer a higher tax rate. Since individual preferences are single peaked, the median voter theorem applies. Majority voting generates a relation on the set of feasible tax rates that is identical with the preference relation of the median voter. With two-party competition both parties would propose the preferred platform of the median voter. For certain classes of distribution functions, more inequality is associated with a poorer median voter and hence with a larger redistributive tax rate.[3]

3 Multidimensional Platforms

3.1 The Policy Space

In this section we introduce our basic model of party competition with multidimensional platforms. The model of the previous section relies on the assumption that all individuals are taxed at the same rate. We now abandon this assumption and permit parties to propose platforms that tax different agents at different rates. In order to permit a graphical exposition we restrict ourselves to the case with three voters, $n = 3$. Taxation now reduces an agent's own income to $(1 \Leftrightarrow t_i)Y_i$ where t_i denotes the individual tax rate. We normalize the economy's aggregate income to one: $\sum_{i=1}^{3} Y_i = 1$. Like in the one-dimensional model we assume that there are efficiency losses that are increasing and convex in the individual tax rate. In particular

[3] An example is given in Benabou (1995): suppose that income is distributed according to the log-normal distribution: $\ln(Y_i) \sim N(m, \sigma^2)$ with a given mean $\bar{Y}$. Mean income satisfies $m = \ln(\bar{Y}) - \sigma^2/2$. An increase of σ^2 both shifts the Lorenz-curve outwards (increases inequality) and raises the difference between median and mean.

we stick to the previous specification of quadratic costs and assume that tax revenues from agent i are given by $(t_i - t_i^2)\, Y_i$. Tax revenues can now be distributed arbitrarily among agents. The set of feasible net income vectors is denoted by $Y \subset R^{3+}$, and the subset of all Pareto-optima in Y by $P(Y)$. Denoting transfers to individual i by $T_i \geq 0$, we may write:

$$Y = \{y \in R^{3+} \mid y_i = (1 - t_i)\, Y_i + T_i \text{ with} \tag{2}$$

$$t_i \in [0,1]\,,\; T_i \geq 0 \text{ and } 0 = \sum_{i=1..n} \left(t_i - t_i^2\right) Y_i - T_i\}.$$

It is then straightforward to find the following characterization of $P(Y)$:

Lemma 1 *A policy leads to a Pareto-optimal allocation of income if and only if (i) no agent simultaneously pays taxes and receive transfers and (ii) no tax rate exceeds* 1/2.

Proof "Only if" follows directly from the convexity of the costs of redistribution and from the fact that tax revenues decline for $t_i > 1/2$. "If": Suppose that an element $\hat{y}$ which fulfills the above conditions is not Pareto optimal. Then there is a Pareto superior element $\tilde{y} \in Y$ that satisfies the above conditions. For at least one agent taxes must be lower or transfers higher with the policy that leads to $\tilde{y}$ than with the one that leads to $\hat{y}$. But this implies that for a second agent the opposite holds. Hence all $\hat{y}$ that satisfy the conditions are Pareto-optima. Q.E.D.

Formally, the set of Pareto-optimal income vectors is:

$$P(Y) := \{y \in R^{3+} \mid y_i = (1 - t_i)\, Y_i + T_i \text{ with} \tag{3}$$

$$\begin{aligned} t_i &\in [0, 1/2] \text{ and } T_i \geq 0, \\ t_i &= 0 \text{ if } T_i > 0, \end{aligned}$$

$$
\begin{aligned}
T_i &= 0 \text{ if } t_i > 0 \text{ and} \\
0 &= \sum \left(t_i \Leftrightarrow t_i^2\right) Y_i \Leftrightarrow T_i \}.
\end{aligned}
$$

- Figure 1 here -

Figure 1 describes the set of possible income allocations and the efficiency frontier in an example with two agents $i = 1, 2$. The set of possible policies is given by tax rates and transfers (t_1, t_2, T_1, T_2) with $t_1, t_2 \in [0, 1]$ and $T_1, T_2 \geq 0$. Efficient policies are either characterized by pure redistribution from agent 1 to agent 2, i.e. $1/2 \geq t_1 \geq 0,\ t_2 = 0,\ T_1 = 0$ or by pure redistribution from agent 2 to agent 1 with $1/2 \geq t_2 \geq 0,\ t_1 = 0,\ T_2 = 0$.

3.2 Players and Time Structure

The political game has the following time structure. There are two parties $I = A, B$. Both parties simultaneously choose a political platform (Stage 1). The political platform of party I generates a vector of payoffs for voters $y^I \in Y$ if implemented. We consider Y as the strategy space for both parties. Parties maximize their expected number of votes. In stage 2, all voters must simultaneously vote for either y^A or y^B. When the agents cast their votes, they know y^A and y^B. After the election, poll results become known, and the party which obtains the majority sees its policy implemented.

3.3 Equilibrium of the One-Shot Game

We now analyze the subgame-perfect Nash equilibria of the game. Without any further restriction, every political platform may be the outcome of the voting game. The reason is that when all voters vote in favor of some platform $y^* \in Y$ then no single voter has an incentive to deviate from this action. Hence:

Proposition 2 *The one-period voting game has an infinity of subgame-perfect Nash equilibria where parties play pure strategies and where voters play weakly dominated strategies. All political platforms $y^* \in Y$ are sustainable as political outcomes.*

PROOF Consider the following strategy profile: all voters vote in favor of the platform that is closest to y^*. Both parties propose y^*. It is easily verified that (i) no player gains if he deviates from his strategy and (ii) the voters' strategies are weakly dominated. Q.E.D.

The strategy profile described in the proof of Proposition 2 is not the most obvious way to play this game: all voters plan to vote in favor of y^* no matter what alternatives are proposed to them. In a situation where a voter is pivotal this means that he plans to vote against his own interest. The above strategy profile is a Nash equilibrium because situations where one voter is pivotal only occur off the equilibrium path. If we would instead restrict attention to strategies that are not weakly dominated then equilibria fail to exist. To see this consider the subgames played among voters when both parties have proposed their platforms. Obviously, each of these subgames has exactly one Nash equilibrium in undominated strategies. In this equilibrium each agent votes in favor of the party that offers himself the highest payoff. Provided that voters do not play weakly dominated strategies, it is a well known result that there is no equilibrium in pure strategies at the first stage, when parties chose their platforms.

Proposition 3 *Suppose that agents do not play weakly dominated strategies in stage 2. The one-period voting game has no subgame-perfect Nash equilibrium where parties play pure strategies.*

PROOF Suppose one party has not more than half of the votes. Let this party copy its opponent's proposal and modify it in the following way: take away some income from one agent and distribute it among two other agents. This new proposal yields the party a majority of votes. Hence there is always a profitable deviation for one party. Q.E.D.

Proposition 3 does not imply that the game has no equilibrium at all. In the Appendix we show for the case of $n = 3$ that the game has a symmetric equilibrium where parties play purely mixed strategies. In such an equilibrium parties randomize over individuals' incomes. Given that there are convex costs of redistribution, it is obvious that a mixed strategies equilibrium is in general not ex-ante Pareto efficient. We have:

Proposition 4 *All equilibria in undominated strategies where an agent is taxed at varying rates or receives transfers of varying magnitude is not ex-ante Pareto-efficient.*

PROOF This follows directly from the convexity of costs of redistribution. Q.E.D.

In a situation where parties randomize over platforms all voters would benefit from a reduction of policy uncertainty. Uncertainty could be reduced if all voters were able to commit to vote in favor of platforms in some subset of Y before stage 1. Such a commitment would induce parties to restrict political competition and to reduce policy uncertainty. However, the voters' commitment would not be credible since voting for a particular platform is a dominated strategy in stage 2.

4 The Repeated Game

In a repeated game voters can improve upon the suboptimal outcome from the one shot game by playing punishment strategies. The repeated game is constructed as follows: Time is divided into periods. There is a set $A = \{1, 2, 3\}$ of agents of infinite life. An agent's initial endowment (think of his earnings ability) in a given period is denoted by Y_i; the endowment is the same in all periods and the aggregate endowment in each period is normalized to one. If an individual is taxed at rate t_i in period τ

then this reduces his income to $y_i = (1 \Leftrightarrow t_i)Y_i$. There are convex efficiency losses from taxation; tax revenues are given by $(t_i \Leftrightarrow t_i^2)\,Y_i$. The set of income vectors achievable is again denoted by $Y \subset R^{3+}$. Agents maximize the discounted sum of incomes, the discount factor is $\delta < 1$.

In each period two parties $I = A, B$ simultaneously offer credible policy platforms $y^I \in Y$ before voters simultaneously cast their votes. Parties live for one period and maximize the number of votes. The assumption that parties live for one period has been chosen in order to concentrate on the emergence of cooperation among voters. The assumption considerably simplifies the analysis of parties equilibrium strategies: in each period both parties must react optimally to the strategy of the voters.

All agents know the history of political platforms and political outcomes, however, votes are secret and individual voting behavior is not observable. The party that gets the majority of votes sees its policy implemented in this period. A strategy of a voter is a plan how to vote in each period given the two parties' platforms and the history of the game. A strategy of a party is characterized by a mapping from the set of histories into the set of political platforms.

5 Consensus

A political outcome for the whole game is a sequence of elected political platforms. From the convexity of redistribution costs it follows that an outcome is efficient if and only if in all periods the same platform $y^* \in P(Y)$ is elected. Which are the efficient outcomes that can be sustained as a subgame-perfect equilibrium? As in the one-shot game we have that - without any further refinement - all efficient outcomes and all the inefficient ones could be sustained as a subgame perfect equilibrium. To see this consider the following strategy profile:

Definition 1 *The following strategy profile is called an rg1-profile: In all periods all voters vote in favor of the platform that comes closest to y^*. In all periods both parties propose y^*.*

Obviously no voter is ever pivotal. Hence no voter (and no party) ever has an incentive to deviate from his (its) equilibrium strategy. Like in the one shot game it is unrealistic to assume that a voter plans to vote in favor of the same platform y^* no matter what alternative is proposed to him. At a knot where a party has proposed an alternative platform that gives a voter more than y^* the voter should vote for the alternative if there is a small probability that the other voters do not stick to their equilibrium strategy. In order to rule out this kind of unrealistic behavior we introduce a refinement that is a weakened form of the perfect equilibrium refinement in agent normal form (c.f. Mas-Colell, Whinston and Green, 1995, p. 299). First let us denote the strategy of party I at date t by σ_t^I and the strategy of voter i by σ^i. The strategy of a voter is composed of plans for each stage $\sigma^i = \sigma_0^i, \sigma_1^i, ...$ Each plan maps the history of the play and the partys' current proposal into a vote. We first define:

Definition 2 *Let $s = \left(\sigma_t^A, \sigma_t^B, \sigma_t^i \,|i = 1, ..\infty, t = 0, ..\infty\right)$ be a strategy profile of our repeated game. We call the profile τ-perfect if the following holds for period τ: assume that for $t < \tau$ the game was played along the path that is described by the profile s. Treat voters that decide in τ as if they were one-period players that they take all future strategies $\left(\sigma_t^A, \sigma_t^B, \sigma_t^1, ..., \sigma_t^n \,|i = 1, ..\infty, t > \tau\right)$ as given. Let them maximize the respective voters' intertemporal utility function. Then the strategies $\sigma_\tau^A, \sigma_\tau^B, \sigma_\tau^1, ..., \sigma_\tau^n$ are a trembling-hand-perfect Nash equilibrium of the stage game at τ.*

Next we define:

Definition 3 *Let* $s = \left(\sigma_t^A, \sigma_t^B, \sigma_t^i \,|i = 1, ..\infty, t = 0, ..\infty\right)$ *be a subgame-perfect Nash equilibrium of our repeated game. We call the equilibrium majority-proof if it is* τ*-perfect for all* $\tau \in \{0, 1, ..\}$.

In our game the τ-perfectness criterion requires that a voter plans to chose the best policy for himself, if the punishment in the subsequent periods is not too severe.[4] To see this consider that if a voter is not pivotal in τ then his action does not affect his payoffs. In a perturbed game however, a voter is pivotal with a positive probability. Hence, in an equilibrium of a perturbed game each voter must vote for the platform that maximizes today's income if the future punishment is not too severe. Hence, a consensus on some platform y^* can only be sustained if there is no alternative platform that makes a majority of voters deviate at the same time. A consensus is majority proof if and only if if a party cannot find a platform $y^J \neq y^*$ that tempts more than half of the voters to vote in favor of it.

Obviously, the rg1-profile defined above is not a majority proof equilibrium since voters' strategies do not foresee a punishment in case that the consensus breaks down. In order to determine the set of possible efficient political outcomes in a majority proof equilibrium we may restrict ourselves to a class of strategy profiles that we call *consensus profiles.* These profiles contain the threat of voters to minimax all other voters forever if the political outcome differs from the desired outcome $y^* \in P(Y)$. Obviously, if a platform cannot be sustained with a minimax punishment, then it cannot be sustained as a majority proof equilibrium at all.

Definition 4 *A strategy profile of the repeated voting game is called a consensus profile if*

(i) voters play according to the following strategy in all periods:

[4]Note that this is only required als long as the previous play in $t < \tau$ has been along the equilibrium path.

1. *If the political outcome in any of the previous periods was not* y^**, then vote for the party with the platform that proposes the largest sum of individual tax rates.*

2. *If* $y^I = y^*$ *and* $y^J \neq y^*$*, then choose* y^J *if the loss from the above punishment is not too large.*

3. *If* $y^I = y^*$ *and* $y^J \neq y^*$*, then choose* y^I *if the loss is too large.*

4. *If both* y^A*,* $y^B = y^*$*, then choose each platform with probability* $1/2$*.*

5. *If both* y^A *and* y^B *are* $\neq y^*$*, then choose the best policy for you.*

(ii) In all periods both parties play the following strategy: Propose $y^I = y^*$ *until another platform than* y^* *wins an election. Propose* $y^I = (0, ..., 0)$ *if another platform than* y^* *has won an election in a previous period.*

Conditions *(i) 1.* and (ii) imply that all voters are minimaxed for the rest of time if a majority of voters deviates in some period τ. The minimax payoff of all agents is zero. Hence, a deviation with payoff d today does not pay iff

$$d \Leftrightarrow y_i^* \leq \frac{\delta}{1 \Leftrightarrow \delta} \cdot y_i^* \Leftrightarrow \tag{4}$$

$$y_i^* \geq (1 \Leftrightarrow \delta) \cdot d. \tag{5}$$

A majority is a subset $m \subset A$ with $\#m \geq 2$. We denote the set of all majorities by M. The criterion of majority proofness is violated if there is a majority m such that taxing the other agent(s) in $A \backslash m$ generates revenues that are sufficient to induce the agents in m to deviate. The maximum amount that can be raised by taxing the agents in $A \setminus m$ is given by $\frac{1}{4} \sum_{A \backslash m} Y_i$. Hence, an allocation y^* is sustainable in a majority-proof subgame-perfect equilibrium if and only if for all majorities $m \in M$:

$$\frac{1}{1 \Leftrightarrow \delta} \sum_{m} y_i^* \geq \sum_{m} Y_i + \frac{1}{4} \sum_{A \backslash m} Y_i \Leftrightarrow \tag{6}$$

$$\sum_{m} y_i^* \geq (1 \Leftrightarrow \delta) \left[\sum_{m} Y_i + \frac{1}{4} \sum_{A \backslash m} Y_i \right]. \tag{7}$$

The proof for the fact that the consensus profile constitutes a subgame perfect equilibrium is straightforward. Subgames either begin when parties move or when voters move. Neither parties nor voters have an incentive to deviate from the equilibrium strategy in any subgame: obviously parties do not deviate given voters' reaction. A voter does not want to deviate because either all other voters plan to vote in favor of y^* or all other voters minimax him.

An alternative to our refinement is to require that equilibria have to be [extensive-form] trembling hand perfect. The consensus profile would then not be an equilibrium profile because the punishment behavior does not satisfy the perfectness criterion. We have chosen our refinement because here the punishment payoffs are well specified. In Appendix 2 we study trembling-hand perfect equilibria where the punishment consists of returning to the mixed strategies equilibrium of the one-shot game.

6 Inequality and Redistribution

We now study the relationship of inequality and redistribution in a special case with three agents in which two (poor) agents own the same amount $Y_1 = Y_2 < Y_3$. Initial inequality is measured by the amount owned by agent 3, $Y_3 = 1 \Leftrightarrow 2Y_1$. We only want to consider the set of sustainable allocations where agents 1 and 2 get the same equilibrium net income, i.e. we consider equilibria where $y_1^* = y_2^*$. In order to proof that an income vector (y_1^*, y_2^*, y_3^*) is an equilibrium, we have to show that a party cannot deviate and induce a majority of voters to deviate from the consensus strategy. We first derive a condition such that a party does not want to redistribute money from one of the poor agents to the other two agents. According to (7) parties can not propose such alternative platforms and get the votes of 1 and 3 or 2 and 3 iff:

$$y_3^* + y_1^* \geq (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow 2Y_1 + Y_1 + \frac{1}{4}Y_1\right) = (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right). \tag{8}$$

Next we can check whether a party wants to redistribute money from the rich agent to the two poor ones. Parties can not propose such alternative platforms iff:

$$2y_1^* \geq (1 \Leftrightarrow \delta)\left(2Y_1 + \frac{1}{4}Y_3\right) = (1 \Leftrightarrow \delta)\left(\frac{3}{2}Y_1 + \frac{1}{4}\right). \tag{9}$$

Condition (8) determines the upper bound on the taxation of agent 3 while condition (9) determines the lower bound on the taxation of agents 1 and 2. The set of feasible income allocations is depicted in Figure 2. Concerning the upper bound on redistribution we find:

Proposition 5 *The maximum redistributive tax rate is decreasing with inequality if and only if agents are sufficiently patient.*

Proof Consider an equilibrium where agent 3 is taxed and where agents 1 and 2 receive identical transfers. In such equilibria the net incomes must satisfy:

$$y_3^* = (1 \Leftrightarrow t_3) \cdot (1 \Leftrightarrow 2Y_1), \tag{10}$$

and

$$y_1^* = y_2^* = Y_1 + \frac{1}{2}\left(t_3 \Leftrightarrow t_3^2\right)(1 \Leftrightarrow 2Y_1). \tag{11}$$

The maximum tax rate on agent 3 can be obtained by substitution of (10) and (11) into (8):

$$\left(1 \Leftrightarrow \frac{t_3}{2} \Leftrightarrow \frac{t_3^2}{2}\right) \cdot (1 \Leftrightarrow 2Y_1) \geq (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right) \Leftrightarrow Y_1. \tag{12}$$

Solving for t yields the upper bound for the taxation of the rich agent:

$$\left(1 \Leftrightarrow \frac{t_3}{2} \Leftrightarrow \frac{t_3^2}{2}\right) \geq \frac{(1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right) \Leftrightarrow Y_1}{1 \Leftrightarrow 2Y_1} \Leftrightarrow \tag{13}$$

$$\frac{t_3^2}{2} + \frac{t_3}{2} \leq 1 \Leftrightarrow \frac{(1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right) \Leftrightarrow Y_1}{1 \Leftrightarrow 2Y_1} \Leftrightarrow \tag{14}$$

$$t_3^2 + t_3 \leq 2 \cdot \frac{1 \Leftrightarrow Y_1 \Leftrightarrow (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right)}{1 \Leftrightarrow 2Y_1}. \tag{15}$$

It remains to be shown that the right-hand side increases with Y_1 if and only if δ is sufficiently large. To see this consider that the derivative of the fraction on the right hand side is:

$$\frac{(1 \Leftrightarrow 2Y_1)\left[\Leftrightarrow 1 + \frac{3}{4}(1 \Leftrightarrow \delta)\right] + 2\left[1 \Leftrightarrow Y_1 \Leftrightarrow (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right)\right]}{(1 \Leftrightarrow 2Y_1)^2} \tag{16}$$

It is positive if

$$\left[\frac{3}{4}(1 \Leftrightarrow 2Y_1) \Leftrightarrow 2\left(1 \Leftrightarrow \frac{3}{4}Y_1\right)\right](1 \Leftrightarrow \delta) > (1 \Leftrightarrow 2Y_1) \Leftrightarrow 2[1 \Leftrightarrow Y_1] \Leftrightarrow \tag{17}$$

$$\left[\frac{3}{4}(1 \Leftrightarrow 2Y_1) \Leftrightarrow 2\left(1 \Leftrightarrow \frac{3}{4}Y_1\right)\right](1 \Leftrightarrow \delta) > \Leftrightarrow 1 \Leftrightarrow \tag{18}$$

$$\left[\frac{3}{4} \Leftrightarrow 2\right](1 \Leftrightarrow \delta) > \Leftrightarrow 1 \Leftrightarrow \tag{19}$$

$$(1 \Leftrightarrow \delta) < \frac{4}{5}. \tag{20}$$

Hence, the maximum tax rate increases with equality if agents are sufficiently patient. Q.E.D.

- Figure 2 here -

High incomes are protected against redistribution because with too much redistribution there is more scope for the rich to politically collaborate with part of the poor. It is interesting to reinterpret this result by relabeling the three voters in our game as three large homogenous groups of voters. More redistribution from the rich group to the poor makes it more likely that a new "coalition" among rich and part of the poor agents emerges. If the lower class owns too much after redistribution then the efficiency losses of redistribution from the rich to the poor are too large. A party can then propose to share the efficiency gain that arises when the rich are taxed at a lower rate among the rich and part of the poor.

Concerning a lower boundary on redistribution results are more conventional. The minimum amount of redistribution from the rich to the poor agents increases with inequality. In the present model redistribution from the poor to the rich is also possible.

Proposition 6 *There are gross income vectors such that redistribution from the poor agents to the rich agent is sustainable as an equilibrium. The maximum amount of redistribution to the rich decreases with inequality.*

PROOF Consider a situation where agents 1 and 2 are taxed at a common rate t_1. The maximum tax rate on agents 1 and 2 can be obtained by substitution of agent 1 and 2's net income into (9):

$$2 \cdot (1 \Leftrightarrow t_1) \cdot Y_1 \geq (1 \Leftrightarrow \delta) \left(\frac{3}{2} Y_1 + \frac{1}{4} \right) \Leftrightarrow \tag{21}$$

$$t_1 \leq 1 \Leftrightarrow \frac{1}{2} (1 \Leftrightarrow \delta) \left(\frac{3}{2} + \frac{1}{4 Y_1} \right). \tag{22}$$

From this it follows that the maximum tax rate on the poor agents increases with Y_1. Q.E.D.

7 Conclusion

The present model provides a simple explanation why inequality may be politically stable: Voters agree that political stability is desirable and believe that excessive redistribution today endangers political stability in the future. A consensus on political outcomes can be reached if voting takes place repeatedly and inequality can be politically sustained as part of this consensus.

According to our analysis policies can only be part of a consensus if a party cannot induce a majority of voters to break the consensus. Inequality may be politically self-sustaining: Policies that are too redistributive can not be part of a consensus because a party may find it profitable to propose a policy that divides the poor and taxes the rich less. Such a policy reduces efficiency losses from taxation of the rich. The resulting surplus can be shared among the rich and part of the poor. Therefore more inequality reduces the maximum amount of redistribution.

Our results challenge the conventional approach to the comparative static analysis in politico-economic models. In the case that we considered the boundaries of the set of equilibrium policies may behave differently from what we know about one-dimensional, one-shot voting games. It may therefore provide new insights to study the robustness of politico-economic results in other models where restrictions on the set of policy options have been imposed.

8 Appendix 1: Mixed Strategies Equilibria

Dasgupta and Maskin (1986, Theorem 6*) provide sufficient conditions for the existence of a mixed strategies equilibrium in a discontinuous game with more than one-dimensional strategies. We here show that our one-shot game of party competition satisfies all the conditions in Dasgupta and Maskin.

First, note that the parties' strategy space Y is a compact set. We begin by characterizing the set of discontinuities of the parties' payoff function $\#_I : Y^2 \to \{0, 1/2, 1, ..., n\}$. Define

$$Y^{2*} = \{(y^A, y^B) \in Y^2 \mid y_k^A = y_k^B \text{ for at least one } k\} \subset Y^2 \tag{23}$$

as the set of strategy profiles at which both party's payoff functions are discontinuous. For each $I = (A, B)$ let Y_I^* be the projection of Y^{2*} onto Y, i.e.

$$Y_I^* = \{y^I \mid \exists y^J \in Y \,\text{s.t.}\, (y^I, y^J) \text{ is a point of discontinuity of } \#_I\}. \tag{24}$$

Note that in our game Y_I^* is equal to Y; that is, any policy chosen by party I is a potential point of discontinuity. We define

$$Y_J^*(\overline{y}^I) := \left\{y^J \in Y \mid \left(\overline{y}^I, y^J\right) \in Y^{2*}\right\} \tag{25}$$

as the set of platforms y^J such that, given $\overline{y}^I$, both payoff functions are discontinuous. We now use a property of the score function which has been introduced in Dasgupta and Maskin (1986, Appendix). Let B^n be the surface of the unit sphere in R^n with the origin as its centre. Let $e \in B^n$ and γ be a positive real.

Property α^* *For each* $\overline{y}^I \in Y_I^*$, $\exists$ *a non-atomic measure* ν *on* B^n *such that for all* $y^J \in Y_J^*(\overline{y}^I)$

$$\int_{B^3} [\lim_{\gamma \to 0} \inf \#_I(\overline{y}^I + \gamma e, y^J) d\nu(e)] \geq \#_I(\overline{y}^I, y^J) \tag{26}$$

where the inequality is strict if $y^J = \overline{y}^I$.

Now, we can state Dasgupta and Maskin's existence theorem:

Theorem 7 (Dasgupta and Maskin (1986))
Suppose $\#_A + \#_B$ *is upper semi-continuous, and for all* I, $\#_I$ *is bounded and satisfies Property* α^*. *Then there is a symmetric mixed-strategy equilibrium* (σ^*, σ^*) *with the property that for each* I *and for each* $\overline{y}^I \in Y_I^*$, $\sigma^*(\{\overline{y}^I\}) = 0$.

PROOF: See Dasgupta and Maskin (1986).

In what follows, we check that the assumptions of Theorem 7 are satisfied. The upper semi-continuity and the boundedness are satisfied since our game is a constant-sum game. Next we have to verify whether Property α^* holds. Consider first any profile $(\overline{y}^I, y^J)$ with $\overline{y}^I \in Y_I^*$ and $y^J \in Y_J^*(\overline{y}^I)$ such that $\overline{y}^I \in Y \setminus P(Y)$. Put all the weight of ν on the sector of B^n that Pareto-dominates $\overline{y}^I$. Property α^* obviously holds since every Pareto-dominating point gets at least as much votes as $\overline{y}^I$. Moreover, each of these points wins all the votes against $\overline{y}^I$. Next consider the case where $\overline{y}^I \in P(Y)$. Denote by $D(y^J)$ the set of strategies that yield player I at least $\#_I(\overline{y}^I, y^J)$ against platform y^J:

$$D(y^J) = \left\{ y \in Y \mid \#_I(y, y^J) \geq \#_I(\overline{y}^I, y^J) \right\} \tag{27}$$

This set is displayed in Figure 3 for the case where $n = 3$. Denote by $\hat{D}(\overline{y}^I)$ the set of strategies that yields player I strictly more than half of the votes against platform $\overline{y}^I$:

$$\hat{D}(\overline{y}^I) = \left\{ y \in Y \mid \#_I(y, \overline{y}^I) > 1/2n \right\} \tag{28}$$

Figure 4 shows why the intersection $D(y^J) \cap \hat{D}(\overline{y}^I)$ is nonempty in the case with three voters. In Figure 4 we consider without restricting generality the case where $\overline{y}_1^I = y_1^J$, $\overline{y}_2^I > y_2^J$ and $\overline{y}_3^I < y_3^J$. Both points $\overline{y}^I, y^J$ are on the efficiency frontier $P(Y)$. The intersection $D(y^J) \cap \hat{D}(\overline{y}^I)$ is given by:

$$D(y^J) \cap \hat{D}(\overline{y}^I) = \left\{ y \in Y \mid y_1 > y_1^I,\ \overline{y}_2^I > y_2 \geq y_2^J,\ \overline{y}_3^I < y_3 \leq y_3^J. \right\}. \tag{29}$$

The set $D(y^J) \cap \hat{D}(\overline{y}^I)$ is nonempty and its closure is a convex set. Convexity implies that it contains a cone C with $\overline{y}^I$ at the peak. Hence, any measure ν which concentrates its weight on the intersection of B^3 with the cone C satisfies (26) where the inequality is strict if $y^J = \overline{y}^I$. The same measure ν satisfies (26) in the case where $y^J \notin P(Y)$ like, e.g. $y^{J\prime}$ in Figure 4.

9 Appendix 2: Punishment as Return to MSE

In this appendix we discuss the robustness of Proposition 1 if we require the punishment to be majority proof too. The consensus profile is majority proof at the off-equilibrium nodes if we replace condition *(i) 4. by: "If the political outcome in any of the previous periods was not y^*, then always choose the best policy for you."* In this case parties will play according to the MSE if a majority of voters deviates in some period τ and all voters get the MSE payoff for the rest of time. Let us call this payoff $v_i\left(Y_1, Y_2, Y_3\right)$. Note that it need not be uniquely defined. A deviation with payoff d today does not pay iff:

$$d \Leftrightarrow y_i^* \leq \frac{\delta}{1 \Leftrightarrow \delta} \cdot \left(y_i^* \Leftrightarrow v_i\right) \Leftrightarrow \tag{30}$$

$$y_i^* \geq \left(1 \Leftrightarrow \delta\right) d + \delta v_i. \tag{31}$$

Now, an allocation y^* is sustainable in a majority-proof subgame-perfect equilibrium if and only if for all majorities $m \in M$:

$$\sum_m y_i^* \geq \left(1 \Leftrightarrow \delta\right) \left[\sum_m Y_i + \frac{1}{4} \sum_{A \backslash m} Y_i\right] + \delta \sum_m v_i(Y_1, .., Y_n). \tag{32}$$

The maximum tax rate on agent 3 can again be obtained by inserting y_1^* and y_3^* into (32):

$$\left(1 \Leftrightarrow \frac{t_3}{2} \Leftrightarrow \frac{t_3^2}{2}\right) \cdot \left(1 \Leftrightarrow 2Y_1\right) \geq \left(1 \Leftrightarrow \delta\right) \left(1 \Leftrightarrow \frac{3}{4} Y_1\right) \Leftrightarrow Y_1 + \delta \left(v_1 + v_3\right). \tag{33}$$

Solving for t yields the new upper bound for the taxation of the rich agent:

$$t_3^2 + t_3 = 2 \cdot \frac{1 \Leftrightarrow Y_1 \Leftrightarrow (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right) \Leftrightarrow \delta \cdot (v_1 + v_3)}{1 \Leftrightarrow 2Y_1} = \tag{34}$$

$$t_3^{\max} = \Leftrightarrow \frac{1}{2} + \sqrt{\frac{1}{4} + 2\frac{1 \Leftrightarrow Y_1 \Leftrightarrow (1 \Leftrightarrow \delta)\left(1 \Leftrightarrow \frac{3}{4}Y_1\right) \Leftrightarrow \delta \cdot (v_1 + v_3)}{1 \Leftrightarrow 2Y_1}}. \tag{35}$$

It remains to be shown that the right-hand side increases with $Y_1$for sufficiently large values of δ. We compare two cases, one where there is no pre-tax-inequality ($Y_1 = 1/3$), and one where there is maximum pre-tax-inequality ($Y_1 = 0$). The maximum tax rate decreases in the former is smaller than in the latter if:

$$\frac{2/3 \Leftrightarrow (1 \Leftrightarrow \delta)\, 3/4 \Leftrightarrow 2\delta \cdot (v_1\,(1/3, 1/3, 1/3))}{1/3} > \delta \Leftrightarrow \delta\,(v_1\,(0,0,1) + v_3\,(0,0,1)) \tag{36}$$

$$[5 \Leftrightarrow 24 v_1\,(1/3,1/3,1/3) + 4\,(v_1\,(0,0,1) + v_3\,(0,0,1))]\,\delta \geq 1. \tag{37}$$

Like in Proposition 4 this would yield us a lower bound for δ, provided that the factor on the left hand-side exceeds 1. This in turn holds if:

$$v_1\,(1/3, 1/3, 1/3) < 1/6 \cdot [1 + v_1\,(0,0,1) + v_3\,(0,0,1)] \tag{38}$$

Hence, the result holds if inefficiency in the MSE with an equal initial distribution is sufficiently strong. Unfortunately we do not have numerical solutions for these MSE payoffs. However, one can easily verify that

Lemma 8 *(i) A lower bound for* $v_1\,(0,0,1) + v_3\,(0,0,1)$ *is given by* $5/8$.

(ii) An upper bound for $v_1\,(1/3,1/3,1/3)$ *is given by* 0.3.

Proof (i) This is so because parties only play policies in the set of Pareto-optima $P(Y)$. In a Pareto-optimum agent 3 gets at least $1/2$. In an equilibrium where agent 2 and 3 are treated symmetrically we have that if agent 3 gets $1/2$ then agent 1 gets an expected value of $1/8$. The rest follows from concavity.

(ii) This bound can be constructed as follows: consider that no

Q.E.D.

To get an idea of how little the upper bound of taxation reacts to inequality we provide a numerical example for the case where $\delta = 0.95$. An upper bound for the maximum tax rate with perfect inequality is given by

$$t_3^{\max}(0,0,1) = \Leftrightarrow \frac{1}{2} + \sqrt{\frac{1}{4} + 2 \cdot \left(1 \Leftrightarrow 0.01 \Leftrightarrow 0.99 \cdot \frac{5}{8}\right)} = .49624$$

A lower bound for the maximum tax rate with complete equality is given by

$$t_3^{\max}(1/3,1/3,1/3) = \Leftrightarrow \frac{1}{2} + \sqrt{\frac{1}{4} + 2\frac{2/3 \Leftrightarrow 0.01\,(0.75) \Leftrightarrow 0.99 \cdot 0.6}{1/3}} : .30062$$

The median voter model would in the former case predict a tax rate of 1/2 in the latter one of zero.

FIGURE 1

The parties' strategy space Y for $n = 2$. $P(Y)$ is the set of Pareto-optimal platforms.

FIGURE 2

The set of platforms for $n = 3$ and $Y_1 = Y_2$ under the restriction that voters 1 and 2 get the same net income: $y_1 = y_2$.

FIGURE 3

FIGURE 4

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